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CHIRAL QUARK MODELS AND THEIR APPLICATIONS

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Dedicated to the memory of Professor Dubravko Tadić

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We give an overview of chiral quark models, both for the pure light sector and the heavy-light sector. We describe how such models can be bosonized to obtain well known chiral Lagrangians which can be inferred from the symmetries of QCD alone. In addition, the coefficients of the various pieces of the chiral Lagrangians can be calculated within these models. We discuss a few applications of the models, in particular, $B-\overline{B}$ mixing and processes of the type $B \to D\bar{D}$, where D might be both pseudoscalar and vector. We suggest how the formalism might be extended to include light vectors (ρ, ω, K^*), and heavy to light transitions like $B \to \pi$.

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1. *Introduction*

While the short-distance (SD) effects in hadronic physics are well understood within perturbative quantum chromodynamics (pQCD), long-distance (LD) effects have been hard to pin down. Even if quark models are not QCD itself, various QCD inspired quark models have been useful to make predictions for a limited class of problems. Lattice QCD and QCD sum rules are on more solid ground theoretically, but are in various cases not so easy to apply. In the light quark sector, low-energy quantities have been studied in terms of the (extended) Nambu-

Jona-Lasinio model (NJL)[1], and also the chiral quark model (χQM)[2], which is the mean-field approximation of NJL.

Within the χQM, the light quarks (u, d, s) couple to the would be Goldstone octet mesons (K, π, η) in a chiral invariant way, such that all effects are in principle calculable in terms of physical quantities and a few model-dependent parameters, namely the quark condensate, the gluon condensate, and the constituent quark mass [3, 4, 5]. More specific, one may calculate the coupling constants of chiral Lagrangians by integrating out the quarks by means of the χQM. In this way chiral quark models bridge between pQCD and chiral perturbation theory (χPT) as indicated in Fig. 1.

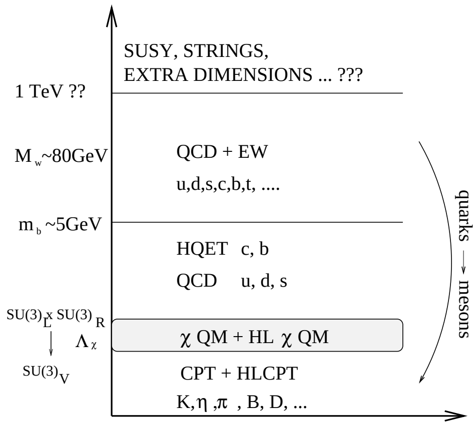


Fig. 1. Energy hierarchy in interactions of elementary particles.

The ideas from the chiral quark model of the pure light sector [2, 3, 4, 5] has been extended to the sector involving a heavy quark (c or b) and thereby to heavy-light mesons [6]. Such models we name heavy-light chiral quark models (HLχQM). Also in this case, one may integrate out the light and heavy quarks and obtain chiral Lagrangians involving light and heavy mesons [7]. That is, we calculate the parameters of chiral Lagrangian terms, where the descriptions of heavy mesons are in accordance with the heavy-quark effective field theory (HQEFT) [8]. In our approach [9], we extended the formalism of [6] to include gluon (vacuum) condensates.

One important motivation for the inclusion of gluon condensates is the possibility to estimate non-factorizable (colour suppressed) contributions in non-leptonic decays. For instance, $K - \bar{K}$-mixing and the $\Delta I = 1/2$ rule for $K \to 2\pi$ can be understood in a reasonable way within the χQM [3, 5] including gluon condensates. Especially, the suppression of the $I = 2$ amplitude found for $K \to 2\pi$ is also in agreement with the generalized factorization [10]. Furthermore, it allows us for instance to consider decays where the gluonic aspect of η' is relevant [11], and some aspects of D-meson decays [12]. The most important application is to calculate non-factorizable contributions to $B - \bar{B}$-mixing [13], where our approach includes $1/m_b$ corrections and chiral corrections both from loops and counterterms. Also processes of the type $B \to D\bar{D}$ are calculable [14]. It should be emphasized that the HLχQM can not, – in its present form, be used for heavy to light transitions like $B \to \pi K$, where QCD factorization [15] or soft collinear theory(SCET) [16] is often applied . Still, in the last section, we suggest how an extension to this case might be performed. We also suggest how the χQM might be extended to include light vectors (ρ, ω, K^*).

2. *Chiral perturbation theory*

2.1. *The pure light sector*

Quarks are the fundamental hadronic matter. However, the particles we observe are those built out of them: baryons and mesons. In the sector of the lowest-mass pseudoscalar mesons (the would-be Goldstone bosons: π, K and η), the interactions can be described in terms of an effective theory, the chiral Lagrangian, that includes only these states. The chiral Lagrangian and chiral perturbation theory (χPT) [17, 18] provide a faithful representation of this sector of the Standard Model after the quark and gluon degrees of freedom have been integrated out. The form of this effective field theory and all its possible terms are determined by the $SU_L(3) \times SU_R(3)$ chiral invariance and Lorentz invariance. Terms which explicitly break chiral invariance are introduced in terms of the quark mass matrix $\mathcal{M}_q$.

The strong chiral Lagrangian is completely fixed to the leading order in momenta by symmetry requirements and the Goldstone boson's decay amplitudes

$$\mathcal{L}^{(2)}_{\rm strong} = \frac{f^2}{4}{\rm Tr}\left(D_\mu \Sigma D^\mu \Sigma^\dagger\right) + \frac{f^2}{2} B_0 {\rm Tr}\left(\mathcal{M}_q \Sigma^\dagger + \Sigma \mathcal{M}_q^\dagger\right) , \tag{1}$$

where the covariant derivative D^μ contains the photon field, and $\mathcal{M}_q$ = diag $[m_u, m_d, m_s]$. The quantity B_0 is defined by $\langle \bar{q}_i q_j \rangle = -f^2 B_0 \delta_{ij}$, where

$$(m_s + m_d)\langle \bar{q} q\rangle = -f_K^2 m_K^2 \ , \quad (m_u + m_d)\langle \bar{q} q\rangle = -f_\pi^2 m_\pi^2 \ , \tag{2}$$

in the PCAC limit. The quantity $\langle \bar{q} q\rangle$ is the quark condensate, being of order (-240 MeV)^3. The $SU_L(3) \times SU_R(3)$ field Σ contains the pseudoscalar octet Π

$$\Sigma \equiv \exp\left(\frac{2{\rm i}}{f}\Pi\right), \ \ \Pi = \frac{\lambda^a}{2}\phi^a(x) = \frac{1}{\sqrt{2}}\left[\begin{array}{ccc} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8 \end{array}\right] . \tag{3}$$

The quantity f is, to lowest order, identified with the pion decay constant f_π (and equal to f_K before chiral loops are introduced).

When the matrix Σ is expanded in powers of f^{-1}, the zeroth-order term obtained from (1) is the free Klein-Gordon Lagrangian for the pseudoscalar particles. From this Lagrangian, one might deduce the (left-handed) current

$$J^n_\mu = -{\rm i}\frac{f^2}{2}{\rm Tr}\left[\lambda^n \Sigma D^\mu \Sigma^\dagger\right] \ , \tag{4}$$

where n is a flavour octet index and λ_n a $SU(3)$ flavour matrix.

For the next-to-leading order Lagrangian $\mathcal{L}^{(4)}_{\rm strong}$, there are ten terms and thereby ten coefficients L_i to be determined [18] either experimentally or by means of some model. Some of these play an important role in the physics of ϵ' in $K \to 2\pi$ decays [19]. As examples, we display the L_5 and L_8 terms in governing much of the penguin physics

$$L_5\, B_0\, {\rm Tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma \left(\mathcal{M}_q^\dagger \Sigma + \Sigma^\dagger \mathcal{M}_q\right)\right] \,, \tag{5}$$

and

$$L_8\, B_0\, {\rm Tr}\left[\mathcal{M}_q^\dagger \Sigma \mathcal{M}_q^\dagger \Sigma + \mathcal{M}_q \Sigma^\dagger \mathcal{M}_q \Sigma^\dagger\right] \,. \tag{6}$$

Under the action of the elements V_R and V_L of the chiral group $SU_R(3)\times SU_L(3)$, the field Σ transforms as

$$\Sigma \to V_L \Sigma V_R^\dagger \,, \tag{7}$$

and accordingly for the conjugated fields. Formally, $\mathcal{M}_q$ is given the same transformation properties as Σ, and $\mathcal{M}_q^\dagger$ as $\Sigma^\dagger$.

2.2. The heavy light sector

The strong chiral Lagrangian for the heavy light sector is [7, 20]:

$$\begin{aligned}\mathcal{L}_{Str} \;&= \mp\, {\rm Tr}\left[\overline{H^{(\pm)}_{vk}}({\rm i}v\cdot \mathcal{D}_{hk})H^{(\pm)}_{vh}\right] \,-\, g_{\mathcal{A}}{\rm Tr}\left[\overline{H^{(\pm)}_{vk}}H^{(\pm)}_{vh}\gamma_\mu\gamma_5 \mathcal{A}^\mu_{hk}\right] \\ &+2\lambda_1 {\rm Tr}\left[\overline{H^{(\pm)}_{vk}}H^{(\pm)}_{vh}(\widetilde{M}_q^V)_{hk}\right] + \frac{e\beta}{4}{\rm Tr}\left[\overline{H^{(\pm)}_{vk}}H^{(\pm)}_{vh}\sigma\cdot F\,(Q_q^\xi)_{hk}\right] + \cdots\end{aligned} \tag{8}$$

where k, h are the $SU(3)$ triplet indices, and v is the velocity of the heavy meson. The ellipses indicate other terms (of higher order, say), and ${\rm i}\mathcal{D}^\mu_{hk} = \delta_{hk}D^\mu + \mathcal{V}^\mu_{hk}$. Moreover, $Q_q^\xi = (\xi^\dagger Q_q \xi + \xi Q_q \xi^\dagger)/2$, where Q_q is the $SU(3)$ charge matrix for light quarks, $Q_q = {\rm diag}(2/3, -1/3, -1/3)$, and F is the electromagnetic field tensor. $H^{(\pm)}_{vk}$ is the heavy meson field containing a spin zero and spin one boson:

$$\begin{aligned}H^{(\pm)}_{vk} \;&\equiv P_\pm(v)(P_k^{(\pm)\mu}\gamma_\mu - {\rm i}\,P_k^{(\pm)5}\gamma_5) \,, \\ \overline{H^{(\pm)}_{vk}} \;&= \gamma^0 (H^{(\pm)}_{vk})^\dagger \gamma^0 = \left[(P_k^{(\pm)\mu})^\dagger \gamma_\mu - {\rm i}\,(P_k^{(\pm)5})^\dagger\gamma_5\right] P_\pm \,,\end{aligned} \tag{9}$$

where

$$P_\pm(v) = (1 \pm \gamma\cdot v)/2 \tag{10}$$

are the projection operators. The fields $P^{(\pm)5}(P^{(\pm)\mu})$ represent heavy-light mesons, $0^-(1^-)$, with velocity v. The signs $\pm$ refer to particles and anti-particles, respectively, and will sometimes be omitted in the following when unnecessary.

The vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are given by

$$\mathcal{V}_\mu \equiv \frac{\mathrm{i}}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \, , \qquad \mathcal{A}_\mu \equiv -\frac{\mathrm{i}}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \, . \tag{11}$$

The fields ξ and H_v transform as

$$\xi \to U \, \xi \, V_R^\dagger = V_L \xi U^\dagger \, , \qquad H_v \to H_v \, U^\dagger \, , \tag{12}$$

where $U \epsilon \, SU(3)_V$, which is the unbroken symmetry group.

