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SPHERICALLY-SYMMETRIC NONSINGULAR MODELS WITH VARIABLE Λ TERM IN RELATIVISTIC COSMOLOGY

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We discuss a class of nonsingular spherically-symmetric cosmological models with radial heat flux and variable cosmological term $\Lambda(t)$. Three different exact solutions of the Einstein's field equations are obtained for both perfect fluid and fluid with bulk viscosity. It turns out that the cosmological term $\Lambda(t)$ is a decreasing function of time, what is consistent with recent observations of type Ia supernovae.

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1. Introduction

One of the main issues to be addressed by quantum gravity is the singularity problem of general relativity. Due to the powerful singularity theorem [1, 2], it was widely believed that cosmological models must have initial singularity. However, in 1990 Senovilla [3] obtained the first singularity-free cosmological perfect-fluid (with a realistic equation of state $3p = \rho$) solution of the Einstein equation and since then the possibility of constructing regular cosmologies was renewed. The interest for regular cosmologies had stifled for nearly 30 years due to the powerful singularity theorems, which seemed to preclude such space-times under very general requirements, such as chronology protecting, energy and generic conditions. The open way to regular cosmologies was found in the violation of some technical premises

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of the theorems. The remarkable feature is the absence of an initial singularity, the curvature and matter invariants being regular and smooth everywhere. This corresponds to a cylindrically-symmetric space-time filled with an isotropic perfect radiation fluid. For instance, it was shown by Chinea et al. [4] that the Senovilla space-time did not possess a compact achronal set without edge and could not have closed trapped surfaces. However, the first results were not encouraging. The extension of the Senovilla solution to a family of space-times left the set of regular models limited to a zero-measure subset surrounded by space-time with Ricci and Weyl curvature singularities [5]. A thorough discussion of models of such type can be found in Senovilla [6]. This family is shown to be included in a wider class of separable cosmological models, which comprises FLRW universe [7]. Other properties of these solutions, such as their inflationary behaviour, generalized Hubble law and the feasibility of constructing a realistic non-singular cosmological model are studied therein.

A large family of non-singular cosmological models and generalization thereof have been considered but they all are cylindrically symmetric [8 – 10]. However, for practical cosmology the spherical symmetry is more appropriate. It is, therefore, pertinent to seek spherically-symmetric nonsingular models. The first model of this kind was obtained by Dadhich [11] with an imperfect fluid with a heat flux. The model satisfied all energy conditions and had no singularity of any kind. Dadhich *et al.* [12] also obtained a non-singular model with zero radiation flux. These models are both inhomogeneous and anisotropic and have a typical behaviour beginning with two densities at $t \to -\infty$, contracting to high density at $t = 0$ and then again expanding to low density at $t \to \infty$. An interesting feature of the space-time metric of these models is that it contains an arbitrary function of time which can be constrained to comply with the demand of non-singularity and energy conditions. Dadhich and Raychaudhuri [13] later showed how a particle choice of this function leads to a model of an ever existing spherically-symmetric universe, oscillating between two regular states, which involves blue shifts as in the quasi steady state cosmological model of Hoyle, Burbige and Narlikar [14] and is filled with a non-adiabatic fluid with anisotropic pressure and radial heat flux. These observations led to the search of spherically-symmetric singularity-free cosmological models with a perfect fluid source characterized by isotropic pressure. Due to this search, Tikekar [15] constructed two spherically-symmetric singularity-free relativistic cosmological models, describing universes filled with non-adiabatic perfect fluid, accompanied by heat flow along radial direction. Recently, many researchers [16 – 20] have studied non-singular cosmological models in different context. From a purely theoretical point of view, the investigation of nonsingular cosmological models gives invaluable insight into the space-time structure, the inherent nonlinear character of gravity and its interaction with matter fields. As a by-product, it also deepens our understanding of the singularity theorem, in particular the assumptions lying in its base [7].

One of the most important and outstanding problems in cosmology is the problem of the cosmological constant. The recent observations indicate that

$\Lambda \sim 10^{-55} cm^{-2}$ while the particle-physics prediction for Λ is greater than this value by a factor of the order of 10^{120}. This discrepancy is known as cosmological-constant problem. Some of the recent discussions on the cosmological-constant "problem" and consequence on cosmology with a time-varying cosmological constant are investigated by Ratra and Peebles [21], Dolgov [20 – 24] and Sahni and Starobinsky [25]. It is remarkable to mention here that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant". However, in the presence of interaction with matter or radiation, a solution of Einstein equations and assumed equation of covariant conservation of stress-energy with time-varying Λ can be found. For these solutions, conservation of energy requires any decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Recent observations by Perlmutter et al. [26] and Riess et al. [27] strongly favour a significant and positive value of Λ. Their finding arises from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \leq z \leq 0.83$ and these suggest Friedmann models with negative pressure matter such as a cosmological constant (Λ), domain walls or cosmic strings (Vilenkin [28], Garnavich et al. [29]). Recently, Carmeli and Kuzmenko [30] have shown that the cosmological relativistic theory (Behar and Carmeli [31]) predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35} s^{-2}$. This value of "Λ" is in excellent agreement with the measurements recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [29]; Perlmutter et al. [26]; Riess et al. [27]; Schmidt et al. [32]). The main conclusion of these observations is that the expansion of the universe is accelerating.

