Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

FLUCTUATION OF PIONS IN RELATIVISTIC AND ULTRARELATIVISTIC NUCLEAR COLLISIONS - SCALE DEPENDENT OR NOT?

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Received 2 August 2006; Revised manuscript received 18 May 2007
Accepted 20 June 2007 Online 12 July 2007

Fluctuation pattern of pions is investigated in a wide range of projectile energy from 4.5 AGeV (^{24}Mg-AgBr interactions) to 200 AGeV (^{32}S-AgBr interactions). Two-dimensional analysis is performed. To obtain the correct phase-space partition condition considering anisotropy of phase space, we use the concept of Hurst exponent H. The analysis is performed in a rigorous way by fitting one-dimensional factorial moment saturation curves. The effective fluctuation strength α_{eff} is calculated. The study reveals that the fluctuation pattern is scale-dependent at both relativistic and ultrarelativistic energies.

PACS numbers: 25.75.-q, 24.60.Ky UDC 539.172

Keywords: nuclear collisions, fluctuation, self-affinity, self-similarity, multi-particle production, scale dependence, phase-space dependence, effective fluctuation strength

1. *Introduction*

The anomalous scaling of scaled factorial moments (SFM's) or intermittency [1] have been studied extensively with the aim of exploring the possible existence of dynamical fluctuations or multifractal structure of multiparticle spectrum in the high-energy collision process. Although the method of intermittency is introduced to understand the unusual "spiky" events in pseudorapidity spectra [2 – 4], the study of particle density fluctuation in different phase space domains can reveal new insights into the underlying dynamics of multi-hadron production process. The pioneers Bialas and Peschansky [1] suggested the behaviour of the scaled factorial moments in analogy with the phenomenon known as intermittency in the hydrodynamics of turbulent fluid flow (Mandelbrot, Ref. [5]). The main advantage of this method is that it disentangles the statistical noise, which contaminates the dynamical fluctuation, and measures only the non-statistical fluctuation. The corresponding experiments have been preformed in various kinds of collisions, e^+–e^-, hadron–hadron, hadron–nucleus and nucleus–nucleus. Most of the studies of intermittency have been performed in one dimension. But the actual process takes places in three dimensions. It is generally believed that the fluctuations should be studied in higher dimensions instead of one dimensional (rapidity) analysis. Ochs [6] pointed out that in the lower-dimensional projection, the fluctuation gets reduced by the averaging process. The projection effect may completely wash out the self-similar nature of fluctuation. Thus, the analysis should be done in higher dimensions to reduce the error due to dimensional reduction.

The usual procedure for analyzing higher dimension, in particular two-dimensional intermittency, is to divide the corresponding phase space subsequently into sub-cells by shrinking equally in each dimension, considering the phase space to be isotropic. This corresponds to *self-similar* fractal structure [7]. However, the phase space in multiparticle production is generally found to be anisotropic [8]. The influence of anisotropic phase space on nonstatistical fluctuation should be considered. For this reason, we cannot simply expect the fluctuation or scaling properties to be the same in both directions. According to Mandelbrot [9], a pattern scaled differently in different direction is called *self-affine* fractal. Since both one- and two-dimensional self-similar evolution are not good candidates for the description of multiparticle production the next possibility is two dimensional self-affinity.

Investigations in high-energy nuclear collisions are generally carried out on the produced pions, because these particles are believed to be the most informative about the collision dynamics. In our earlier works, we have investigated the fluctuation pattern of ^{24}Mg-AgBr interaction at 4.5 AGeV and ^{32}S-AgBr interaction at 200 AGeV in two dimension and the fluctuation pattern was found to be self-affine with Hurst exponent $H = 0.4$ and 0.7, respectively [10]. However, the question arises, is the fluctuation pattern different at different scales? Analysis is carried out at two different projectile energies to find out whether the scale dependence of fluctuation pattern of pions is a general observation and whether the dependence is different at different projectile energies.

