Economic Research-Ekonomska Istraživanja

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

A probabilistic linguistic thermodynamic method based on the water-filling algorithm and regret theory for emergency decision making

Wenting Xue, Zeshui Xu & Wuhui Lu

To cite this article: Wenting Xue, Zeshui Xu & Wuhui Lu (2023) A probabilistic linguistic thermodynamic method based on the water-filling algorithm and regret theory for emergency decision making, Economic Research-Ekonomska Istraživanja, 36:1, 2076141, DOI: 10.1080/1331677X.2022.2076141

To link to this article: https://doi.org/10.1080/1331677X.2022.2076141

Published online: 23 May 2022.

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2023, VOL. 36, NO. 1, 2076141
https://doi.org/10.1080/1331677X.2022.2076141

A probabilistic linguistic thermodynamic method based on the water-filling algorithm and regret theory for emergency decision making

Wenting Xue[a], Zeshui Xu[b] and Wuhui Lu[a]

[a]School of Statistics and Mathematics, Zhejiang Gongshang University, Hangzhou, China; [b]Business School, Sichuan University, Chengdu, Sichuan, China

ABSTRACT

Since thermodynamics can describe the energy of matter and its form of storage or transformation in the system, it is introduced to resolve the uncertain decision-making problems. The paper proposes the thermodynamic decision-making method which considers both the quantity and quality of the probabilistic linguistic decision information. The analogies for thermodynamical indicators: energy, exergy and entropy are developed under the probabilistic linguistic circumstance. The probabilistic linguistic thermodynamic method combines the regret theory which captures decision makers' regret-aversion and the objective weight of criterion obtained by the water-filling algorithm. The proposed method is applied to select the optimal solution to respond to the floods in Chongqing, China. The self-comparison is conducted to verify the effectiveness of the objective weight obtained by the water-filling algorithm and regret theory in the probabilistic linguistic thermodynamic method. The reliability and feasibility of the proposed method are verified by comparative analysis with other decision-making methods by some simulation experiments and non-parametric tests.

ARTICLE HISTORY
Received 10 April 2021
Accepted 4 May 2022

KEYWORDS
Probabilistic linguistic term set; thermodynamics; regret theory; water-filling algorithm; emergency decision making

JEL CODES
I18; C44; D81

1. Introduction

In recent years, more and more emergency events are constantly emerging, such as the American flu pandemic and COVID-19 in 2019 and the Australia fires and the east African locust plague in 2020, which seriously threaten the safety of people's life and property. How to develop the emergency-rescue plan and respond to the emergency quickly and efficiently becomes the top priorities for local government and related institution or organisation (Chanamool & Naenna, 2016; Tian et al., 2018; Zhang et al., 2022; Zshou et al., 2018). The response of emergency is a group emergency decision making (EDM) which usually involves many intricate and complex

CONTACT Zeshui Xu xuzeshui@263.net

Nomenclature

EDM	emergency decision making	λ	parameter to adjust the importance degree of $v(x)$ and $R(\Delta v)$
HFLTSs	hesitant fuzzy linguistic term sets	w_j^k	criterion weight of the k th decision maker
PLTS	probabilistic linguistic term set	E	PLEN
PLRDM	probabilistic linguistic regret decision matrix	r	PLP
MCDM	multiple criteria decision making	w	PLF
PLDM	probabilistic linguistic decision matrix	q^k	quality of the PLP for the k th decision maker
PLP	probabilistic linguistic potential	$E(r^k)$	score function of the k th decision maker
PLEN	probabilistic linguistic energy	$E(\bar{r})$	averaging score function of the PLP r^k
PLF	probabilistic linguistic force	B	PLEX
PLEX	probabilistic linguistic exergy	$\dot{S}$	probabilistic linguistic entropy
S	a linguistic term set	$\dot{R}^k$	PLDM of the decision maker D_k
$s^{i(l)}(p^{(l)})$	the l th linguistic term $s^{i(l)}$ with the probability $p^{(l)}$	w^k	objective weight vector assigned by the k th decision maker
$s^{i(l)}$	a linguistic term	ϖ	hybrid weight
$h_S^i(p)$	probabilistic linguistic element	$\overrightarrow{R}^k$	PLRDM of the decision maker D_k
$\#L$	the number of different $s^{i(l)}$ in $h_S^i(p)$	$\dot{U}^k$	PLEN matrix
$\dot{h}_S^i(p)$	normalised PLTS	$\dot{Q}^k$	quality matrix of the PLP
$E(h_S(p))$	score function of $h_S(p)$	$\dot{X}^k$	PLEX matrix
$\sigma(h_S(p))$	deviation function of $h_S(p)$	$\dot{U}_{i\times k}$	averaging PLEN
$d(h_S^1(p), h_S^2(p))$	deviation degree between $h_S^1(p)$ and $h_S^2(p)$	$\dot{X}_{i\times k}$	averaging PLEX
$v(x)$	utility function	$\dot{U}_i$	PLEN indicator of alternative A_i
α	risk aversion coefficient	$\dot{X}_i$	PLEX indicator of alternative A_i
$R(\Delta v)$	regret-rejoice function	$\dot{S}_i$	entropy indicator of alternative A_i
δ	regret aversion coefficient		
$U(A)$	perceived utility value of A		

factors. The decision makers should select proper decision-making method in accordance with the specific decision information (Tian et al., 2017, 2020; Zheng et al., 2020). In the early stages of the emergency, it is difficult and impractical to collect complete decision information, in view of the complexity and urgency of decision-making situations. In such instances, decision makers prefer to utilise more flexible ways to express their qualitative assessments within limited time.

In the practical decision-making problems, the decision makers usually adopt some linguistic terms, such as 'poor', 'fair', 'good' to depict their preferences over the criterion. Therefore, it is quite important for decision makers to choose suitable forms to portray their evaluations (Wang, Xu, et al., 2018). Considering that decision makers may be hesitant among several adjacent linguistic terms, Rodríguez et al. (2012) proposed the hesitant fuzzy linguistic term sets (HFLTSs) which combined hesitant fuzzy sets (Torra, 2010) and linguistic variables (Zadeh, 1975). However, each linguistic term in the HFLTS is assigned equal probability or weight by default. In addition, HFLTSs fail to express the possible linguistic terms which are not adjacent. In the individual and group decision making, the decision makers may prefer to use some possible linguistic terms instead of the adjacent ones to express their assessments and the probability or weight of each linguistic term. To overcome these drawbacks, Pang et al. (2016) proposed the probabilistic linguistic term set (PLTS), which consisted of several possible linguistic terms with corresponding probabilities (or weights). In the paper, the PLTS is utilised to depict the qualitative assessments considering its strong ability of describing the vagueness and reserving original information.

As a powerful technique to express vague information, the theories and decision-making methods have been studied widely, such as PL-TOPSIS (Pang et al., 2016), PL-DEA (Pan et al., 2021), PL-GLDS (Wu & Liao, 2019), PL-MULTIMOORA (Chen et al., 2019), PL-TODIM (Wei & Wu, 2019) and PL-QUALIFLEX (Tian et al., 2019). The existing decision-making methods usually rank the alternatives by means of decision transformation and information fusion. Due to the randomness and complexity of decision-making situations, it is difficult for decision makers to obtain enough information and extract the key characteristic of the uncertain information. Thus, we should improve the existing decision-making methods from multiple angles and make the utmost of the existing uncertain information.

The macroscopic theory of thermodynamics studies thermal properties of matter in an energy transformation perspective and explains the macroscopic law that should be followed when energy is transformed into another form (Dincer & Cengel, 2001). Thermodynamics is a system theory based on experimental results and it does not involve microscopic structure and specific nature of matter. Hence, the thermodynamics theory is with high reliability and universality and applied to the decision domain based on the thermodynamical indicators: energy, exergy and entropy. Introducing thermodynamics into the decision-making fully considers the numerical size and distribution characteristics of the uncertain decision information.

Prathap (2011) extended the analogies of thermodynamics to the domain of bibliometric research and energy–exergy–entropy sequences were introduced to rank the scientist's performance. Verma and Rajasankar (2017) further proposed the thermodynamical indicators under the crisp and fuzzy circumstance to solve the multi-criteria decision making problems. Combing the quantity and quality of intuitionistic fuzzy information, thermodynamics is applied to assist the hierarchical medical system by connecting with the descriptive characters of physical thermodynamic parameters (Ren, Xu, Liao, et al., 2017). Subsequently, based on the modified prospect theory, the thermodynamic decision-making methods with the hesitant fuzzy information and hesitant fuzzy linguistic information are presented to measure the quantity and quality of the uncertain information (Liao et al., 2018; Ren, Xu, & Hao, 2017). Wang, Liang, et al. (2020) proposed a probabilistic linguistic belief thermodynamic method to evaluate the mobile health apps based on psychological perception.

In the practical decision-making problems, there exists the phenomenon that decision makers compare the result of selected alternative with unselected ones. If the results of other alternatives perform better than the selected one, they feel regretful, or they will be delighted. Bell (1982) and Loomes and Sugden (1982) proposed the regret theory considering decision makers' regret-aversion. In the regret theory, the decision makers not only focus on the direct outcome, but also concern the results if they select other solutions. Regret theory has been extended to hesitant fuzzy sets (Xia, 2018), fuzzy complex spherical fuzzy sets (Akram et al., 2021), type-2 fuzzy sets

(Wang, Pan, et al., 2020), etc. After extending the perceived utility function of regret theory to the probabilistic linguistic information, Xue et al. (2021) presented probabilistic linguistic dynamic reference point method to select the optimal response strategy for the EDM of COVID-19. In the paper, the perceived utility function in Xue et al. (2021) is adopted to depict decision makers' regret-aversion.

In the decision-making methods, there are two ways to determine the weight of each criterion. (1) The weight is endowed by the decision makers subjectively according to their experiences in a specialised field or by convention (Ren, Xu, & Hao, 2017; Ren, Xu, Liao, et al., 2017; Tian et al., 2019; Zhang, 2017). Consider that decision makers are bounded rational individuals, the weight may be biased inevitably which has a negative impact on the decision-making results. (2) The weight is calculated by the existing decision information. For example, the entropy method is applied to many decision-making methods to determine the weight of criterion (Chen, 2021; Wang, Liu, et al., 2018; Xue et al., 2018). In addition, the weight of criterion can be calculated by solving the optimisation model according to the specific characteristics of decision-making methods. For example, the weight of criterion can be computed by solving the single-objective optimisation model according to the maximising deviation method (Pang et al., 2016).

The water-filling theory is a classical method of solving the channel power optimal allocation problems in the communication field (Zhao et al., 2015). The allocation of sub-channel power in the wireless communication can be analogised to the determination of the criterion weight in the decision-making problems. The criterion is regarded as the sub-channel, and corresponding weight can be considered as the power assigned in the sub-channel. How to allocate the sub-channel power reasonably is of significance to improve system performance. If each sub-channel is allocated to same power according to the traditional method, it may lead to a waste of system resources, and even cause system transmission errors in severe cases. The adaptive power allocation to each sub-channel provides an effective way to solve this problem. The system performance can be further optimised by adaptively distributing power to each sub-channel according to the actual channel conditions. Similarly, in order to improve the quality of information utilisation, the water-filling algorithm can be introduced to assign the weight for each criterion based on the given probabilistic linguistic information and practical needs.

