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CONSTANT CUTOFF APPROACH TO HYPERON RADIATIVE DECAYS IN CHK SU(3)-SOLITON MODEL

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We give a brief review a quantum stabilization method for the SU(2) σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna et al., which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We then study the radiative decays of hyperons in the constant-cutoff approach to the bound state soliton model, developed by Callan, Hornbostel and Klebanov. The results for the total decay widths and the E2/M1 ratios, corresponding to decouplet-to-octet electromagnetic transitions, are obtained in a qualitative agreement with the results obtained using constant-cutoff approach to the SU(3) collective model, the complete Skyrme models and quark-based models.

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1. *Introduction*

It was shown by Skyrme [1] that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral SU(2) σ-model is given by

$$L = \frac{F_\pi^2}{16}\ \mathrm{Tr}\ \partial_\mu U \partial^\mu U^+ \tag{1}$$

where

$$U = \frac{2}{F_\pi}(\sigma + \mathrm{i}\ \vec{\tau}\cdot\vec{\pi}) \tag{2}$$

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is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (2), $\sigma = \sigma(\vec{r})$ is a scalar meson field and $\vec{\pi} = \vec{\pi}(\vec{r})$ is the pion-isotriplet.

The classical stability of the soliton solution to the chiral σ-model Lagrangian requires the additional ad-hoc term, proposed by Skyrme [1], to be added to (1)

$$L_{sk} = \frac{1}{32e^2}\mathrm{Tr}\ [U^+\partial_\mu U,\ U^+\partial_\nu U]^2 \tag{3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors [2] that, after the collective coordinate quantization using the spherically symmetric ansatz

$$U_0 = \exp[\mathrm{i}\vec{\tau} \cdot \vec{r}_0 F(r)]\,, \qquad \vec{r}_0 = \vec{r}/r\,, \tag{4}$$

the chiral model, with both (1) and (3) included, gives a good agreement with the experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1) and (3), from a more fundamental theory like QCD. On the other hand, it is not easy to generate the terms like (3) and give a clear physical meaning to the dimensionless constant e in (3) using QCD.

Mignaco and Wulck (MW) [3] indicated therefore a possibility to build a stable single baryon ($n = 1$) quantum state in the simple chiral theory, with Skyrme stabilizing term (3) omitted. Despite the non-existence of the stable classical soliton solution to the non-linear σ-model, it is possible, to build a stable chiral soliton at the quantum level, after the collective coordinate quantization, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e. $F(0) = -n\pi$ and $F(\infty) = 0$.

However, as pointed out by Iwasaki and Ohyama [4], the quantum stabilization method in the form proposed by MW [3] can not be used, since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) = -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

In Ref. [8], the present author suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the Ansatz (4), as follows

$$U(\vec{r}) = \exp[\mathrm{i}\ \vec{\tau} \cdot \vec{r_0} F(r) + \mathrm{i}\varphi(r)]\,. \tag{5}$$

Such a method provides a stable chiral quantum soliton but the resulting model is an entirely non-covariant chiral model, different from the original chiral σ-model.

In the present paper, we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna, Sanyuk, Schechter and Subbaraman [6] to construct a stable chiral quantum soliton within the original chiral σ-model. We then study the radiative decays of hyperons in the constant-cutoff approach to the Callan, Hornbostel and Klebanov (CHK) bound state soliton model [7]. Thus the results for the total decay widths and the E2/M1 ratios corresponding to decouplet-to-octet electromagnetic transitions are obtained in a qualitative agreement with the results obtained using the constant-cutoff approach to the SU(3) collective model [14] (DCOL), the complete SU(3) collective model [9] (CCOL), the complete CHK bound state soliton model [10] (CCHK), non-relativistic quark model [11] (NRQM) and the quenched lattice model [12] (QLM).

The reason why the cutoff-approach to the problem of chiral quantum soliton works is connected to the fact that the solution $F = F(r)$, which satisfies the boundary condition $F(\infty) = 0$, is singular at $r = 0$.

From the physical point of view, the chiral quantum model is not applicable to the region about the origin since in the physical world in that region there is a quark-dominated 'bag' of the soliton. However, in the constant-cutoff approach employed here, the 'cavity' in the middle of the soliton is not assumed to carry any quark degrees of freedom.