Several Ansätze have been proposed in which the Λ term decays with time (see Gasperini [33, 34], Berman [35], Freese et al. [36], Özer and Taha [36], Peebles and Ratra [37], Chen and Hu [38], Abdussattar and Viswakarma [39], Gariel and Le Denmat [40], Pradhan et al. [41]). Of the special interest is the Ansatz $\Lambda \propto a^{-2}$ (where a is the scale factor of the Robertson-Walker metric) by Chen and Wu [38], which has been considered/modified by several authors (Abdel-Rahaman [42], Carvalho et al. [43], Waga [44], Silveira and Waga [45], Vishwakarma [46]). This motivates us to study the cosmological models, where Λ varies with time.

Recently Tikekar [15] obtained two spherically-symmetric singularity-free relativistic cosmological models, describing universes filled with non-adiabatic perfect fluid accompanied by heat flow along the radial direction. In this paper, motivated by the situation discussed above, we shall focus on the problem with varying cosmological term in the presence of perfect fluid and also fluid with bulk viscosity. We do this by extending the work of Tikekar [15] by including a varying cosmological term. The remainder of this paper is organized as follows. In Section 2 we revisit the solutions of Tikekar and give a description of the cosmological models with their dynamical equations and solve them. We find three different cosmological models to different values of the function $g(t)$ and discuss results for these regimes. Among these solutions there appear the above mentioned singularity-free family of solutions, and we describe its properties in detail. Section 3 comprises bulk viscous universe. Finally, we devote in Section 4 to expose a brief discussion of the results.

2. A perfect fluid universe revisited

In this section, we review the solutions obtained by Tikekar [15]. Space-time of the singularity-free cosmological model is described by the metric

$$ds^2 = (1+\alpha r^2)dt^2 - g(t)\left[\left(\frac{1+2\alpha r^2}{1+\alpha r^2}\right)dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right], \qquad (1)$$

where $\alpha > 0$, is a constant.

The physical content of the space-time of these models is stipulated to be a non-adiabatic perfect fluid accompanied with heat flux along the radial direction with the energy-momentum tensor

$$T_i^j = (\rho + p)u_i u^j - p\delta_i^j + q_i u^j + u_i q^j, \quad u^i u_i = 1\,, \qquad (2)$$

and the coordinates are chosen to follow the motion of the fluid so that the four velocity u^i and the heat flux vector q^i take on the respective expressions

$$u^i = \left(0, 0, 0, \frac{1}{\sqrt{\alpha r^2 + 1}}\right), \qquad (3)$$

$$q^i = (q, 0, 0, 0)\,. \qquad (4)$$

The Einstein field equations with time-dependent cosmological constant

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda(t)g_i^j = -T_i^j, \qquad (5)$$

relate the dynamical variable ρ, p, Λ and q corresponding to the matter density, fluid pressure, cosmological constant and heat flux parameters with the metric potentials and imply

$$8\pi\rho = \frac{\alpha(2\alpha r^2+3)}{g(2\alpha r^2+1)^2} + \frac{3}{4(\alpha r^2+1)}\frac{\dot{g}^2}{g^2} + \Lambda(t)\,, \qquad (6)$$

$$8\pi p = \frac{\alpha}{g(2\alpha r^2+1)} - \frac{1}{(\alpha r^2+1)}\left(\frac{\ddot{g}}{g} - \frac{\dot{g}^2}{4g^2}\right) - \Lambda(t)\,, \qquad (7)$$

$$8\pi q = -\frac{\alpha r}{(2\alpha r^2+1)(\alpha r^2+1)}\frac{\dot{g}}{g^2}\,. \qquad (8)$$

We have freedom of choosing the function $g(t)$ so that the metric and the above dynamical variables remain finite everywhere all the time and the respective model gives a non-singular behaviour. If $g(t) = 1$, the space-time metric (1) has the interesting features [47], describes a static cosmological of **EFEs** with a perfect fluid source which is not conformally flat and can be embedded in a five-dimensional flat space-time. It should be noted here that the space-time of the **FRW**-model of the universe can be embedded in a five dimensional flat space-time but geometry conformal to a flat space-time.

In this paper, we choose the the three cases as $g(t) = at^2 + b,\ a + e^{-bt}$, $a + b\cos\omega t,\ a > 0,\ b > 0$. For all these choices it is observed that all physical and kinematic parameters remain regular and finite for all entire range of variables.

For complete determinacy of the system, we assume an equation of state of the form

$$p = \gamma\rho\,, \quad 0 \le \gamma \le 1\,, \tag{9}$$

where γ is a constant.

