ISSN 1330–0016
CODEN FIZBE7

WHAT DO WE KNOW ABOUT TIME REVERSAL INVARIANCE VIOLATION

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Dedicated to Professor Kseno Ilakovac on the occasion of his 70$^{\mathrm{th}}$ birthday

Received 9 March 2001; Accepted 5 September 2001 Online 30 December 2001

The time reversal (TR) transformation and its consequences are reviewed. Recent experiments and theories are discussed and limits on time reversal invariance (TRI)-breaking are given. Finally, future experiments are cited. We discuss TRI independently of the CP symmetry.

PACS numbers: 11.30.Er UDC 535.12

Keywords: violation of time reversal invariance, limits, future experiments

1. Introduction

The breaking of CP invariance has been known since the classic experiment of Christenson, Cronin, Fitch and Turley [1] in 1967. Despite 30 years of progress, it is only recently [2] that a direct violation of time reversal invariance (TRI) has been seen experimentally. To-date, the only system which shows CP noninvariance is that of neutral kaons. However, experiments at the B-factories soon should provide further clues as to the underlying theory or theories. [2]

The CPT theorem [3] suggests that TRI is violated if CP invariance does not hold. There are numerous tests that have shown the validity of CPT; these include the mass difference of the K^0 and $\bar{K}^0$, and magnetic moment difference of the e^+, e^- and μ^+, μ^- [4] (see however Ref. [5] for comments), as well as more esoteric tests [6]. However, the CPT theorem requires local forces, and recent (nonlocal) superstring theories, which try to unite all known forces, predict that the CPT theorem should break down at some level, perhaps related to the Planck mass (10^{19} GeV). So far, there is no experimental confirmation, but searches are continuing [5].

[1]Supported in part by the Department of Energy

[2]CP invariance violation has now been observed at B factories.

2. *Time reversal in quantum mechanics*

Because the Schrödinger equation is a diffusion equation and is first order in time, the time reversal information in quantum mechanics requires an antiunitary transformation [7]. Under time reversal, a wave function $\psi(\vec{r},t) \to T\psi(\vec{r},t) = \psi_T(\vec{r},-t) = K\psi(\vec{r},-t) = \psi^*(\vec{r},-t)$. Here K is a complex conjugation operator. The complex conjugate Schrödinger equation is

$$-\mathrm{i}\frac{\partial\psi^*(t)}{\partial t} = H^*\psi^*(t)\,. \tag{1}$$

If we let $t \to t' = -t$, we obtain

$$-\mathrm{i}\frac{\partial\psi^*(-t)}{\partial(-t)} = \mathrm{i}\frac{\partial\psi^*(-t)}{\partial(t)} = \mathrm{i}\frac{\partial\psi_T(-t)}{\partial t} = H^*\psi_T\,. \tag{2}$$

If H is real, $KHK = H^* = H$ and we have the usual Schrödinger equation for ψ_T. Under the time reversal transformation, momentum $\vec{p}$ should change sign, since it is first order in time, and so should the angular momentum $\vec{J}$. In quantum mechanics, $\vec{p} = -\mathrm{i}\vec{\nabla}$ changes sign because of the operation of K. For a plane wave, for instance, we have

$$T\psi(\vec{r},t) = T\mathrm{e}^{\mathrm{i}(\vec{p}\cdot\vec{r}-Et)} = \psi^*(\vec{r},-t) = \mathrm{e}^{-\mathrm{i}(\vec{p}\cdot\vec{r}+Et)} = \mathrm{e}^{\mathrm{i}(-\vec{p}\cdot\vec{r}-Et)}\,, \tag{3}$$

which also shows that $\vec{p}$ changes sign under the time reversal transformation. For the spherical harmonics it follows that

$$TY_\ell^m = KY_\ell^m = Y_\ell^{m*} = (-1)^m Y_\ell^{-m}\,. \tag{4}$$

For many purposes, and as we will see, it is useful to introduce spherical harmonics with a different phase when discussing time reversal, namely

$$\begin{aligned} \mathcal{Y}_\ell^m &= \mathrm{i}^\ell Y_\ell^m\,, \\ T\mathcal{Y}_\ell^m &= (-1)^{\ell-m}\mathcal{Y}_\ell^{-m}\,. \end{aligned} \tag{5}$$

It is the complex conjugation which makes TRI more difficult and the time reversal operation non-unitary.

For the case of a fermion, or more generally, we have

$$T\psi(t) = \psi_T(-t) = U_T K\psi(-t) = U_T\psi^*(-t)\,, \tag{6}$$

where U_T is a unitary transformation. We can see the non unitarity of T by examining the scalar product of two states

$$\begin{aligned} \langle T\psi(t)|T\phi(t)\rangle = \langle U_T\psi^*(-t)|U_T\phi^*(-t)\rangle &= \langle\psi^*(-t)|\phi^*(-t)\rangle \\ \langle\psi(-t)|\phi(-t)\rangle^* &= \langle\phi(-t)|\psi(-t)\rangle\,, \end{aligned} \tag{7}$$

$$\langle\psi_T|\phi_T\rangle = \langle\psi|\phi\rangle^* = \langle\phi|\psi\rangle\,. \tag{8}$$