The vector and axial fields transform as

$$\mathcal{V}_\mu \to U \, \mathcal{V}_\mu \, U^\dagger \; + \; iU \partial_\mu \, U^\dagger \, , \qquad \mathcal{A}_\mu \to U \mathcal{A}_\mu \, U^\dagger \, . \tag{13}$$

The vector field $\mathcal{V}^\mu$ is seen to transform as a gauge field under local $SU(3)_V$, and can only appear in combination with a derivative as a covariant derivative $(\mathrm{i}\,\partial^\mu + \mathcal{V}^\mu)$. The quantity $\widetilde{M_q}^V$ (as well as the orthogonal combination $\widetilde{M_q}^A$) is related to the current mass term

$$\widetilde{M_q^V} \equiv \frac{1}{2}(\xi^\dagger \mathcal{M}_q \xi^\dagger \; + \xi \mathcal{M}_q^\dagger \xi) \, , \qquad \widetilde{M_q^A} \equiv -\frac{1}{2}(\xi^\dagger \mathcal{M}_q \xi^\dagger - \xi \mathcal{M}_q^\dagger \xi) \, . \tag{14}$$

The heavy-light weak current, to zeroth order in $1/m_Q$ and chiral counting, is represented by

$$J_k^\alpha(0) = \frac{\alpha_H}{2} \mathrm{Tr} \left[\xi_{hk}^\dagger \Gamma^\alpha H_{vh} \right] \, , \tag{15}$$

and under $SU(3)_L$ it transforms as

$$J_k^\alpha \to J_h^\alpha \left(V_L^\dagger \right)_{hk} \, . \tag{16}$$

This current has also (counter) terms, of higher order in the chiral counting, needed to make the chiral loops finite,

$$J_k^\mu(\mathcal{M}) = \frac{\omega_1}{2} \, \mathrm{Tr} \left[\xi_{hk}^\dagger \, \Gamma^\mu H_{vl} \, \widetilde{\mathcal{M}_{lh}^V} \right] + \frac{\omega_1'}{2} \, \mathrm{Tr} \left[\xi_{kh}^\dagger \, \Gamma^\mu H_{vh} \right] \, \widetilde{\mathcal{M}_{ll}^V} \, , \tag{17}$$

where the parameters ω_1 and ω_1' are commented on in Sec. 4.3. To the leading order, $\Gamma^\alpha = \gamma^\alpha L$, where L is the left-handed projector in Dirac space, $L = (1 - \gamma_5)/2$. However, this is slightly modified by perturbative QCD for μ below m_Q, which gives [8]

$$\Gamma^\alpha \equiv C_\gamma(\mu) \, \gamma^\alpha \, L \; + \; C_v(\mu) \, v^\alpha \, R \, , \tag{18}$$

where R is the right-handed projector, $R = (1+\gamma_5)/2$. The coefficients $C_{\gamma,v}(\mu)$ are determined by QCD renormalization for $\mu < m_Q$. They have been calculated to NLO and the result is the same in MS and $\overline{MS}$ scheme [21]. (C_γ is close to one and C_v is rather small). Corrections to the weak current of order $1/m_Q$ will be discussed in Sec. 5.

Before closing this section, we write down the bosonized $b \to c$ transition current in terms of the heavy fields

$$\overline{Q_{v_b}^{(+)}}\,\gamma^\mu L Q_{v_c}^{(+)} \;\longrightarrow\; -\zeta(\omega)\mathrm{Tr}\left[\overline{H_c^{(+)}}\gamma^\alpha L H_b^{(+)}\right], \tag{19}$$

where $\zeta(\omega)$ is the Isgur-Wise function for the $\bar{B} \to D$ transition [22]. The indices on the heavy fields here refer to the the b- and c-quarks with velocities v_b and v_c, with $\omega \equiv v_b \cdot v_c$. The current for $D\overline{D}$ production is

$$\overline{Q_{v_c}^{(+)}}\,\gamma^\mu\, L Q_{\bar{v}}^{(-)} \;\longrightarrow\; -\zeta(-\lambda)\mathrm{Tr}\left[\overline{H_c^{(+)}}\gamma^\alpha L H_{\bar{c}}^{(-)}\right], \tag{20}$$

where the Isgur-Wise function $\zeta(-\lambda)$ is (in general) complex. We have $\lambda = v_c \cdot \bar{v}$, where $\bar{v}$ is the velocity of $\bar{c}$.

3. *The chiral quark model (χQM)*

3.1. *The Lagrangians for χQM*

The light quark sector is described by the chiral quark model (χQM), having a standard QCD term and a term describing interactions between quarks and (Goldstone) mesons [1−5]

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{q}_L \mathrm{i}\,\gamma\cdot D\, q_L + \bar{q}_R \mathrm{i}\,\gamma\cdot D\, q_R - \bar{q}_L \mathcal{M}_q\, q_R - \bar{q}_R \mathcal{M}_q^\dagger\, q_L - m(\bar{q}_R \Sigma^\dagger q_L + \bar{q}_L \Sigma q_R)\,, \tag{21}$$

where m is the ($SU(3)$-invariant) constituent quark mass for light quarks $q^T = (u,d,s)$. The left- and right-handed projections q_L and q_R are transforming after $SU(3)_L$ and $SU(3)_R$, respectively,

$$q_L \to V_L\, q_L \quad \text{and} \qquad q_R \to V_R\, q_R\,. \tag{22}$$

From (21), we deduce the Feynman rules. For instance, the $Pq\bar{q}$ coupling is $(m\gamma_5/f)$ times some $SU(3)$ factor (P is a pseudoscalar meson, π, K, η). From such Feynman rules, and including the quark propagator $S(p) = (\gamma \cdot p - M_q)^{-1}$, we can calculate aplitudes for, say, $\pi - \pi$ scattering in the strong sector. Here $M_q = m + m_q$ is the total mass. Alternatively, one might keep only the constituent mass m in the propagator, and take the current mass m_q as a coupling. Incuding also the Feynman rules for weak vertices, one might calculate [5, 4] amplitudes for non-leptonic decays in terms of quark loops representing f_π and the semileptonic form factors $f_\pm$, but also for more complicated cases.

Also, as a more exotic example, one may calculate the effect of the electroweak $s \to d$ self-energy transition contribution to $K \to 2\pi$ as shown in Fig. 2. This is an off-shell effect which vanishes in the free quark case, but is non-zero for bound quarks and proportional to m within our framework [4].

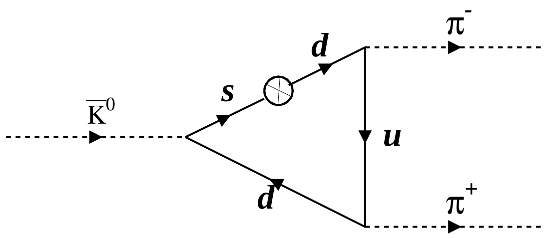


Fig. 2. Contribution to the process $K \to 2\pi$ from the non-diagonal $s \to d$ transition.

If one wants to deduce chiral Lagrangians from the χQM, it is more transparent to consider the flavour rotated version of (21) by defining the flavour rotated quark fields χ given by

$$\chi_L = \xi^\dagger q_L \;, \qquad \chi_R = \xi q_R \;, \qquad \xi \cdot \xi = \Sigma \,. \tag{23}$$

The constituent quark fields χ_L and χ_R transform in a simple way under $SU(3)_V$

$$\chi_L \to U\,\chi_L \;, \qquad \chi_R \to U\,\chi_R \;. \tag{24}$$

In the rotated version, the chiral interactions are rotated into the kinetic term, while the interaction term proportional to m in (21) becomes a pure (constituent) mass term [2, 5]

$$\mathcal{L}_{\chi\mathrm{QM}} = \overline{\chi}\,[\gamma^\mu(\mathrm{i}\,D_\mu + \mathcal{V}_\mu + \gamma_5\mathcal{A}_\mu) - m]\,\chi - \overline{\chi}\widetilde{M}_q\chi \,, \tag{25}$$

which is manifestly invariant under $SU(3)_V$. Moreover,

$$\widetilde{M}_q \equiv \widetilde{M}_q^V + \widetilde{M}_q^A\gamma_5 \,, \tag{26}$$

where $\widetilde{M}_q^{V,A}$ are given in (14).

In the light sector, the various pieces of the strong Lagrangian in Sec. 2.1 can now be obtained by integrating out the constituent quark fields χ, and these pieces can be written in terms of the fields $\mathcal{A}_\mu$, $\widetilde{M}_q^V$ and $\widetilde{M}_q^A$. This can easily be seen by using the relation

$$\mathcal{A}_\mu \;=\; -\frac{1}{2i}\xi\,(D_\mu\Sigma^\dagger)\,\xi \;=+\;\frac{1}{2i}\xi^\dagger\,(D_\mu\Sigma)\,\xi^\dagger \;. \tag{27}$$

The same method can be used for chiral Lagrangians in the weak sector.

In our model, the hard gluons are assumed to be integrated out and we are left with soft gluonic degrees of freedom. These gluons can be described using the external field technique, and their effect will be parameterized by vacuum expectation values, i.e. the gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$. Gluon condensates with higher dimension could also be included, but we truncate the expansion by keeping only the condensate with the lowest dimension.

When calculating the soft-gluon effects in terms of the gluon condensate, we follow the prescription given in [23]. The calculation is easily carried out in the Fock–Schwinger gauge. In this gauge, one can expand the gluon field as

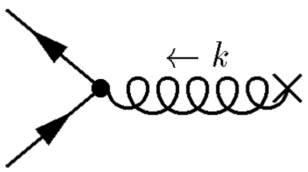
$$A_\mu^a(k) = -\frac{\mathrm{i}\,(2\pi)^4}{2} G_{\rho\mu}^a(0) \frac{\partial}{\partial k_\rho} \delta^{(4)}(k) + \cdots . \tag{28}$$

In some simple cases, one may also use the light-quark propagator in a gluonic background (to the first order in the gluon field)

$$S_1(p,G) = -\frac{g_s}{4} G_{\alpha\beta}^b t^b \left[\sigma^{\alpha\beta}(\gamma \cdot p + m) \;+\; (\gamma \cdot p + m)\sigma^{\alpha\beta}\right] (p^2 - m^2)^{-2} \,, \tag{29}$$

where g_s is the strong coupling constant, a and b are the colour octet indices, and t^a are the colour matrices. In general, one should stick to the prescription in Ref. [23] in order to get correct results. Since each vertex in a Feynman diagram is accomplished with an integration, we get the Feynman rule given in Fig 3. The gluon condensate contributions are obtained by the replacement

$$g_s^2 G_{\mu\nu}^a G_{\alpha\beta}^b \;\rightarrow\; \frac{4\pi^2}{(N_c^2 - 1)} \delta^{ab} \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \,. \tag{30}$$



$$-\frac{1}{2} g_s t^a \gamma^\mu G_{\mu\rho}(0) \frac{\partial}{\partial k^\rho}\bigg|_{k^\rho = 0} \qquad + \cdots$$

Fig. 3. Feynman rule for the light quark – soft gluon vertex.

3.2. Bosonization of the χQM

The Lagrangians (21) or (25) from the previous section can now be used for bosonization, i.e. to integrate out the quark fields. This can be done in the path integral formalism, or as we do here, by expanding in terms of Feynman diagrams. Within the χQM, with Feynman rules obtained from (21), one may calculate the simple quark-loop amplitude for $\pi \rightarrow W$ which defines f (the bare f_π) in terms of a logarithmicaly divergent integral I_2 times the coupling $\sim m/f$. Including also the gluon condensate contribution, one obtains [3, 4, 5]

$$f^2 = -i4m^2 N_c I_2 + \frac{1}{24m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle \, , \tag{31}$$

where I_2 is the following logarithmically divergent integral (d is the dimension of space within dimensional regularization)

$$I_2 \quad \equiv \quad \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)^2} \, . \tag{32}$$

Equivalently, one may obtain the (kinetic part of the) strong Lagrangian in (1) by attaching two axial fields $\mathcal{A}_\mu$ to a vacuum-polarization-like quark loop diagram by using (25). Then one obtains

$$\mathrm{i}\,\mathcal{L}^{(2)}_{str} = -N_c \int \frac{\mathrm{d}^d p}{(2\pi)^d} \mathrm{Tr}\, [(\gamma_\sigma\gamma_5 \mathcal{A}^\sigma)\ S(p)\ (\gamma_\rho\gamma_5\mathcal{A}^\rho)\ S(p)] \ = \ f^2 \mathrm{Tr}\,[\mathcal{A}_\mu \mathcal{A}^\mu] \, , \tag{33}$$

where the trace is both in flavour and Dirac spaces (a similar diagram with gluons should also be added). This is easily seen by using the relation (27), provided f^2 is given by (31). Equation (33) gives the Lagrangian (1) in the light sector by applying (27).