In the paper, the water-filling algorithm is utilised to compute the weight of each criterion as the objective weight. Compared with other methods of determining weight, the water-filling algorithm can optimise the weight allocation of criteria adaptively. It assigns the criterion weight based on the nonlinear programming model to maximise the total capacity of criteria. The importance of the criterion is reflected by the ratio of the standard deviation and the mean value of the criterion. The large the ratio, indicating that the criterion has a less impact on the results, the smaller the corresponding weight. The hybrid weight is obtained by combing decision makers' subjective weight and the decision makers can flexibly adjust the proportion according to actual requirements. The water-filling algorithm provides an innovative idea for determining the criterion weight and can make full use of the probabilistic linguistic information.

The thermodynamics theory is with high reliability and universality and applied to the uncertain decision problems domain (Ren, Xu, & Hao, 2017; Ren, Xu, Liao, et al., 2017). In the paper, we introduce the thermodynamical indicators: energy, exergy and entropy to describe the feature of uncertain decision-making information, which can be analogous to the energy, quality, effectiveness and imbalance of probabilistic linguistic information. Since the decision makers are bounded rational, the probabilistic linguistic thermodynamic method combines the regret theory which captures decision makers' regret-aversion. The modified utility function and the regret-rejoice function in the regret theory fully consider the characteristics of the probabilistic linguistic information. In order to maximise the total capacity of criteria when determining the objective weight, the nonlinear programming model is established by the water-filling algorithm. The revised water-filling algorithm considers the specific characteristic of the PLTSs and the nonlinear programming model can be solved by the genetic algorithm. The probabilistic linguistic thermodynamic method is effective to deal with the probabilistic linguistic multiple criteria decision making (MCDM) problems according to the actual situation.

The merits of the paper are summarised as follows:

1. The paper proposes a probabilistic linguistic thermodynamic method from the perspective of both the quantity and quality of the probabilistic linguistic decision information.
2. The proposed method combines the regret theory which depicts decision makers' regret-aversion and the objective weight of criterion obtained by the water-filling algorithm.
3. The probabilistic linguistic thermodynamic method is applied to select the optimal solution to respond to the floods in Chongqing, China.
4. A series of simulation experiments and non-parametric tests are conducted to verify the applicability and effectiveness of the proposed method.

The remainder of the paper is organised as follows: In Section 2, we recall some definitions about PLTSs, classic thermodynamic method and regret theory in the probabilistic linguistic environment. In Section 3, based on the probabilistic linguistic regret decision matrix (PLRDM) and the objective weight of criterion obtained by the water-filling algorithm, the probabilistic linguistic thermodynamic method is developed to solve the EDM problems. Section 4 is the application of the probabilistic linguistic thermodynamic method in the EDM. Sections 5 and 6 are the self-comparison and comparative analysis parts based on a series of simulation experiments and non-parametric tests. Conclusions and future directions are presented in Section 7.

2. Preliminaries

In this section, we recall some concepts about PLTSs, and then introduce the original thermodynamic method and regret theory in the probabilistic linguistic environment which build a basic framework for the probabilistic linguistic thermodynamic method.

2.1. PLTSS

To describe the uncertain quantitative information, Pang et al. (2016) defined the PLTSs which included several linguistic terms with relevant probabilities. Let $x_i \in X$ be fixed and $S = \{s_{-\tau}, \ldots, s_{-1}, s_0, s_1, \ldots, s_\tau\}$ be a linguistic term set, a PLTS on S is $H_S(p) = \{< x_i, h_S^i(p) > |x_i \in X\}$ with $h_S^i(p) = \{s^{i(l)}\ (p^{(l)})|s^{i(l)} \in S, p^{(l)} \geq 0, l = 1, 2, \ldots, \#L, \sum_{l=1}^{\#L} p^{(l)} \leq 1\}$, where $s^{i(l)}(p^{(l)})$ is the l th linguistic term $s^{i(l)}$ with the probability $p^{(l)}$. The linguistic term $s^{i(l)}$ is arranged in ascending order. $h_S^i(p)$ is called the probabilistic linguistic element and $\#L$ is the number of different $s^{i(l)}$ in $h_S^i(p)$.

When $\sum_{l=1}^{\#L} p^{(l)}<1$, the missing probability can be assigned to the linguistic terms appearing in $s^{i(l)}$. Assume that the unknown probability of $s^{i(l)}$ is equal in this paper, then $\dot{p}^{(l)} = p^{(l)}/\sum_{l=1}^{\#L} p^{(l)}, l = 1, 2, \ldots, \#L$ and $\sum_{l=1}^{\#L} \dot{p}^{i(l)} = 1$ (Pang et al., 2016). To compute and compare the PLTSs with different numbers of linguistic terms, an extension method (Pang et al., 2016) is proposed as follows: For any two PLTSs $h_S^1(p)$ and $h_S^2(p)$, if $\#L_1>\#L_2$, then we add $\#L_1-\#L_2$ linguistic terms to $h_S^2(p)$, where corresponding probabilities of the added linguistic terms are equal to 0. The extension method is developed without changing any previous information of the PLTSs. Then we obtain the normalised PLTS: $\dot{h}_S^i(p) = \left\{\dot{s}^{i(l)}(\dot{p}^{(l)})|l = 1, 2, \ldots, \#L\right\}$.

The score function and the deviation degree of PLTSs are introduced as follows:

Definition 1 (Pang et al., 2016). Let $h_S(p) = \left\{s^{(l)}(p^{(l)})|l = 1, 2, \ldots, \#L\right\}$ be a PLTS, and $r^{(l)}$ be the subscript of the linguistic term $s^{(l)}$. The score function of $h_S(p)$ is:

$$E\left(h_S(p)\right) = s_{\bar{r}} \tag{1}$$

where $\bar{r} = \sum_{l=1}^{\#L} r^{(l)}p^{(l)}/\sum_{l=1}^{\#L} p^{(l)}$.

The deviation function of $h_S(p)$ is defined as:

$$\sigma\left(h_S(p)\right) = \sqrt{\sum_{l=1}^{\#L} \left(p^{(l)}\left(r^{(l)} - \bar{r}\right)\right)^2 / \sum_{l=1}^{\#L} p^{(l)}} \tag{2}$$

Definition 2 (Pang et al., 2016). Let $h_S^1(p) = \left\{s^{1(l)}(p_1^{(l)})|l = 1, 2, \ldots, \#L_1\right\}, h_S^2(p) = \left\{s^{2(l)}(p_2^{(l)})|l = 1, 2, \ldots, \#L_2\right\}$ be two normalised PLTSs with $\#L_1 = \#L_2$. Then the deviation degree between $h_S^1(p)$ and $h_S^2(p)$ can be defined as:

$$d\left(h_S^1(p), h_S^2(p)\right) = \sqrt{\sum_{l=1}^{\#L_1} \left(p_1^{(l)}r_1^{(l)} - p_2^{(l)}r_2^{(l)}\right)^2 / \#L_1} \tag{3}$$

where $r_1^{(l)}$ and $r_2^{(l)}$ are the subscripts of the linguistic terms $s^{1(l)}$ and $s^{2(l)}$, respectively.

2.2. Thermodynamic decision-making method

Thermodynamics is a subject concerning the interaction between the system and external environment as the state of matter changes (Charles & Herbert, 1990). The laws of thermodynamics show that the total energy is constant, and the entropy does not decrease over time in an isolated system (Wikipedia, 2021a). The entropy increase theory indicates that the entropy does not decrease in an isolated thermodynamic system, it always increases or remains unchanged (Wikipedia, 2021b). It means that an isolated system cannot move to a low entropy state and become orderly. Due to the increasing randomness of emergency and the complexity of uncertain decision-making environment, it is rather difficult to obtain complete and accurate decision information. It is of great significance to extend the existing decision-making method and extract valid information from the uncertain decision-making information with the thermodynamical indicators. Prathap (2011) applied thermodynamics combining with energy, exergy and entropy to the bibliometric research to rank the scientist's performance. The exergy indicator can reflect the amount of energy which can be converted to useful work. The entropy indicator reflects the unevenness of the decision potential in the rating of alternatives. Afterwards, Verma and Rajasankar (2017) proposed a thermodynamic MCDM method based on the exergy indicator under the crisp and fuzzy circumstance.

In the MCDM problem, assume that m alternatives are represented by $A = \{A_i | i = 1, 2, \ldots, m\}$ and n criteria are expressed by $C = \{C_j | j = 1, 2, \ldots, n\}$. The decision makers are denoted by $D_k (k = 1, 2, \ldots, h)$ and they provide the decision matrix $R^k = (r_{ij}^k)_{m \times n}$ to represent the decision values in regard to C_j by the real number. The weight of C_j assigned to D_k can be expressed by $w = \{w_j^k | j = 1, 2, \ldots, n, \ k = 1, 2, \ldots, h\}$. The thermodynamic decision-making method proposed by Verma and Rajasankar (2017) in the crisp environment is concisely introduced as follows:

Step 1: Identify the decision matrices $R^k = (r_{ij}^k)_{m \times n} (k = 1, 2, \ldots, h)$ of the alternatives $A_i \ (i = 1, 2, \ldots, m)$ is the potential energy in regard to the criterion C_j and the weight w_j^k is corresponding driving force.

Step 2: Calculate the energy matrices $U^k = (u_{ij}^k)_{m \times n} (k = 1, 2, \ldots, h)$ and the quality matrices $Q^k = (q_{ij}^k)_{m \times n} (k = 1, 2, \ldots, h)$ for each decision maker, where

$$u_{ij}^k = w_j^k \cdot r_{ij}^k, q_{ij}^k = 1 - \frac{\left| r_{ij}^k - \frac{1}{h} \sum_{k=1}^h r_{ij}^k \right|}{\frac{1}{h} \sum_{k=1}^h r_{ij}^k}$$

Step 3: Construct the exergy matrix $X^k = (x_{ij}^k)_{m \times n}$, where $x_{ij}^k = q_{ij}^k \cdot u_{ij}^k, (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \ k = 1, 2, \ldots, h)$.

Step 4: Compute the average energy and exergy of A_i with respect to D_k as follows:

$$u_i^k = \frac{1}{n} \sum_{j=1}^n u_{ij}^k, x_i^k = \frac{1}{n} \sum_{j=1}^n x_{ij}^k$$

Step 5: Obtain the energy indicator u_i and the exergy indicator x_i by:

$$u_i = \frac{1}{h}\sum_{k=1}^{h} u_i^k, x_i = \frac{1}{h}\sum_{k=1}^{h} x_i^k$$

Step 6: Determine the entropy indicator $S_i = u_i - x_i$ of A_i, then rank the alternatives. The smaller value S_i is, the better performance A_i is.