In our earlier works, we have optimized the value of H by finding the minimum of $\chi^2/d.o.f.$ of the linear fit to the plots of $\ln\langle F_q\rangle$ to $\ln M$ [11]. But in that case the corresponding errors should be estimated with utmost care keeping in mind that the data points are correlated since they stem from the same data sample which is quite difficult. Though different approaches for error calculation have been prescribed, none of them has been claimed to give a correct estimation. The estimation of errors is nontrivial and the accuracy of the method of determination of depends on how accurately the errors are determined. The robustness of the method of determination of H was questioned by some physicists, hence, in this paper we deal with the problem in a more rigorous manner by fitting one-dimensional F_2 vs. M saturation curves as done by Agababyan et al. [12]. The value of H is extracted, two-dimensional analysis is performed and values of intermittency exponent and effective fluctuation strength are calculated. The study reveals that the fluctuation pattern is scale-dependent as well as phase-space dependent.

2. Experimental details

The data set used in this analysis were obtained by exposing NIKFI BR2 nuclear-emulsion plates to ^{24}Mg beam at 4.5 AGeV at JINR Dubna, Russia and G5 nuclear emulsion plates to S beam with energy 200 AGeV from CERN SPS. The particles emitted after interaction are classified as :

(i) Black particles which are constituted of target fragments. They have range < 3 mm and velocity $< 0.3\,c$.

(ii) Grey particles which are constituted of recoil protons with energy upto 400 MeV. They have range > 3 mm and velocity between 0.3 c and 0.7 c.

(iii) Shower particles which are constituted mainly of produced pions. They have velocity $> 0.7\,c$.

Along with these tracks there are few projectile fragments. They generally lie within 3° w.r.t. the mean beam direction. Great care is taken to identify the projectile fragments.

A total 800 events for ^{24}Mg beam ($\langle n_s \rangle = 9.99$) and 140 events for ^{32}S beam ($\langle n_s \rangle = 93.12$) with $n_h \geq 8$ were selected for the present analysis as genuine ^{24}Mg-AgBr and ^{32}S-AgBr interactions ($n_h = n_b + n_g$) where n_s, n_b and n_g are average multiplicities of the shower, black and grey particles, respectively. The details of the data, scanning and measurement procedure is given in our earlier papers [11, 13]. It is worthwhile to mention that the emulsion technique possesses a very high spatial resolution which makes it a very effective detector, though with a limited statistics.

3. Method of study

For two-dimensional analysis of fluctuation pattern of pions, we first consider the two-dimensional factorial moment defined by the relation (1). In the two-dimensional phase space, if the two phase space variables are x_1 and x_2, the factorial moment of order q, F_q is given by the relation [1]

$$F_q(\delta x_1, \delta x_2) = \frac{1}{M'} \sum_{m=1}^{M} \frac{n_m(n_m-1)\cdots(n_m-q+1)}{\langle n_m \rangle^q} \qquad (1)$$

where $\delta x_1 \times \delta x_2$ is the size of a two-dimensional cell, n_m is the multiplicity in the m^{th} cell, M' is the number of two-dimensional cells into which the considered phase-space has been divided.

In the two-dimensional space (say denoted as (x_1, x_2)) we make self-affine transformation $\delta x_1 = \Delta x_1 / M_1$ and $\delta x_2 = \Delta x_2 / M_2$, where $M_1 \neq M_2$ and $M' = M_1 \cdot M_2$. Here M_1 and M_2 are the scale factors that satisfy the equation

$$M_1 = M_2^H \qquad (2)$$

where the parameter H is called the Hurst exponent [14]. The roughness factor or the Hurst exponent H is given by

$$H_{12} = \frac{\ln M_1}{\ln M_2} \tag{3}$$

with $M_1 \le M_2$ and $0 \le H \le 1$.

It is the parameter which is characterizing the anisotropy of the system under study. For $H = 0$, $M_1 = 1$, and the scaling property does not exist in that direction. For $H = 1$, $M_1 = M_2$, the self-affine transform reduces to a self-similar one, i.e., the system is isotropic in these two directions. For $0 < H < 1$, the nontrivial self-affine fractality exists. The Hurst exponent can be deduced from the data by fitting the one-dimensional second-order factorial moment saturation curves [6],

$$F_2^{(i)}(M_i) = a_i - b_i M_i^{-c_i} \qquad \text{where} \qquad i = 1, 2 \tag{4}$$

as prescribed by Agababyan et al. [12].