The quark condensate is

$$\langle\,\overline{q}q\,\rangle = -\mathrm{i}\,N_c \mathrm{Tr} \int \frac{\mathrm{d}^d p}{(2\pi)^d} S(p) \ = \ -4\mathrm{i}\,m N_c I_1 - \frac{1}{12m}\langle\frac{\alpha_s}{\pi}G^2\rangle \, , \tag{34}$$

where I_1 is the quadratically divergent integral

$$I_1 \quad \equiv \quad \int \frac{\mathrm{d}^d k}{(2\pi)^d} \, \frac{1}{k^2 - m^2} \, . \tag{35}$$

In Eqs. (33) and (34), $S(p)$ has to be understood as the quark propagator in an external gluon field up to the second order in g_s. The a priori divergent integrals $I_{1,2}$ have to be interpreted as the regularized ones. The physical values of $I_{1,2}$ are determined by the physical values of f and $\langle\,\overline{q}q\,\rangle$. In general, by coupling the fields $\mathcal{A}_\mu$ and $\widetilde{M}_q^{V,A}$ to quark loops, the chiral Lagrangian terms and their coefficients within the light sector can be obtained.

Similarly, we may bosonize the weak currents. The left-handed current can be written

$$\bar{q}_L\gamma^\mu\lambda^n q_L \ = \ \bar{\chi}_L\gamma^\mu\Lambda^n\,\chi_L \, , \qquad \Lambda^n \, \equiv \, \xi^\dagger\lambda^n\,\xi \, . \tag{36}$$

The lowest-order term $\mathcal{O}(p)$ is obtained when the vertex Λ^n from (36) and axial vertex ($\sim \mathcal{A}_\mu$) from (25) are entering a quark loop (see Fig. 4)

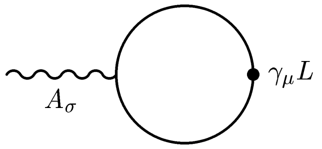


Fig. 4. Feynman diagram for bosonization of the left-handed current to order $\mathcal{O}(p)$.

$$j^n_\mu(\mathcal{A}) \;=\; -\,\mathrm{i}\,N_c \int \frac{\mathrm{d}^d p}{(2\pi)^d}\,\mathrm{Tr}\,\left[(\gamma_\mu L\,\Lambda^n)\,S(p)\;(\gamma_\sigma\gamma_5\mathcal{A}^\sigma)\;S(p)\right] \;\sim\; \mathrm{Tr}\,\left[\Lambda^n\,\mathcal{A}_\mu\right]\;, \tag{37}$$

which coincides with (4) when (27) is used.

As a more non-trivial example, to obtain a non-zero non-factorizable contribution to $D^0 \to K^0\bar{K}^0$ at tree level, one has to consider the coloured current $j^{n,a}_\mu$ to $\mathcal{O}(p^3)$, involving insertions of the "mass fields" $\widetilde{M}_q$ in (26) [12]. (This coloured current is obtained by Fierz transformations of the relevant four-quark operator). From Fig. 5, one obtains the contribution

$$j^{n,a}_\mu(\mathrm{Fig.}\ 5) \;=\mathrm{i}\int \frac{\mathrm{d}^d p}{(2\pi)^d}\,\mathrm{Tr}\,\Big[(\gamma_\mu L\,\Lambda^n t^a)\;S(p)\;(\gamma_\sigma\gamma_5\mathcal{A}^\sigma)\;S(p)\,\widetilde{M}_q\,S_1(p,G)\Big]\,. \tag{38}$$

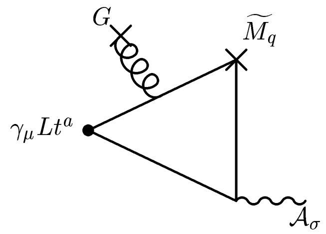


Fig. 5. Diagram for bosonization of the colour current to $\mathcal{O}(p^3)$.

Summing all six diagrams with permutated vertices compared to the one in Fig. 5, we obtain in total

$$j^{n,a}_\mu(G,\xi,\mathcal{A},\widetilde{\mathcal{M}_q}) \;=\; \frac{g_s}{12m}\,\frac{1}{16\pi^2}\,G^{a,\kappa\lambda}\left[\mathrm{i}\,\varepsilon_{\mu\rho\kappa\lambda}\,T^{n,\rho}_\varepsilon + (g_{\mu\kappa}g_{\rho\lambda} - g_{\mu\lambda}g_{\rho\kappa})\,T^{n,\rho}_g\right], \tag{39}$$

where (we have used the analytical computer program FORM [24])

$$T^{n,\rho}_\varepsilon \;=\; 4\,S^K_\rho \;-3(S^L_\rho + S^R_\rho)\;,\;\; T^{n,\rho}_g \;=S^L_\rho \;-\; S^R_\rho\;. \tag{40}$$

The $S's$ are chiral Lagrangian terms

$$S^L_\rho \;\;\equiv\;\; \mathrm{Tr}\left[\Lambda^n\mathcal{A}_\rho\,\widetilde{M}^L_q\right] \;=\; \frac{1}{2i}\mathrm{Tr}\left[\lambda^n(D_\rho\Sigma)\,\mathcal{M}^\dagger_q\right]\;, \tag{41}$$

$$S^R_\rho \;\;\equiv\;\; \mathrm{Tr}\left[\Lambda^n\,\widetilde{M}^R_q\,\mathcal{A}^\rho\right] \;=\; \frac{-1}{2i}\mathrm{Tr}\left[\lambda^n\,\mathcal{M}_q\,(D_\rho\Sigma^\dagger)\right]\;, \tag{42}$$

$$S_\rho^K \equiv \frac{1}{2}\mathrm{Tr}\left[\Lambda^n \left(\mathcal{A}^\rho \widetilde{M}_q^R + \widetilde{M}_q^L \mathcal{A}^\rho\right)\right] \tag{43}$$

$$= \frac{1}{4i}\mathrm{Tr}\left[\lambda^n \left((D_\rho\Sigma)\,\Sigma^\dagger \mathcal{M}_q \Sigma^\dagger - \Sigma \mathcal{M}_q^\dagger \Sigma (D_\rho \Sigma^\dagger)\right)\right] . \tag{44}$$

The current (39) has to be combined with the left-handed colour current for D-meson decay, later given in (93), to obtain a contribution to $D^0 \to K^0 \bar{K}^0$ [12].

4. *The heavy - light chiral quark model (HLχQM)*

4.1. *The Lagrangian for HLχQM*

Our starting point is the following Lagrangian containing both quark and meson fields

$$\mathcal{L} = \mathcal{L}_{\mathrm{HQEFT}} + \mathcal{L}_{\chi\mathrm{QM}} + \mathcal{L}_{\mathrm{Int}} \,, \tag{45}$$

where [8]

$$\mathcal{L}_{\mathrm{HQEFT}} = \pm\overline{Q_v^{(\pm)}}\,\mathrm{i}\,v\cdot D\,Q_v^{(\pm)} + \frac{1}{2m_Q}\overline{Q_v^{(\pm)}}\left(-C_M \frac{g_s}{2}\sigma\cdot G + (\mathrm{i}\,D_\perp)^2_{\mathrm{eff}}\right) Q_v^{(\pm)} + \mathcal{O}(m_Q^{-2}) \tag{46}$$

is the Lagrangian for the heavy-quark effective field theory (HQEFT). The heavy quark field $Q_v^{(+)}$ annihilates a heavy quark with velocity v and mass m_Q. Similarly, $Q_v^{(-)}$ annihilates a heavy anti-quark. Moreover, D_μ is the covariant derivative containing the gluon field (eventually also the photon field), and $\sigma\cdot G = \sigma^{\mu\nu} G^a_{\mu\nu} t^a$, where $\sigma^{\mu\nu} = \mathrm{i}[\gamma^\mu, \gamma^\nu]/2$, and $G^a_{\mu\nu}$ is the gluonic field tensor. This chromo-magnetic term has a factor C_M, being one at tree level, but slightly modified by perturbative QCD. (When the covariant derivative also contains the photon field, there is also a corresponding magnetic term $\sim \sigma\cdot F$, where $F^{\mu\nu}$ is the electromagnetic tensor). Furthermore, $(\mathrm{i}D_\perp)^2_{\mathrm{eff}} = C_D(\mathrm{i}D)^2 - C_K(\mathrm{i}v\cdot D)^2$. At tree level, $C_D = C_K = 1$. Here, C_D is not modified by perturbative QCD, while C_K is different from one due to perturbative QCD corrections for $\mu < m_Q$ [25]. We observe that soft gluons coupling to a heavy quark are suppressed by $1/m_Q$, since to leading order, the vertex is proportional to $v_\mu v_\nu G^{a\mu\nu} = 0$, v_μ being the heavy quark velocity.

In the heavy – light case, the generalization of the meson – quark interactions in the pure light sector χQM is given by the following $SU(3)$ invariant Lagrangian

$$\mathcal{L}_{\mathrm{Int}} = -G_H\left[\overline{\chi}_k\,\overline{H_{vk}^{(\pm)}}\,Q_v^{(\pm)} + \overline{Q_v^{(\pm)}}\,H_{vk}^{(\pm)}\,\chi_k\right], \tag{47}$$

where k is a triplet $SU(3)$-index and G_H is a coupling constant of dimension mass to the power (-1/2). In Ref. [6], one used instead $G_H = 1$, but compensated with a renormalization factor for the heavy meson fields H_v, which is equivalent. The interaction Lagrangian (47) can, as for the χQM, be obtained from a NJL model. This has been done in Ref. [6] (as for the light sector in Ref. [1]).

4.2. *Bosonization within the HLχQM*

The interaction term $\mathcal{L}_{\text{Int}}$ in (47) can now be used to bosonize the model, i.e. to integrate out the quark fields. This can be done in terms of Feynman diagrams as we do here, by attaching the external fields $H_v^a, \overline{H_v^a}, \mathcal{V}^\mu, \mathcal{A}^\mu$ and $\widetilde{M}_q^{V,A}$ of Sec. 2.1 to quark loops, and using (25) and (47). In this way, one obtains the strong chiral Lagrangian (8) and terms of higher order in the heavy light sector. Some of the loop integrals will be divergent and have to be related to physical parameters, as for the pure light sector [2, 3, 4, 5]. As the pure light sector is a part of our model, we have to keep the relations in (31) and (34) from the pure light sector within the heavy light case studied here. The $1/m_Q$ terms will not be discussed in this section, but will be considered later in Sec. 5.

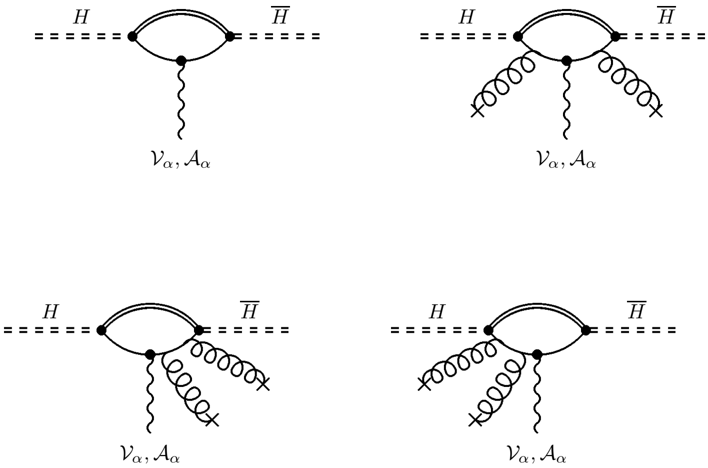


Fig. 6. Bosonization in the strong sector to obtain Eq. (8).

