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# N$\pi$ SCATTERING AND ELECTROMAGNETIC CORRECTIONS IN THE EXTENDED LINEAR SIGMA MODEL

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Recent analysis of pion–nucleon scattering and nucleon magnetic moments are investigated in an extended linear sigma model. The field equations are solved in the mean-field approximation. Good results have been obtained in comparison with previous work and experimental data.

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## 1. Introduction

One of the effective models in describing hadron properties is the linear sigma model which has been suggested earlier by Gell-Mann and Levy [1] to describe nucleons interacting via sigma ($\sigma$) and pion ($\pi$) exchanges. Some of the consequences of this model, however, are not supported by observations. Notably, the isoscalar pion–nucleon ($\pi$N) scattering length predicted by the model is larger than the experimental value by an order of magnitude (see, e.g. Refs. [2] and [3]). Many solutions for this model have already been suggested. Birse and Banerjee [2] constructed equations of motion treating both the $\sigma$- and $\pi$-fields as time-independent classical fields and quarks in the hedgehog spinor state. Birse [3] generalized this mean-field model to include angular momentum and isospin projection. Goeke et al. [4] investigated hadron properties in a chiral model for the nucleon based on the linear sigma model with scalar-isoscalar and scalar-isovector mesons coupled to quarks, using the coherent pair approximation. Their results for $\sigma(\pi\mathrm{N})$ differ considerably from the experimental data. That work has been reexamined by Aly et al. [5].

Recently, the mesons were found to play a very important role in improving the nucleon properties in chiral quark models. Horvat et al. [6] applied the Tamm-Dancof method which extended the sigma model to include nine scalar and nine pseudoscalar degrees of freedom, to reproduce the nucleon magnetic moments. On the other hand, Rashdan et al. [7] also investigated the effect of higher-order mesonic interactions in the sigma model to improve nucleon properties. Sahu et al. [8] considered the effect of higher-order mesonic interactions in the sigma model to obtain better description of nuclear matter. In the same direction, Broniowski and Golli [9] added a new term to the Langrangian of the type of the linear sigma model to investigate the dynamical consequences of this term and its relevance to the phenomenology of the soliton models of the nucleon.

The present work aims to demonstrate the importance of mesonic correlations of higher order. The order is taken to be higher than other soliton models adopted in the previous works [2−5].

This paper is organized as follows. A brief review of the linear sigma model with higher-order mesonic interactions is given in Sec. 2. Numerical calculations and discussion are presented in Sec. 3.

## 2. *Chiral quark-sigma model with higher-order mesonic interactions*

### 2.1. *The linear sigma model*

In this subsection, we summarize the original sigma model of Gell-Mann and Levy [1] and Birse and Banerjee [2].

The Lagrangian density of the linear sigma model, which describes the interactions between quarks via the $\sigma$- and $\vec{\pi}$-mesons is written as [2]

$$L(r) = \mathrm{i}\overline{\Psi}\gamma_\mu\partial^\mu\Psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) + g\overline{\Psi}(\sigma + \mathrm{i}\gamma_5\vec{\tau}\cdot\vec{\pi})\Psi - U_1(\sigma,\vec{\pi})\,, \quad (1)$$

where

$$U_1(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - \nu^2)^2 + m_\pi^2 f_\pi \sigma \quad (2)$$

is the meson–meson interaction potential, and $\Psi$, $\sigma$ and $\vec{\pi}$ are the quark, sigma and pion fields, respectively. In the mean-field approximation, the meson fields are treated as time-independent classical fields. This means that we are replacing power and products of the meson fields by corresponding powers and products of their expectation values. The meson–meson interactions in Eq. (2) lead to the hidden chiral $SU(2)\times SU(2)$ symmetry with $\sigma(r)$ taking on the expectation value

$$\langle\sigma\rangle = -f_\pi\,, \tag{3}$$

where $f_\pi = 93$ MeV is the pion decay constant. The last term in Eq. (2) is included to break the chiral symmetry. It leads to partial conservation of axial-vector isospin current (PCAC). The parameters $\lambda^2$ and $\nu^2$ can be derived in terms of $f_\pi$ and the masses of $\sigma$- and $\vec{\pi}$-mesons. One expands the vacuum expectation values, which minimize the potential, about the $\sigma = -f_\pi,\ \vec{\pi} = 0$,

$$\nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}\,, \tag{4}$$

and

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}\,. \tag{5}$$

For details, see Ref. [2].