For an operator A, such as the momentum, $\vec{p}$, the time reversal transformation is

$$A_T = TAT^{-1} = U_T A^* U_T^\dagger\,. \tag{9}$$

For the matrix element of the operator A we have [7]

$$\begin{aligned}\langle T\psi(t)|TAT^{-1}|T\phi(t)\rangle &= \langle U_T\psi^*(-t)|U_T A^* U_T^\dagger|U_T\phi^*(-t)\rangle \\ &= \langle\psi(-t)|A|\phi(-t)\rangle^* = \langle\phi(-t)|A^\dagger|\psi(-t)\rangle\,.\end{aligned} \tag{10}$$

If A is a Hermitian observable $A_H = A_H^\dagger$, we have

$$\langle\psi_T|A_{H,T}|\phi_T\rangle = \langle\phi|A_H|\psi\rangle\,. \tag{11}$$

I have not yet used time reversal symmetry. If $A_T = A$ (e.g., $H_T = H$), then an invariance is implied. If, furthermore, ψ and ϕ are eigenstates of T, e.g., $T\phi = \phi$, then it follows that $\langle\phi|A|\phi\rangle$ is purely real (purely imaginary if $A_T = -A$, e.g., $\vec{p}_T = -\vec{p}$).

For a non-relativistic fermion, $U_T = \mathrm{i}\sigma_y$; this follows because σ_y is imaginary and anticommutes with σ_x and σ_z. Relativistically, we can take $U_T = \mathrm{i}\gamma_1\gamma_3$ in the Bjorken-Drell metric [8]. With this transformation we obtain for the electromagnetic charge and current and the vector potentials

$$Tj^0T^{-1} = j^0, \quad T\vec{j}T^{-1} = -\vec{j}\,, \tag{12}$$

$$TA^0T^{-1} = A^0, \quad T\vec{A}T^{-1} = -\vec{A}\,. \tag{13}$$

We also find

$$T|\vec{p},\frac{1}{2},m\rangle = (-1)^{\frac{1}{2}-m}\,|-\vec{p},\frac{1}{2},-m\rangle\,, \tag{14}$$

where m is the magnetic quantum number of the spin. The usefulness of the phase introduced in the spherical harmonic $\mathcal{Y}_\ell^m$ now becomes apparent because we then can readily generalize this transformation by substituting s for the spin $\frac{1}{2}$,

$$T|\vec{p},s,m) = (-1)^{s-m}|-\vec{p},s,-m\rangle\,. \tag{15}$$

3. *Tests of time reversal symmetry and their basis*

In the following, we discuss some leading tests of time reversal invariance. The experiments cited are not intended to be a complete list of those that have been carried out [9,10].

Time invariance implies that [7]

$$THT^{-1} = H\,. \tag{16}$$

In general, tests of TRI require a comparison of two reaction rates; the exception is a process that is sufficiently weak that perturbation with a Hermitian interaction Hamiltonian can be used and *there are no strong interactions among the final or initial particles*. Consider an electromagnetic or weak transition from a state $|i\rangle = |a, j_i, m_i\rangle$ to a state $|f\rangle = |b, j_f, m_f\rangle$, where a and b stand for all other quantum numbers than the angular momenta, with a Hamiltonian H' responsible for the transition. If $H'_T = H'$, it follows from Eq. (11) that

$$\langle f_T|H'|i_T\rangle = (-1)^{j_i+j_f-m_i-m_f}\langle b, j_f, -m_f|H'|a, j_i, -m_i\rangle = \langle f|H'|i\rangle^*\,. \tag{17}$$

If H' is a scalar or pseudoscalar operator, then $j_i = j_f$ and $m_i = m_f$. Since $2(j-m)$ is even, we find that the matrix element must be real. Thus, the matrix element of a Hermitian and rotationally invariant operator must be real.

3.1. Electromagnetic transitions

The above reality can be generalized for electromagnetic transition multipoles, $T_q^{(k)}$ [see Eq. (7)]

$$T\,T_q^{(k)}T^{-1} = \eta_k\,(-1)^{k-q}\;T_{-q}^{(k)}\,, \tag{18}$$

where k is the tensor order, q the spherical component, and $\eta_k = \pm 1$. After some algebra [7] and use of the symmetries of Clebsch-Gordan coefficients, we obtain

$$\begin{aligned} \langle f_T|T\,T_q^{(k)}T^{-1}|i_T\rangle &= \\ \eta_k(-1)^{j_f+j_i+k-m_f-m_i-q}\;\langle f, j_f, -m_f|T_{-q}^{(k)}|i, j_i, -m_i\rangle\,, \end{aligned} \tag{19}$$

$$\langle f_T|T\,T_q^{(k)}T^{-1}|i_T\rangle = \eta_k\langle f|T_q^{(k)}|i\rangle = \langle f|T_q^{(k)}|i\rangle^*\,. \tag{20}$$

Thus, the matrix elements of the tensor operators are either real or imaginary, depending on whether $\eta_k = \pm 1$, but it is always possible to choose phases such that $\eta_k = +1$, and all matrix elements are then real. Most experiments that have been carried out make use of a search for a phase different from 0° or 180° in an interference of an E2 (quadrupole) and M1 (magnetic dipole) transition matrix elements. Such a phase would indicate a time reversal invariance violation. Experimentally, to search for a TRI violation in an electromagnetic transition, one can look for a term proportional to $\vec{k}\cdot\vec{j}_f\vec{k}\cdot\vec{j}_f\times\vec{j}_i$, where $\vec{j}_i$ ($\vec{j}_f$) is the polarization of the initial (final) state, and $\vec{k}$ is the momentum of the photon emitted in the transition. The correlation sought is even under the parity (P) transformation but odd under time reversal. You can also search for a time reversal violation that is odd under a P,