2.3. Regret theory in the probabilistic linguistic environment

In the EDM, to minimise the losses as much as possible, decision makers need to select the optimal alternative among the rescue plans quickly. Since the decision information is uncertain and incomplete in the early stages of the emergency, most EDM problems are always the risk decision-making problems. It is quite necessary to consider the bounded rational characteristic of decision makers, such as reference dependence, loss aversion and regret aversion. Bell (1982) and Loomes and Sugden (1982) proposed regret theory which described decision makers' regret-aversion. In the regret theory, decision makers not only focus on the direct outcome, but also concern about the results if they select other solutions. In addition to the utility of the selected alternative, decision makers' regret-aversion should be considered in the behaviour decision-making theory. Therefore, the perceived utility of regret theory consists of two parts: the current results of the utility function and the regret-rejoice function compared with others.

Definition 3 (Zhang et al., 2016). Let x be the criterion value, then the utility function $v(x)$ can be defined as follows:

$$v(x) = x^{\alpha}, 0<\alpha<1 \tag{4}$$

where the first and second derivative satisfy $v'(x)>0$, $v''(x)<0$, and α is the risk aversion coefficient of the decision maker.

Definition 4 (Zhang et al., 2016). The regret-rejoice function $R(\Delta v)$ can be defined as follows:

$$R(\Delta v) = 1 - e^{-\delta \Delta v}, \delta>0 \tag{5}$$

where Δv denotes the utility difference of two alternatives, and $R(\Delta v)$ represents the regret-rejoice function of Δv. Similar to $v(x)$, the first and second derivatives of $R(\Delta v)$ satisfy $R'(\Delta v)>0$ and $R''(\Delta v)<0$, and δ is the regret aversion coefficient of the decision maker.

Definition 5 (Zhang et al., 2016). Let x and y denote the evaluation values of the alternatives A and B, respectively. The perceived utility value of A is obtained by the utility function and the regret-rejoice function as follows:

$$U(A) = v(x) + R(\Delta v) = x^{\alpha} + 1 - e^{-\delta \Delta v}, \Delta v = v(x) - v(y) \tag{6}$$

Definition 6 (Xue et al., 2021). Let $\dot{h}_S^1(p) = \left\{ \dot{s}^{1(l)}(\dot{p}_1^{(l)}) | l = 1, 2, \ldots, \#L \right\}$ and $\dot{h}_S^2(p) = \left\{ \dot{s}^{2(l)}(\dot{p}_2^{(l)}) | l = 1, 2, \ldots, \#L \right\}$ denote the evaluation value of the alternatives A and B expressed by normalised PLTSs, respectively. The perceived utility value of A is defined as follows:

$$\begin{aligned} U(A) &= \lambda \bar{v}(r^{(l)}) + (1-\lambda) R(\Delta H) \\ &= \lambda \tau \cdot \left[\left(\frac{r^{(l)}}{\tau} + 1 \right)^{\alpha} - 1 \right] + (1-\lambda) \frac{\tau}{e^{2\delta\tau} - 1} \cdot (1 - e^{-\delta \Delta H}), 0 \le \lambda \le 1 \end{aligned} \tag{7}$$

$$U\left(\dot{h}_S^1(p)\right) = \left\{ s_{\lambda \bar{v}(r^{(l)}) + (1-\lambda) R(\Delta H)}(\dot{p}_1^{(l)}) | l = 1, 2, \ldots, \#L \right\} \tag{8}$$

where $r^{(l)}$ is the subscript of the linguistic term $\dot{s}^{1(l)}$ and $\Delta H = \bar{v}(r_1^{(l)}) - \bar{v}(r_2^{(l)})$. λ is a parameter to adjust the importance degree of the utility function and the regret-rejoice function. Since the utility function and the regret-rejoice function account for same proportion in the classical regret theory, we set $\lambda = 0.5$ in the following part of the paper. Refer to some literature about regret theory (Wang et al., 2021; Xue et al., 2021; Zhang et al., 2016), $\alpha = 0.88, \delta = 0.3$ in this paper.

According to Equation (8), the probabilistic linguistic decision matrix (PLDM) is transformed into the PLRDM, which considers the regret-aversion characteristic of bounded rational decision makers. Compared with original decision matrix, the PLRDM can effectively reflect the actual behaviours of decision makers in the decision-making process.

3. Probabilistic linguistic thermodynamic method

In this section, we extend the thermodynamic decision-making method to the probabilistic linguistic environment based on the water-filling algorithm and regret theory. In the thermodynamic decision-making method, the weights of criteria in regard to decision makers are endowed subjectively. Because of the unexpectedness and uncertainty of the emergency, it is unrealistic for decision makers to provide the exact weight of each criterion in a short time. Due to the similarity between the water-filling theory of wireless communication area (Zhao et al., 2015) and the weight assignment method in the MCDM problems, the water-filling algorithm is applied to determine the objective weight of criterion. Combing with the subjective weight given by decision makers, the hybrid weight of each criterion can be determined.

3.1. Solving the objective weight of criterion by the water-filling algorithm

The water-filling algorithm is a classic algorithm for solving capacity maximisation problems in the multi-channel wireless communication systems. To maximise the

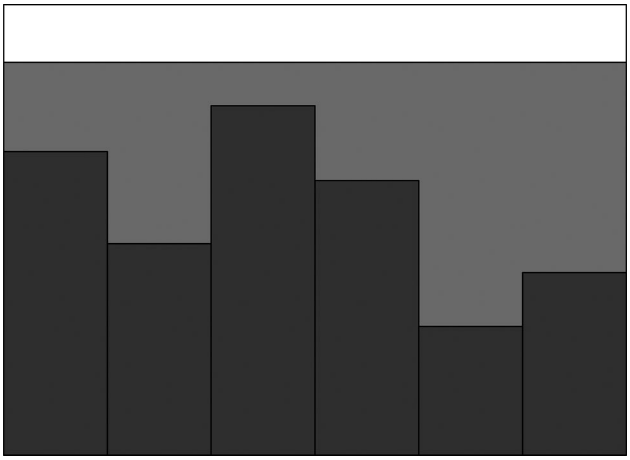

Figure 1. The schematic diagram of water-filling theory. Source: Authors' own research.

channel capacity, the transmitted power is adaptive allocation according to the signal-to-noise ratio of each sub-channel in the water-filling algorithm. As shown in Figure 1, when filling the container whose bottom is uneven with water, the height of the convex part at the bottom is inversely proportional to the amount of water. The higher the projecting at the bottom, the less water is injected, meaning the low power is allocated when corresponding signal-to-noise ratio of the sub-channel is small. Combining Shannon equation, the criterion weight of the k th decision maker w_j^k is derived by:

$$W_C = \sum_{k=1}^{h} \log_2\left(\tau + 1 + \frac{\bar{r}_j^k w_j^k}{\sigma_j^k}\right) \tag{9}$$

where $\bar{r}_j^k = \frac{1}{m}\sum_{i=1}^{m} r_{ij}^k$ and $\sigma_j^k = \sqrt{\frac{1}{m}\sum_{i=1}^{m} (r_{ij}^k - \bar{r}_j^k)^2}$.

To obtain the optimal weight, the optimisation model of maximising the total capacity of criteria is established:

$$\max W_C = \sum_{j=1}^{n} \log_2\left(\tau + 1 + \frac{\bar{r}_j^k w_j^k}{\sigma_j^k}\right)$$
$$s.t.\quad \sum_{j=1}^{n} w_j^k = 1, w_j^k \geq 0$$

where $\bar{r}_j^k = \frac{1}{m}\sum_{i=1}^{m} r_{ij}^k$ and $\sigma_j^k = \sqrt{\frac{1}{m}\sum_{i=1}^{m} (r_{ij}^k - \bar{r}_j^k)^2}$.

3.2. The probabilistic linguistic thermodynamic method

In this section, first, we define the probabilistic linguistic potential (PLP) and the probabilistic linguistic energy (PLEN), which lay a foundation for the thermodynamic decision-making method with the probabilistic linguistic information.

Definition 7. The PLP is the potential energy of an alternative towards a criterion, which can be represented by the value in the PLRDM, and the probabilistic linguistic force (PLF) is corresponding weight of a criterion.

Definition 8. The PLEN indicates the energy that an alternative possesses with respect to a criterion in the decision-making process, which can be defined by:

$$E = w \otimes r = \left\{ s_{wr^{i(l)}}(p^{(l)}) | s^{i(l)} \in S, p^{(l)} \geq 0, l = 1, 2, \ldots, \#L, \sum_{l=1}^{\#L} p^{(l)} \leq 1 \right\} \tag{10}$$

where $r = \left\{ s^{i(l)}(p^{(l)}) | s^{i(l)} \in S, p^{(l)} \geq 0, l = 1, 2, \ldots, \#L, \sum_{l=1}^{\#L} p^{(l)} \leq 1 \right\}$ is the PLP of an alternative, $r^{i(l)}$ is the subscript of the linguistic term $s^{i(l)}$, and w is corresponding PLF.

Example 1. If the probabilistic linguistic assessment value of a decision maker towards an alternative with respect to a criterion is $\{s_1(0.2), s_2(0.8)\}$ $(\tau = 3)$ and the weight of criterion is 0.2, then the PLP can be represented by $\{s_1(0.2), s_2(0.8)\}$ and the PLF is 0.2. According to Def. 8, the alternative's PLEN is calculated by:

$$E = w \otimes r = 0.2 \otimes \{s_1(0.2), s_2(0.8)\} = \{s_{0.2}(0.2), s_{0.4}(0.8)\}$$

Based on the defined PLP, PLF and PLEN, some classical decision operators, such as the probabilistic linguistic weighted averaging operator (Pang et al., 2016) and the probabilistic linguistic Choquet integral operator (Chen et al., 2019), can aggregate the decision information of alternatives and obtain the ranking result. However, these methods only concern the quantity of the decision information but neglect the quality of the data. The quality of decision information is an important characteristic of the PLRDM, since it describes the divergence degree among alternatives in the MCDM problems. If all the decision makers have a consensus on the ranking result, then the quality is equal to 1. When the averaging of alternatives is same, the smaller the variance is, the more reliable the result becomes. Inspired by the definition of the deviation degree of PLTSs, we define the quality of the PLP as follows:

Definition 9. The quality of the PLP for the k th decision maker can be measured by the similarity degree between itself and the averaging PLP of all decision makers, which can be defined by:

$$q^k = 1 - \frac{|E(r^k) - E(\bar{r})|}{2\tau} \tag{11}$$

where $E(r^k)$ is the score function of the k th decision maker and $E(\bar{r})$ is the averaging score function of the PLP $r^k (k = 1, 2, \ldots, h)$.

Example 2. Assume that the probabilistic linguistic assessment values of three decision makers with respect to the object are $r^1 = \{s_2(0.6), s_3(0.2)\}, r^2 = \{s_2(0.2), s_4(0.8)\}, r^3 = \{s_3(0.4), s_4(0.6)\}$ $(\tau = 4)$, respectively. According to Definition 9, their qualities can be computed as:

$$E(\bar{r}) = \frac{E(r^1) + E(r^2) + E(r^3)}{3} = 3.15,$$

$$q^1 = 1 - \frac{|E(r^1) - E(\bar{r})|}{2\tau} = 0.8875,$$

$$q^2 = 1 - \frac{|E(r^2) - E(\bar{r})|}{2\tau} = 0.9438,$$

$$q^3 = 1 - \frac{|E(r^3) - E(\bar{r})|}{2\tau} = 0.9438.$$

Remark 1. The range of q^k is from 0 to 1. When all the PLPs $r^k(k = 1, 2, \ldots, h)$ are equal, the qualities $q^k = 1(k = 1, 2, \ldots, h)$.