2.1. Model I

We set $g(t) = at^2 + b,\ \ a > 0,\ \ b > 0$. In this case the matter density ρ, the fluid pressure p, the heat flux parameter q and the kinematic parameter of expansion θ are found to have the following expressions:

$$8\pi\rho = \frac{\alpha(2\alpha r^2 + 3)}{(2\alpha r^2 + 1)^2(at^2 + b)} + \frac{3a^2t^2}{(\alpha r^2 + 1)(at^2 + b)^2} + \Lambda(t)\,, \tag{10}$$

$$8\pi p = \frac{\alpha(\alpha - 4a)r^2 + \alpha - 2a}{(2\alpha r^2 + 1)(\alpha r^2 + 1)(at^2 + b)} + \frac{a^2t^2}{(\alpha r^2 + 1)(at^2 + b)^2} - \Lambda(t)\,, \tag{11}$$

$$8\pi q = -\frac{2\alpha r}{(2\alpha r^2 + 1)(\alpha r^2 + 1)}\frac{at}{(at^2 + b)^2}\,, \tag{12}$$

$$\theta = \frac{3}{\sqrt{(\alpha r^2 + 1)}}\frac{at}{(at^2 + b)}\,. \tag{13}$$

Using (9) in (10) and (11), we obtain

$$(1+\gamma)\Lambda(t) = \frac{1}{(2\alpha r^2 + 1)(at^2 + b)}\left[\frac{\alpha(\alpha - 4a)r^2 + \alpha - 2a}{(\alpha r^2 + 1)} - \frac{\alpha\gamma(2\alpha r^2 + 3)}{(2\alpha r^2 + 1)}\right] + \frac{a^2t^2(1 - 3\gamma)}{(\alpha r^2 + 1)(at^2 + b)^2}\,. \tag{14}$$

From Eqs. (10) and (11), it is observed that the matter density is always and everywhere positive while positivity of pressure is ensured if $\alpha \ge 4a$. The heat flux parameter $q > 0$ for $t < 0$, $q = 0$ for $t = 0$ and $q < 0$ for $t > 0$. Equation (13) implies that the model describes a contracting universe for $t < 0$ with $q > 0$ and an expanding universe for $t > 0$ with $q < 0$, the switching from contracting phase of expansion occurring at $t = 0$.

If T denotes the temperature distribution of the fluid, the heat flux vector q^i is related with it through the phenomenological heat conduction equation

$$q^i = -K(g^{ij} - u^i u^i)(T_{,j} + T u_{j,k} u^k)\,,$$

which ensures positivity of entropy flux production with K denoting the conduction which is expected to be proportional to temperature.

If we stipulate the dependence of K on T in the universe of the model under following consideration, then

$$K = \kappa T^{m+1} = \kappa F\,,$$

where $m > 0$ is a constant.

The phenomenological conduction integrates out to give

$$T^{m+1} = \beta(\alpha r^2 + 1)^{\frac{m+1}{2}} + \frac{2(m+1)}{\kappa(m+3)} \frac{at}{(at^2 + b)(\alpha r^2 + 1)}\,,$$

where β is an arbitrary constant of integration. If $\beta = 0$, a finite temperature is measured everywhere for all time.

The overall evolution of the universe of this model can be described as follows. The universe begins to contract from a state of infinite dilution with vanishing small matter density at infinitely remote past and remains in the contracting phase during the period $(-\infty < t < 0)$. The switching from the phase of contraction to that of expansion occurs at $t = 0$, the epoch when matter density at the centre has reached its maximum value $\rho_{\max} = 3\alpha/b$, which can be made as large as desired by choosing α and b. The universe then expands forever to reach the state of infinite dilution at infinitely remote future, ρ, p and T remaining regular everywhere all the time. Thus this model describes a singularity-free universe which exits all the time, contracting for $t < 0$ and there after expanding forever.

From Eq. (14), we observe that the cosmological term Λ is a decreasing function of t and it approaches a small negative value at late time. A negative cosmological constant adds to the attractive gravity of matter, therefore universes with a negative cosmological constant are invariably doomed to re-collapse [56]. If we set $\Lambda = 0$, we recover the first model obtained by Tikekar [15].

2.2. Model II

We set $g(t) = a + e^{-bt^2}$, $a > b > 0$. In this case the matter density ρ, the fluid pressure p, the heat flux parameter q and the kinematic parameter of expansion θ are found to have the following expressions:

$$8\pi\rho = \frac{\alpha(2\alpha r^2 + 3)}{(2\alpha r^2 + 1)^2(e^{-bt^2} + a)} + \frac{3b^2t^2e^{-2bt^2}}{(\alpha r^2 + 1)(e^{-bt^2} + a)^2} + \Lambda(t)\,, \tag{15}$$

$$8\pi p = \frac{\alpha}{(2\alpha r^2+1)(e^{-bt^2}+a)} - \frac{be^{-bt^2}\left[2a(bt^2-1)+e^{-bt^2}(3bt^2-2)\right]}{(\alpha r^2+1)(e^{-bt^2}+a)^2} - \Lambda(t)\,, \quad (16)$$

$$8\pi q = \frac{2\alpha r b t e^{-bt^2}}{(2\alpha r^2+1)(\alpha r^2+1)(e^{-bt^2}+a)^2}\,, \quad (17)$$

$$\theta = -\frac{3}{\sqrt{(\alpha r^2+1)}}\frac{bte^{-bt^2}}{(e^{-bt^2}+a)}\,. \quad (18)$$

Using (9) in (15) and (16), we obtain

$$(1+\gamma)\Lambda(t) = \frac{2\alpha^2 r^2(1-\gamma)+\alpha(1-3\gamma)}{(2\alpha r^2+1)^2(e^{-bt^2}+a)} -$$

$$\frac{be^{-bt^2}\left[2a(bt^2-1)+e^{-bt^2}\{(3bt^2(1-\gamma)-2)\}\right]}{(\alpha r^2+1)(e^{-bt^2}+a)^2}\,. \quad (19)$$

From Eq. (19), we observe that the cosmological term is a decreasing function of time and it approaches a small positive value at late time which is a supported by the results from recent type Ia supernovae observations. From Fig. 1, it is seen that in the beginning, the value of Λ goes to a negative value, then it gradually approaches to small and positive value at late time.