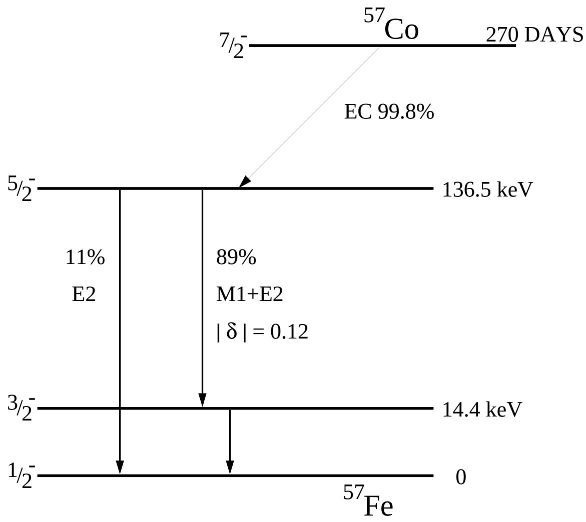

Fig. 1. Decay scheme for ^{57}Co*, showing the mixed decay and subsequent gamma-ray transition.*

e.g., $\vec{k} \cdot \vec{j}_f \times \vec{j}_i$. The alignment of the final state (f) is detected by a subsequent electromagnetic transition, as illustrated in Fig. 1.

The most significant limit on a TRI violation for a parity-even test has perhaps been found in the study of ^{57}Fe, with a decay scheme shown in Fig. 1. The sine of the interference angle, η, betwen the E2 and M1 matrix elements was found to be $\sin\eta = (2.9 \pm 6.6) \times 10^{-4}$ [9]. Parity-odd triple correlations have also been sought [9], but no violation of TRI has been found. There are other T-odd correlations than those mentioned above which can be used, but the limits are not as good as those described [9]. One has to be careful in that any interaction of the photon with atomic shell electrons spoils the Hermiticity of the operator and thus the test. It is easy to see where this comes from. In the first order perturbation theory, the matrix element is that of a Hermitian Hamiltonian, H'. Already in the next order, we have the matrix element of the operator $H'(E - H_0 + \mathrm{i}\epsilon)^{-1}H'$. The denominator can be written as $P(E - H_0)^{-1} - \mathrm{i}\epsilon\delta(E - H_0)$. The last term, which gets a contribution from the final state rescattering, introduces an imaginary phase, which mimics a TRI-violating term. These effects are small except in very heavy nuclei; they can be estimated and corrections made [9]. This was done in the above experiments and in ^{191}Ir tests, where the final state interaction phase was measured to be $-(4.0 \pm 0.2) \times 10^{-3}$ in agreement with the theory, $-(3.9 \pm 4.7) \times 10^{-3}$ [9].

3.2. Beta decay tests

Correlation tests of TRI can also be used for beta decays, but here parity is not conserved. One can therefore search for a phase in the interference of a Fermi and Gammow-Teller transition matrix element. TR-odd terms would be of the form $D\vec{j}\cdot\vec{p}_e\times\vec{p}_\nu$ and $R\vec{j}\cdot\vec{s}_e\times\vec{p}_\nu$, where $\vec{j}$ refers to the polarization of the parent nucleus, and the other terms to the momenta of the subscribed particle and polarization (s_e) of the electron. These terms may violate P as well as T. Measurements of D and R again are limited by final state interactions of the emitted electron and nucleus, which spoils the Hermiticity od H' by introducing an imaginary part to the amplitude. You then can get effects which mimic a TRI violation but is really just a final state interaction. There have been searches for both the D and R terms. The most precise measurement of R depends primarily on an interference of the axial coupling and a tensor one, $C_T\sigma_{\mu\nu}q^\nu\gamma_5$, proportional to Im $(C_A C_T^*)$; it has been carried out for the β-decay of ^{8}Li [9,11] with $R = -(0.2 \pm 4.0)\times 10^{-3}$. The D-coefficient has been sought, particularly in the decay of ^{19}Ne and for the decay of polarized neutrons. For ^{19}Ne, $D = (0.1\pm 6)\times 10^{-3}$, or $\mathrm{Im}\,(C_A/C_T^*) < 2\times 10^{-3}$. The neutron decay provides the best limit on the interference of the vector and axial vector terms, $D = (0.5\pm 1.4)\times 10^{-3}$ or $\mathrm{Im}\,(C_A/C_T^*) < 1.4\times 10^{-3}$ [9,10]. Recently, a new experiment on the decay of the neutron gave $D = (-0.6 \pm 1.7)\times 10^{-3}$ [12]. Combined with the previous results, this gives $D = (-5.5\pm 9.5)\times 10^{-4}$, or $\mathrm{Im}\,(C_A/C_T^*) < 1\times 10^{-3}$, e.g., a phase that differs from 180° by less than 1 part in 10^3.

Efforts are under way to improve these results. Neither of these experiments is sufficiently accurate to probe final state interaction effects, although these corrections are taken into account. No violation of TRI has been found in beta decay tests [9].