From the diagrams in Fig. 6, we obtain the identification for the kinetic term, which by Ward identities is the same as for the term with the vector field $\mathcal{V}_\mu$ attached to the light quark

$$-\mathrm{i}\, G_H^2 N_c \left(I_{3/2} + 2m I_2 + \mathrm{i}\, \frac{(8-3\pi)}{384 N_c m^3} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) = 1 \;, \tag{48}$$

where I_2 is given in (32) and

$$I_{3/2} \quad \equiv \quad \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(v \cdot k)(k^2 - m^2)} \;, \tag{49}$$

which formally depends on v^2 which is equal to one. From the same diagram, with the axial field $\mathcal{A}_\mu$ attached, we obtain the following identification for the axial vector coupling $g_{\mathcal{A}}$

$$g_{\mathcal{A}} \equiv \mathrm{i}\, G_H^2 N_c \left(\frac{1}{3} I_{3/2} - 2m I_2 - \mathrm{i}\, \frac{m}{12\pi} - \mathrm{i}\, \frac{(8-3\pi)}{384 N_c m^3} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right), \tag{50}$$

As I_1 and I_2 are related to the quark condensate and f_π, respectively, the (formally) linearly divergent integral $I_{3/2}$ is related to $\delta g_{\mathcal{A}} \equiv 1 - g_{\mathcal{A}}$, which is found by eliminating I_2 from Eqs. (48) and (50)

$$\delta g_{\mathcal{A}} = -\frac{4}{3} \mathrm{i}\, G_H^2 N_c \left(I_{3/2} - \mathrm{i}\, \frac{m}{16\pi} \right). \tag{51}$$

Note that the gluon condensate drops out here. Within a primitive cut-off regularization, $I_{3/2}$ is (in the leading approximation) proportional to the cut-off in the first power

$$I_{3/2} = \mathrm{i}\, \frac{\Lambda}{16\pi} \left(1 + \mathcal{O}(\frac{m}{\Lambda}) \right), \tag{52}$$

where the cut-off Λ is of the same order as the chiral symmetry breaking scale Λ_χ. In contrast, $I_{3/2}$ is finite and proportional to m in dimensional regularization. Note that the cut-off Λ is only used in qualitative considerations here and in subsection 4.4. Anyway, $I_{3/2}$ is determined by the physical value of $g_{\mathcal{A}}$.

When attaching $\widetilde{M}_q^V$ like in Fig. 6 instead of vector or axial vector fields, one finds for the mass correction term in (8)

$$2\lambda_1 \equiv \mathrm{i}\, G_H^2 N_c \left(I_{3/2} - 2m I_2 - \mathrm{i}\, \frac{m}{8\pi} - \frac{\mathrm{i}\,(3\pi - 4)}{192 N_c m^3} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) . \tag{53}$$

The electromagnetic β term in (8) is obtained by considering diagrams like those in Fig. 6, but with the vector and axial vector fields $\mathcal{V}_\mu$ or $\mathcal{A}_\mu$ replaced by a photon field tensor

$$\beta = \frac{G_H^2}{2} \left\{ -4\mathrm{i}\, N_c I_2 + \frac{N_c}{4\pi} - \left(\frac{32 + 3\pi}{576 m^4} \right) \langle \frac{\alpha_s}{\pi} G^2 \rangle \right\}. \tag{54}$$

Within the full theory (SM) at the quark level, the weak current is

$$J_k^\alpha = \overline{q_k}_L\, \gamma^\alpha\, Q\,, \tag{55}$$

where Q is the heavy quark field in the full theory. Within HQEFT this current will, below the renormalization scale $\mu = m_Q\ (= m_b, m_c)$, be modified in the following way

$$J_k^\alpha = \overline{\chi}_h \xi_{hk}^\dagger \Gamma^\alpha Q_v \ + \mathcal{O}(m_Q^{-1})\,, \tag{56}$$

The operator in Eq. (56) can be bosonized by calculating the Feynman diagrams shown in Fig. 7, which gives the bosonized current in (15) with

$$\alpha_H \equiv -2\mathrm{i}\, G_H N_c \left(-I_1 + m I_{3/2} + \frac{\mathrm{i}\,(3\pi - 4)}{384 N_c m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) . \tag{57}$$

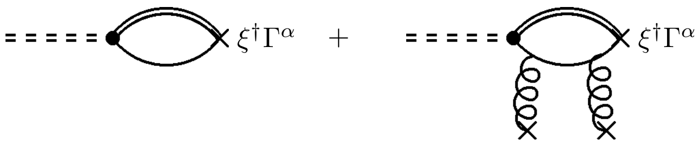


Fig. 7. Diagrams for bosonization of the left-handed quark current.

To the first order in the chiral expansion, we obtain the bosonized current obtained by attaching one extra field $\mathcal{A}_\nu$ to the quark loops in Fig. 7

$$J_k^\alpha(1) \;=\; \frac{1}{2}\; \mathrm{Tr}\left[\xi_{hk}^\dagger \Gamma^\alpha H_{vh} (\alpha_{H\gamma}^{(1)} \gamma^\nu \gamma_5 + \alpha_{Hv}^{(1)} v^\nu \gamma_5) \mathcal{A}_\nu \right] , \tag{58}$$

where the quantities $\alpha_H^{(1)}$ are given by expressions similar to (57).

The coupling α_H in (15) is related to the physical decay constants f_H and f_{H^*} in the following way (for $H = B, D$)

$$\langle 0 | \overline{u} \gamma^\alpha \gamma_5 b | H \rangle = -2\, \langle 0 | J_a^\alpha | H \rangle \; = \mathrm{i}\, M_H f_H v^\alpha \; . \tag{59}$$

Taking the trace over the gamma matrices in (15), we obtain a relation for α_H and the relations between the heavy meson decay constants f_H and f_{H^*} (for $H = B, D$)

$$\alpha_H = \frac{f_H \sqrt{M_H}}{C_\gamma(\mu) + C_v(\mu)} = \frac{f_{H^*} \sqrt{M_{H^*}}}{C_\gamma(\mu)} \; , \tag{60}$$

where the model dictates us to put $\mu = \Lambda_\chi$. Later, in Sec. 5.2, we will see how chiral corrections and $1/m_Q$ corrections modify this relation.

4.3. *Constraining the parameters of the HLχQM*

The gluon condensate can be related to the chromomagnetic interaction

$$\mu_G^2(H) = \frac{1}{2M_H} C_{\mathrm{M}}(\mu) \langle H | \bar{Q}_v \frac{1}{2} \sigma \cdot G Q_v | H \rangle \, , \tag{61}$$

where the coefficient $C_{\mathrm{M}}(\mu)$ contains the short-distance effects down to the scale μ and has been calculated to next to the leading order (NLO) [26]. The chromomagnetic operator is responsible for the splitting between the 1^- and 0^- state, and is known from spectroscopy,

$$\mu_G^2(H) \;=\; \frac{3}{2} m_Q (M_{H^*} - M_H) \; . \tag{62}$$

An explicit calculation of the matrix element in Eq. (61) gives

$$\mu_G^2 = \eta_2 \frac{G_H^2}{m} \langle \frac{\alpha_s}{\pi} G^2 \rangle \,, \quad \text{where} \quad \eta_2 \equiv \frac{(\pi+2)}{32} C_M(\Lambda_\chi) \,. \tag{63}$$

Combining Eqs. (31), (34) and (63), we obtain the following relations

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = \frac{\mu_G^2 f^2}{2\eta_2} \frac{1}{\rho} \,, \qquad G_H^2 = \frac{2m}{f^2} \rho \,, \tag{64}$$

where the quantity ρ is of the order one and is given by

$$\rho \equiv \frac{(1+3g_{\mathcal{A}}) + \dfrac{\eta_1 \mu_G^2}{\eta_2 m^2}}{4(1 + \dfrac{N_c m^2}{8\pi f^2})} \,, \tag{65}$$

where $\eta_1 \equiv \pi/32$. In the limit where only the leading logarithmic integral I_2 is kept in (48), we obtain

$$g_{\mathcal{A}} \to 1 \,, \qquad \rho \to 1 \,, \qquad \beta \to \frac{1}{m} \,, \qquad G_H \to G_H^{(0)} \equiv \frac{\sqrt{2m}}{f} \,, \tag{66}$$

which for $g_{\mathcal{A}}$ and β correspond to the non-relativistic values [7].

From Eqs. (31), (53), and (63), we find

$$2\lambda_1 = \frac{1}{2}(3g_{\mathcal{A}} - 1) + \frac{(9\pi - 16)\mu_G^2}{384\eta_2 m^2} \,. \tag{67}$$

In the limit (66), we obtain $2\lambda_1 \to 1$, as expected. The parameter λ_1 is related to the mass difference $M_{H_s} - M_{H_d}$. To the leading order, we obtained the following expression for the β-term

$$\beta = \frac{\rho}{m} \left\{ 1 + \frac{N_c m^2}{4\pi f^2} - \left(\frac{56 + 3\pi}{576 f^2 m^2} \right) \langle \frac{\alpha_s}{\pi} G^2 \rangle \right\} , \tag{68}$$

which is rather sensitive to m. Choosing m in the range 250 − 300 MeV, we find [27] $\beta = (2.5 \pm 0.6)$ GeV^{-1} to be compared with $\beta = (2.7 \pm 0.20)$ GeV^{-1} extracted from experiment.

Using Eqs. (48) and (34) we may write α_H as

$$\alpha_H = \frac{G_H}{2} \left(- \frac{\langle \overline{q} q \rangle}{m} - 2 f_\pi^2 (1 - \frac{1}{\rho}) + \frac{(\pi - 2)}{16 m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) , \tag{69}$$

Combining (60) with (69), in the leading limit (taking into account the logarithmic and quadratic divergent integrals only, and let $C_\gamma \to 1$, $C_v \to 0$ and $g_{\mathcal{A}} \to 1$ as in (66)) we obtain [12] the "Goldberger-Treiman like" relation

$$f_H \sqrt{M_H} \;\to\; -\frac{\langle \overline{q}q \rangle}{f_\pi \sqrt{2m}} \,, \tag{70}$$

which gives the scale for f_H. (It is, however, numerically by the factor 2 off for the B-meson.)

Using the relations (48), (50) and (65), we obtain for $\alpha^{(1)}_{H\gamma}$ and $\alpha^{(1)}_{Hv}$ in (58)

$$\alpha^{(1)}_{H\gamma} = \frac{2 g_{\mathcal{A}}}{G_H} \,, \tag{71}$$

$$\alpha^{(1)}_{Hv} = \frac{4}{3} G_H \left(\frac{f_\pi^2}{2m} \left(\frac{1}{\rho} - 1 \right) + \left[\frac{m N_c}{8\pi} + \frac{(\pi+8)}{256 m^3} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right] \right) . \tag{72}$$

Moreover, for the mass correction to the weak current given in (17), we find that $\omega_1 = -4\lambda_1/G_H$, where λ_1 is given in Eqs. (53) or (67). The term ω_1' is subleading in $1/N_c$.

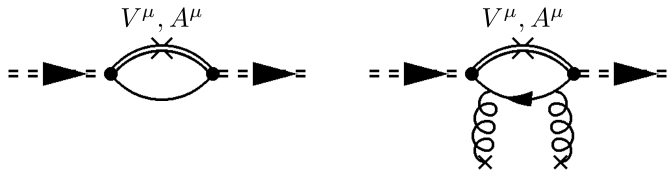


Fig. 8. Loop diagrams for bosonizing the $b \to c$ current, $V^\mu = \gamma^\mu$, $A^\mu = \gamma^\mu \gamma_5$.

The Isgur-Wise function $\zeta(\omega)$ in (19) relates all form factors describing the processes $B(B^*) \to D(D^*)$ in the heavy-quark limit. This function can be calculated from the diagrams shown in Fig. 8. The result before $1/m_Q$ and chiral corrections is

$$\zeta(\omega) = \frac{2}{1+\omega}\,(1-\rho) + \rho\, r(\omega) \,, \tag{73}$$

where ρ is given in (65) and $r(\omega)$ is the same function that is appearing in loop calculations of the anomalous dimension in HQEFT

$$r(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln\left(\omega + \sqrt{\omega^2 - 1} \right) . \tag{74}$$

Note that $\zeta(1) = 1$ as it should be.

4.4. *The formal limit $m \to 0$*

In this subsection, we will discuss the limit of restauration of chiral symmetry, i.e. the limit $m \to 0$ [27]. In order to do this, we have to consider the various constraints obtained when constructing the HLχQM [9].