Therefore, the present model differs from the hybrid models [13], where in the CHK bound-state SU(3)-soliton model a cavity populated with quarks is introduced in the center of the soliton. The present model is fully analogous to the original Skyrme model and our soliton is a topological soliton with the winding number equal to the baryon number. The total baryon number is determined by the soliton degrees of freedom from the region where r is larger than the cutoff ϵ, and there are no contributions from any quark degrees of freedom in the 'bag'. Thus, in the constant cutoff model there is no problem with the balance of the baryon number of hyperons.

However, as argued in Ref. [6], when a cutoff ϵ is introduced, then the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$, can be satisfied. In Ref. [6], an interesting analogy with the damped pendulum has been discussed, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by Eq. (2.2) in Ref. [6]. From these asymptotic solutions, we immediately see that for $\epsilon \to 0$, the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach have been discussed in Ref. [14].

2. *Constant-cutoff stabilization*

The chiral soliton with baryon number $n = 1$ is given by (4), where $F = F(r)$ is the radial chiral phase function satisfying the boundary conditions $F(0) = -\pi$ and $F(\infty) = 0$.

Substituting (4) into (1), we obtain the static energy of the chiral baryon

$$M = \frac{\pi}{2} F_\pi^2 \int\limits_{\epsilon(t)}^{\infty} \mathrm{d}r\, [r^2 (\frac{\mathrm{d}F}{\mathrm{d}r})^2 + 2\sin^2 F]\,. \tag{6}$$

In (6), we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e. by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced following Ref. [6] as a dynamic time-dependent variable. From (6), we obtain the following differential equation for the profile function $F = F(r)$

$$\frac{\mathrm{d}}{\mathrm{d}r}(r^2 \frac{\mathrm{d}F}{\mathrm{d}r}) = \sin(2F)\,, \tag{7}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus from the nonlinear σ-model Lagrangian (1), following the derivation given in Ref. [15], we obtain

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu\,, \tag{8}$$

where $x(t) = [\epsilon(t)]^{3/2}$, $y = r/\epsilon$ and $\mathrm{Tr}(\partial_0 A \partial_0 A^+) = 2\dot{\alpha}_\nu \dot{\alpha}^\nu$ with α_ν ($\nu = 0, 1, 2, 3$) being the collective coordinates defined as in Ref. [16]. In (8) the integrals a , b and c are defined as follows [15]

$$a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \mathrm{d}y\, [y^2 (\frac{\mathrm{d}F}{\mathrm{d}y})^2 + 2\sin^2 F], \quad b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \mathrm{d}y\, y^2 \sin^2 F\,, \tag{9}$$

$$c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty \mathrm{d}y\, y^2 (\frac{\mathrm{d}F}{\mathrm{d}y})^2 y^2\,.$$

In the limit of a time-independent cutoff ($\dot{x} \to 0$), we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1)\,, \tag{10}$$

where $\vec{J}^2 = J(J+1)$ is the eigenvalue of the square of the soliton laboratory angular momentum. A minimum of (10) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{-3/8} \quad \Rightarrow \quad \epsilon^{-1} = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{1/4}. \tag{11}$$

The energy obtained by substituting (11) into (10) is given by

$$E = \frac{4}{3} \left[\frac{3}{2} \frac{a^3}{b} J(J+1)\right]^{1/4}. \tag{12}$$

This result is identical to the result obtained by Mignaco and Wulck which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$,

$b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-2/3}F_\pi$. However, in the present approach, as shown in Ref. [6], there is a profile function $F = F(y)$ with the proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$, and the integrals a, b and c in (9) exist and are shown in Ref. [6] to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2 and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

A discussion about the quantitative predictions for some empirical results for nucleons and Δ particles in the present model can be found in Ref. [15].

3. The constant-cutoff approach to CHK bound-state soliton model

3.1. The effective Lagrangian

Following Ref. [14], we write the effective SU(3) Lagrangian as follows

$$L = \frac{F_\pi^2}{16}\ \mathrm{Tr}\ \partial_\mu U \partial^\mu U^+ + \frac{F_\pi^2}{16} m_\pi^2\ \mathrm{Tr}\ (U + U^+ - 2)$$

$$+ \frac{F_K^2 - F_\pi^2}{48}\ \mathrm{Tr}\ (1 - \sqrt{3}\lambda_8)(U\partial_\mu U \partial^\mu U^+ + \partial_\mu U \partial^\mu U^+ U^+) \tag{13}$$

$$+ \frac{F_K^2 m_K^2 - F_\pi^2 m_\pi^2}{24}(1 - \sqrt{3}\lambda_8)\ \mathrm{Tr}\ (U + U^+ - 2)$$