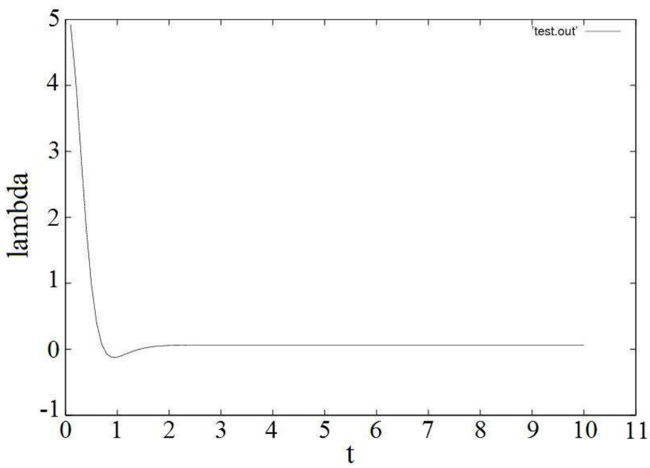

Fig. 1. Variation of Λ with time for 2.2 Model II. The values of parameters are: $\alpha = 1,\ \gamma = 0.25,\ a = 1,\ b = 2$ and $r = 1$.

2.3. Model III

We set $g(t) = a + b\cos\omega t, \quad a > b > 0$. In this case the matter density ρ, the fluid pressure p and the heat flux parameter for the non-adiabatic matter content accompanying it are found to have the following respective expressions:

$$8\pi\rho = \frac{\alpha(2\alpha r^2 + 3)}{(2\alpha r^2 + 1)(b\cos\omega t + a)} + \frac{3b^2\omega^2\sin^2\omega t}{4(\alpha r^2 + 1)(b\cos\omega t + a)^2} + \Lambda(t)\,, \tag{20}$$

$$8\pi p = \frac{\alpha}{(2\alpha r^2 + 1)(b\cos\omega t + a)} + \frac{b\omega^2(3b\cos^2\omega t + 4a\cos\omega t + b)}{4(\alpha r^2 + 1)(b\cos\omega t + a)^2} - \Lambda(t)\,, \tag{21}$$

$$8\pi q = \frac{\alpha r}{(\alpha r^2 + 1)(2\alpha r^2 + 1)}\frac{b\omega t}{(at^2 + b)}\,. \tag{22}$$

Using (9) in (20) and (21), we obtain

$$(1+\gamma)\Lambda(t) = \frac{2\alpha^2 r^2(1-\gamma) + \alpha(1-3\gamma)}{(2\alpha r^2 + 1)^2(b\cos\omega t + a)} +$$

$$\frac{b^2\omega^2(1-3\gamma) + b\omega^2[4a + 3b(1+\gamma)\cos\omega t]\cos\omega t}{4(\alpha r^2 + 1)(b\cos\omega t + a)^2}\,. \tag{23}$$

The expression for the expansion parameter

$$\theta = -\frac{3b\omega\sin\omega t}{2\sqrt{(1+\alpha r^2)}}(b\cos\omega t + a) \tag{24}$$

indicates that the universe in this model is in the phase of contraction for $2n\pi < \omega t < (2n+1)\pi$ and in the phase of expansion $(2n+1)\pi < \omega t < 2(n+1)\pi$ where n takes on integral values only. During the phase of contraction $q < 0$, while during the expansion phase $q > 0$ with q vanishing when switching from contraction to expansion or vice a versa occurs. The integration of phenomenological equation for conduction of heat as indicated in the discussion of the model I in this case also leads to similar conclusions about the regularity of the temperature throughout the space-time.

The overall behaviour of the universe of this model may be described as follows. The universe oscillates between two regular states with oscillation period $t = 2\pi/\omega$, with the central matter density attaining its maximum value $\rho_{\max} = 3\alpha/8\pi(a-b)$ at $t = (2n+1)\pi/\omega$ and its minimum value $\rho_{\min} = 3\alpha/8\pi(a+b)$ at $t = 2(n+1)\pi/\omega$. The model permits as low and as high values for the central matter density as desired for appropriate choice of the parameters a, b. The model involves three parameters a, b and ω, which are related to the maximum and minimum values of matter density at the centre and the duration of an oscillation cycle. Since the model is centrally symmetric, it accommodates additional parameters r_0, t_0 corresponding to the radial coordinate and time of observation for the observer. Thus the model has a number of free parameters which can be suitably chosen to bring it closer to observations.

The most interesting feature of the oscillatory model is that the universe according to it, like that of the steady state cosmology has no beginning and no end. Accordingly, as in the quasi steady state cosmology, this model would also lead to the prediction of blue shifts. The oscillatory model here arises strictly within the framework of general relativity, without violating the conservation of matter energy and positivity of energy. Unlike the oscillation model of Dadhich and Raychuadhuri, the universe of this model is filled with a perfect fluid characterized by pressure isotropy. The apparent simplicity of the space-time metric is indicative of geodesic completeness and causal stability. The model, therefore, opens up new avenues for further studies which may bear promise for observational cosmology.