Looking at the Eqs. (31) and (34), one may worry that $\langle \overline{q}q \rangle$ and f behave like $1/m$ in the limit $m \to 0$ unless one assumes that $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ also go to zero in this limit. We should stress that the exact limit $m = 0$ cannot be taken because our loop integrals will then be meaningless. Still, we may let m approach zero without going to this exact limit. In the pure light sector (at least when vector mesons are not included), there are no restrictions on how $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ might go to zero. In the heavy light sector, we have in addition to (31) and (34) also the relations (48) and (50), which put restrictions on the behaviour of the gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ for small masses. As $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ has dimension mass to the fourth power, we find that $\langle \overline{q}q \rangle$ and f^2 may go to zero if $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ goes to zero as m^4 or $m^3\Lambda$ (eventually combined with $\ln(m/\Lambda)$). However, the behaviour of $m^3\Lambda$ is inconsistent with the additional equations (64) and (65). Still, from all equations (48), (64), (65), we find the possible solution

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = \hat{c}\, N_c m^4 K(m) \ , \ \text{where} \quad K(m) \equiv (-4\mathrm{i}\, I_2 + \frac{1}{8\pi}) \ , \tag{75}$$

and $\hat{c}$ is some constant. Then we must have the following behaviour for G_H^2, $g_{\mathcal{A}}$ and μ_G^2 when m approaches zero

$$G_H^2 \sim \frac{1}{N_c \Lambda} \ , \quad (1 + 3g_{\mathcal{A}}) \sim \frac{m}{\Lambda} K(m) \ , \quad \mu_G^2 \sim \frac{m^3}{\Lambda} K(m) \ , \tag{76}$$

with some restrictions on the proportionality factors. Here, the regularized I_2 is such that for small m, $K(m) = (c_1 + c_2 \ln m/\Lambda)$, c_1 and c_2 being constants. The behaviour of G_H^2 is in agreement with Nambu-Jona-Lasinio models [1]. Note that in our model, $\delta g_{\mathcal{A}} \to 4/3$ (corresponding to $g_{\mathcal{A}} \to -1/3$) for $m \to 0$, in contrast to $\delta g_{\mathcal{A}} \to 2/3$ for a free Dirac particle with $m = 0$. Note that in Ref. [9], we gave the variation of the gluon condensate with m for a fixed value of μ_G^2. For the considerations in this subsection, we have to let μ_G^2 go to zero with m in order to be consistent. When $m \to 0$, we also find that $\beta \to 1/\Lambda$, provided that the coefficient $\hat{c}$ in (75) is fixed to a specific value (which is $\hat{c} = 576/(3\pi + 32) \simeq (1.93)^4$).

5. $1/m_Q$ corrections within the HLχQM

5.1. Bosonization of the strong sector

To order $1/m_Q$, one obtains further contributions to chiral Lagrangians (see ref. [9] and references therein)

$$\begin{aligned}
\mathcal{L}_{\mathrm{Str}} \ &= -\frac{\varepsilon_1}{m_Q} \ \mathrm{Tr}\left[\overline{H_k}(\mathrm{i}\, v \cdot D) H_k\right] \ + \frac{\varepsilon_1}{m_Q} \ \mathrm{Tr}\left[\overline{H_k} H_h v_\mu \mathcal{V}^\mu_{hk}\right] \\
&+ \frac{g_1}{m_Q} \mathrm{Tr}\left[\overline{H_k} H_h \gamma_\mu \gamma_5 \mathcal{A}^\mu_{hk}\right] + \frac{\varepsilon_2}{m_Q} \mathrm{Tr}\left[\overline{H_k} \sigma^{\alpha\beta} \mathrm{i} v \cdot D H_k \sigma_{\alpha\beta}\right] \\
&- \frac{\varepsilon_2}{m_Q} \mathrm{Tr}\left[\overline{H_k} \sigma^{\alpha\beta} v_\mu \mathcal{V}^\mu_{hk} \sigma_{\alpha\beta} H_h\right] + \frac{g_2}{m_Q} \mathrm{Tr}\left[\overline{H_k} \gamma_\mu \gamma_5 \mathcal{A}^\mu_{hk} H_h\right] \ + \cdots ,
\end{aligned} \tag{77}$$

where the ellipses indicate other terms (of higher order, say), and D_μ contains the photon field. The new terms of order $1/m_Q$ in (77) are a consequence of the chromomagnetic interaction O_{mag} (the second term in equation (46)), and the kinetic interaction O_{kin} (the third term in (46)). Calculating the diagrams of Fig. 9 and

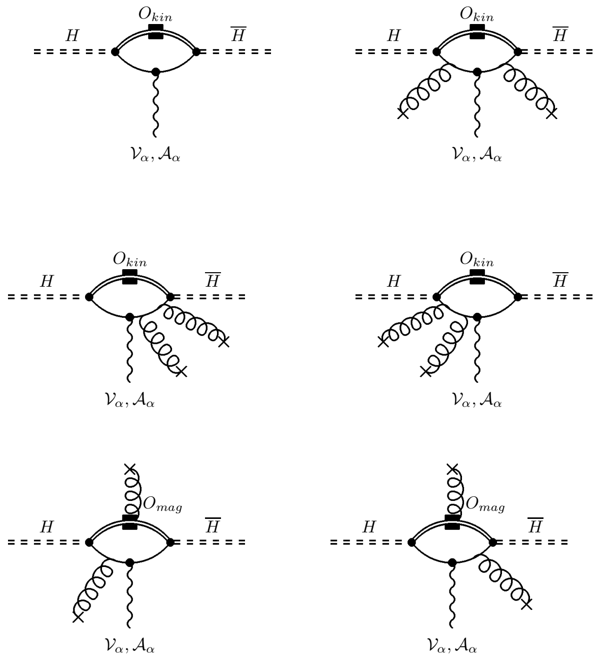


Fig. 9. Diagrams responsible for $1/m_Q$ terms in the chiral Lagrangian.

eliminating the divergent integrals and using equations (32), (34) and (51), gives for example

$$g_1 = m - G_H^2 \left(\frac{\langle \overline{q}q \rangle}{12m} + \frac{f_\pi^2}{6} + \frac{N_c m^2 (3\pi + 4)}{48\pi} - \frac{C_K}{16} (\frac{\langle \overline{q}q \rangle}{m} + 3 f_\pi^2) + \frac{(C_K - 2\pi)}{64 m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right), \tag{78}$$

$$g_2 = \frac{(\pi + 4)}{(\pi + 2)} \frac{\mu_G^2}{6m} . \tag{79}$$

As the $1/m_Q$ terms break heavy-quark spin symmetry, the chiral Lagrangian in (77) will split in $H(0^-)$ and $H^*(1^-)$ terms.

5.2. The weak current to order $1/m_Q$

In HQEFT, the weak vector current at order $1/m_Q$ is [8]

$$J^\alpha = \sum_{i=1,2} C_i(\mu) J_i^\alpha + \frac{1}{2m_Q} \sum_j B_j(\mu) O_j^\alpha + \frac{1}{2m_Q} \sum_k A_k(\mu) T_k^\alpha , \tag{80}$$

where the first terms are given in (18) and (56), the B_j's and A_j's are the Wilson coefficients, and the O_j^α's are two quark operators

$$\begin{aligned}
O_1^\alpha &= \bar{q}_L \gamma^\alpha \mathrm{i}\, \not{D} Q_v \,, \qquad & O_4^\alpha &= \bar{q}_L \gamma^\alpha (-\mathrm{i}\, v \cdot \overleftarrow{D}) Q_v \,, \\
O_2^\alpha &= \bar{q}_L v^\alpha \mathrm{i}\, \not{D} Q_v \,, \qquad & O_5^\alpha &= \bar{q}_L v^\alpha (-\mathrm{i}\, v \cdot \overleftarrow{D}) Q_v \,, \\
O_3^\alpha &= \bar{q}_L \mathrm{i}\, D^\alpha Q_v \,, \qquad & O_6^\alpha &= \bar{q}_L (-\mathrm{i}\, \overleftarrow{D}^\alpha) Q_v \,,
\end{aligned} \tag{81}$$

The operators T_k are nonlocal and are a combination of the leading order currents J_i and a term of order $1/m_Q$ from the effective Lagrangian (46).

Combining (80) with (15), and adding chiral corrections and the $1/m_Q$ corrections indicated in (56), we obtain for $H = B, D$

$$f_H = \frac{1}{\sqrt{M_H}} \left[(C_\gamma(\mu) + C_v(\mu))\, \alpha_H + \frac{\eta_Q}{m_Q} + \frac{\eta_\chi}{32\pi^2 f_\pi^2} \right], \tag{82}$$

where $C_{\gamma,v}$ are defined in (18). Here the model dictates us to put $\mu = \Lambda_\chi$. The quantities η_Q and η_χ are given in Ref. [9]. One should note that the quantities η_Q for $Q = b, c$ depend on the Wilson coefficients C_i, B_i, A_i in (80) and some hadronic parameters, for instance $\varepsilon_{1,2}$ from (77). The Wilson coefficients entering f_H depend on m_Q through $\ln(m_Q/\mu)$, and therefore f_H is a complicated function of m_Q, $\langle \overline{q}q \rangle$, $g_{\mathcal{A}}$, f_π, and the constituent light-quark mass m. Note that $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4}$ is fixed to be around 320 MeV. In Fig. 10, f_B is plotted as function of m for standard values of the other parameters. One should note that bigger values of $|\langle \overline{q}q \rangle|$ give higher values of f_B. For a discussion of the numerical values of our parameters, see Refs. [9] and [27].

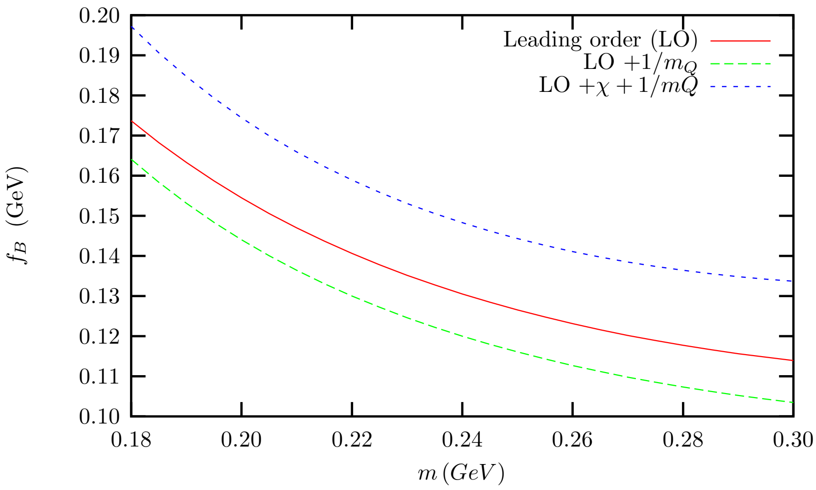


Fig. 10. f_B as a function of m for $\langle \overline{q}q \rangle^{1/3} = -240$ MeV.

6. Applications

In this section, we focus on the chiral quark model aspects, especially contributions proportional to the gluon condensate. There are always additional chiral loop corrections which can be found in Refs. [5, 9, 13, 14] and references therein.