We have performed our analysis in two dimensional (η, ϕ) space. The Hurst exponent can be determined from the from the parameter c_i as

$$H_{ij} = \frac{1 + c_j}{1 + c_i} \tag{5}$$

As mentioned earlier, $0 \le H \le 1$, and

$$\text{if} \quad c_\eta > c_\phi, \quad \text{then} \quad H = \frac{1 + c_\phi}{1 + c_\eta} \quad \text{and} \quad M_\eta = M_\phi^H \tag{6}$$

$$\text{if} \quad c_\eta < c_\phi, \quad \text{then} \quad H = \frac{1 + c_\eta}{1 + c_\phi} \quad \text{and} \quad M_\phi = M_\eta^H$$

It is clear from Eq. (2) that the scale factors M_η and M_ϕ cannot simultaneously be integer, so that the size of elementary phase space cell can be take continuously varying values.

To perform the analysis with non-integral value of scale factor (M), we adopt the following method. For simplicity, consider one-dimensional space (y) and let

$$M = N + a \tag{7}$$

where N is an integer and $0 \le a < 1$. When we use the elementary bin of width $\delta y = \Delta y / M$ as 'scale' to 'measure' the region Δy, we get N of them and a smaller bin of width $a \Delta y / M$ left. Putting the smaller bin at the last (or first) place of the region and doing the average with only the first (or last) N bins, we have

$$F_q(\delta x) = \frac{1}{N} \sum_{m=1}^{N} \frac{\langle n_m (n_m - 1) \cdots (n_m - q + 1) \rangle}{\langle n_m \rangle^q} \tag{8}$$

M determined by Eq. (7) can be any positive real number and so can vary continuously.

Our work has been performed in two-dimensional pseudorapidity − azimuthal angle (w.r.t. the beam direction) space. As shape of this distribution influences the scaling behaviour of the factorial moments, we have used the "cumulative" variables X_η and X_ϕ instead of η and ϕ [15]. The corresponding region of investigation for both variables then becomes $(0, 1)$. The cumulative variable $X(x)$ is given by the relation

$$X(x) = \frac{\int_{x_1}^{x} \rho(x')\partial x'}{\int_{x_1}^{x_2} \rho(x')\partial x'} \tag{9}$$

where x_1 and x_2 are two extreme points in the distribution $\rho(x)$, between which X varies from 0 to 1. However, to avoid confusion we have called the variables X_η and X_ϕ as η and ϕ, and later in this paper, wherever we mention η and ϕ, we actually mean X_η and X_ϕ.

The intermittent behaviour of multiplicity distribution manifests itself as the power-law dependence of factorial moment on the cell size as the cell size $\to 0$,

$$\langle F_q \rangle \propto (\delta\eta\delta\phi)^{-\alpha_q} . \tag{10}$$

The exponent α_q is the slope characterizing linear rise of $\langle \ln F_q \rangle$ with $-\ln(\delta\eta\delta\phi)$. The strength of the intermittency is characterized by the exponent α_q, and can be obtained from a linear fit of the form

$$\ln\langle F_q \rangle = -\alpha_q \ln(\delta\eta\delta\phi) + A \tag{11}$$

where A is a constant.

The above anomalous scaling of F_q with non vanishing indices α_q is an evidence for the existence of the dynamical fluctuation.