3.3. Strong interaction reactions

Time reversal invariance can also be tested in reactions, whether due to strong interactions or weaker forces, by means of the principle of reciprocity. We define the scattering matrix S by

$$\begin{aligned} S\,|in\rangle &= |out\rangle\,, \\ SS^\dagger &= S^\dagger S = 1\,, \\ S_T &= TST^{-1} = S^\dagger\,. \end{aligned} \tag{21}$$

The rate for reaching a final state $|f\rangle$ from an initial one $|i\rangle$ is

$$\mathcal{R}_{fi} = const\ |\langle f|S|i\rangle|^2\rho_f \equiv const\ |S_{fi}|^2\rho_f\,, \tag{22}$$

where ρ_f is the final phase space density. If TRI holds, we have

$$\frac{\mathcal{R}_{fi}}{\mathcal{R}_{i_T f_T}} = \frac{\rho_f}{\rho_i}\,. \tag{23}$$

This is the principle of reciprocity, with

$$\begin{aligned} S_{fi} = \langle f|S|i\rangle &= \langle b, \vec{p}, j_f, m_f|S|a, \vec{p}, j_i, m_i\rangle \,, \\ S_{i_T f_T} = \langle i_T|S|f_T\rangle &= (-1)^{j_i+j_f-m_i-m_f}\langle a_T, -\vec{p}_i, j_i, -m_i|S|b_T, -\vec{p}_f, j_f, -m_f\rangle \,. \end{aligned} \tag{24}$$

Here a and b stand for all other quantum numbers. The time reversed states have both their momenta and spins reversed. This relation tells us that the polarization in a given reaction should equal the analyzing power in the reverse reaction. Such tests have been carried out and show that TRI holds to better than 1% in nuclear reactions. If parity conservation holds, we can change $-\vec{p}$ into $\vec{p}$ in both initial and final states. If we average over the spins (do not measure polarizations), we obtain the principle of detailed balance

$$\sum_{m_i m_f} |\langle b, \vec{p}_f, j_f, m_f|S|a, \vec{p}_i, j_i, m_i\rangle|^2 = \sum_{m_i m_f} |\langle a_T, \vec{p}_i, j_i, m_i|S|b_T, \vec{p}_f, j_f, m_f\rangle|^2. \tag{25}$$

The best detailed balance experiment is the reaction d + ^{27}Al $\leftrightarrow$ ^{4}He + ^{24}Mg and shows no asymmetry at the level $\leq 5 \times 10^{-4}$ [13]. Reactions which proceed via compound nuclei are particularly sensitive to odd-TRI forces due to the high density of energy levels; they give comparable limits on TRI violation [10].

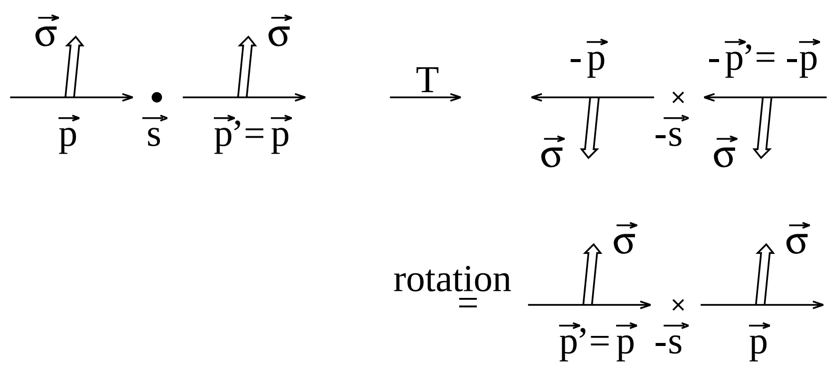

Fig. 2. Reciprocity test of TRI with polarized neutron ($\vec{\sigma}$) transmision through a polarized ($\vec{s}$) target.

A clever null-type experiment was suggested by Stodolsky [14]. It can be carried out in either a parity conserving or parity-violating mode. It is the forward elastic scattering of slow polarized neutrons from transversally polarized nuclei (e.g., Fe) as shown in Fig. 2. Although it is a null experiment and appears to require a weak interaction with a Hermitian interaction H', the search for terms (in forward elastic scattering) $A\vec{\sigma}\cdot\vec{s}\times\vec{p}$, which is P-odd and T-odd, or $B\vec{\sigma}\cdot\vec{s}\times\vec{p}\,\vec{p}\cdot\vec{s}$, which is P-even and T-odd, actually makes use of reciprocity. This is illustrated in Fig. 2. From this figure it is seen that the time reversed situation for forward elastic scattering is identical to the initial one, except for the reversal of the target spin ($\vec{s}$). Thus, if time reversal is a valid symmetry, the terms A and B, above are not allowed. An experimental search was carried out in the parity-conserving mode for a ^{165}Ho target by Huffman et al. [15]. Their null result can be interpreted as a TRI violation $\leq 7.1 \times 10^{-4}$ [15]. There has been discussion of carrying out the test for a nucleus such as ^{139}La, where a large ($\sim 10^5$) enhancement was found for parity nonconservation due to the mixing of very closely spaced compound nucleus resonances.