### 2.2. *Higher-order mesonic interactions*

The extended linear sigma model is described in details in Ref. [7]. In the following, we give a brief summary of a new version of the meson–meson interaction. We assume the following form of the mesonic potential

$$U_n(\sigma,\pi) = \frac{1}{4}\left(\frac{\lambda}{a^{2n-2}}\right)^2 \left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^{2n} + m_\pi^2 f_\pi \sigma\,, \tag{6}$$

where $a$ is a scale parameter taken to be 1 MeV. It is evident that this potential is also chiral symmetric. For $n = 1$, we get the original potential $U_1(\sigma,\vec{\pi})$ defined by Eq. (2). Here we consider the case $n = 2$ [7, 8]. We are going to use an effective model, approximating the underlying quark theory; the model need not and should not be renormalizable [7−9]. Applying the PCAC and taking $\lambda/a^2 = \lambda_1$, we get

$$\nu^2 = f_\pi^2 - \left(\frac{m_\pi^2}{2\lambda_1^2}\right)^{1/3}, \tag{7}$$

and

$$\lambda_1^2 = \frac{(m_\sigma^2 - m_\pi^2)^3}{27(2m_\pi)^4 f_\pi^6}\,. \tag{8}$$

Now, we expand the extremum with the shifted field defined as

$$\sigma = \sigma' - f_\pi\,. \tag{9}$$

Substituting Eq. (9) into Eq. (1) and using Eq. (6), we get

$$L(r) = \mathrm{i}\overline{\Psi}\gamma_\mu\partial^\mu\Psi + \frac{1}{2}(\partial_\mu\sigma'\partial^\mu\sigma' + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) - g\overline{\Psi}f_\pi\Psi + g\overline{\Psi}\sigma'\Psi + \mathrm{i}g\overline{\Psi}\gamma_5\vec{\tau}\cdot\vec{\pi}\Psi - U_2(\sigma', \vec{\pi}), \tag{10}$$

where

$$U_2(\sigma', \vec{\pi}) = \frac{\lambda_1^2}{4}\left((\sigma' - f_\pi)^2 + \vec{\pi}^2 - \nu^2\right)^4 + m_\pi^2 f_\pi\sigma' - m_\pi^2 f_\pi^2 . \tag{10}$$

The time-independent fields $\sigma'$ and $\pi(r)$ are to satisfy the Euler-Lagrangian equation, and the quark wave function satisfies the Dirac eigenvalue equation. The mesonic fields are written as

$$\Box\,\sigma' = g\overline{\Psi}\Psi - 2\lambda_1^2(\sigma' - f_\pi)\left((\sigma' - f_\pi)^2 + \vec{\pi}^2 - \nu^2\right)^3 - m_\pi^2 f_\pi , \tag{12}$$

$$\Box\,\vec{\pi} = \mathrm{i}g\overline{\Psi}\gamma_5\cdot\vec{\tau}\Psi - 2\lambda_1^2\vec{\pi}\left((\sigma' - f_\pi)^2 + \vec{\pi}^2 - \nu^2\right)^3 , \tag{13}$$

where $\vec{\tau}$ refers to Pauli isospin matrices and $\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. We used the the hedgehog Ansatz [2],

$$\vec{\pi}(r) = \hat{r}\pi(r) . \tag{14}$$

The Dirac equation for the quarks is

$$\frac{\mathrm{d}u}{\mathrm{d}r} = -p(r)u + (E - m_q + S(r))w , \tag{15}$$

where $S(r) = g\langle\sigma'\rangle$, $P(r) = \langle\vec{\pi}, \hat{r}\rangle$ and E are the scalar potential, pseudoscalar potential and the eigenvalue of the quarks spinor $\Psi$, respectively, and

$$\frac{\mathrm{d}w}{\mathrm{d}r} = -(E - m_q + S(r))u + \left(\frac{2}{r} - p(r)\right)w . \tag{16}$$