The multifractal dimension D_q is defined by

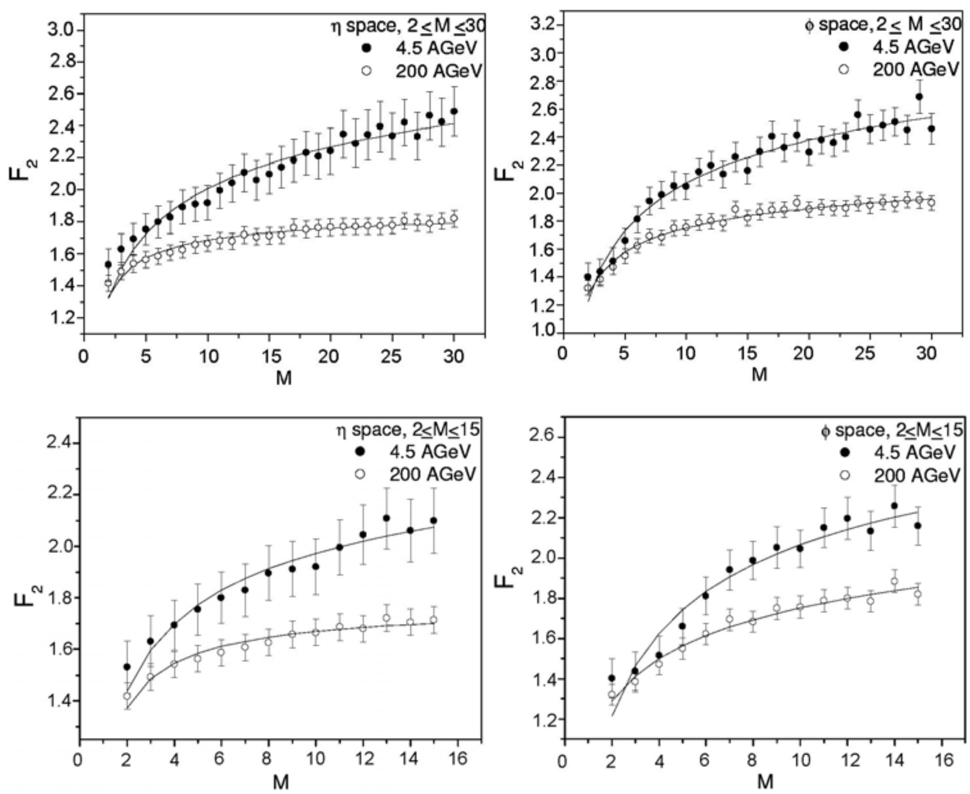
$$D_q = 1 - \alpha_q/(q - 1) \tag{12}$$

Both the intermittency exponent α_q and multifractal dimension D_q are related to the strength of the dynamical fluctuations. But they have their own physical meaning and, hence, cannot be taken directly as a measure of the fluctuation strength. Liu Lianshou, Fu Jinghua and Wu Yuanfang [16] proposed a method of determining the strength of fluctuation from the intermittency exponent α_q and multifractal dimension D_q. The strength of the fluctuation is given by $\sqrt{\frac{6 \ln 2}{q}(1 - D_q)}$. However, for $q = 2$ this formula reduces to the simpler form $\sqrt{3 \ln 2(1 - D_2)} \approx \sqrt{2(1 - D_2)}$. Thus the strength of the fluctuation can be obtained from above formula. The formula for the strength of fluctuation is obtained from the α cascading model. For the general case, when the underlying dynamics is unclear, it is really difficult to obtain the expression for the strength of the fluctuation. However, we can obtain an approximate estimation of the fluctuation strength. For an arbitrary process, the effective fluctuation strength is given by

$$\alpha_{\text{eff}} = \sqrt{2(1 - D_2)} = \sqrt{2\alpha_2} \qquad (13)$$

4. Results and discussion

To perform the two-dimensional analysis of fluctuation pattern of pions, we must first find the correct partition condition in η and ϕ directions with the Hurst exponent taking care of the anisotropy of phase space. For extracting the value of H we calculate first the one-dimensional factorial moment F_2 for η and ϕ space for $2 \le M \le 30$, $2 \le M \le 15$, $15 \le M \le 30$, $10 \le M \le 20$ and $12 \le M \le 25$ for both the interactions. The parameters a, b and c can be calculated by fitting the data in Fig. 1 using Eq. (4). We have used the Marquardt-Levenberg algorithm to find the parameters that give the best fit of Eq. (4) to the data. This algorithm seeks the values of the parameters that minimize the sum of the squared differences between