3.4. Electric dipole moments

Perhaps the most sensitive test of TRI is the search for an electric dipole moment (d_E) of an electron, a neutron, or an atom. Parity non-conservation is required to have a non-vanishing d_E, but TRI violation is *also* necessary. The argument is illustrated in Fig. 3. When time is reversed, the angular momentum changes sign, but the electric dipole moment does not; hence if d_E does not vanish, we can tell the difference between a movie running forwards and backwards, i.e., TRI is violated. A similar argument holds for parity.

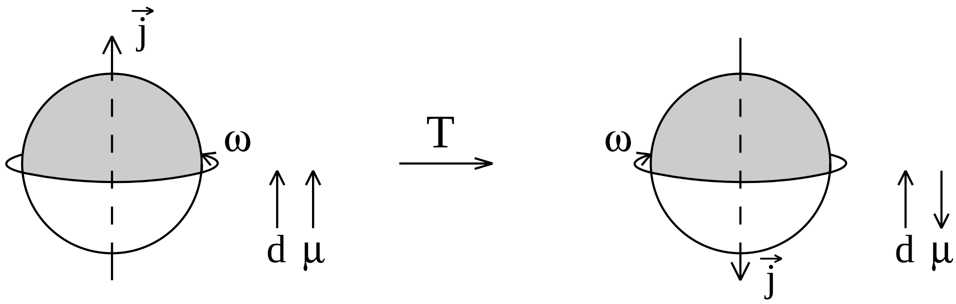

Fig. 3. Behavior of electric and magnetic dipole moments under time reversal; the angular momentum defines a direction in space.

Even more accurate experiments have attempted to find a non-vanishing d_E and the best present upper limits are [16–18]

$$\begin{aligned} d_E(\mathrm{n}) &\leq 6.3 \times 10^{-26}\ e\,\mathrm{cm}, \\ d_E(^{199}\mathrm{Hg}) &\leq 8 \times 10^{-28}\ e\,\mathrm{cm}, \\ d_E(^{129}\mathrm{Xe}) &\leq 4 \times 10^{-27}\ e\,\mathrm{cm}, \\ d_E(\mathrm{e}^-) &\leq 4 \times 10^{-27}\ e\,\mathrm{cm}. \end{aligned} \tag{26}$$

^{199}Hg has paired electrons but the spin of the nucleus is 1/2. The numbers for d_E are very small. You might expect $d_E(\mathrm{n}) \sim e\times$ neutron size $\times$ relative strength of weak interaction × reduction due to need of TRI violation, i.e., $\sim e \times (10^{-13} - 10^{-14})$ cm $\times (10^{-6} - 10^{-7}) \times 10^{-3} \sim (10^{-22} - 10^{-24})\, e\,$cm. You may thus say that the test shows no TRI violation to $\sim 10^{-5}$ of the weak interaction strength.

Measurements of the electric dipole moment also limit T-odd, P-even interactions through the parity violation of the weak forces [17].

3.5. The K^0, $\bar{\mathrm{K}}^0$ *system*

Since CP is known to be violated in the $\mathrm{K}^0\bar{\mathrm{K}}^0$ system to $\sim 2 \times 10^{-3}$, the CPT theorem leads us to expect a TRI violation of about the same magnitude. However, until 1999, no direct test of TRI had shown any measurable effect.

In 1999, the CPLEAR team at CERN reported on the neasurement of the asymmetry

$$a = \frac{\mathcal{R}(\bar{\mathrm{K}}^0 \to \mathrm{e}^+\pi^-\nu) - \mathcal{R}(\mathrm{K}^0 \to \mathrm{e}^-\pi^+\bar{\nu})}{\mathcal{R}(\bar{\mathrm{K}}^0 \to \mathrm{e}^+\pi^-\nu) + \mathcal{R}(\mathrm{K}^0 \to \mathrm{e}^-\pi^+\bar{\nu})} = (6.6 \pm 1.3 \pm 1.0) \times 10^{-3}\,, \qquad (27)$$

where $\mathcal{R}$ is a rate, the first error is statistical and the second one systematic. Why is this a test of TRI [19]? Only if TRI holds will

$$\mathcal{R}\big[\bar{\mathrm{K}}^0(t=0) \to \mathrm{K}^0(t=T)\big] = \mathcal{R}\big[\mathrm{K}^0(t=0) \to \bar{\mathrm{K}}^0(t=T)\big]\,. \qquad (28)$$

These experimental results are in agreement with expectation based on the CP invariance violation in the K^0, $\bar{\mathrm{K}}^0$ system. The test does assume CPT invariance, but only in that the semileptonic kaon decays obey it; alternatively, the Bell-Steinberger relation needs to be saturated by the known decays. Moreover, Ellis and Mavromatos [19] show that the test cannot be interpreted as a maintenance of T invariance and violation of the CPT theorem. The CPT symmetry would have to be broken at the level of $\sim 10^{-3}$, and it has been shown to hold many orders of magnitude better than this limit [27].

Another experiment which shows a possible violation of T invariance is that carried out by the KTeV collaboration [21]. Here one measures $\vec{n}_\mathrm{e} \times \vec{n}_\pi \cdot \vec{p}_\pi\, \vec{n}_\mathrm{e} \cdot \vec{n}_\pi$ in the decay $\mathrm{K}_L \to \pi^+\pi^-\mathrm{e}^+\mathrm{e}^-$, with $\vec{n}$ being a vector perpendicular to the plane of the electron or pions, and $\vec{p}_\pi$ being a unit vector in the direction of the center-of-mass of the two pions. This correlation is T-odd and CP-odd, but it could come about from a CPT violation [22,23]; furthermore, there are strong final state interactions of the pions which can affect the effect observed.