6.1. $B - \overline{B}$ mixing and heavy-quark effective theory

At quark level, the standard effective Lagrangian describing $B - \overline{B}$ mixing is [28]

$$\mathcal{L}_{\rm eff}^{\Delta B=2} \; = \; - \frac{G_F^2}{4\pi^2} M_W^2 \left(V_{tb}^* V_{tq}\right)^2 \; S_{IL}\left(x_t\right) \, \eta_B \, b(\mu) \, Q_B \, , \tag{83}$$

where G_F is Fermi's coupling constant, the V's are KM factors (for which $q = d$ or s for B_d and B_s, respectively) and S_{IL} is the Inami-Lim function due to the short-distance electroweak loop effects for the box diagram. The quantity $Q_B \equiv Q(\Delta B = 2)$ is a four-quark operator

$$Q_B = \overline{q_L} \, \gamma^\alpha \, b_L \; \overline{q_L} \, \gamma_\alpha \, b_L \, , \tag{84}$$

where q_L (b_L) is the left-handed projection of the q- (b)-quark field. The quantities $\eta_B = 0.55 \pm 0.01$ and $b(\mu)$ are calculated in perturbative quantum chromodynamics (pQCD). At the renormalization scale $\mu = m_b$ ($\simeq 4.8$ GeV), one has $b(m_b) \simeq 1.56$ in the naive dimension-regularization scheme. The matrix element of the operator Q_B between the meson states is parameterized by the bag parameter B_{B_q}

$$\langle B | Q_B | \overline{B} \rangle \equiv \frac{2}{3} f_B^2 M_B^2 B_{B_q}(\mu) \, , \tag{85}$$

where by definition, $B_{B_q} = 1$ within the factorized limit. In general, the matrix element of the operator Q_B is dependent on μ, and thereby B_{B_q} also depends on μ. As for the $K - \overline{K}$ mixing, one defines a renormalization-scale-independent quantity

$$\hat{B}_{B_q} \equiv b(\mu) B_{B_q}(\mu) \, . \tag{86}$$

For $\Lambda_\chi < \mu < m_b$, the $\Delta B = 2$ operator in Eq. (84) can be written [29, 30]

$$Q_B = \; C_1 \, Q_1 + C_2 \, Q_2 + \frac{1}{m_b} \sum_i \, h_i X_i \; + \mathcal{O}(1/m_b^2) \, . \tag{87}$$

The operator Q_1 is Q_B for b replaced by $Q_v^{(\pm)}$, while Q_2 is generated within perturbative QCD for $\mu < m_b$. The operators X_i are taking care of the $1/m_b$ corrections.

The quantities C_1, C_2, h_i are Wilson coefficients. The operators are given by

$$Q_1 = \; 2\, \overline{q_L}\, \gamma^\mu\, Q_v^{(+)} \;\; \overline{q_L}\, \gamma_\mu\, Q_v^{(-)} \;, \tag{88}$$

$$Q_2 = \; 2\, \overline{q_L}\, v^\mu\, Q_v^{(+)} \;\; \overline{q_L}\, v_\mu\, Q_v^{(-)} \;, \tag{89}$$

$$X_1 = \; 2\, \overline{q_L}\, iD^\mu\, Q_v^{(+)}\, \overline{q_L}\, \gamma_\mu Q_v^{(-)} \;\; + \cdots . \tag{90}$$

The explicit expressions for the operators X_i are given in Ref. [13]. There are also non-local operators constructed as time-ordered products of $Q_{1,2}$ and the first order HQEFT Lagrangian in (46). The Wilson coefficients C_1 and C_2 have been calculated to NLO [29] and for $\mu = \Lambda_\chi$ one has $C_1(\Lambda_\chi) = 1.22$ and $C_2(\Lambda_\chi) = -0.15$. The coefficients h_i have been calculated to leading order (LO) in Ref. [30].

In order to find all matrix element of $Q_{1,2}$, one uses the following relation between the generators of $SU(3)_c$ (i, j, l, n are colour indices running from 1 to 3)

$$\delta_{ij}\delta_{ln} = \frac{1}{N_c}\delta_{in}\delta_{lj} \; + \; 2\, t^a_{in}\, t^a_{lj} \;, \tag{91}$$

where a is an index running over the eight gluon charges. This means that using the Fierz transformation, the operator Q_1 in (88) may also be written in the following way (there is a similar expression for Q_2)

$$Q_1^F \;=\; \frac{1}{N_c} Q_1 \;+\; 4\, \overline{q_L}\, t^a\, \gamma^\mu\, Q_v^{(+)}\, \overline{q_L}\, t^a\, \gamma_\mu\, Q_v^{(-)} \,. \tag{92}$$

The first (naive) step to calculate the matrix element of a four-quark operator, like Q_1, is to insert vacuum states between the two currents. This factorized limit means to bosonize the two currents in Q_1 and multiply them (see Eq. (56)). The second operator in (92) is genuinely non-factorizable. In the approximation where only the lowest gluon condensate is taken into account, the last term in (92) can be written in a *quasi-factorizable* way by bosonizating the heavy-light colour current with an extra colour matrix t^a inserted and with an extra gluon emitted as shown in Fig. 11.

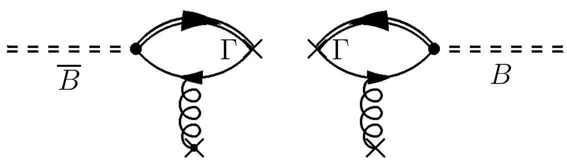


Fig. 11. Non-factorizable soft gluonic contribution to the bag-parameter. (Here $\Gamma \equiv t^a\, \gamma^\mu\, L$.)

We find the bosonized colour current

$$\left(\overline{q_L} t^a\, \gamma^\alpha Q_v^{(\pm)}\right)_{1G} \to -\frac{G_H g_s}{8}\, G^a_{\mu\nu} \mathrm{Tr}\left[\xi^\dagger \gamma^\alpha L\, H^{(\pm)} \left(\pm \mathrm{i}\, I_2\, \{\sigma^{\mu\nu}, \gamma \cdot v\} + \frac{1}{8\pi}\, \sigma^{\mu\nu}\right)\right], \tag{93}$$

where $\{\,,\,\}$ symbolizes an anti-commutator. The result for the right part of the diagram with $\bar{B}$ replaced by B is obtained by changing the sign of v and letting $P_5^{(+)} \to P_5^{(-)}$. Multiplying the coloured currents, we obtain the non-factorizable parts of Q_1 and Q_2 to the first order in the gluon condensate by using Eq. (30).

Now the bag parameter can be extracted and may be written in the form

$$\hat{B}_{B_q} = \frac{3}{4}\widetilde{b}\left[1 + \frac{1}{N_c}(1-\delta_G^B) + \frac{\tau_b}{m_b} + \frac{\tau_\chi}{32\pi^2 f^2}\right], \tag{94}$$

where the parameter $\widetilde{b}$ also involves the Wilson coefficients $C_{\gamma,v}$ defined in (18)

$$\widetilde{b} \;=\; b(m_b)\left[\frac{C_1 - C_2}{(C_\gamma + C_v)^2}\right]_{\mu=\Lambda_\chi}. \tag{95}$$

The soft gluonic non-factorizable effects are given by

$$\delta_G^B \;=\; \frac{N_c\langle\frac{\alpha_s}{\pi}G^2\rangle}{32\pi^2 f^2 f_B^2}\,\frac{m}{M_B}\,\kappa_B\left[\frac{C_1}{C_1 - C_2}\right]_{\mu=\Lambda_\chi}, \tag{96}$$

where κ_B is a dimensionless hadronic parameter which depends on m, f, μ_G^2 and $g_{\mathcal{A}}$, and is of the order 2. Note that we are qualitatively in agreement with Ref. [31], where also a negative contribution to the bag factor from soft gluon effects was found. Numerically, f and f_B are of the same order of magnitude, and δ_G^B is, therefore, suppressed like m/M_B compared to the corresponding quantity

$$\delta_G^K \;=\; N_c\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{32\pi^2 f^4} \tag{97}$$

found for $K - \overline{K}$ mixing [5].

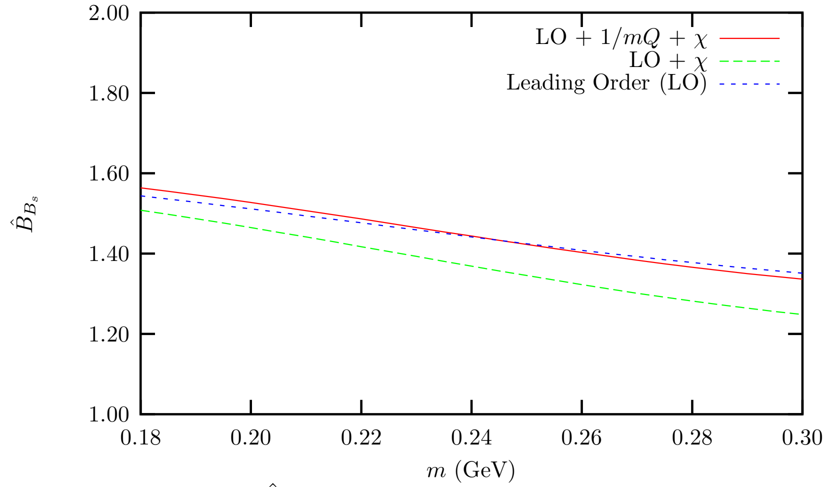


Fig. 12. The bag parameter $\hat{B}$ for B_s as a function of m.

However, one should note that f_B scales as $1/\sqrt{M_B}$ within HQEFT, and therefore δ_G^B is still formally of the order of $(m_b)^0$. The quantity τ_b represents the $1/m_b$ corrections due to the operators X_i. Furthermore, the quantity τ_χ represents the chiral corrections (including counterterms) to the bosonized versions of $Q_{1,2}$ [13]. The bag parameter $\hat{B}$ is plotted as function of m in Fig. 12 for the case B_s. Our results are numerically in agreement with the lattice results [32].

6.2. *The processes $B \to D^{(*)}\overline{D^{(*)}}$*

It has been observed [14] that the processes $\bar{B}^0_{d,s} \to D_{s,d}\bar{D}_{s,d}$, $\bar{B}^0_{d,s} \to D^*_{s,d}\bar{D}_{s,d}$, $\bar{B}^0_{d,s} \to D_{s,d}\bar{D}^*_{s,d}$ and $\bar{B}^0_{d,s} \to D^*_{s,d}\bar{D}^*_{s,d}$, have no factorized contribution from the spectator mechanism. If one or two of the D-mesons in the final state are vectors, there are relatively small contributions from the annihilation mechanism. The effective non-leptonic Lagrangian at the quark level has the usual form [28]

$$\mathcal{L}_W = -4\frac{G_F}{\sqrt{2}} V_{cb} V^*_{cq} \sum_i a_i \, \hat{Q}_i(\mu) \, . \tag{98}$$

In our case, there are only two numerically relevant operators (for $q = d, s$)

$$\hat{Q}_1 = (\overline{q}_L \gamma^\alpha b_L)\,(\overline{c}_L \gamma_\alpha c_L) \ ; \ \hat{Q}_2 = (\overline{c}_L \gamma^\alpha b_L)\,(\overline{q}_L \gamma_\alpha c_L) \, . \tag{99}$$

At $\mu = m_b$, one has $a_2 \sim 1$ and $a_1 \sim 1/10$.

Using (91), we obtain the Fierzed version of the operators $\hat{Q}_{1,2}$

$$\begin{aligned}
\hat{Q}_1^F &= \frac{1}{N_c}\hat{Q}_2 + 2(\overline{c}_L \gamma^\alpha t^a b_L)\,(\overline{q}_L \gamma_\alpha t^a c_L) \\
\hat{Q}_2^F &= \frac{1}{N_c}\hat{Q}_1 + 2(\overline{q}_L \gamma^\alpha t^a b_L)\,(\overline{c}_L \gamma_\alpha t^a c_L)
\end{aligned} \tag{100}$$

The genuine non-factorizable $1/N_c$ chiral Lagrangian terms from "coloured-quark operators" can be estimated within the HLχQM. However, in order to do this, we have to treat the effective weak non-leptonic Lagrangian in (98) within heavy-quark effective theory (HQEFT) [8]. Then b, c, and $\overline{c}$ quarks are replaced by their corresponding operators in HQEFT,

$$b \to Q_{v_b}^{(+)} \, , \quad c \to Q_{v_c}^{(+)} \, , \quad \overline{c} \to Q_{\bar{v}}^{(-)} \, , \tag{101}$$

up to $1/m_b$ and $1/m_c$ corrections. Then the effective weak non-leptonic Lagrangian (98) can be evolved down to the scale $\mu \sim \Lambda_\chi \sim$1 GeV [33]. At $\mu = 1$ GeV, we have $a_2 \simeq 1.29 + 0.08\,\mathrm{i}$, and $a_1 \simeq -0.35 - 0.07\,\mathrm{i}$. Note that $a_{1,2}$ are complex for $\Lambda_\chi < \mu < m_c$ [33].