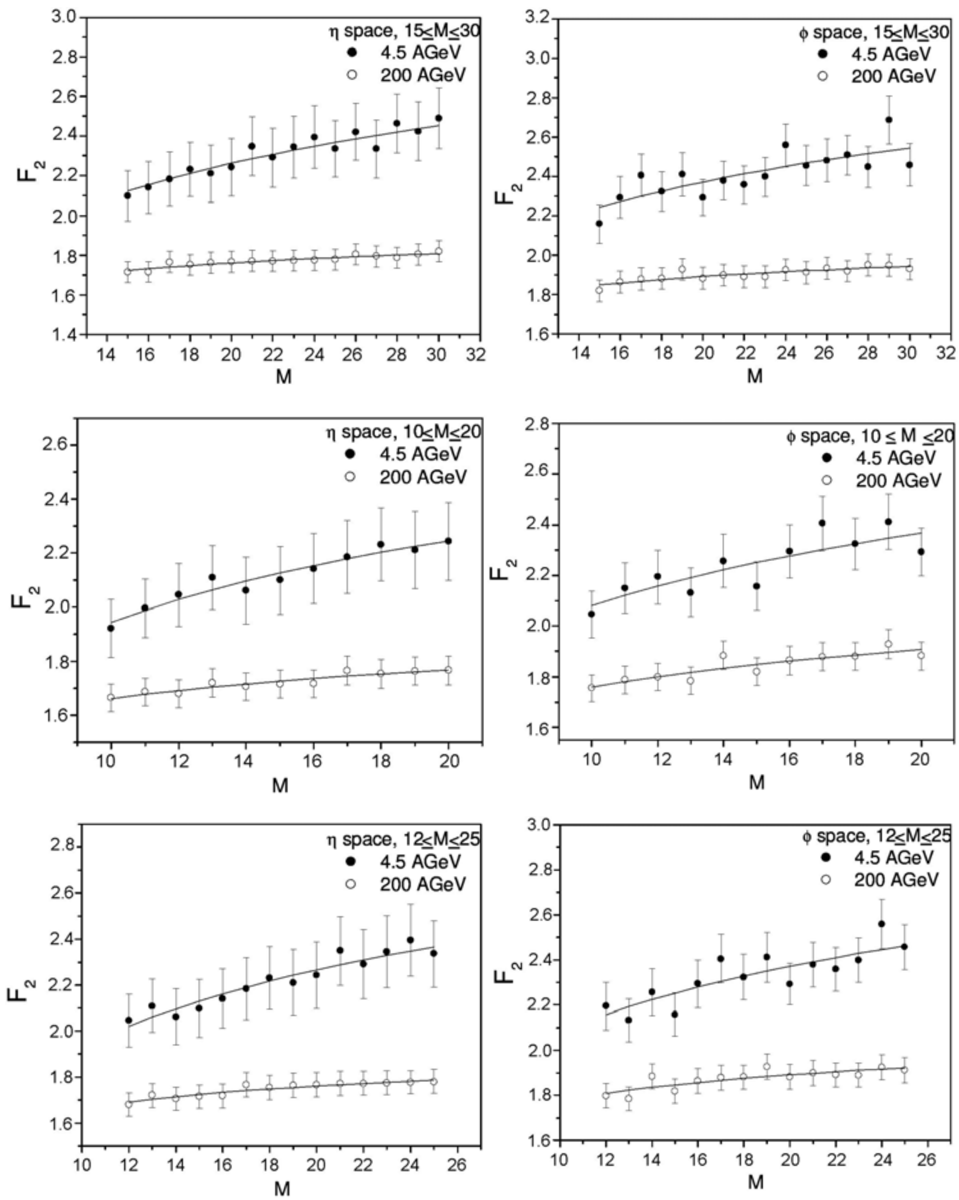


Fig. 1. Plot of F_2 vs. $\ln M$ for both interactions in η and ϕ space in bin ranges: Page at left (top) $2 \le M \le 30$, (bottom) $2 \le M \le 15$, this page (top) $15 \le M \le 30$, (middle) $10 \le M \le 20$ and (bottom) $12 \le M \le 25$.