4. Theories of TRI violation

There are many phenomenological models of TRI violation. Some of these have been ruled out by the ever more precise searches for an electric dipole moment [16,17]. Among those ruled out are some models with several charged Higgs bosons;

survivors are left-right symmetry models and R-parity violation models in supersymmetric theories [20]. I will not describe these models, but concentrate on the standard model and a QCD-based theory. Both of these predict only TRI- and parity-violating interactions. However, the weak forces can introduce T-odd P-even ones. This has been discussed by several authors see, e.g., Ref. [17]).

In the standard model, the quarks of the strong interaction are not appropriate eigenstates of the weak interaction; the strong interaction quarks are mass eigenstates which respect P, C and CP. Kobayashi and Maskawa [24] proposed a matrix (now called the CKM matrix, after Cabibbo, Kobayashi and Maskawa) to connect these eigenstates. By convention, the up, charmed and top quarks are unmixed, but the down, strange and bottom quarks are mixed by a unitary matrix, the CKM natrix U. The charged current weak interaction for the three families can be written as

$$\mathcal{L} = \frac{g}{2\sqrt{2}}(\bar{u}\bar{c}\bar{t})\gamma_\mu(1-\gamma_5)U \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\mu + h.c.. \tag{29}$$

with

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}; \tag{30}$$

this can be approximated as [4]

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - \mathrm{i}\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-\mathrm{i}\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{31}$$

The diagonal elements of the CKM matrix are close to 1. The matrix element V_{us} is known as the sine of the Cabibbo angle $\simeq 0.22 \equiv \lambda$. Wolfenstein [4] has shown that this approximation provides an order of magnitude for various matrix elements. The measurable phase can be introduced because we have three families. Because the matrix U is unitary, we have, for instance

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1\,, \tag{32}$$

which can be tested [25].

The standard model and CKM matrix leads to the prediction of a very small electric dipole moment of the neutron. The reason is that flavor is conserved in the matrix element of the neutron dipole moment, so that the CKM matrix must be invoked twice. This leads to a reduction of $10^{-6} - 10^{-7}$ in the order of magnitude expected for $d_E(\mathrm{n})$, for an expected $d_E \sim (10^{-28} - 10^{-31})\ e\,\mathrm{cm}$. Calculations based on the CKM matrix give an even smaller number, $\sim (10^{-31} - 10^{-33})\ e\,\mathrm{cm}$ [20].

The standard model need not, and probably cannot be the sole source of TRI violation. One reason is that it fails to be sufficiently large to allow us to understand the lack of antibarions in our Universe [26].

It is noteworthy that QCD allows a T-odd interaction term. The most general QCD Lagrangian density, consistent with Lorentz invariance, Hermiticity, and gauge and chiral invariances is (color indices are omitted, except where required) [27]

$$\mathcal{L} = \bar{\psi}\mathrm{i}\,\not{D}\psi - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{g_s^2}{32\pi^2}\theta G_{\mu\nu}\bar{G}^{\mu\nu} = \mathcal{L}_{QCD} + \mathcal{L}_\theta\,, \tag{33}$$

where θ is an angle parameter, g_s is the strong coupling constant,

$$\begin{aligned} \not{D} &\equiv (\partial_\mu + \mathrm{i}g_s A_\mu^a \lambda^a/2)\gamma^\mu\,, \\ \bar{G}^{\mu\nu} &\equiv \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}\,, \\ G_{\mu\nu} &\equiv \left[\partial_\mu + \mathrm{i}g_s A_\mu,\, \partial_\nu + \mathrm{i}g_s A_\nu\right], \end{aligned} \tag{34}$$

where A_μ is the gloun field and a is a color index. The θ term violates both P and T, and is therefore CP-odd, but it is gauge invariant and renormalizable. It is a total derivative and therefore does not contribute classically or in perturbation theory, but only to non-perturbative effects. It arises because there are an infinite number of nontrivial degenerate QCD vacuua with gluon fields in their lowest state [28]. One can also write higher-dimensional terms, but they are expected to be small. The angle variable θ is used because the S-matrx has a periodicity of 2π in θ. The problem is complicated by the possible presence of a complex mass term in the QCD Lagrangian

$$m_q\bar{\psi}\mathrm{e}^{\mathrm{i}\theta_m\gamma_5}\psi\,, \tag{35}$$

which has a P- and T-odd part $\mathrm{i}m_q\sin\theta_m\bar{\psi}\gamma_5\psi$. A chiral rotation can shift some of the θ term into the θ_m mass term or vice versa, but you cannot rotate both away simultaneously. If either θ or θ_m were alone, then it could be rotated away by a chiral rotation,

$$\begin{aligned} q_i &\rightarrow \exp(\mathrm{i}\gamma_5\alpha_i/2)q_i \\ \theta &\rightarrow \theta - 2\alpha_i\,. \end{aligned} \tag{36}$$

Thus, it is only $\tilde{\theta} \equiv (\theta + \theta_m)$, which is measurable and meaningful. For low energy calculations, it is simpler to shift all of the $\tilde{\theta}$ to the θ_m term [28,29].