Definition 10. The probabilistic linguistic exergy (PLEX) is a rating indicator of the PLP, which considers the quantity and quality of the PLP and can be represented by:

$$B = QE = \left\{ s_{qwr^{i(l)}}(p^{(l)}) | s^{i(l)} \in S, p^{(l)} \geq 0, l = 1, 2, \ldots, \#L, \sum_{l=1}^{\#L} p^{(l)} \leq 1 \right\} \tag{12}$$

Definition 11. The probabilistic linguistic entropy can depict the unevenness of the PLP, which can be measured by:

$$\dot{S} = E \ominus B \tag{13}$$

The probabilistic linguistic entropy is different from classical Shannon's entropy which assumes a prior distribution. It is an indicator that reflects how close the evaluation information of an alternative is to the overall information. In other words, the probabilistic linguistic entropy is smaller when the PLP is more consistent with other PLPs. The PLEX can effectively measure both the quantity and quality of the PLP, which makes the ranking results of the MCDM problems more credible and rational.

3.3. The procedure of the probabilistic linguistic thermodynamic method

In this section, we introduce the procedure of the probabilistic linguistic thermodynamic method based on the water-filling algorithm and regret theory as follows:

Step 1. Collect the PLDMs $\dot{R}^k = (\dot{r}_{ij}^k)_{m \times n}(k = 1, 2, \ldots, h)$ of the decision makers $D_k(k = 1, 2, \ldots, h)$ according to the identified alternatives $A_i(i = 1, 2, \ldots, m)$ and the criteria $C_j(j = 1, 2, \ldots, n)$ of the MCDM problem.

Step 2. Determine the objective weight vector of the j th criterion assigned by the k th decision maker, denoted as $w^k, k = 1, 2, \ldots, h$ by Equation (9) based on the PLDMs $\dot{R}^k = (\dot{r}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$. Combining the subjective weight given by decision makers, the ratio of the subjective and objective weight is determined by the decision maker, then the hybrid weight is denoted by $\varpi = \{\varpi^k | k = 1, 2, \ldots, h\}$.

Step 3. Calculate the PLRDMs $\overrightarrow{R}^k = (\overrightarrow{r}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$ based on Equation (8), after converting the PLDM into normalised PLDM.

Step 4. Compute the PLEN matrices of each decision maker: $\dot{U}^k = (\dot{u}_{ij}^k)_{m\times n} = (\varpi_j^k \overrightarrow{r}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$ by Equation (10).

Step 5. Based on the PLDMs $\dot{R}^k = (\dot{r}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$ and Equation (11), obtain the quality matrices of the PLPs: $\dot{Q}^k = (\dot{q}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$.

Step 6. According to Equation (12), construct the PLEX matrices $\dot{X}^k = (\dot{x}_{ij}^k)_{m\times n} = (\dot{q}_{ij}^k \dot{u}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$.

Step 7. Calculate the averaging PLEN and the averaging PLEX:

$$\dot{U}_{i\times k} = \frac{1}{n}\left(E(\dot{u}_{i1}^k) \oplus E(\dot{u}_{i2}^k) \oplus \cdots \oplus E(\dot{u}_{in}^k)\right)$$

$$\dot{X}_{i\times k} = \frac{1}{n}\left(E(\dot{x}_{i1}^k) \oplus E(\dot{x}_{i2}^k) \oplus \cdots \oplus E(\dot{x}_{in}^k)\right)$$

Step 8. Compute the PLEN indicator and the PLEX indicator of each A_i:

$$\dot{U}_i = \frac{1}{h}\left(\tilde{U}_i^1 \oplus \tilde{U}_i^2 \oplus \cdots \oplus \tilde{U}_i^h\right)$$

$$\dot{X}_i = \frac{1}{h}\left(\tilde{X}_i^1 \oplus \tilde{X}_i^2 \oplus \cdots \oplus \tilde{X}_i^h\right)$$

Step 9. Obtain the entropy indicator of each alternative $\dot{S}_i = \dot{U}_i \ominus \dot{X}_i$. Calculate the comprehensive score of $\dot{S}_i$, the smaller the comprehensive score, the better the alternative.

Remark 2. The subscript of the virtual linguistic terms in the PLEN matrices $\dot{U}^k(k = 1, 2, \ldots, h)$ and the PLEX matrices $\dot{X}^k(k = 1, 2, \ldots, h)$ become very small after a series of calculations of scalar multiplication with $\varpi^k(0 \le \varpi^k \le 1)$ and $q^k(0 \le q^k \le 1)$. Although the semantics of virtual linguistic terms in the PLEN matrix and the PLEX matrix have changed completely, they are still valid when sorting by numerical values.

Remark 3. In the specific procedure of the probabilistic linguistic thermodynamic method, the objective weight vectors $w^k(k = 1, 2, \ldots, h)$ and the quality matrices: $\dot{Q}^k = (\dot{q}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$ are obtained by the original PLDMs $\dot{R}^k = (\dot{r}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$. Other aggregating information is based on the PLRDMs $\overrightarrow{R}^k = (\overrightarrow{r}_{ij}^k)_{m\times n}(k = 1, 2, \ldots, h)$ considering decision makers' regret-aversion.

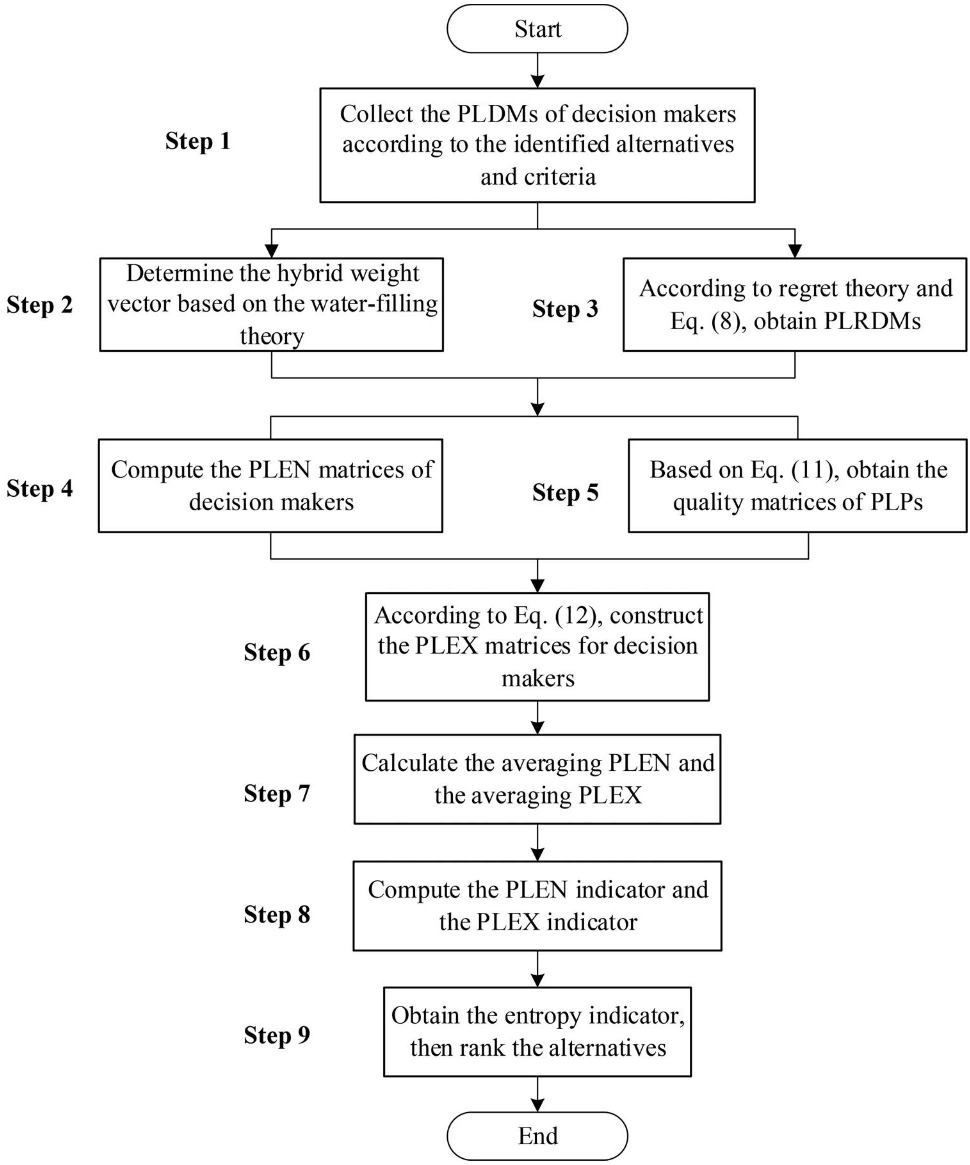

Figure 2. A framework of the probabilistic linguistic thermodynamic method. Source: Authors' own research.

The concrete framework of the probabilistic linguistic thermodynamic method is constructed in Figure 2.

4. Applications in the EDM of floods in Chongqing, China

The Yangtze, China's longest river, recorded the fifth flood after the heavy rainfall in the upstream. Along with the second flood of Jialing River, the floods passed through the central city of Chongqing, located in southwest China, on August 18, 2020.

Although the upstream outflow of Xiangjiaba Reservoir was reduced from 6300 to 4000 m^3/s and the downstream outflow of Three Gorges Reservoir was increased from 42,000 to 46,000 m^3/s (www.news.cn, 2020). As five floods occurred in the upper reaches of the Yangtze since July 2020 and the water level of the Yangtze was unusually high in Chongqing section after the floods accumulation. On August 17, Chongqing Cuntan hydrologic station recorded a water flow of 50,100 m^3/s. According to the report released by the Upper Hydrology Bureau of the Yangtze River Commission, this round of floods will cause the water level of the Yangtze and Jialing River in Chongqing section to exceed the guaranteed water level seriously, and the water level in the main urban area of Chongqing will reach the highest level since 1981.

In view of the urgency of situation, on August 18, 2020, Chongqing activated a top-level response in the country's emergency response system. It is quite necessary to strengthen the management of funds and materials for the flood control and implement some effective and timely measures to respond to the floods. The local government should monitor the areas which are prone to disasters, such as reservoir dikes, floods storage and detention areas, urban waterlogging points. In the key areas, the relevant departments should strengthen monitoring and inspection, continue to check the safety hazards, and rectify hidden perils. It is quite important to dispatch scientifically the flood control projects and prevent the water level from exceeding the warning line effectively. The relevant departments should promptly issue early warning about landslides, mud-rock flows and other geological disasters, strengthen safety precautions in the road traffic, tourist attractions and construction sites, and resolutely avoid accidents involving mass casualties. The emergency rescue forces should be fully mobilised to relocate and resettle the disaster-hit people, and strive to achieve an overall victory in the flood relief. The floods caused the emergency evacuation of 251,000 people and inundated 23,700 stores, fortunately, no deaths (People.com.cn, 2020).