From Eq. (23), we observe that the cosmological term Λ is oscillating due to properties of sinusoidal functions. It is also worth noting that the average value (with respect to one period) of Λ is positive. Here we will have negative equation of state at late times required to support the current acceleration of the universe. If we set $\Lambda = 0$, we recover the second model obtained by Tikekar [15].

3. *Bulk viscous universe*

The realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well known that in an earlier stage of the universe, when the radiation in the form of photon as well as neutrino decoupled from matter, it behaved like a viscous fluid. Misner [48] has studied the effect of viscosity on the evolution of cosmological models. A number of authors have discussed cosmological solutions with bulk viscosity in various context. It is, therefore, of considerable interest to study also nonsingular cosmological models for viscous fluid distributions.

The equation of the bulk viscosity can be obtained from the general relativistic field equation, we introduce the effective pressure [49],

$$\bar{p} = p - \xi\theta \,, \tag{25}$$

where p is the pressure due to the perfect fluid present, ξ is the coefficient of bulk viscosity and θ is the expansion scalar. Thus, given $\xi(t)$, we can solve for cosmological parameters. In most investigations involving bulk viscosity, it is assumed to be a simple power function of the energy density (Pavan [50], Maartens [51], Zimdahl [52])

$$\xi(t) = \xi_0 \rho^n, \tag{26}$$

where ξ_0 and n are constants. For a large value of ρ, n is quite small and Santos et al. [53] suggested to get more realistic models if n lies in the regime $0 \le n \le \frac{1}{2}$. For small density, n may even be equal to unity as used in Murphy's work for simplicity [54]. Also if $n = 1$, Eq. (26) may correspond to a radiative fluid [55].

Introducing (25) and (26) into (7), we obtain

$$8\pi p = 8\pi\xi_0\rho^n\theta + \frac{\alpha}{g(2\alpha r^2+1)} - \frac{1}{(\alpha r^2+1)}\left(\frac{\ddot{g}}{g} - \frac{\dot{g}^2}{4g^2}\right) - \Lambda(t)\,. \tag{27}$$

For simplicity and realistic models for physical importance, we consider the following two cases ($n = 0, 1$).

3.1. *Model I*

We set $g(t) = at^2 + b$, $a > 0$, $b > 0$. In this case we consider the two following cases.

3.1.1. **Case I: solution for** $\xi = \xi_0$

When $n = 0$, Eq. (26) reduces to $\xi = \xi_0$ (constant) and hence Eq. (27) with the help of (9), (10) and (13) reduces to the form

$$8\pi(1+\gamma)\rho = 8\pi T_1 + \frac{4a^2t^2}{(\alpha r^2+1)(at^2+b)^2} + \frac{1}{(2\alpha r^2+1)(at^2+b)}\times$$
$$\left[\frac{\alpha(2\alpha r^2+3)}{(2\alpha r^2+1)} + \frac{\alpha(\alpha-4a)r^2+\alpha-2a}{(\alpha r^2+1)}\right], \tag{28}$$

where

$$T_1 = \frac{3\xi_0}{\sqrt{(\alpha r^2+1)}}\frac{at}{(at^2+b)}\,. \tag{29}$$

Eliminating $\rho(t)$ between Eqs. (10) and (28), we get

$$(1+\gamma)\Lambda(t) = 8\pi T_1 + \frac{(1-3\gamma)a^2t^2}{(\alpha r^2+1)(at^2+b)^2} + \frac{1}{(2\alpha r^2+1)(at^2+b)}\times$$
$$\left[\frac{\alpha(\alpha-4a)r^2+\alpha-2a}{(\alpha r^2+1)} - \frac{\alpha(2\alpha r^2+3)\gamma}{(2\alpha r^2+1)}\right]. \tag{30}$$

3.1.2. **Case II: solution for** $\xi = \xi_0\rho$

When $n = 1$, Eq. (26) reduces to $\xi = \xi_0\rho$ and hence Eq. (27) with the help of (9), (10) and (13) reduces to the form

$$8\pi[1+\gamma-T_1]\rho = \frac{4a^2t^2}{(\alpha r^2+1)(at^2+b)^2} + \frac{1}{(2\alpha r^2+1)(at^2+b)}\times$$

$$\left[\frac{\alpha(2\alpha r^2+3)}{(2\alpha r^2+1)}+\frac{\alpha(\alpha-4a)r^2+\alpha-2a}{(\alpha r^2+1)}\right]. \tag{31}$$

Eliminating $\rho(t)$ between Eqs. (10) and (31), we get

$$[2+\gamma-T_1]\Lambda(t)=\frac{1}{(2\alpha r^2+1)(at^2+b)}\left[\frac{\alpha(\alpha-4a)r^2+\alpha-2a}{(\alpha r^2+1)}-\frac{\alpha(2\alpha r^2+3)}{(2\alpha r^2+1)}\right]$$

$$-\frac{2a^2t^2}{(\alpha r^2+1)(at^2+b)^2}-(\gamma-T_1)\left[\frac{\alpha(2\alpha r^2+3)}{(2\alpha r^2+1)^2(at^2+b)}+\frac{3a^2t^2}{(\alpha r^2+1)(at^2+b)^2}\right]. \tag{32}$$

From Eqs. (30) and (32), we observe that the cosmological term Λ is a decreasing function of time and it approaches a small negative value at late time which is similar to the previously discussed Model I (Section 2.1).