The absence of a neutron electric dipole moment at the measured level places a stringent limit on $\tilde{\theta}$. The first calculations made use of the dominance of the coupling of the photon to the pion in a pion loop (see Fig. 4) to compute the

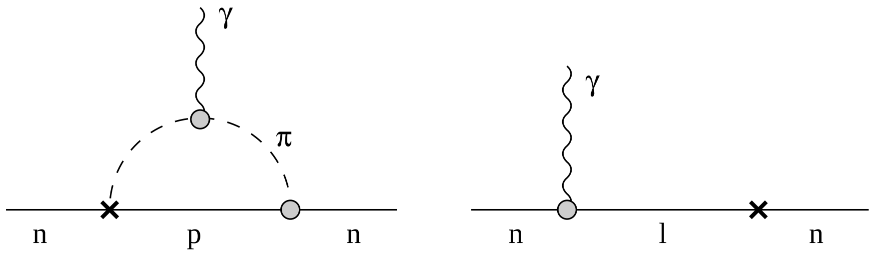

Fig. 4. Feinman diagram contributing to d_E(n). The cross corresponds to a CP-violating πNN *vertex,* $ig'_{\pi NN}\bar{N}\tau N\cdot\pi$.

neutron electric dipole moment [30], with a CP-violating coupling $ig_{\pi NN}\bar{\psi}\tau\psi\cdot\pi$, with $g'_{\pi NN} = 0.028\,\tilde{\theta}\; e\,\mathrm{cm}$. This gives

$$d_E(\mathrm{n}) = \frac{e g_{\pi NN} g'_{\pi NN}}{4\pi^2 m_N} \ln\frac{m_N}{m_\pi}\,,$$

with $g_{\pi NN}$ being the strong pion nucleon coupling. There have also been calculations with various bag models [28], with QCD sum rules [29,31] and by other means [28]. They all give approximately the same result

$$d_E \simeq -(3-6)\times 10^{-10}\;\tilde{\theta}\; e\;\mathrm{cm}, \quad \text{so that} \quad \tilde{\theta} \leq 2\times 10^{-10}\,.$$

We thus have what is often called the strong CP problem (it could also be called a T-problem) as to why $\tilde{\theta}$ is so small.

As mentioned earlier, there are other models that have been considered [20]. We need a sufficiently large violation of TRI to explain the barion asymmetry of the Universe [26] and thus there may well be other sources of T-violation.

5. *Outlook*

There are efforts under way to improve some of the TRI tests described herein. Some of these proposed improvements have already been mentioned. In addition, forbidden beta decays have been proposed for added enhancement [32]. Higher static multipole moments, e.g., M2, have also been suggested in nuclei where the effect could be enhanced. Measurements of the electric dipole moment of the electron, neutron, and atoms have already seen many orders of magnitude improvement. New techniques have been suggested to allow further improvements by more orders of magnitude [33,34]. In a few years, it is likely that these experiments will get close to values predicted by the standard model.

Measurements of the muon transverse (to the scattering plane) polarization, e.g., $\vec{\sigma}_\mu\cdot\vec{k}_\mu\times\vec{k}_\nu$, where the subscripts refer to the particles, in kaon decay, K $\to \pi+\mu+\nu$ is being carried out at KEK [35]. In addition, it would be worth while to seek TRI violation in the neutrino sector. This system is similar to that of three quarks and a meaningful TRI-violating phase clearly can be introduced. One way to search for a T-violation would be in the comparison of oscillation rate $\nu_\mu \to \nu_e$ with that for $\nu_e \to \nu_\mu$ [36,37].

In interactions involving hadrons, the decay $\Lambda \to p+\pi^-$ can be used for a search of a TRI violation. However, there is a large final state interaction which need to be measured to a high accuracy and subtracted from any measured asymmetry. One can also use $\bar{p}p \to \bar{\Lambda}\Lambda \to \bar{p}\pi^+ p\pi^-$, with the correlation $\vec{p}_i \times \vec{\Lambda} \cdot (\vec{p}_f - \vec{\bar{p}}_f)$.

Charge symmetry-breaking forces lead to T-odd P-even terms in neutron-proton scattering. Transmission experiments have been proposed to take advantage of correlations similar to those suggested by Stodolsky [14] (see Sect. 3.3). Both neutron scattering on protons (at TRIUMF) and protons on deuterium (at COSY) have been suggested [35].

There are many other suggestions, see, e.g., Refs. [17,38] for TRI-breaking tests and there will surely be such tests at the B-factories.

6. *Summary*

In this paper we have reviewed the theory of TRI, discussed some tests of TRI that have been carried out, and presented some of the ones being planned or proposed. So far, only one experiment, namely that of the $K^0\bar{K}^0$ system has shown a non-zero effect.

References

[1] J. H. Christensen, J. W. Cronin, W. L. Fitch and R. Turlay, Phys. Rev. Letters **13** (1964) 138.

[2] A. Angelopoulos et al., CPCLEAR Collaboration, Phys. Lett. B **444** (1998) 43.

[3] J. Schwinger, Phys. Rev. **82** (1951) 914 and **91** (1951) 713; G. Lueders, Z. Phys. **133** (1952) 325; W. Pauli, in *Niels Bohr and the Development of Physics*, Pergamon, London (1955). J. S. Bell, Proc. Roy. Soc. A (London) **231** (1955) 479.