Tongnan County, located in the one-hour economic circle of Chongqing, was hit by the fifth flood of Yangtze combined with the impact of the last flood. By 10 am on August 18, 2020, the water level in Tongnan was 244.81 m exceeding the guaranteed water level by 4.81 m and the flow reached 20,700 m/s. The local government made some preparations for relocation and resettlement of local residents before the floods flowed down. The criteria of the MCDM problem are C_1 : the number of casualties caused by the floods, C_2 : the property loss caused by the floods, C_3 : the input costs of the strategy, C_4 : the public satisfaction degree of the strategy. There are three alternatives:

A_1 : Evacuate people from low-lying areas, close schools, and cancel all outdoor gatherings.

A_2 : Based on A_1, check wiring and equipment for security failures and rectify hidden perils.

A_3 : Based on A_2, impose temporary traffic controls and set up the rescue team and epidemic prevention team.

Suppose that S is a linguistic term set, whose linguistic terms are s_{-3} : extremely poor, s_{-2} : poor, s_{-1} : slightly poor, s_0 : fair, s_1 : slightly good, s_2 : good and s_3 :

extremely good. Three decision makers $D_k (k = 1, 2, 3)$ with same weight are invited to assess the MCDM problem. The subjective weights of criteria are same and the importance degrees of objective and subjective weights are equal in the MCDM problem. Hence, the weight of C_j of D_k is $\varpi_j^k = w_j^k/2 + 1/2n, j = 1, 2, \ldots, n, k = 1, 2, \ldots, h$. The PLDM of D_k is as follows:

$$R^1 = \begin{pmatrix} \{s_1(0.3), s_2(0.7)\} & \{s_0(0.5), s_1(0.5)\} & \{s_{-1}(0.6), s_1(0.4)\} & \{s_1(0.5), s_2(0.4)\} \\ \{s_0(0.8)\} & \{s_{-1}(0.4), s_0(0.6)\} & \{s_1(0.3), s_2(0.4), s_3(0.3)\} & \{s_0(0.6), s_1(0.4)\} \\ \{s_1(0.7), s_2(0.3)\} & \{s_2(1)\} & \{s_0(0.2), s_1(0.8)\} & \{s_{-2}(0.7), s_{-1}(0.3)\} \end{pmatrix}$$

$$R^2 = \begin{pmatrix} \{s_2(0.8), s_3(0.2)\} & \{s_{-1}(0.5), s_0(0.5)\} & \{s_0(0.7), s_2(0.3)\} & \{s_{-1}(0.6), s_0(0.4)\} \\ \{s_1(0.7), s_2(0.3)\} & \{s_2(0.5), s_3(0.4)\} & \{s_{-1}(0.4), s_0(0.6)\} & \{s_2(0.8), s_3(0.1)\} \\ \{s_2(1)\} & \{s_{-1}(0.4), s_1(0.6)\} & \{s_{-1}(0.5), s_0(0.4), s_1(0.1)\} & \{s_2(0.9)\} \end{pmatrix}$$

$$R^3 = \begin{pmatrix} \{s_0(0.3), s_1(0.7)\} & \{s_2(0.8), s_3(0.1)\} & \{s_{-1}(0.5), s_0(0.5)\} & \{s_0(0.4), s_1(0.6)\} \\ \{s_2(0.5), s_3(0.5)\} & \{s_{-1}(0.3), s_0(0.5), s_1(0.2)\} & \{s_2(0.7), s_3(0.3)\} & \{s_{-1}(0.8), s_0(0.2)\} \\ \{s_1(0.9), s_2(0.1)\} & \{s_2(0.5), s_3(0.5)\} & \{s_{-2}(0.6), s_0(0.3)\} & \{s_0(0.75), s_1(0.25)\} \end{pmatrix}$$

Step 1. Determine the objective weight vectors $w^k (k = 1, 2, 3)$ of $D_k (k = 1, 2, 3)$ according to Equation (9):

$$w^1 = (0.6560, 0, 0.3440, 0), \quad w^2 = (0.9282, 0, 0, 0.0718), \quad w^3 = (0.6106, 0.3894, 0, 0)$$

Combining the same subjective weight assigned to each criterion, we obtain the weight vectors $\varpi^k (k = 1, 2, 3)$ of D_k $(k = 1, 2, 3)$ as follows:

$$\begin{aligned} \varpi^1 &= (0.4530, 0.1250, 0.2970, 0.1250), \varpi^2 \\ &= (0.5891, 0.1250, 0.1250, 0.1609), \varpi^3 = (0.4303, 0.3197, 0.1250, 0.1250) \end{aligned}$$

Step 2. Obtain the PLRDMs $\overrightarrow{R}^k (k = 1, 2, 3)$ of D_k $(k = 1, 2, 3)$ by Equation (8):

$$\overrightarrow{R}^1 = \begin{pmatrix} \{s_{0.4373}(0), s_{0.4373}(0.3), s_{0.8910}(0.7)\} & \{s_{-0.0352}(0), s_{-0.0352}(0.5), s_{0.4373}(0.5)\} & \{s_{-0.5334}(0), s_{-0.5334}(0.6), s_{0.4373}(0.4)\} & \{s_{0.4373}(0), s_{0.4373}(0.55), s_{0.8910}(0.45)\} \\ \{s_{-0.0201}(0), s_{-0.0201}(0), s_{-0.0201}(1)\} & \{s_{-0.5161}(0), s_{-0.5161}(0.4), s_{-0.0201}(0.6)\} & \{s_{0.4506}(0.3), s_{0.9028}(0.4), s_{1.3403}(0.3)\} & \{s_{-0.0201}(0), s_{-0.0201}(0.6), s_{0.4506}(0.4)\} \\ \{s_{0.4470}(0), s_{0.4470}(0.7), s_{0.8996}(0.3)\} & \{s_{0.8996}(0), s_{0.8996}(0), s_{0.8996}(1)\} & \{s_{-0.0242}(0), s_{-0.0242}(0.2), s_{0.4470}(0.8)\} & \{s_{-1.0571}(0), s_{-1.0571}(0.7), s_{-0.5208}(0.3)\} \end{pmatrix}$$

$$\overrightarrow{R}^2 = \begin{pmatrix} \{s_{0.9051}(0), s_{0.9051}(0.8), s_{1.3425}(0.2)\} & \{s_{-0.5125}(0), s_{-0.5125}(0.5), s_{-0.0170}(0.5)\} & \{s_{-0.0170}(0), s_{-0.0170}(0.7), s_{0.9051}(0.3)\} & \{s_{-0.5125}(0), s_{-0.5125}(0.6), s_{-0.0170}(0.4)\} \\ \{s_{0.4177}(0), s_{0.4177}(0.7), s_{0.8737}(0.3)\} & \{s_{0.8737}(0), s_{0.8737}(0.55), s_{1.3146}(0.45)\} & \{s_{-0.5590}(0), s_{-0.5590}(0.4), s_{-0.0576}(0.6)\} & \{s_{0.8737}(0), s_{0.8737}(0.85), s_{1.3146}(0.15)\} \\ \{s_{0.8881}(0), s_{0.8881}(0), s_{0.8881}(1)\} & \{s_{-0.5377}(0), s_{-0.5377}(0.4), s_{0.4340}(0.6)\} & \{s_{-0.5377}(0.5), s_{-0.0390}(0.4), s_{0.4340}(0.1)\} & \{s_{0.8881}(0), s_{0.8881}(0), s_{0.8881}(1)\} \end{pmatrix}$$

$$\overrightarrow{R}^3 = \begin{pmatrix} \{s_{-0.0299}(0), s_{-0.0299}(0.3), s_{0.4420}(0.7)\} & \{s_{0.8951}(0), s_{0.8951}(0.85), s_{1.3336}(0.15)\} & \{s_{-0.5274}(0), s_{-0.5274}(0.5), s_{-0.0299}(0.5)\} & \{s_{-0.0299}(0), s_{-0.0299}(0.4), s_{0.4420}(0.6)\} \\ \{s_{0.8873}(0), s_{0.8873}(0.5), s_{1.3267}(0.5)\} & \{s_{-0.5389}(0.3), s_{-0.0400}(0.5), s_{0.4331}(0.2)\} & \{s_{0.8873}(0), s_{0.8873}(0.7), s_{1.3267}(0.3)\} & \{s_{-0.5389}(0), s_{-0.5389}(0.8), s_{-0.0400}(0.2)\} \\ \{s_{0.4456}(0), s_{0.4456}(0.9), s_{0.8984}(0.1)\} & \{s_{0.8984}(0), s_{0.8984}(0.5), s_{1.3365}(0.5)\} & \{s_{-1.0591}(0), s_{-1.0591}(0.65), s_{-0.0258}(0.35)\} & \{s_{-0.0258}(0), s_{-0.0258}(0.75), s_{0.4456}(0.25)\} \end{pmatrix}$$

Step 3. Compute the PLEN matrices $\dot{U}^k (k = 1,2,3)$ of D_k $(k = 1,2,3)$ based on Equation (10):

$$\dot{U}^1 = \begin{pmatrix} \{s_{0.1981}(0), s_{0.1981}(0.3), s_{0.4036}(0.7)\} & \{s_{-0.0044}(0), s_{-0.0044}(0.5), s_{0.0547}(0.5)\} & \{s_{-0.1584}(0), s_{-0.1584}(0.6), s_{0.1299}(0.4)\} & \{s_{0.0547}(0), s_{0.0547}(0.55), s_{0.1114}(0.45)\} \\ \{s_{-0.0091}(0), s_{-0.0091}(0), s_{-0.0091}(1)\} & \{s_{-0.0645}(0), s_{-0.0645}(0.4), s_{-0.0025}(0.6)\} & \{s_{0.1338}(0.3), s_{0.2681}(0.4), s_{0.3981}(0.3)\} & \{s_{-0.0025}(0), s_{-0.0025}(0.6), s_{0.0563}(0.4)\} \\ \{s_{0.2025}(0), s_{0.2025}(0.7), s_{0.4075}(0.3)\} & \{s_{0.1125}(0), s_{0.1125}(0), s_{0.1125}(1)\} & \{s_{-0.0072}(0), s_{-0.0072}(0.2), s_{0.1328}(0.8)\} & \{s_{-0.1321}(0), s_{-0.1321}(0.7), s_{-0.0651}(0.3)\} \end{pmatrix}$$