where m_π and m_K are pion and kaon masses, respectively, and F_K is the kaon weak-decay constant with the empirical ratio to pion decay constant $F_K/F_\pi \approx 1.23$. The first term in (13) is the usual σ-model Lagrangian, while the remaining terms are chiral-symmetry-breaking terms present in the mesonic sector of the model. In addition to the action obtained using the Lagrangian (13), the Wess-Zumino action in the form

$$S = -\frac{\mathrm{i}N_C}{240\pi^2}\int \mathrm{d}^5x\, \mathrm{e}^{\mu\nu\alpha\beta\gamma}\ \mathrm{Tr}\ [U^+\partial_\mu U U^+ \partial_\nu U U^+ \partial_\alpha U U^+ \partial_\beta U U^+ \partial_\gamma U] \tag{14}$$

must be included in the total action of the model, where N_C is the number of colours in the underlying QCD. The Wess-Zumino action defines the topological properties of the model important for the quantization of the solitons. In the SU(2) case, the Wess-Zumino action vanishes identically and was therefore not present in the discussions of the previous two sections.

In the present approach, the meson-soliton field is written in the form

$$U = \sqrt{U_\pi}\ U_K \sqrt{U_\pi} \tag{15}$$

where U_π is a SU(3) extension of the usual SU(2) skyrmion field used to describe the nucleon spectrum, and U_K is the field used to decribe the kaons.

$$U_\pi = \begin{bmatrix} u_\pi & 0 \\ 0 & 1 \end{bmatrix}, \qquad U_K = \exp\left\{ \mathrm{i}\, \frac{2^{3/2}}{F_K} \begin{bmatrix} 0 & K \\ K^+ & 0 \end{bmatrix} \right\}. \tag{16}$$

In (16), u_π is the usual SU(2)-skyrmion field given by (4) and K is the two-dimensional kaon doublet given by

$$K = \begin{bmatrix} K^+ \\ K^0 \end{bmatrix}, \qquad K^+ = [K^- \quad \overline{K}^0]. \tag{17}$$

Substituting now (15) with (16) into the total action of the kaon-soliton system and expanding U_K to the second order in kaon fields (17), we obtain the Lagrangian consisting of the pure SU(2) Lagrangian depending on the soliton field only and the effective interaction-Lagrangian between the soliton and the kaon fields given in [14]. From the effective interaction-Lagrangian, we obtain the following eigenvalue equation for kaon modes [14]

$$\nabla^2 K(\vec{r}) + [v_0(r) - 2\,\frac{1-\cos F}{r^2}\vec{I}\cdot\vec{L}]\, K(\vec{r}) - m_K^2 K(\vec{r}) + 2\omega\lambda(r)K(\vec{r}) + \omega^2 K(\vec{r}) = 0 \tag{18}$$

where $\vec{L}$ is the kaon orbital momentum, $\vec{I}$ is the total angular momentum of the rotating soliton and ω is the bound-state energy of the kaon-soliton system. In (18), we have introduced the quantities $v_0(r)$ and $\lambda(r)$ as follows

$$v_0(r) = \frac{1}{4}(\frac{\mathrm{d}F}{\mathrm{d}r})^2 + \frac{\cos F(1-\cos F)}{r^2} + \frac{F_\pi^2 m_\pi^2}{2F_K^2}(1-\cos F) \tag{19}$$

$$\lambda(r) = -\frac{N_C}{2\pi^2 F_K^2}\frac{\sin^2 F}{r^2}\frac{\mathrm{d}F}{\mathrm{d}r}. \tag{20}$$

3.2. The hyperon spectrum

Expanding the kaon wave functions $K(\vec{r})$ in terms of vector spherical harmonics [14] as follows

$$K(\vec{r}) = \sum_{\alpha,L} k_{\alpha L} Y_{\alpha I L}, \tag{21}$$

the wave equation (18) becomes a one-dimensional differential equation. The form of the interaction makes the P-state ($\alpha = \frac{1}{2},\ L = 1$) the lowest bound state corresponding to the octet and decouplet hyperons. The splittings among the hyperons with different spins and/or isospins are described by the rotational corrections, obtained after applying the following rotations to the kaon and soliton fields, respectively

$$K \to a(t)K \;, \qquad U \to A(t)UA^{+}(t) \tag{22}$$

where

$$A(t) = \begin{bmatrix} a(t) & 0 \\ 0 & 1 \end{bmatrix} \tag{23}$$

is a SU(2) subgroup of SU(3). The SU(2) rotational operator $A(t)$ adds extra time-derivative terms to the Lagrangian, so that, after Ref. [14], we obtain the following formula for the hyperon spectrum