[4] *Review of Particle Data*, Particle Data Group, Europ. Phys. J. C **15** (2000) 1.

[5] I. I. Bigi, LANL Archive, hep-ph-0011231 (2000).

[6] R. Blum, V. A. Kostalecky and C. D. Lane, Phys. Rev. Lett. **84** (2000) 1098; D. Bear et al., LANL Archive, phys-0007049 (2000).

[7] H. Frauenfelder and E. M. Henley, *Nuclear and Particle Physics*, W. A. Benjamin, Reading, MA (1975).

[8] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill, New York, (1964).

[9] F. Boehm, in *Symmetries and Fundamental Interactions in Nuclei*, eds. W. C. Haxton and E. M. Henley, World Scientific, Singapore (1995) p. 67.

[10] P. Herczeg, in *Symmetries and Fundamental Interactions in Nuclei*, eds. W. C. Haxton and E. M. Henley, World Scientific, Singapore (1995) p. 89.

[11] J. Sromicki et al., Phys. Rev. C **53** (1996) 932.

[12] L. J. Lising et al., Phys. Rev. C **62** (1996) 055501.

[13] E. Blanke et al. Phys. Rev. Lett. **51** (1983) 355;

[14] L. Stodolsky, in *Tests of Time Reversal Invariance in Neutron Physics*, eds. N. Roberson, C. R. Gould and J. D. Bowman, World Scientific, Singapore (1987) p. 12.

[15] P. R. Huffman et al., Phys. Rev. C **55** (1997) 2684.

[16] P. G. Harris et al., Phys. Rev. Lett. **82** (1999) 904; J. P. Jacobs et al., Phys. Rev. A **52** (1995) 3521.

[17] A. Kurylov, G. C. McLaughlin and M. J. Ramsey-Musolf, LANL Archive, hep-ph-0011185 (15 Nov. 2000).

[18] M. A. Rosenberry and T. E. Chupp, Phys. Rev. Lett. **86** (2001) 22.

[19] J. Ellis and N. E. Navromatos, Phys. Rep. **320** (1999) 341.

[20] L. Wolfenstein, Ann. Rev. Nucl. Part. Sci. **36** (1986) 137; X. G. He, B. H. J. McKellar and S. Pakvasa, Int. J. Mod. Phys. A **4** (1986) 5; N. F. Ramsay, Ann. Rev. Nucl. Part. Sci. **40** (1990) 1.

[21] The KTeV Collaboration, A. Halavi-Harati et al., Phys. Lett. **84** (2000) 408; B. Winstein, LANL Archiv, hep-ex/9911031 (1999).

[22] I. I. Bigi and A. I. Sanda, LANL Archiv, hep-ph/9904484 (1999).

[23] L. Wolfenstein, Phys. Rev. Lett. **83** (1999) 911.

[24] M. Kobayashi and T. Maskawa, Progr. Theor. Phys. **49** (1973) 652; N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531;

[25] I. S. Towner and J. C. Hardy, in *Symmetries and Fundamental Interactions in Nuclei*, eds. W. C. Haxton and E. M. Henley, World Scientific, Singapore (1995) p. 183.

[26] A. Riotto and M. Troddent, Ann. Rev. Nucl. Part. Sci. **49** (1999) 35.

[27] E. M. Henley, *CP, T, and CPT Symmetries*, to be published in *Proc. Sch. on Violation of CPT Symmetries and Related Processes*, Prerow, Germany Oct. 1–8, 2000.

[28] H.-Y. Cheng, Phys. Rep. **158** (1988) 1.

[29] C.-T. Chan, Ph. D. thesis, Univ. of Washington (1996); C.-T. Chan, E. M. Henley and T. Meissner, LANL Archive, hep-ph/9905317 (1999).

[30] V. Baluni, Phys. Rev. D **19** (1979) 2227; R. J. Crewther et al., Phys. Lett. B **88** (1979) 123 and Erratum, ibid **91** (1980) 487.

[31] M. Pospelov and A. Ritz, Phys. Rev. Lett. **83** (1999) 2526, and Nucl Phys. B **573** (2000) 177.

[32] E. M. Henley and I. B. Khriplovich, Phys. Lett. B **289** (1992) 223.

[33] E. N. Fortson, private communication.

[34] E. Hinds, to appear in *Conf. on CP Violation Physics*, Sept. 18–22, 2000, Ferrara, Italy.

[35] W. T. H. van Oers, Int. J. Mod. Phys. E **8** (1999) 513.

[36] L. Wolfenstein, Int. J. Mod. Phys. E **8** (1999) 501.

[37] H. Yokomakura, K. Kimura and A. Takamura, Phys. Lett. B **496** (2000) 175.

[38] R. Kitano and Y. Okada, LANL Archive, hep-ph/0012040 (Dec. 5, 2000).

ŠTO ZNAMO O KRŠENJU INVARIJANTNOSTI OBRATA VREMENA

Razmatra se pretvorba vremenskog obrata (TR) i njene posljedice. Raspravljaju se nedavna mjerenja i teorije i daju granice kršenja invarijantnosti obrata vremena (TRI). Na kraju, navode se buduća mjerenja. TRI se raspravlja neovisno o CP simetriji.