$$\dot{U}^2 = \begin{pmatrix} \{s_{0.5332}(0), s_{0.5332}(0.8), s_{0.7908}(0.2)\} & \{s_{-0.0641}(0), s_{-0.0641}(0.5), s_{-0.0021}(0.5)\} & \{s_{-0.0021}(0), s_{-0.0021}(0.7), s_{0.1131}(0.3)\} & \{s_{-0.0825}(0), s_{-0.0825}(0.6), s_{-0.0027}(0.4)\} \\ \{s_{0.2460}(0), s_{0.2460}(0.7), s_{0.5147}(0.3)\} & \{s_{0.1092}(0), s_{0.1092}(0.55), s_{0.1643}(0.45)\} & \{s_{-0.0699}(0), s_{-0.0699}(0.4), s_{-0.0072}(0.6)\} & \{s_{0.1406}(0), s_{0.1406}(0.85), s_{0.2115}(0.15)\} \\ \{s_{0.5232}(0), s_{0.5232}(0), s_{0.5232}(1)\} & \{s_{-0.0672}(0), s_{-0.0672}(0.4), s_{0.0543}(0.6)\} & \{s_{-0.0672}(0.5), s_{-0.0049}(0.4), s_{0.0543}(0.1)\} & \{s_{0.1429}(0), s_{0.1429}(0), s_{0.1429}(1)\} \end{pmatrix}$$

$$\dot{U}^3 = \begin{pmatrix} \{s_{-0.0129}(0), s_{-0.0129}(0.3), s_{0.1902}(0.7)\} & \{s_{0.2862}(0), s_{0.2862}(0.85), s_{0.4264}(0.15)\} & \{s_{-0.0659}(0), s_{-0.0659}(0.5), s_{-0.0037}(0.5)\} & \{s_{-0.0037}(0), s_{-0.0037}(0.4), s_{0.0552}(0.6)\} \\ \{s_{0.3818}(0), s_{0.3818}(0.5), s_{0.5709}(0.5)\} & \{s_{-0.1723}(0.3), s_{-0.0128}(0.5), s_{0.1385}(0.2)\} & \{s_{0.1109}(0), s_{0.1109}(0.7), s_{0.1658}(0.3)\} & \{s_{-0.0674}(0), s_{-0.0674}(0.8), s_{-0.0050}(0.2)\} \\ \{s_{0.1918}(0), s_{0.1918}(0.9), s_{0.3866}(0.1)\} & \{s_{0.2872}(0), s_{0.2872}(0.5), s_{0.4273}(0.5)\} & \{s_{-0.1324}(0), s_{-0.1324}(0.65), s_{-0.0032}(0.35)\} & \{s_{-0.0032}(0), s_{-0.0032}(0.75), s_{0.0557}(0.25)\} \end{pmatrix}$$

Step 4. Obtain the quality matrices $\dot{Q}^k (k = 1,2,3)$ of PLPs based on Equation (11):

$$\dot{Q}^1 = \begin{pmatrix} 0.8257 & 0.9743 & 0.8576 & 0.8674 \\ 0.8910 & 0.8243 & 0.7757 & 0.9576 \\ 0.8924 & 0.7757 & 0.9757 & 0.6076 \end{pmatrix}$$

$$\dot{Q}^2 = \begin{pmatrix} 0.7861 & 0.7639 & 0.9472 & 0.7472 \\ 0.9361 & 0.7444 & 0.7806 & 0.7944 \\ 0.8194 & 0.8806 & 0.7806 & 0.8194 \end{pmatrix}$$

$$\dot{Q}^3 = \begin{pmatrix} 0.9861 & 0.7722 & 0.7861 & 0.9694 \\ 0.7139 & 0.8528 & 0.7472 & 0.7361 \\ 0.9472 & 0.7139 & 0.6528 & 0.9111 \end{pmatrix}$$

Step 5. According to Equation (12), calculate the PLEX matrices $\dot{X}^k (k = 1,2,3)$ of D_k $(k = 1,2,3)$ as follows:

$$\dot{X}^1 = \begin{pmatrix} \{s_{0.1636}(0), s_{0.1636}(0.3), s_{0.3333}(0.7)\} & \{s_{-0.0043}(0), s_{-0.0043}(0.5), s_{0.0533}(0.5)\} & \{s_{-0.1359}(0), s_{-0.1359}(0.6), s_{0.1114}(0.4)\} & \{s_{0.0474}(0), s_{0.0474}(0.55), s_{0.0966}(0.45)\} \\ \{s_{-0.0081}(0), s_{-0.0081}(0), s_{-0.0081}(1)\} & \{s_{-0.0532}(0), s_{-0.0532}(0.4), s_{-0.0021}(0.6)\} & \{s_{0.1038}(0.3), s_{0.2080}(0.4), s_{0.3088}(0.3)\} & \{s_{-0.0024}(0), s_{-0.0024}(0.6), s_{0.0539}(0.4)\} \\ \{s_{0.1807}(0), s_{0.1807}(0.7), s_{0.3636}(0.3)\} & \{s_{0.0872}(0), s_{0.0872}(0), s_{0.0872}(1)\} & \{s_{-0.0070}(0), s_{-0.0070}(0.2), s_{0.1295}(0.8)\} & \{s_{-0.0803}(0), s_{-0.0803}(0.7), s_{-0.0396}(0.3)\} \end{pmatrix}$$

$$\dot{X}^2 = \begin{pmatrix} \{s_{0.4192}(0), s_{0.4192}(0.8), s_{0.6217}(0.2)\} & \{s_{-0.0489}(0), s_{-0.0489}(0.5), s_{-0.0016}(0.5)\} & \{s_{-0.0020}(0), s_{-0.0020}(0.7), s_{0.1072}(0.3)\} & \{s_{-0.0616}(0), s_{-0.0616}(0.6), s_{-0.0020}(0.4)\} \\ \{s_{0.2323}(0), s_{0.2323}(0.7), s_{0.4818}(0.3)\} & \{s_{0.0813}(0), s_{0.0813}(0.55), s_{0.1223}(0.45)\} & \{s_{-0.0545}(0), s_{-0.0545}(0.4), s_{-0.0056}(0.6)\} & \{s_{0.1117}(0), s_{0.1117}(0.85), s_{0.1680}(0.15)\} \\ \{s_{0.4287}(0), s_{0.4287}(0), s_{0.4287}(1)\} & \{s_{-0.0592}(0), s_{-0.0592}(0.4), s_{0.0478}(0.6)\} & \{s_{-0.0525}(0.5), s_{-0.0038}(0.4), s_{0.0423}(0.1)\} & \{s_{0.1171}(0), s_{0.1171}(0), s_{0.1171}(1)\} \end{pmatrix}$$

$$\dot{X}^3 = \begin{pmatrix} \{s_{-0.0127}(0), s_{-0.0127}(0.3), s_{0.1875}(0.7)\} & \{s_{0.2210}(0), s_{0.2210}(0.85), s_{0.3292}(0.15)\} & \{s_{-0.0518}(0), s_{-0.0518}(0.5), s_{-0.0029}(0.5)\} & \{s_{-0.0036}(0), s_{-0.0036}(0.4), s_{0.0536}(0.6)\} \\ \{s_{0.2726}(0), s_{0.2726}(0.5), s_{0.4075}(0.5)\} & \{s_{-0.1469}(0.3), s_{-0.0109}(0.5), s_{0.1181}(0.2)\} & \{s_{0.0829}(0), s_{0.0829}(0.7), s_{0.1239}(0.3)\} & \{s_{-0.0496}(0), s_{-0.0496}(0.8), s_{-0.0037}(0.2)\} \\ \{s_{0.1816}(0), s_{0.1816}(0.9), s_{0.3662}(0.1)\} & \{s_{0.2050}(0), s_{0.2050}(0.5), s_{0.3050}(0.5)\} & \{s_{-0.0864}(0), s_{-0.0864}(0.65), s_{-0.0021}(0.35)\} & \{s_{-0.0029}(0), s_{-0.0029}(0.75), s_{0.0508}(0.25)\} \end{pmatrix}$$

Step 6. Compute the averaging PLEN $\dot{U}_{i\times k}$ and the averaging PLEX $\dot{X}_{i\times k}$ of A_i with regard to D_k :

$$\dot{U}_{i\times k} = \begin{pmatrix} 0.4042 & 0.5335 & 0.4333 \\ 0.2514 & 0.5796 & 0.5184 \\ 0.3692 & 0.6416 & 0.4928 \end{pmatrix}$$

$$\dot{X}_{i\times k} = \begin{pmatrix} 0.3394 & 0.4273 & 0.3680 \\ 0.1965 & 0.5005 & 0.3689 \\ 0.3570 & 0.5273 & 0.4087 \end{pmatrix}$$

Step 7. Compute the PLEN indicators $\dot{U}_i (i = 1, 2, 3)$ and the PLEX indicators $\dot{X}_i (i = 1, 2, 3)$:

$$\dot{U}_1 = 0.4570, \dot{U}_2 = 0.4498, \dot{U}_3 = 0.5012$$

$$\dot{X}_1 = 0.0787, \dot{X}_2 = 0.0945, \dot{X}_3 = 0.0702$$

Step 8. Obtain the entropy indicators $\dot{S}_1 = 0.3783, \dot{S}_2 = 0.3553, \dot{S}_3 = 0.4310$. The smaller the entropy indicator, the better the alternative. Thus, the ranking is: $A_2 \succ A_1 \succ A_3$. The alternative A_2 is the best option. It indicates the flood is hazardous in Tongnan County and the government and public should pay more attention to potential security threat.

5. Decision support

In this section, the self-comparison is conducted to verify the effectiveness of the objective weight obtained by the water-filling algorithm and regret theory in the probabilistic linguistic thermodynamic method. A series of simulation experiments are conducted to verify the influence of the water-filling algorithm and regret theory.

The proposed method, the proposed method without regret theory and the proposed method without water-filling algorithm are denoted by Method 1, Method 2 and Method 3, respectively. First, we acquire the ranking results of 1000 MCDM problems randomly of three experts with m alternatives and n criteria by the three proposed methods. Then we record the times of same optimal and worst alternatives calculated by the three proposed methods and any two decision methods. The ratios of same optimal and worst alternatives of the three methods and any two methods are shown in Figure 3.

Figure 3 indicates that: (1) The ranking result of Method 1 is obviously similar to that of Method 2. Therefore, regret theory only slightly adjusts the PLDM to adapt the need of individual's regret-aversion. Regret theory would not greatly affect the

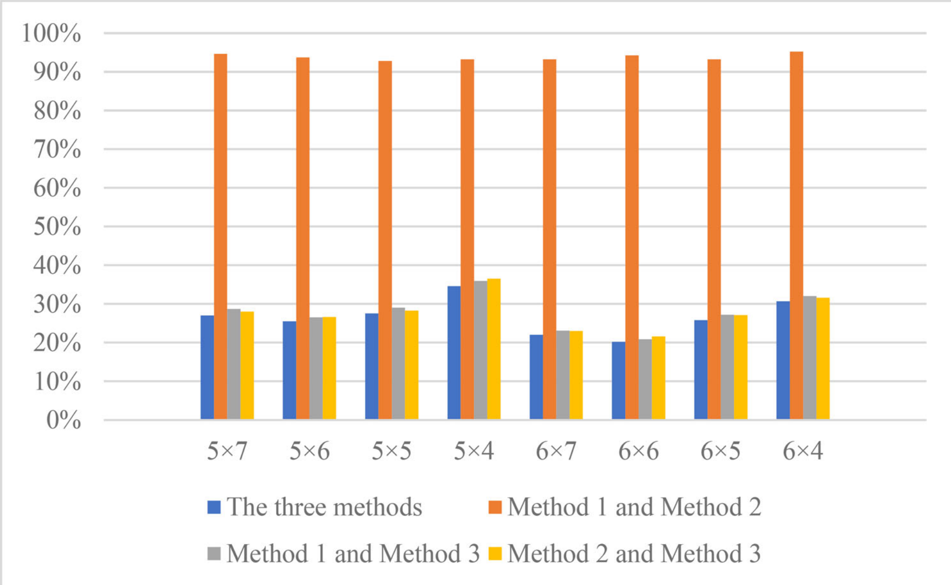

Figure 3. The ratios of same optimal and worst alternatives. Source: Authors' calculation.

ranking results of the proposed method. (2) There is difference between ranking results of Method 1 and Method 3. (3) Similarly, there is difference between ranking results of Method 2 and Method 3. The leading factor is that the weight of criterion changes caused by the water-filling algorithm in the proposed method. To further compare the significance differences among the three proposed methods, we conduce the following simulation experiments by the nonparametric tests.