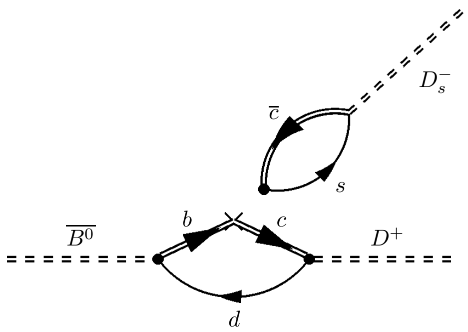


Fig. 13. Factorized contribution for $\overline{B^0} \to D^+ D_s^-$ through the spectator mechanism, which does not exist for the decay mode $\overline{B^0} \to D_s^+ D_s^-$. The double dashed lines represent heavy mesons, the double lines represent heavy quarks, and the single lines light quarks.

The bosonized factorized weak Lagrangian corresponding to Fig. 13 and the operator $\hat{Q}_2$ (with the dominating Wilson coefficient a_2) is obtained from (15), (19) and (98)

$$\mathcal{L}^{\rm Bos}_{\rm W-Fact}(Q_2) = 4\frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* (a_2 + \frac{a_1}{N_c})\, \zeta(\omega) \frac{\alpha_H}{2} \,\mathrm{Tr}\Big[\overline{H_c^{(+)}} \gamma^\alpha L H_b^{(+)}\Big] \cdot \mathrm{Tr}\Big[\xi^\dagger \gamma^\alpha L H_{\bar{c}}^{(-)}\Big], \tag{102}$$

where $\omega \equiv v_b \cdot v_c = v_b \cdot \bar{v} = M_B/(2M_D)$. This Lagrangian (corresponding to the spectator mechanism) contributes to the factorized amplitude for the process $\overline{B^0} \to D^+ D_s^-$, and is the starting point for chiral loop contributions to the process $B \to D^{(*)} \overline{D^{(*)}}$. These chiral loop contributions, suppressed by $1/N_c$, are of order $(m_K g_A / 4\pi f)^2$ times the factorizable contribution obtained from (102).

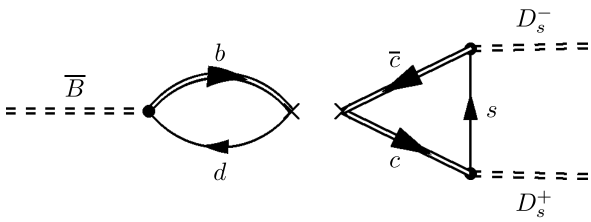


Fig. 14. Factorized contribution for $\overline{B^0} \to D_s^+ D_s^-$ through the annihilation mechanism, which give zero contributions if both D_s^+ and D_s^- are pseudo-scalars.

The bosonized factorized weak Lagrangian corresponding to Fig. 14 and the non-dominating Wilson coefficient a_1 is

$$\mathcal{L}^{\rm Bos}_{\rm W-Fact}(Q_1) = 4\frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* (a_1 + \frac{a_2}{N_c})\, \zeta(-\lambda) \frac{\alpha_H}{2} \,\mathrm{Tr}\Big[\xi^\dagger \gamma^\mu L\, H_b^{(+)}\Big] \cdot \mathrm{Tr}\Big[\overline{H_c^{(+)}} \gamma^\alpha L H_{\bar{c}}^{(-)}\Big], \tag{103}$$

where $\lambda \equiv \bar{v} \cdot v_c = (M_B^2/(2M_D^2) - 1)$. Unless one or both of the D-mesons in the final state are vector mesons, this matrix element is zero due to the current conservation, which is analogous to the decay mode $\overline{D^0} \to K^0 \overline{K^0}$ [12].

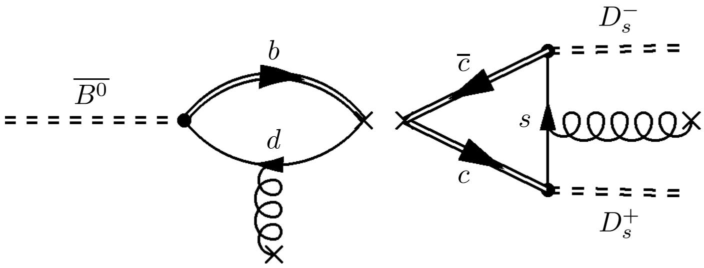


Fig. 15. Non-factorizable contribution for $\overline{B^0} \to D_s^+ D_s^-$ through the annihilation mechanism with additional soft gluon emission. The wavy lines represent soft gluons ending in vacuum to make gluon condensates.

The genuine non-factorizable part for $\overline{B^0} \to D_s^+ D_s^-$ at quark level can, by means of Fierz transformations and the identity (91), be written in terms of colour currents. The left part in Fig. 15 with gluon emission gives us the bosonized colour current which is the same as for $B - \overline{B}$ mixing in eq. (93). For the creation of a $D\bar{D}$ pair in the right part of Fig. 15, there is an analogue of (93), which can be written

$$\left(\overline{Q_{v_c}^{(+)}}\, t^a\, \gamma^\alpha\, L Q_{\bar{v}}^{(-)}\right)_{1G} \longrightarrow \frac{G_H^2\, g_s}{32\pi m}\, G^a_{\mu\nu}\, \mathrm{Tr}\Big[\, \overline{H_c^{(+)}}\, \gamma^\alpha\, L\, H_{\bar{c}}^{(-)} X^{\mu\nu} \Big]\,, \tag{104}$$

where

$$X^{\mu\nu} \equiv \frac{r(-\lambda)}{\pi}\, \sigma^{\mu\nu} \;+\; \frac{1}{4(\lambda-1)}\, \{\sigma^{\mu\nu}, \gamma\cdot t\}$$

and $t \equiv v_c - \bar{v}$. Multiplying the currents and using (30), we obtain a bosonized effective Lagrangian as the product of two traces. Note that our non-factorizable amplitudes (proportional to the gluon condensate) are proportional to the numerically favourable Wilson coefficient a_2.

The gluon condensate contribution obtained from (93) and (104) is a linear combination of terms like:

$$\mathrm{Tr}\left[\xi^\dagger \sigma^{\mu\alpha} L\, H_b^{(+)}\right] \cdot \mathrm{Tr}\left[\overline{H_c^{(+)}} \gamma_\alpha L H_{\bar{c}}^{(-)} \gamma_\mu\right],$$

$$\mathrm{Tr}\left[\xi^\dagger \gamma^{\mu} L\, H_b^{(+)}\right] \cdot \mathrm{Tr}\left[\overline{H_c^{(+)}} \gamma^\alpha L H_{\bar{c}}^{(-)} \sigma_{\mu\alpha} R\right],$$

$$\mathrm{Tr}\left[\xi^\dagger \gamma^{\mu} L\, H_b^{(+)}\right] \cdot \mathrm{Tr}\left[\overline{H_c^{(+)}} \gamma_5 L H_{\bar{c}}^{(-)} \gamma_\mu\right],$$

$$\mathrm{Tr}\left[\xi^\dagger L\, H_b^{(+)}\right] \cdot \mathrm{Tr}\left[\overline{H_c^{(+)}}\gamma^\alpha L H_{\bar{c}}^{(-)}\gamma_\alpha\right],$$

$$\mathrm{Tr}\left[\xi^\dagger \sigma^{\mu\alpha} L\, H_b^{(+)}\right] \cdot \mathrm{Tr}\left[\overline{H_c^{(+)}}\gamma_\alpha L H_{\bar{c}}^{(-)}\right](\bar{v}-v_c)_\mu,$$

$$\varepsilon^{\mu\nu\alpha\lambda}(v_c+\bar{v})_\nu \mathrm{Tr}\left[\xi^\dagger \gamma^\mu L\, H_b^{(+)}\right] \cdot \mathrm{Tr}\left[\overline{H_c^{(+)}}\gamma^\alpha L H_{\bar{c}}^{(-)}\gamma_\lambda\right]. \tag{105}$$

These terms might have been written down based on the heavy-quark symmetry, but the HLχQM selects a certain linear combination to be realized.

Our amplitudes for $B \to D\bar{D}$, in terms of the chiral loop and gluon condensate contributions, are sensitive to the $1/m_c$ corrections and counterterms which have not yet been calculated [14]. Operators suppresssed by $1/m_Q$ are obtained by the replacements

$$Q_v^{(\pm)} \to \frac{1}{m_Q}\mathrm{i}\,\gamma\cdot D_\perp Q_v^{(\pm)}\,, \qquad D_\perp^\nu = D^\nu - v^\nu(v\cdot D)\,, \tag{106}$$

for one of the heavy quarks in (99) and (100). Some new quark operators of the order of $1/m_Q$ might also be generated by pQCD for $\mu < m_Q$. Counterterms correspond to mass insertion of $\widetilde{M_q}$, given by (14) and (26), at light quark lines in the diagrams for $B \to D\bar{D}$ in Figs. 14 and 15.

6.3. Other applications

Within the HLχQM, the process $B \to D\eta'$ has been estimated [11]. This is done in two steps. First we calculate the subprocess $B \to Dgg^*$. Then the virtual gluon g^* is attached to the $\eta' gg^*$-vertex, and the other end in vacuum and make a gluon condensate together with one of the other soft gluons (g) from the $\eta' gg^*$-vertex. Using Fierz transformations for the four quark operators for $b \to cd\bar{u}$, we obtain contributions corresponding to Fig. 16.

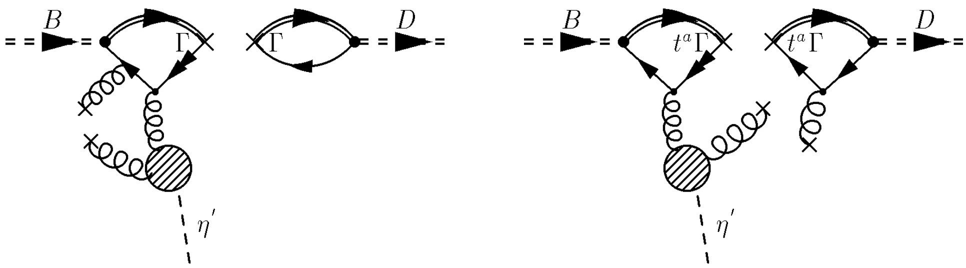


Fig. 16. Gluon condensate contributions to $B \to D\eta'$.

We have used the existing parameterizations of the $\eta' g g^*$-vertex form factor and assumed that the current for $B \to g^*$ is related to the better known case $B \to \rho$. It turns out that the "factorizable" diagram to the left in Fig. 16 can be neglected compared to the non-factorizable diagram to the right. For m in the range 230 – 270 MeV, we obtained the result [11] $Br(B \to D\eta') = (2.2 \pm 0.4) \times 10^{-4}$. Here $1/m_Q$ and chiral corrections are not included.

Heavy to light non-leptonic processes like $B, D \to K\pi$ cannot in general be treated within the HLχQM in its present form. (See, however, Sect 7.2). Still, semileptonic heavy to light processes might be treated at the "no recoil point" [9]. The form factors $f_+(q^2)$ and $f_-(q^2)$ are defined as:

$$\langle \pi^+(p_\pi)|\overline{u}\gamma^\alpha b|H\rangle = 2\,\langle \pi^+(p_\pi)|J_f^\alpha|H\rangle = f_+(q^2)(p_H + p_\pi)^\alpha + f_-(q^2)(p_H - p_\pi)^\alpha \tag{107}$$

where $p_H^\alpha = M_H v^\alpha$, the index a corresponds to quark flavour u and $q^\mu = p_H^\mu - k_\pi^\mu$. The form factors get contributions from $J_f^\alpha(0)$ in (15) and $J_f^\alpha(1)$ in (58) close to the "no recoil point" where $v \cdot p_\pi$ is small

$$f_+(q^2) + f_-(q^2) = \frac{-1}{\sqrt{2M_H}f_\pi}\,(C_\gamma + C_v - g_{\mathcal{A}} C_\gamma)\,\alpha_H\,, \tag{108}$$

$$f_+(q^2) - f_-(q^2) = -C_\gamma \frac{\sqrt{M_H}}{\sqrt{2}f_\pi}\left(\frac{g_{\mathcal{A}}\alpha_H}{v\cdot p_\pi} + \alpha_{H\gamma}^{(1)}\right), \tag{109}$$

where we have neglected terms of the first order in $v \cdot p_\pi$ (where $\alpha_{Hv}^{(1)}$ contributes). The $1/v \cdot p_\pi$ term in (109) is due to the H^* pole. From Eqs. (108) and (109), we see that

$$(f_+(q^2) + f_-(q^2))/(f_+(q^2) - f_-(q^2)) \sim 1/M_H\,, \tag{110}$$

which is the well known Isgur-Wise scaling law [34]. The equations for the two form factors f_+ and f_- should be studied further, and chiral corrections and $1/m_Q$ corrections should be added.