$$E = E_0 + \omega \mid S \mid + \frac{1}{2\Omega}\left[cJ(J+1) + (1-c)I(I+1) + \frac{1}{4}c(c-1) \mid S \mid (\mid S \mid +2)\right] \tag{24}$$

where E_0 is the soliton mass, Ω is the moment of inertia of the soliton, given by

$$\Omega = \frac{2\pi}{3}F_\pi^2 \int_\epsilon^\infty \mathrm{d}r\, r^2 \sin^2 F \tag{25}$$

and c is the hyperfine splitting constant, given by

$$c = 1 - \frac{8\omega}{3}\int_\epsilon^\infty \mathrm{d}r\, r^2 k_P^*(r) \cos^2\frac{F}{2} K_p(r)\,. \tag{26}$$

In the constant-cutoff approach, using the formula (24), we obtain the following hyperon spectrum [14]

$$E = \omega \mid S \mid + \frac{4}{3}\left\{\frac{3}{2}\frac{a^3}{b}\left[cJ(J+1) + (1-c)I(I+1) + \frac{1}{4}c(c-1) \mid S \mid (\mid S \mid +2)\right]\right\}^{1/4} \tag{27}$$

and the following expression for the inertia of the soliton [14]

$$\Omega = b\left\{\frac{3}{2}\frac{1}{ab}\left[cJ(J+1) + (1-c)I(I+1) + \frac{1}{4}c(c-1) \mid S \mid (\mid S \mid +2)\right]\right\}^{3/4} \tag{28}$$

with a and b defined by (9). In Ref. [14], it was shown that the formula (27) gives a good agreement with the empirical spectrum of hyperons.

3.3. Radiative decays of hyperons

In the present paper, we consider the radiative decays of hyperons, i.e. the processes

$$\Sigma^* \to \Lambda\gamma \,, \qquad \Sigma^* \to \Sigma\gamma \,, \qquad \Xi^* \to \Xi\gamma \,. \tag{29}$$

For the processes listed in (29), both M1 and E2 transitions are allowed, and using the usual multipole expansion of the electromagnetic field, we have [14]

$$\Gamma_{E2}(B^* \to B\gamma) = \frac{675}{8}\alpha_{em} q \, |< B \mid \widehat{E}(q) \mid B^* >|^2 \,, \tag{30}$$

$$\Gamma_{M1}(B^* \to B\gamma) = 18\alpha_{em} q \, |< B \mid \widehat{M}(q) \mid B^* >|^2 \tag{31}$$

where [14]

$$\widehat{E}(q) = \frac{1}{2} \int\limits_{r>\epsilon} \mathrm{d}^3\vec{r}\, j_2(qr)(\frac{z^2}{r^2} - \frac{1}{3}) J_0^{em} \,, \tag{32}$$

$$\widehat{M}(q) = \frac{1}{2} \int\limits_{r>\epsilon} \mathrm{d}^3\, \vec{r} j_1(qr) \varepsilon^{3ij} r_{0i} J_j^{em} \,. \tag{33}$$

In these equations, J_μ^{em} is the electromagnetic current operator obtained explicitly in Ref. [14], $\alpha_{em} = 1/137$ is the electromagnetic fine structure constant and q is the photon momentum. In (32), (33) $j_2(qr)$ and $j_1(qr)$ are spherical Bessel functions of the order $l = 2$ and $l = 1$, respectively.

In the present paper the constant-cutoff approach to the CHK bound state soliton model gives [14]

$$\widehat{M}(q) = m_1(q) J_c^3 - 2[m_2(q) + m_3(q) \mid S \mid] R_{33} + m_4(q) J^{-3} \,, \tag{34}$$

$$\widehat{E}(q) = e_1(q) \, \left[J_c^3 R_{33} + \frac{I_3}{3} \right] + e_2(q) \, \left[J^{-3} R_{33} - \frac{J^{-a} R_{3a}}{3} \right] \,, \tag{35}$$

where J_c is the collective angular momentum, J^- is the bound kaon isospin and $R_{ij} = \frac{1}{2}\mathrm{Tr}[\tau_i A \tau_j A^+]$. In (34) and (35), $m_i (i = 1, 2, 3, 4)$ and $e_i (i = 1, 2)$, respectively, are obtained explicitly in [14] in the following form