3.2. Model II

We set $g(t)=a+e^{-bt^2}$, $a>b>0$. In this case we consider two following cases.

3.2.1. **Case I: solution for** $\xi=\xi_0$

When $n=0$, Eq. (26) reduces to $\xi=\xi_0$ (constant) and hence Eq. (27) with the help of (9), (15) and (18) reduces to the form

$$4\pi(1+\gamma)\rho=4T_2+\frac{2\alpha(\alpha r^2+1)}{(2\alpha r^2+1)^2(e^{-bt^2}+a)}+\frac{be^{-bt^2}(e^{-bt^2}-abt^2+a)}{(\alpha r^2+1)(e^{-bt^2}+a)^2}, \tag{33}$$

where

$$T_2=\frac{3\xi_0}{\sqrt{(\alpha r^2+1)}}\frac{bte^{-bt^2}}{(e^{-bt^2}+a)}. \tag{34}$$

Eliminating $\rho(t)$ between (33) and (15), we obtain

$$(1+\gamma)\Lambda(t)=8\pi T_2+\frac{\alpha[(2\alpha r^2+1)-(2\alpha r^2+3)\gamma]}{(2\alpha r^2+1)^2(e^{-bt^2}+a)}$$

$$-\frac{be^{-bt^2}}{(\alpha r^2+1)(e^{-bt^2}+a)^2}[2a(bt^2-1)-\{2-3b(1-\gamma)t^2\}e^{-bt^2}]. \tag{35}$$

3.2.2. **Case II: solution for** $\xi=\xi_0\rho$

When $n=1$, Eq. (26) reduces to $\xi=\xi_0\rho$ and hence Eq. (27) with the help of (9), (15) and (18) reduces to the form

$$4\pi[1+\gamma-T_2]\rho=\frac{2\alpha(\alpha r^2+1)}{(2\alpha r^2+1)^2(e^{-bt^2}+a)}+\frac{be^{-bt^2}(e^{-bt^2}-abt^2+a)}{(\alpha r^2+1)(e^{-bt^2}+a)^2}. \tag{36}$$

Eliminating $\rho(t)$ between (36) and (15), we obtain

$$[2+\gamma-T_2]\Lambda(t) = -\frac{2\alpha(1+\gamma-T_2)}{(2\alpha r^2+1)^2(e^{-bt^2}+a)} - \frac{be^{-bt^2}}{(\alpha r^2+1)(e^{-bt^2}+a)^2}\times$$

$$[2a(bt^2-1)-2e^{-bt^2}-3bt^2e^{-bt^2}(\gamma-T_2)]\,. \tag{37}$$

From Eqs. (35) and (37), we observe that Λ is a decreasing function of time and it approaches a small positive value at late time which matches with recent type Ia supernovae observations.

3.3. Model III

We set $g(t) = a + b\cos(\omega t)$, $a > 0$, $b > 0$. In this case we consider the two following cases.

3.3.1. Case I: solution for $\xi = \xi_0$

When $n = 0$, Eq. (26) reduces to $\xi = \xi_0$ (constant) and hence Eq. (27) with the help of (9), (20) and (24) reduces to the form

$$8\pi(1+\gamma)\rho = 8\pi T_3 + \frac{2\alpha(\alpha r^2+2)}{(2\alpha r^2+1)(b\cos\omega t+a)} + \frac{b\omega^2(a\cos\omega t+b)}{(\alpha r^2+1)(b\cos\omega t+a)^2}\,, \tag{38}$$

where

$$T_3 = \frac{3\xi_0 b\omega}{2\sqrt{(\alpha r^2+1)}}\sin\omega(b\cos\omega t+a)\,. \tag{39}$$

Eliminating $\rho(t)$ between (38) and (20), we obtain

$$(1+\gamma)\Lambda(t) = 8\pi T_3 + \frac{\alpha\{1-(2\alpha r^2+3)\gamma\}}{(2\alpha r^2+1)(b\cos\omega t+a)}$$

$$+\frac{b\omega^2\{4(a\cos\omega t+b)-3b(1+\gamma)\sin^2\omega t\}}{4(\alpha r^2+1)(b\cos\omega t+a)^2}\,. \tag{40}$$

3.3.2. Case II: solution for $\xi = \xi_0\rho$

When $n = 1$, Eq. (26) reduces to $\xi = \xi_0\rho$ and hence Eq. (27) with the help of (9), (20) and (24) reduces to the form

$$8\pi[1+\gamma-T_3]\rho = \frac{2\alpha(\alpha r^2+2)}{(2\alpha r^2+1)(b\cos\omega t+a)} + \frac{b\omega^2(a\cos\omega t+b)}{(\alpha r^2+1)(b\cos\omega t+a)^2}\,. \tag{41}$$

Eliminating $\rho(t)$ between (41) and (20), we obtain

$$[1+\gamma-T_3]\Lambda(t) = \frac{\alpha\{1-(2\alpha r^2+3)(\gamma-T_3)\}}{(2\alpha r^2+1)(b\cos\omega t+a)}$$

$$+\frac{b\omega^2\{4(a\cos\omega t+b)-3b(1+\gamma-T_3)\sin^2\omega t\}}{4(\alpha r^2+1)(b\cos\omega t+a)^2}\,. \tag{42}$$

From Eqs. (40) and (42), we observe that the cosmological term Λ oscillates with time due to the properties of sinusoidal functions, present in these equations. The nature of these models are same as already discussed in Model III (Section 2.3).