The ranking results of the three methods are recorded for 1000 MCDM problems randomly of three experts with m alternatives and n criteria. The ranking results of the three methods can be converted into three sample sequences. Since the three sequences might not be normal distribution, the non-parametric test: Wilcoxon signed-rank test is introduced to compare the ranking results of the three methods by pairwise comparison. The results of Wilcoxon signed-rank test among the three methods with m alternatives and n criteria are shown in Table 1. The results of Wilcoxon signed-rank test are obtained by SPSS and the confidence level is set as 0.05 by convention in statistics.

As shown in Table 1, the concomitant probabilities of all the tests are greater than 0.05 indicating that the ranking results have no significant differences with the above three methods. Therefore, the probabilistic linguistic thermodynamic method is stable and reasonable. The water-filling algorithm only slightly changes the weight to extract useful information from the uncertain decision-making information. Regret theory and the objective weight obtained by the water-filling algorithm only flexibly adjust the ranking results according to the practical needs.

6. Comparative analysis

The comparison analysis is composed of two parts: (1) Compare the ranking results of the case in Pan et al. (2021) obtained by the probabilistic linguistic thermodynamic

Table 1. Wilcoxon signed-rank test results among the three methods with *m* alternatives and *n* criteria.

	$m \times n$	The proposed method without regret theory	The proposed method without water-filling algorithm
The proposed method	6×4	Sig. .950	Sig. .992
The proposed method without regret theory			Sig. .988
The proposed method	6×5	Sig. .999	Sig. .939
The proposed method without regret theory			Sig. .917
The proposed method	6×6	Sig. .894	Sig. .938
The proposed method without regret theory			Sig. .912
The proposed method	6×7	Sig. .987	Sig. .981
The proposed method without regret theory			Sig. .993
The proposed method	5×4	Sig. .976	Sig. .900
The proposed method without regret theory			Sig. .942
The proposed method	5×5	Sig. .991	Sig. .867
The proposed method without regret theory			Sig. .960
The proposed method	5×6	Sig. .997	Sig. .966
The proposed method without regret theory			Sig. .988
The proposed method	5×7	Sig. .998	Sig. .956
The proposed method without regret theory			Sig. .978

Source: Authors' calculation.

method, the PL-TOPSIS method (Pang et al., 2016), the PL-QUALIFLEX method (Tian et al., 2019), the PL-DEA method (Pan et al., 2021) and the PL-TODIM method (Wei & Wu, 2019) and analyse the comparison results. (2) According to a series of simulation experiments and non-parametric tests, further compare the probabilistic linguistic thermodynamic method, the PL-TOPSIS method (Pang et al., 2016) and the PL-QUALIFLEX method (Tian et al., 2019) and present the comparative conclusions.

Part 1. The case result comparisons with the probabilistic linguistic thermodynamic method, PL-TOPSIS method, PL-QUALIFLEX method, PL-DEA method and PL-TODIM method

We calculate the ranking results of five probabilistic linguistic decision-making methods by the decision-making data of Pan et al. (2021), and then compare the decision making results of the five decision making methods shown in Table 2.

As shown in Table 2, the ranking results of the PL-TOPSIS method and the PL-QUALIFLEX method are exactly the same. The optimal alternative obtained by the above two methods is the alternative DMU_3. Although the optimal alternative obtained by the probabilistic linguistic thermodynamic method, the PL-DEA method and the PL-TODIM method is the same, the specific ranking results are different. The worst alternative of the probabilistic linguistic thermodynamic method and the PL-TODIM method is DMU_4.

Table 2. The ranking results with the five decision-making methods.

	The ranking result
The proposed method	$DMU_1 \prec DMU_5 \prec DMU_2 \prec DMU_3 \prec DMU_6 \prec DMU_4$
PL-TOPSIS	$DMU_3 \prec DMU_1 \prec DMU_5 \prec DMU_2 \prec DMU_4 \prec DMU_6$
PL-QUALIFLEX	$DMU_3 \prec DMU_1 \prec DMU_5 \prec DMU_2 \prec DMU_4 \prec DMU_6$
PL-DEA	$DMU_1 \prec DMU_2 \prec DMU_5 \prec DMU_3 \prec DMU_4 \prec DMU_6$
PL-TODIM	$DMU_1 \prec DMU_3 \prec DMU_5 \prec DMU_2 \prec DMU_6 \prec DMU_4$

Source: Authors' calculation.

Due to the complexity of the decision-making environment and the limitations of human thinking, it is difficult for decision makers to provide precise evaluations for alternatives. How to measure the quantity and quality of the uncertain decision-making information is a critical factor that affects decision-making results, especially when the evaluation information given by decision makers differs greatly. The thermodynamical indicators are introduced to the probabilistic linguistic environment to extract the main features of the uncertain information. Besides, the proposed probabilistic linguistic thermodynamic method not only considers decision makers' regret-aversion, but also assigns the criterion weight adaptively by the water-filling algorithm. Although the PL-TODIM method also takes the bounded rationality of decision makers into account, the proposed method introduces the water-filling algorithm to determine the objective weight adaptively. The water-filling algorithm assigns the criterion weight adaptively, aiming to maximise the total ability of criteria. The above reasons explain the ranking difference between the proposed method and other probabilistic linguistic decision-making methods from the methodological principle.

Part 2. The simulation result comparisons with the probabilistic linguistic thermodynamic method, the PL-TOPSIS method and the PL-QUALIFLEX method

In order to further compare the ranking results of the probabilistic linguistic thermodynamic method, the PL-TOPSIS method and the PL-QUALIFLEX method, Friedman test is utilised to examine the significant differences among the three methods by SPSS. Friedman test and Wilcoxon signed-rank test are nonparametric test methods that test the distribution of samples whether there is a significant difference. Different from Wilcoxon signed-rank test, Friedman test can deal with multiple related samples. Since the PL-TOPSIS method and the PL-QUALIFLEX method are not group decision-making methods, the ranking results of three sequences of the three methods are recorded for 1000 random MCDM problems of an expert with m alternatives and n criteria. Each criterion is endowed the same subjective weight and the confidence level is taken as 0.05. Likewise, the ranking results are converted into three related sequence samples. The concomitant probabilities of Friedman test among the proposed method, the PL-TOPSIS method and the PL-QUALIFLEX method are presented in Table 3.

Table 3 shows that the ranking results of the proposed method, the PL-TOPSIS method and the PL-QUALIFLEX method have significant differences, that is, the concomitant probability is less than the confidence level 0.05, except the case that

Table 3. Friedman test results among the three methods with m alternatives and n criteria.

$m \times n$	4×4	5×4	5×5	6×4	6×5	6×6
Sig.	.816	0	0	0	0	0

Source: Authors' calculation.

Table 4. Wilcoxon signed-rank test results among the three methods with m alternatives and n criteria.

	$m \times n$	PL-TOPSIS	PL-QUALIFLEX
The proposed method	4×4	Sig. .912	Sig. .894
PL-TOPSIS			Sig. .993
The proposed method	5×4	Sig. .000	Sig. .000
PL-TOPSIS			Sig. .985
The proposed method	5×5	Sig. .000	Sig. .000
PL-TOPSIS			Sig. .998
The proposed method	6×4	Sig. .000	Sig. .000
PL-TOPSIS			Sig. .930
The proposed method	6×5	Sig. .000	Sig. .000
PL-TOPSIS			Sig. .968
The proposed method	6×6	Sig. .000	Sig. .000
PL-TOPSIS			Sig. .994

Source: Authors' calculation.

$m = 4, n = 4$. The main reason is that the number of alternatives is too few when $m = 4, n = 4$. To further study the significance difference among the three methods, we conduct pair-wise comparisons by Wilcoxon signed-rank test with same ranking samples in Table 3. The concomitant probabilities among the three methods are presented in Table 4.

According to Table 4: (1) The concomitant probabilities of Wilcoxon signed-rank test are greater than 0.05 when $m = 4, n = 4$, which means that the ranking results have no significant differences with the above three methods. (2) The concomitant probabilities of Wilcoxon signed-rank test between the proposed method and the PL-TOPSIS method are less than 0.05 except when $m = 4, n = 4$, which means that their ranking results vary enormously. (3) There is significant difference between the proposed method and the PL-QUALIFLEX method except $m = 4, n = 4$. (4) There is no significant difference between the PL-TOPSIS method and the PL-QUALIFLEX method. Since the number of alternatives is too few, we neglect the case of $m = 4, n = 4$.

The ranking results of the PL-TOPSIS method and the PL-QUALIFLEX method are quite similar, while the proposed probabilistic linguistic thermodynamic method is significantly different from them. The probabilistic linguistic thermodynamic method is a new decision method based on the water-filling algorithm and regret theory. It applies thermodynamics combining with energy, exergy and entropy indicators to aggregate the probabilistic linguistic information. Hence, there are significance differences in the ranking results of the proposed method and other two methods. Although the ranking results of the PL-TOPSIS method and the PL-QUALIFLEX method do not differ significantly, the time complexity of the PL-TOPSIS method is lower than the PL-QUALIFLEX method. In addition, to avoid complex calculations, the QUALIFLEX method should be applied to the decision-making problems where the number of criteria is much more than the number of alternatives (Chen et al., 2013).

7. Conclusions

Thermodynamics is a subject that studies the laws of equilibrium system of energy and material, as well as the interaction between the system and external environment when the state changes. In the paper, the probabilistic linguistic thermodynamic method is proposed based on the water-filling algorithm and regret theory. Thermodynamics is introduced to the decision-making method based on the proposed thermodynamical indicators: energy, exergy and entropy under the probabilistic linguistic circumstance. The PLDM can be converted into the PLRDM which portrays decision makers' regret-aversion. The hybrid weight of criterion combines the objective weight calculated by the water-filling algorithm and decision makers' subjective weight. The self-comparison and comparison with other probabilistic linguistic methods verify the effectiveness of the proposed method by a series of simulation experiments and non-parametric tests.