$$m_1(q) = -\frac{1}{3\pi\Omega} \int\limits_\epsilon^\infty \mathrm{d}r\, r j_1(qr) \sin^2 F \frac{\mathrm{d}F}{\mathrm{d}r} \,, \tag{36}$$

$$m_2(q) = \frac{\pi}{6} F_\pi^2 \int\limits_\epsilon^\infty \mathrm{d}r\, r j_1(qr) \sin^2 F \,, \tag{37}$$

$$m_3(q) = \frac{1}{6}\int_{\epsilon}^{\infty} \mathrm{d}r\, r j_1(qr) k^2 \cos^2 F (1 - 4\sin^2 F) \tag{38}$$

$$+\frac{N_C}{18}\frac{\omega}{F_K^2 \pi^2}\int_{\epsilon}^{\infty} \mathrm{d}r\, r j_1(qr)\left(k^2 \sin^2 F \frac{\mathrm{d}F}{\mathrm{d}r} + k\frac{\mathrm{d}k}{\mathrm{d}r}\sin 2F\right),$$

$$m_4(q) = c m_1(q) - \frac{2}{3}\int_{\epsilon}^{\infty} \mathrm{d}r\, r j_1(qr) k^2 \cos^2 \frac{F}{2}, \tag{39}$$

$$e_1(q) = -\frac{2\pi F_\pi^2}{15\Omega}\int_{\epsilon}^{\infty} \mathrm{d}r\, r^2 j_2(qr) \sin^2 F, \tag{40}$$

$$e_2(q) = c e_1(q) + \frac{8}{15}\int_{\epsilon}^{\infty} \mathrm{d}r\, r^2 j_2(qr)[\omega k^2 \cos^2 \frac{F}{2} \tag{41}$$

$$-\frac{N_C}{12\pi^2 F_K^2}\frac{1}{r^2}\cos^2\frac{F}{2}\left(k^2 \frac{\mathrm{d}F}{\mathrm{d}r}\cos^2\frac{F}{2} - k\frac{\mathrm{d}k}{\mathrm{d}r}\sin F\right).$$

The matrix elements of the operators $\widehat{M}(q)$ and $\widehat{E}(q)$ between the relevant baryon states are obtained, following Refs. [10] or [14], using the standard angular momentum techniques, as follows

$$< \Lambda \mid \widehat{M}(q) \mid \Sigma_0^* >= \frac{2\sqrt{2}}{3}[m_2(q) + m_3(q)], \tag{42}$$

$$< \Sigma_0 \mid \widehat{M}(q) \mid \Sigma_0^* >= \frac{\sqrt{2}}{3}[m_1(q) - m_4(q)], \tag{43}$$

$$< \Sigma_\pm \mid \widehat{M}(q) \mid \Sigma_\pm^* >= \frac{\sqrt{2}}{3}[m_1(q) - m_4(q) \pm (m_2(q) + m_3(q))], \tag{44}$$

$$< \Xi_{\underline{0}} \mid \widehat{M}(q) \mid \Xi_{\underline{0}}^* >= \frac{\sqrt{2}}{3}[m_1(q) - m_4(q) \pm \frac{4}{3}(m_2(q) + 2m_3(q))], \tag{45}$$

$$< \Lambda \mid \widehat{E}(q) \mid \Sigma_0^* >= -\frac{\sqrt{2}}{6}e_1(q), \tag{46}$$

$$< \Sigma_0 \mid \widehat{E}(q) \mid \Sigma_0^* >= 0, \tag{47}$$

$$< \Sigma_{\pm} \mid \widehat{E}(q) \mid \Sigma^{*}_{\pm} >= \mp\frac{\sqrt{2}}{6}[e_1(q) - \frac{1}{6}e_2(q)]\,, \tag{48}$$

$$< \Xi_{\underline{0}} \mid \widehat{E}(q) \mid \Xi^{*}_{\underline{0}} >= \mp\frac{4\sqrt{2}}{27}e_2(q)\,. \tag{49}$$

Finally we give the expression for the E2/M1 ratios. Following Refs. [10] or [14], we may write

$$\frac{E2}{M1} = \frac{5}{4}\frac{< \widehat{E}(q) >}{< \widehat{M}(q) >} \tag{50}$$

such that the following relation is satisfied

$$\frac{\Gamma_{E2}}{\Gamma_{M1}} = 3[\frac{E2}{M1}]^2\,. \tag{51}$$

4. *Numerical results*

The numerical results for the hyperon radiative decay widths in keV obtained in the present paper are presented in Table 1. In Table 1, the present results are also compared with the results obtained using DCOL, CCOL, CCHK, NRQM and QLM.