7. *Possible extensions of chiral quark models*

In this section we consider two possible extensions of the chiral quark models which have not yet been worked out in detail. The descriptions are therefore sketchy.

7.1. *Inclusion of light vectors*

One might include vectors in the chiral perturbation theory [35] and thus it should be possible to use the chiral quark model also in this case. We suggest a Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{mass}} + \mathcal{L}_{\chi\text{QM}} + \mathcal{L}_{\text{IVA}}\,, \tag{111}$$

where the interaction between quarks and vectors and axial vectors is given by

$$\mathcal{L}_{\mathrm{IVA}} = \overline{\chi}\,[h_V \gamma^\mu V_\mu + h_A \gamma^\mu \gamma_5 A_\mu]\,\chi \,. \tag{112}$$

Here V are given as Π in (3) with π replaced by ρ etc., and similarly for the axial vector A where π is replaced by a_1. The (bare) mass term is

$$\mathcal{L}_{\mathrm{mass}} = \bar{m}_V^2 \mathrm{Tr}\,[V_\mu V^\mu] + \bar{m}_A^2 \mathrm{Tr}\,[A_\mu A^\mu] \ . \tag{113}$$

After quarks are integrated out, the masses are modified and identified with the physical ones. Then a kinetic term is also generated

$$\mathcal{L}_{\mathrm{kin}} = -\frac{1}{2}\,[V_{\mu\nu} V^{\mu\nu}] \, - \, \frac{1}{2}\,[A_{\mu\nu} A^{\mu\nu}] \ , \tag{114}$$

where for $X = V, A$

$$X_{\mu\nu} \, = \, \nabla_\mu X_\nu \, - \, \nabla_\nu X_\mu \, , \tag{115}$$

and similarly for the axial vector. Here ∇ is a covariant derivative including the Goldstones

$$\nabla_\mu X_\nu \, \equiv \partial_\mu X_\nu \, + \, \mathrm{i}\,[\mathcal{V}_\mu, X_\nu] \, . \tag{116}$$

For the left-handed current for $vac \to X = V, A$, we find the $SU(3)$ octet current

$$J_\mu^n(vac \to X) = \frac{1}{2} k_X \mathrm{Tr}\,[\Lambda^n X_\mu] \ , \tag{117}$$

where the quantity Λ^n is given by (36).

As previuosly, bosonization gives constraints on the parameters of the vectorial sector. From normalization of the kinetic term(s) we obtain

$$\frac{f^2 h_V^2}{3m^2}\left[1 \, - \, \frac{1}{15m^2 f^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right] \, = \, 1 \, , \tag{118}$$

where $h_A = h_V$ before chiral corrections. For the currents we obtain

$$k_V \, = \, \frac{1}{2} h_V \left(-\frac{\langle\,\overline{q}q\,\rangle}{m} + f^2 - \frac{1}{8m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle \right) , \tag{119}$$

and similarly, for the axial case

$$k_A \, = \, \frac{1}{2} h_A \left(-\frac{\langle\,\overline{q}q\,\rangle}{m} - 3f^2 + \frac{1}{8m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle \right) . \tag{120}$$

The formalism suggested in this subsection might, for instance, give a reasonable description of the weak current for D-meson decays $D \to V$ [36], when combined with HLχQM. This might also be the case for processes like $D \to VP$, where V is a vector meson and P is a pseudoscalar. In the last case, non-factorizable contributions can be calculated in terms of chiral loops and gluon condensates. However, one should keep in mind that a limitation in this case is that V is (also formally) light compared to D.

7.2. Heavy to light transitions

As emphasized in the Introduction, the HLχQM is not suited to describe $B \to \pi$ transitions except for semileptonic transitions close to the no recoil point. It might therefore be surprising that we consider a formalism for chiral perturbation theory for $B \to \pi$ transitions (and more general $B \to P$ for $P = \pi, K, \eta$), because the involved pion is hard. However, in general, in a transition $B \to$ pions, we might have a configuration where one pion is hard and one (or more) is soft. For such cases, we split the pseudoscalar sector in hard and soft pseudoscalars. The soft pseudoscalars are represented as before, while the hard pseudoscalars are represented by an octet 3×3 matrix M given as Π in Eq. (3), but transforming as Σ under $SU(3)_L \times SU(3)_R$.

Starting with a γ_5 coupling for quarks coupling to pseudoscalars, we represent the hard light quark with a quark field q_n [37, 16] and the soft light quarks by the flavour rotated fields χ in Sec. 3. Then we arrive at the interaction Lagrangian

$$\mathcal{L}_n = G_M \bar{\chi} \left[\xi^\dagger M R - \xi M^\dagger L \right] q_n \tag{121}$$

for a hard light quark q_n entering a hard pion (kaon) with momentum $p_M = E\, n$ where n is a lightlike vector and E is the energy of the hard pion(kaon). The hard quark has then momentum $p_q = E\, n + k$, where k is of the order of $\Lambda_\chi \sim 1$ GeV or smaller. G_M is a coupling which has to be determined by some physical requirements. For an outgoing hard quark we have

$$\mathcal{L}_{\bar{n}} = G_M \bar{q_n} \left[M \xi^\dagger R - M^\dagger \xi L \right] \chi \, . \tag{122}$$

Now one might combine (121) and (122) with HLχQM and use some version of a large energy effective field theory (LEET) [37] to describe the light hard quarks. Using the LEET propagator

$$\frac{\gamma \cdot n}{2 n \cdot k} \tag{123}$$

for the light hard quark, we can write down a quark loop diagram for $B \to P$ with a corresponding amplitude for the heavy-light weak current (to leading order)

$$J_k^\alpha (B \to P) \;=\; K \, \mathrm{Tr} \left[\Gamma^\alpha H_{vh} \gamma \cdot n \, \xi M^\dagger \right] \; . \tag{124}$$

Given the transformation properties in (7), (12), and (24), the current (124) transforms as in (16).

The behaviour of the quantity K (form factor) is known from theoretical considerations within LEET [37] and soft collinear effective theory (SCET) [16]

$$K \;\sim\; E \, \zeta^{(v)}(M_B, E) \, , \tag{125}$$

where $\zeta^{(v)}$ is expected to scale as

$$\zeta^{(v)}(M_B, E) \;\sim\; \frac{\sqrt{M_B}}{E^2} \; . \tag{126}$$

Note that the factor $\sqrt{M_B}$ is associated with the heavy (B) meson wave function, and similarly a factor $\sqrt{E}$ with the wave function of the hard pseudoscalar meson. Within our framework, K will contain the product of the couplings G_H and G_M, and some loop integrals involving the heavy quark propagator, the ordinary Dirac propagator for the soft quark, and the LEET propagator in (123). However, it has been pointed out that the LEET propagator is too singular to give meaningful loop integrals [38], and that the LEET is incomplete [16]. Therefore, the simple expression in (123) has to be modified in some way, by keeping $n^\mu n_\mu = \delta^2 \neq 0$ with $\delta \sim 1/E$, by adding a small quantity in the LEET propagator denominator, or by modifying the formalism in other ways. This modification has to be done in such way that one does not come in conflict with the known scaling properties of $\zeta^{(v)}$. Keeping $\delta \neq 0$ and $\delta \sim 1/E$, some of the involved loop integrals have the same mathematical form as those involved in the Isgur-Wise function in (73), but with $\omega \to 1/\delta$. The most plausible scenario is that $G_M \sim E^{-3/2}$. Anyway, knowledge of $\zeta^{(v)}$ will put restrictions on G_M.

The $W \to \pi$ transition is in Refs. [15, 16] represented by an integral over a momentum distribution proportional to $x(1-x)$ dominated at $x \sim 1/2$. However, there are also suppressed contributions (for $E \gg \Lambda_\chi$) from momentum configurations where one quark (anti-quark) is hard and the anti-quark (quark) is soft. The left-handed current is in this case given by

$$j^l_\mu \; = \bar{q}_n \, \gamma_\mu L \, (\lambda^l \xi) \, \chi \; , \quad \text{or} \qquad j^l_\mu \; = \bar{\chi} \, (\xi^\dagger \lambda^l) \, \gamma_\mu L \, q_n \; , \tag{127}$$

where l is an $SU(3)$ octet index. These quark currents will, when combined with (121) and (122), generate a bosonized current

$$\Delta J^l_\mu \; = \; N \, \tilde{n}^\mu \mathrm{Tr} \left[\lambda^l (\Sigma M^\dagger + M \Sigma^\dagger) \right] , \tag{128}$$

where $\tilde{n}$ is another (almost) lightlike vector with opposite three momentum compared to n such that $\tilde{n}^2 = \delta^2$ and $\tilde{n} \cdot n = 2 - \delta^2$. In the most plausible scenario mentioned above, N scales as a constant (E^0), which is suppressed by $1/E$ compared with the leading order current $\sim E \, f_P^{(0)} \, \tilde{n}^\mu$. The physical decay constant f_P (for $P = \pi, K, \eta$) is within this scheme given by $f_P^{(0)}$ plus the suppressed contribution $\sim N/E$ from (128).

Now, the product of the currents in (124) and (128) will give a factorized $1/E$ suppressed contribution to $B \to K\pi$ corresponding to the diagram in Fig. 13, with $\bar{c}$ and c replaced by energetic (anti) quarks, D by π and D_s by K. Of course, this contribution can not be distinguished from the standard factorized contribution. However, pulling out soft pseudoscalars from ξ and Σ in the currents (124) and (128), we obtain $1/E$ suppressed non-factorizable chiral loop contributions to $B \to K\pi$. Similarly, there will be $1/E$ suppressed gluon condensate contributions. Such suppressed terms are not in conflict with QCD factorization [15].

8. *Conclusion*

We have presented the main features of chiral quark models, both in the pure light and the heavy-light sector. Especially, the HLχQM seem to work well. In that case, it is possible to systematically calculate the $1/m_Q$ corrections as well as chiral corrections. The model may be used to give predictions for many quantities. Especially, it is suitable for calculation of the B-parameter for $B-\bar{B}$ mixing [9], and for a study of processes of the type $B \to D\bar{D}$. For heavy to light transitions ($B \to K\pi$, say) the HLχQM cannot be used in its present form. It remains to be seen if the extension indicated in Sec. 7.2 to incorporate light energetic quarks will lead to some understanding of such decays.

In our version of the chiral quark models (pure light and heavy light cases) soft gluon effects are truncated to include only the secord order gluon condensate. It has worked reasonably well up to now, but one may wonder if this is enough to accomodate all effects, for instance when light vectors are included. Maybe for instance higher order gluon condensates could be included, but then our simple model will be replaced by a more complicated one.

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KIRALNI KVARKOVSKI MODELI I NJIHOVE PRIMJENE

Dajemo pregled kiralnih kvarkovskih modela, kako za laki, tako i za teško-laki sektor. Opisujemo kako se u tim modelima može provesti bozonizacija radi dobivanja poznatih kiralnih Lagrangiana, koji se mogu izvesti iz samih simetrija QCD-a. Nadalje, u tim modelima možemo računati koeficijente raznih članova kiralnih Lagrangiana. Raspravljamo neke primjene modela, posebice miješanje $\mathrm{B} - \bar{\mathrm{B}}$ i procese tipa $\mathrm{B} \to \mathrm{D}\bar{\mathrm{D}}$, gdje D može biti pseudoskalar ili vektor. Navodimo kako se taj formalizam može proširiti radi uključivanja lakih vektora $(\rho, \omega, \mathrm{K}^*)$, i prijelaza teških u lake mezone, kao $\mathrm{B} \to \pi$.