4. *Conclusions*

We have obtained a new class of nonsingular spherically-symmetric cosmological models with variable Λ term in the presence of a perfect fluid and also fluid with bulk viscosity with a radial heat flux. We have derived three types of models choosing different form of the function $g(t)$. For these three choices, it is clear that all physical and kinematic parameters remain regular and finite for the entire range of the variables. The Models I (Sections 2.1 and 3.1) describe a singularity-free universe which exists all time, contracting for $t < 0$ and there after expanding forever. The Models III (Sections 2.3 and 3.3) admit an interesting oscillation behaviour in time between two regular states with oscillation period $t = 2\pi/\omega$. This model has three parameters a, b and ω, which can be suitably chosen to bring it closer to observations.

The cosmological term is a parameter describing the energy density of the vacuum (empty space), and a potentially important contribution to the dynamical history of the universe. The physical interpretation of the cosmological constant as vacuum energy is supported by the existence of the "zero point" energy predicted by quantum mechanics. In quantum mechanics, particle and antiparticle pairs are consistently created out of the vacuum. Even though these particles exist for only a short amount of time before annihilating each other, they do give the vacuum a non-zero potential energy. In general relativity, all form of energy should gravitate, including the energy of vacuum, hence the cosmological constant. A negative cosmological constant adds to the attractive gravity of matter, therefore universes with a negative cosmological constant are invariably doomed to re-collapse [56]. A positive cosmological constant resists the attractive gravity of matter due to its negative pressure. For most universes, the positive cosmological constant eventually dominates over the attraction of matter and drives the universe to expand exponentially [57].

The cosmological terms in all models in Section 2.2 and 3.2 are found to be decreasing functions of time and they all approach a small and positive value at late times which are supported by the results from recent type Ia supernovae observations recently obtained by High-z Supernova Team and Supernova Cosmological

Project (Garnavich et al. [29], Perlmutter et al. [26], Riess et al. [27], Schmidt et al. [32]). Our models provide a good agreement with these observational results. We have derived value for the cosmological term and attempted to formulate a physical interpretation for it. The coefficient of bulk viscosity is assumed to be a power function of mass density. The effect of bulk viscosity is to introduce a change in the perfect fluid model. It is seen that the solutions obtained by Tikekar [15] are particular cases of our solutions.

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References

[1] R. Penrose, Phys. Rev. Lett. **14** (1965) 57.

[2] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-time*, Cambridge University Press, Cambridge (1973).

[3] J. M. M. Senovilla, Phys. Rev. Lett. **64** (1990) 2219.

[4] F. J. Chinea, L. Fernández-Jambrina and J. M. M. Senovilla, Phys. Rev. D **45** (1990) 481.

[5] E. Ruiz and J. M. M. Senovilla, Phys. Rev. D **45** (1992) 1995.

[6] J. M. M. Senovilla, *El Escorial Summer School on Gravitation and Relativity* (1992); *Rotating Objects and Other Topics*, ed. F. J. Chinea and L. M. ález-Romero Gonz, Springer, Berlin (1993).

[7] J. M. M. Senovilla, Phys. Rev. D **53** (1996) 1799.

[8] N. Dadhich, L. K. Patel and R. Tikekar, Pramana-J. Phys. **44** (1995) 303.

[9] L. Fernández-Jambrina, Class. Quantum Grav. **14** (1997) 3407.

[10] J. M. M. Senovilla, Gen. Rel. Grav. **30** (1998) 701.

[11] N. Dadhich, J. Astrophys. Astr. **18** (1997) 343.

[12] N. Dadhich, L. K. Patel and R. Tikekar, Math. Today **17** (1999) 17.

[13] N. Dadhich and A. K. Raychaudhuri, Mod. Phys. Lett. A **14** (1999) 2135.

[14] F. Hoyle, G. Burbidge and J. V. Narlikar, Not. R. Astron. Soc. **267** (1994) 1007.

[15] R. Tikekar, Pramana-J. Phys. **55** (2000) 623.

[16] J. M. Overduin, Astrophys. J. **517** (1999) L1.

[17] K. A. Bronnikov, A. Dobosz and I. G. Dymnikova, Class. Quantum Grav. **20** (2003) 397.

[18] L. Fernández-Jambrina and L. M. González-Romero, Int. J. Mod. Phys. D **7** (2004) 1; J. Math. Phys. **45** (2004) 2113; A. K. Raychaudhuri, Gen. Rel. Grav. **36** (2004) 36; L. Fernández-Jambrina, Gen. Rel. Grav. **37** (2005) 421; A. K. Raychaudhuri, Gen. Rel. Grav. **37** (2005) 425.

[19] G. V. Vereshchagin, preprint, gr-qc/0309036 (2004).

[20] A. Pradhan, K. Srivastava and A. L. Ahuja, gr-qc/0408057 (2004).

[21] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37** (1988) 3406.