Table 1. Hyperon radiative decay widths in keV.

	This work	DCOL [14]	CCOL [9]		CCHK [10]		NRQM [11]	QLM [12]
			Set 1	Set 2	Set 1	Set 2		
$\Sigma^{*0} \to \Lambda\gamma$	227	178	180	194	243	170	232	-
$\Sigma^{*-} \to \Sigma^{-}\gamma$	1	2	1	2	1	1	2	3
$\Sigma^{*0} \to \Sigma^{0}\gamma$	18	16	15	12	19	11	18	17
$\Sigma^{*+} \to \Sigma^{+}\gamma$	85	80	78	71	91	59	100	100
$\Xi^{*-} \to \Xi^{-}\gamma$	4	4	3	4	5	5	3	4
$\Xi^{*0} \to \Xi^{0}\gamma$	135	121	115	108	148	97	137	129

The numerical results for the ratios E2/M1 in (%) obtained in the present paper are presented in Table 2. In Table 2, the comparison of present results with the results obtained using DCOL, CCOL and CCHK is also given.

From Tables 1 and 2, we see that the present results are in relatively good qualitative agreement with the results obtained using the complete CHK model in Ref. [10]. There is also a qualitative agreement with the results obtained using the other models. For the E2/M1 ratio, we see that our results are closer to those obtained in Ref. [10] than to those obtained in Refs. [14] and [9], which is expected due to the fact that the present approach is based on the bound-state soliton model used in Ref. [10].

Table 2. Ratios E2/M1 in (%).

	This work	DCOL [14]	CCOL [9]		CCHK [10]	
			Set 1	Set 2	Set 1	Set 2
$\Sigma^{*0} \to \Lambda\gamma$	-4.4	-3.9	-3.8	-3.7	-4.7	-5.4
$\Sigma^{*-} \to \Sigma^{-}\gamma$	-37.9	-6.0	-7.3	-4.3	-57.7	-51.1
$\Sigma^{*0} \to \Sigma^{0}\gamma$	0	-1.4	-1.5	-1.9	0	0
$\Sigma^{*+} \to \Sigma^{+}\gamma$	-4.1	-2.0	-2.2	-2.3	-4.8	-7.6
$\Xi^{*-} \to \Xi^{-}\gamma$	-12.6	-4.6	-6.1	-4.3	-17.8	-18.5
$\Xi^{*0} \to \Xi^{0}\gamma$	-3.0	-2.0	-2.4	-2.6	-3.1	-4.4

5. *Conclusions*

We have shown the possibility of using the Skyrme model for the study of the radiative decays of hyperons in the constant-cutoff approach to the CHK bound state soliton model [7], without the use of the Skyrme stabilizing term proportional to e^{-2}, which makes the practical calculations more complicated and introduces the problem of the choice of the stabilizing term. Thus the results for the total decay widths and the E2/M1 ratios corresponding to decouplet-to-octet electromagnetic transitions are obtained in a qualitative agreement with the results obtained using the constant-cutoff approach to the SU(3) collective model [14] (DCOL), the complete SU(3) collective model [9] (CCOL), the complete CHK bound state soliton model [10] (CCHK), non-relativistic quark model [11] (NRQM) and the quenched lattice model [12] (QLM).

For such a simple model with only one arbitrary dimensional constant F_π, chosen to be equal to its empirical value $F_\pi = 186$ MeV, we find that the results obtained here are in good qualitative agreement with those obtained using the complete CHK model in Ref. [10].

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PRISTUP STALNIM ODREZOM RADIJATIVNIM RASPADIMA HIPERONA U CHK SU(3)-SOLITONSKOM MODELU

Daje se kratak pregled metode kvantne stabilizacije za SU(2) σ-model, zasnovan na granici stalnog odreza odrezno-kvantizacijske metode od Balakrishne i sur., koja izbjegava teškoće s običnim graničnim uvjetima koje su našli Iwasaki i Ohyama. Razmatraju se radijativni raspadi hiperona pristupom stalnog odreza modelu vezanog solitona koji su razvili Callan, Hornbostel i Klebanov. Ishodi za ukupne širine i omjere E2/M1, koji odgovaraju elektromagnetskim prijelazima dekuplet-oktet, su u kvalitativnom suglasju s ishodima postignutim pristupom stalnog odreza kolektivnom SU(3) modelu, potpunim Skyrmovim modelima i modelima zasnovanim na kvarkovima.