However, the paper still exists some drawbacks: (1) It is necessary to improve the aggregating process of the PLTSs in the proposed method, since the obtained virtual linguistic terms loss original semantic feature. Although the obtained virtual linguistic terms do not have any effect on the ranking of alternatives. (2) How to adjust the proportion of subjective and objective weight is an interesting topic which should be further investigated according to large-scale scenario simulations and actual situation analysis.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The work was partly supported by the National Natural Science Foundation of China (No. 71771155).

ORCID

Zeshui Xu http://orcid.org/0000-0003-3547-2908

References

Akram, M., Kahraman, C., & Zahid, K. (2021). Group decision-making based on complex spherical fuzzy VIKOR approach. *Knowledge-Based Systems*, *216*, 106793. https://doi.org/10.1016/j.knosys.2021.106793

Bell, D. E. (1982). Regret in decision-making under uncertainty. *Operations Research*, *30*(5), 961–981. https://doi.org/10.1287/opre.30.5.961

Chanamool, N., & Naenna, T. (2016). Fuzzy FMEA application to improve decision-making process in an emergency department. *Applied Soft Computing*, *43*, 441–453. https://doi.org/10.1016/j.asoc.2016.01.007

Charles, K., & Herbert, K. (1990). *Thermal Physics*. W. H. Freeman.

Chen, P. Y. (2021). Effects of the entropy weight on TOPSIS. *Expert Systems with Applications*, *168*, 114186. https://doi.org/10.1016/j.eswa.2020.114186
Chen, S. X., Wang, J. Q., & Wang, T. L. (2019). Cloud-based ERP system selection based on extended probabilistic linguistic MULTIMOORA method and Choquet integral operator. *Computational and Applied Mathematics*, *38*(2), 88. https://doi.org/10.1007/s40314-019-0839-z
Chen, T. Y., Chang, C. H., & Lu, J. F. R. (2013). The extended QUALIFLEX method for multiple criteria decision analysis based on interval type-2 fuzzy sets and applications to medical decision making. *European Journal of Operational Research*, *226*(3), 615–625. https://doi.org/10.1016/j.ejor.2012.11.038
Dincer, I., & Cengel, Y. A. (2001). Energy, entropy and exergy concepts and their roles in thermal engineering. *Entropy*, *3*(3), 116–149. https://doi.org/10.3390/e3030116
Liao, H. C., Wu, D., Huang, Y. L., Ren, P. J., Xu, Z. S., & Verma, M. (2018). Green logistic provider selection with a hesitant fuzzy linguistic thermodynamic method integrating cumulative prospect theory and PROMETHEE. *Sustainability*, *10*(4), 1291. https://doi.org/10.3390/su10041291
Loomes, G., & Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *The Economic Journal*, *92*(368), 805–824. https://doi.org/10.2307/2232669
Pan, L., Xu, Z. S., & Ren, P. J. (2021). DEA cross-efficiency framework for efficiency evaluation with probabilistic linguistic term sets. *Journal of the Operational Research Society*, *72*(5), 1191–1206. https://doi.org/10.1080/01605682.2020.1848360
Pang, Q., Wang, H., & Xu, Z. S. (2016). Probabilistic linguistic term sets in multi-attribute group decision making. *Information Sciences*, *369*, 128–143. https://doi.org/10.1016/j.ins.2016.06.021
People.com.cn. (2020). *Fresh floods hit China's Chongqing*. http://english.people.com.cn/n3/2020/0727/c90000-9715259.html
Prathap, G. (2011). The Energy-Exergy-Entropy (or EEE) sequences in bibliometric assessment. *Scientometrics*, *87*(3), 515–524. https://doi.org/10.1007/s11192-011-0367-0
Ren, P. J., Xu, Z. S., & Hao, Z. N. (2017). Hesitant fuzzy thermodynamic method for emergency decision making based on prospect theory. *IEEE Transactions on Cybernetics*, *47*(9), 2531–2543. https://doi.org/10.1109/TCYB.2016.2638498
Ren, P. J., Xu, Z. S., Liao, H. C., & Zeng, X. J. (2017). A thermodynamic method of intuitionistic fuzzy MCDM to assist the hierarchical medical system in China. *Information Sciences*, *420*, 490–504. https://doi.org/10.1016/j.ins.2017.08.070
Rodríguez, R. M., Martínez, L., & Herrera, F. (2012). Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems*, *20*(1), 109–119. https://doi.org/10.1109/TFUZZ.2011.2170076
Tian, G. D., Hao, N. N., Zhou, M. C., Pedrycz, W., Zhang, C. Y., Ma, F. W., & Li, Z. W. (2020). Fuzzy grey Choquet integral for evaluation of multicriteria decision making problems with interactive and qualitative indices. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *51*(3), 1–1868. https://doi.org/10.1109/TSMC.2019.2906635
Tian, G. D., Zhang, H. H., Feng, Y. X., Jia, H. F., Zhang, C. Y., Jiang, Z. G., Li, Z. W., & Li, P. G. (2017). Operation patterns analysis of automotive components remanufacturing industry development in China. *Journal of Cleaner Production*, *164*, 1363–1375. https://doi.org/10.1016/j.jclepro.2017.07.028
Tian, G. D., Zhang, H. H., Feng, Y. X., Wang, D. Q., Peng, Y., & Jia, H. F. (2018). Green decoration materials selection under interior environment characteristics: A grey-correlation based hybrid MCDM method. *Renewable and Sustainable Energy Reviews*, *81*, 682–692. https://doi.org/10.1016/j.rser.2017.08.050
Tian, X. L., Xu, Z. S., Wang, X. X., Gu, J., & Alsaadi, F. E. (2019). Decision models to find a promising start-up firm with QUALIFLEX under probabilistic linguistic circumstance. *International Journal of Information Technology & Decision Making*, *18*(4), 1379–1402. https://doi.org/10.1142/S0219622019500226

Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems*, *25*(6), 529–539. https://doi.org/10.1002/int.20418

Verma, M., & Rajasankar, J. (2017). A thermodynamical approach towards group multi-criteria decision making (GMCDM) and its application to human resource selection. *Applied Soft Computing*, *52*, 323–332. https://doi.org/10.1016/j.asoc.2016.10.033

Wang, H., Xu, Z. S., & Zeng, X. J. (2018). Modeling complex linguistic expressions in qualitative decision making: An overview. *Knowledge-Based Systems*, *144*, 174–187. https://doi.org/10.1016/j.knosys.2017.12.030

Wang, H. D., Pan, X. H., Yan, J., Yao, J. L., & He, S. F. (2020). A projection-based regret theory method for multi-attribute decision making under interval type-2 fuzzy sets environment. *Information Sciences*, *512*, 108–122. https://doi.org/10.1016/j.ins.2019.09.041

Wang, M. W., Liang, D. C., Xu, Z. S., & Ye, D. J. (2020). The evaluation of mobile health apps: A psychological perception-based probabilistic linguistic belief thermodynamic multiple attribute decision making method. *Journal of the Operational Research Society*, *72*(11), 2596–2610. https://doi.org/10.1080/01605682.2020.1801361

Wang, W. J., Tian, G. D., Zhang, T. Z., Jabarullah, N. H., Li, F. Y., Fathollahi-Fard, A. M., Wang, D. Q., & Li, Z. W. (2021). Scheme selection of design for disassembly (DFD) based on sustainability: A novel hybrid of interval 2-tuple linguistic intuitionistic fuzzy numbers and regret theory. *Journal of Cleaner Production*, *281*, 124724. https://doi.org/10.1016/j.jclepro.2020.124724

Wang, W. Z., Liu, X. W., Qin, Y., & Fu, Y. (2018). A risk evaluation and prioritization method for FMEA with prospect theory and Choquet integral. *Safety Science*, *110*, 152–163. https://doi.org/10.1016/j.ssci.2018.08.009

Wei, C., & Wu, J. (2019). TODIM method for probabilistic linguistic multiple attribute group decision making based on the similarity measures and entropy. *Journal of Intelligent & Fuzzy Systems*, *37*(5), 7025–7037. https://doi.org/10.3233/JIFS-191164

Wikipedia. (2021a). *First law of thermodynamics*. https://en.wikipedia.org/wiki/First_law_of_thermodynamics

Wikipedia. (2021b). *Second law of thermodynamics*. https://en.wikipedia.org/wiki/Second_law_of_thermodynamics

Wu, X. L., & Liao, H. C. (2019). A consensus-based probabilistic linguistic gained and lost dominance score method. *European Journal of Operational Research*, *272*(3), 1017–1027. https://doi.org/10.1016/j.ejor.2018.07.044

www.news.cn. (2020). *Fighting the fifth flood of Yangtze*. https://www.thepaper.cn/newsDetail_forward_8793266

Xia, M. M. (2018). A hesitant fuzzy linguistic multi-criteria decision-making approach based on regret theory. *International Journal of Fuzzy Systems*, *20*(7), 2135–2143. https://doi.org/10.1007/s40815-018-0502-7

Xue, W. T., Xu, Z. S., Mi, X. M., & Ren, Z. L. (2021). Dynamic reference point method with probabilistic linguistic information based on the regret theory for public health emergency decision-making. *Economic Research-Ekonomska Istraživanja*, *34*(1), 3355–3381. https://doi.org/10.1080/1331677X.2021.1875254

Xue, W. T., Xu, Z. S., Zhang, X. L., & Tian, X. L. (2018). Pythagorean fuzzy LINMAP method based on the entropy theory for railway project investment decision making. *International Journal of Intelligent Systems*, *33*(1), 93–125. https://doi.org/10.1002/int.21941

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences*, *8*(3), 199–249. https://doi.org/10.1016/0020-0255(75)90036-5

Zhang, C., Tian, G. D., Fathollahi-Fard, A. M., Wang, W. J., Wu, P., & Li, Z. W. (2022). Interval-valued intuitionistic uncertain linguistic cloud Petri net and its application to risk assessment for subway fire accident. *IEEE Transactions on Automation Science and Engineering*, *19*(1), 163–177. https://doi.org/10.1109/TASE.2020.3014907

Zhang, S. T., Zhu, J. J., Liu, X. D., & Chen, Y. (2016). Regret theory method-based group decision-making with multidimensional preference and incomplete weight information. *Information Fusion*, *31*, 1–13. https://doi.org/10.1016/j.inffus.2015.12.001

Zhang, X. L. (2017). A closeness index-based TODIM method for hesitant qualitative group decision making. *Informatica*, *28*(3), 565–581. https://doi.org/10.15388/Informatica.2017.145

Zhao, H., Yan, A. J., & Wang, P. (2015). Feature reduction method based on threshold optimization for case-based reasoning classifier. *Control Theory and Applications*, *32*(4), 533–539. https://doi.org/10.7641/CTA.2015.40541

Zheng, J., Wang, Y. M., Zhang, K., & Liang, J. (2020). A dynamic emergency decision-making method based on group decision making with uncertainty information. *International Journal of Disaster Risk Science*, *11*(5), 667–679. https://doi.org/10.1007/s13753-020-00308-4

Zhou, L., Wu, X. H., Xu, Z. S., & Fujita, H. (2018). Emergency decision making for natural disasters: An overview. *International Journal of Disaster Risk Reduction*, *27*, 567–576. https://doi.org/10.1016/j.ijdrr.2017.09.037