[22] A. D. Dolgov, in *The Very Early Universe*, Eds. G.W. Gibbons, S.W. Hawking and S. T. C. Siklos, Cambridge Univerity Press, Cambridge (1983).

[23] A. D. Dolgov, M. V. Sazhin and Ya. B. Zeldovich, *Basics of Modern Cosmology*, Editions Frontiers (1990).

[24] A. D. Dolgov, Phys. Rev. D **55** (1997) 5881.

[25] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9** (2000) 373, gr-qc/9904398.

[26] S. Perlmutter et al., Astrophys. J. **483** (1997) 565, astro-ph/9608192; Nature **391** (1998) 51, astro-ph/9712212; Astrophys. J. **517** (1999) 565, astro-ph/9608192.

[27] A. G. Riess et al., Astron. J. **116** (1998) 1009, astro-ph/9805201.

[28] A. Vilenkin, Phys. Rep. **121** (1985) 265.

[29] P. M. Garnavich et al., Astrophys. J. **493** (1998a) L53, astro-ph/9710123; Astrophys. J. **509** (1998b) 74, astro-ph/9806396.

[30] M. Carmeli and T. Kuzmenko, Int. J. Theor. Phys. **41** (2002) 131.

[31] S. Behar and M. Carmeli, Int. J. Theor. Phys. **39** (2002) 1375, astro-ph/0008352.

[32] B. P. Schmidt et al., Astrophys. J. **507** (1998) 46, astro-ph/9805200.

[33] M. Gasperini, Phys. Lett. B **194** (1987) 347.

[34] M. Gasperini, Class. Quant. Grav. **5** (1988) 521.

[35] M. S. Berman, Int. J. Theor. Phys. **29** (1990) 567; Int. J. Theor. Phys. **29** (1990) 1419; Phys. Rev. D **43** (1991) 75; M. S. Berman and M. M. Som, Int. J. Theor. Phys. **29** (1990) 1411; M. S. Berman, M. M. Som and F. M. Gomide, Gen. Rel. Grav. **21** (1989) 287; M. S. Berman and F. M. Gomide, Gen. Rel. Grav. **22** (1990) 625.

[36] M. Özer and M. O. Taha, Nucl. Phys. B **287** (1987) 776.

[37] P. J. E. Peebles and B. Ratra, Astrophys. J. **325** (1988) L17.

[38] W. Chen and Y. S. Wu, Phys. Rev. D **41** (1990) 695.

[39] Abdussattar and R. G. Vishwakarma, Pramana-J. Phys. **47** (1996) 41.

[40] J. Gariel and G. Le Denmat, Class. Quant. Grav. **16** (1999) 149.

[41] A. Pradhan and A. Kumar, Int. J. Mod. Phys. D **10** (2001) 291.
A. Pradhan and V. K. Yadav, Int. J. Mod. Phys. D **11** (2002) 983.

[42] A. -M. M. Abdel-Rahaman, Gen. Rel. Grav. **22** (1990) 655; Phys. Rev. D **45** (1992) 3492.

[43] J. C. Carvalho, J. A. S. Lima and I. Waga, Phys. Rev. D **46** (1992) 2404.

[44] I. Waga, Astrophys. J. **414** (1993) 436.

[45] V. Silveira and I. Waga, Phys. Rev. D **50** (1994) 4890.
V. Silviera and I. Waga, ibid. D **50** (1994) 4890.

[46] R. G. Vishwakarma, Class. Quant. Grav. **17** (2000) 3833.

[47] R. S. Tikekar, Curr. Sci. **39** (1970) 460.

[48] C. W. Misner, Nature **124** (1967) 40; Astrophys. J. **151** (1968) 431.

[49] Ya. B. Zeldovich, Sov. Phys. -Uspekhi **11** (1968) 381.

[50] D. Pavon, J. Bafaluy and D. Jou, Class. Quant. Grav. **8** (1991) 357; in *Proc. Hanno Rund Conf. on Relativity and Thermodynamics*, ed. S. D. Maharaj, University of Natal, Durban (1996) p. 21.

[51] R. Maartens, Class. Quant. Grav. **12** (1995) 1455.

[52] W. Zimdahl, Phys. Rev. D **53** (1996) 5483.

[53] N. O. Santos, R. S. Dias and A. Banerjee, J. Math. Phys. **26** (1985) 878.

[54] G. L. Murphy, Phys. Rev. D **8** (1973) 4231.

[55] S. Weinberg, *Gravitation and Cosmology*, Wiley, New Yark (1972).

[56] S. M. Carroll, W. H. Press and E. L. Turner, ARA&A **30** (1992) 499.

[57] C. S. Kochanek, Astrophys. J. **384** (1992) 1.

SFERNO SIMETRIČNI NESINGULARNI MODELI S PROMJENLJIVIM ČLANOM Λ U RELATIVISTIČKOJ KOZMOLOGIJI

Raspravljamo niz nesingularnih sferno-simetričnih kozmoloških modela s radijalnim tokom topline i promjenljivim kozmološkim članom $\Lambda(t)$. Postigli smo tri različita egzaktna rješenja Einsteinovih jednadžbi polja za perfektnu tekućinu i tekućinu s volumnim trenjem. Ishodi računa pokazuju da je kozmološki član padajuća funkcija vremena što je u suglasju s nedavnim opažanjima supernova tipa Ia.