Including the colour degrees of freedom, one has $g\overline{\Psi}\Psi \to N_c\overline{\Psi}\Psi$, where $N = 3$ colours and $g$ is the coupling constant. The Dirac wave functions $\overline{\Psi}$ and $\Psi$ are given by

$$\Psi(r) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) \\ \mathrm{i}w(r) \end{bmatrix} \qquad \overline{\Psi} = \frac{1}{\sqrt{4\pi}}\Big[u(r)\ \mathrm{i}w(r)\Big], \tag{17}$$

and the sigma, pion and vector densities are given by

$$\rho_s = N_c g\overline{\Psi}\Psi = \frac{3g}{4\pi}(u^2 - w^2) , \tag{18}$$

$$\rho_p = \mathrm{i}N_c g\overline{\Psi}\gamma_5\vec{\tau}\Psi = \frac{3g}{4\pi}(-2uw) , \tag{19}$$

$$\rho_v = \frac{3g}{4\pi}(u^2 + w^2)\,. \tag{20}$$

These equations are subject to the boundary conditions which require that the fields asymptotically approach their vacuum values,

$$\sigma(r) \sim -f_\pi \qquad \text{and} \qquad \pi(r) \sim 0 \quad \text{at} \quad r \to \infty\,. \tag{21}$$

## 3. Numerical calculations

We have solved the field equations (15) and (16) using the fourth-order Runge-Kutta method. Due to the nonlinearity of the equations, it is necessary to iterate the solution until self-consistency is achieved. To start this iteration process, we used the chiral circle form for the meson fields,

$$S(r) = m_q(1 - \cos\theta)\,, \tag{22}$$

$$P(r) = -m_q \sin\theta\,, \tag{23}$$

where

$$\theta = \pi \tanh r\,. \tag{24}$$

The mean fields mediate an instantaneous interaction among the quarks. The meson mean fields [2] don't carry momentum, angular momentum, charge, or isospin, since the operator corresponding to these observables depends on the time derivatives of the meson fields. However, there are other operators which do receive contributions from the mean fields. These include the magnetic moment, $\sigma(\pi, N)$ given by

$$\sigma(\pi, N) = 4\pi f_\pi m_\pi^2 \int_0^\infty \mathrm{d}r\; r^2(\sigma(r) - f_\pi), \tag{25}$$

For a review of this quantity, see Ref. [5].

## 4. Results

The field equations (12) to (16) have been solved by the iteration method as in Refs. [2] and [7] for different values of the quark and sigma masses. Tables 1, 2 and 3 show the nucleon observables for $m_q = 400$, 500 and 600 MeV, respectively. The sigma mass was assumed $m_\sigma \geq 700$ MeV. As seen from these tables, a stronger mesonic interaction significantly modified the nucleon observables. In the linear sigma model of Gell-Mann and Levy [1] and Birse and Banerjee [2], the quark and sigma masses were taken 500 MeV and 1200 MeV, respectively, that are larger than what is commonly used [10−12]. This is due to the fact that in Ref. [2]

the bound solutions have only been obtained for $3.9 < g < 4.55$ and a lower order of the mesonic interaction (it was taken in the lowest order, $n = 1$). By taking the higher order ($n = 2$), this problem was removed. So we obtain good results for more reasonable values for the quark and sigma masses, $m_q = 400 - 600$ MeV and $m_\sigma \geq 700$ MeV, which are consistent with several soliton models. Good results are obtained for $m_q = 400$ MeV and $m_\sigma = 1370$ MeV, as seen in Table 4, where the nucleon observables are better reproduced than by Birse and Banerjee [2] who used mesonic potential of the order $n = 1$ (see Eq. (6)). In particular, we obtain reasonable values for $\sigma(\pi, N)$. It is a very attractive feature of our model in comparison with the results of Refs. [2], [5] and [7] (see Table 4).

*TABLE 1. Values of magnetic moments of the nucleons (in nuclear magnetons) and $\sigma(\pi, N)$ (in MeV) for $m_q = 400$ MeV.*

| $m_\sigma$ (MeV) | 700 | 900 | 1100 | 1300 | 1370 | Exptl. [3] |
|---|---|---|---|---|---|---|
| Sigma term $\sigma(\pi N)$ | 132 | 106 | 93 | 82 | 51 | $35 \pm 10$ |
| $\mu_{\text{proton}}$ | 2.653 | 2.711 | 2.74 | 2.771 | 2.776 | 2.79 |
| $\mu_{\text{neutron}}$ | –1.959 | –2.064 | –2.12 | –2.164 | –2.172 | -1.91 |

*TABLE 2. Values of magnetic moments of the nucleons (in nuclear magnetons) and $\sigma(\pi, N)$ (in MeV) for $m_q = 500$ MeV.*

| $m_\sigma$ (MeV) | 700 | 900 | 1100 | 1300 | 1370 | Exptl. [3] |
|---|---|---|---|---|---|---|
| Sigma term $\sigma(\pi N)$ | 144 | 122 | 108 | 94 | 68 | $35 \pm 10$ |
| $\mu_{\text{proton}}$ | 2.854 | 2.867 | 2.874 | 2.877 | 2.876 | 2.79 |
| $\mu_{\text{neutron}}$ | –2.188 | –2.234 | –2.266 | –2.287 | –2.290 | -1.91 |

*TABLE 3. Values of magnetic moments of the nucleons (in nuclear magnetons) and $\sigma(\pi, N)$ (in MeV) for $m_q = 600$ MeV.*

| $m_\sigma$ (MeV) | 700 | 900 | 1100 | 1300 | 1370 | Exptl. [3] |
|---|---|---|---|---|---|---|
| Sigma term $\sigma(\pi N)$ | 133 | 124 | 114 | 104 | 76 | $35 \pm 10$ |
| $\mu_{\text{proton}}$ | 2.964 | 2.942 | 2.930 | 2.924 | 2.921 | 2.79 |
| $\mu_{\text{neutron}}$ | –2.344 | –2.334 | –2.341 | –2.351 | –2.353 | -1.91 |

*TABLE 4. Values of magnetic moments of the nucleons (in nuclear magnetons) and $\sigma(\pi, N)$ (in MeV), calculated by Birse and Banerjee ([2]), T. S. T. Aly ([5]) and Rashdan et al. ([7]), compared with our calculations (for $m_q = 400$ MeV)*

| Quantity | [2] | [5] | [7] | Present work | Exptl. [3] |
|---|---|---|---|---|---|
| $\sigma(\pi N)$ | 92.5 | 88.9 | 88 | 51 | $35 \pm 10$ |
| $\mu_{\mathrm{proton}}$ | 2.76 | 1.71 | 2.797 | 2.776 | 2.79 |
| $\mu_{\mathrm{neutron}}$ | –2.06 | –1.31 | –2.06 | –2.172 | -1.91 |

## 5. Conclusion

The present calculation show the importance of mesonic correlations which may be of the higher order than what is normally used in most soliton models. The breaking of the chiral symmetry is verified, so the model has shifted from the chiral limit.

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# N$\pi$ RASPRŠENJE I ELEKTROMAGNETSKE POPRAVKE U PROŠIRENOM LINEARNOM SIGMA MODELU

Proučavamo nedavne analize raspršenja pion–nukleon i magnetske momente nukleona u proširenom linearnom sigma modelu. Jednadžbe polja riješili smo u približenju srednjeg polja. Postigli smo bolji sklad s eksperimentalnim podacima nego raniji radovi.