the values of the observed and predicted values of the dependent variable. This process is iterative. We begin with aguess of the parameters, the process contin-ues until convergence is achieved. We have performed an un-weighted least-squares curve fit. The values of the parameters a, b, c and the calculated values of H are provided in Table 1. The correct partition condition in η and ϕ directions can be obtained according to Eq. (6). Two-dimensional factorial moments of the second order are then calculated. The value of intermittency exponent α_2 can be extracted from the slope of the linear fit in the plot of $\ln\langle F_2\rangle$ vs. $-\ln(\delta\eta\delta\phi)$. $\chi^2/d.o.f.$ is

TABLE 1. Values of the parameters a, b, c and H for different ranges of M for both interactions.

^{24}Mg-AgBr interaction at 4.5 GeV					
M	Space	a	b	c	H
$2 \leq M \leq 30$	η	5.803±0.423	4.814±0.413	0.103±0.011	0.960±0.011 → 1.0
	ϕ	5.816±0.219	4.393±0.207	0.149±0.010	
$2 \leq M \leq 15$	η	2.882±0.084	1.762±0.072	0.288±0.022	0.988±0.033 → 1.0
	ϕ	3.621±0.168	2.907±0.148	0.272±0.025	
$15 \leq M \leq 30$	η	7.282±0.481	6.675±0.453	0.095±0.010	0.968±0.027 → 1.0
	ϕ	5.726±0.546	4.961±0.485	0.131±0.022	
$10 \leq M \leq 20$	η	3.959±0.715	3.450±0.507	0.233±0.009	0.886±0.025 → 0.9
	ϕ	6.698±1.104	5.707±1.101	0.092±0.023	
$12 \leq M \leq 25$	η	4.245±0.854	3.936±0.587	0.229±0.009	0.876±0.021 → 0.9
	ϕ	7.750±1.267	6.753±1.230	0.076±0.019	
^{32}S-AgBr interaction at 200 GeV					
M	Space	a	b	c	H
$2 \leq M \leq 30$	η	1.905±0.045	0.865±0.042	0.599±0.011	0.836±0.032 → 0.9
	ϕ	2.412±0.149	1.436±0.105	0.337±0.007	
$2 \leq M \leq 15$	η	1.785±0.056	0.715±0.062	0.791±0.023	0.630±0.097 → 0.6
	ϕ	3.777±0.256	2.718±0.249	0.128±0.016	
$15 \leq M \leq 30$	η	2.079±0.092	1.026±0.062	0.393±0.012	0.979±0.069 → 1.0
	ϕ	2.221±0.129	1.168±0.138	0.423±0.017	
$10 \leq M \leq 20$	η	2.110±0.131	1.089±0.156	0.385±0.013	0.935±0.088 → 0.9
	ϕ	2.565±0.451	1.592±0.216	0.295±0.018	
$12 \leq M \leq 25$	η	2.017±0.064	1.056±0.089	0.473±0.011	0.980±0.066 → 1.0
	ϕ	2.218±0.157	1.232±0.146	0.443±0.020	

also calculated. The $\chi^2/d.o.f.$ values are obtained by considering the statistical errors calculated independently of each point and they form the diagonal terms of the complete covariance matrix. The off-diagonal terms of the covariance matrix arise due to the correlation between data points. To have a detailed idea of the full covariance matrix, this correlation between data points should be taken into account. However, it is seen in different works [12] that the contributions to the $\chi^2/d.o.f.$ values come mainly from the diagonal terms of the complete covariance matrix. The changes in the values of $\chi^2/d.o.f.$ are insignificant when the effects of the off-diagonal terms are considered.

$\alpha_{\rm eff}$ is calculated using the Eq. (13). The respective values are provided in Table 2. The variation of $\alpha_{\rm eff}$ with the different bins (scales) for both the interactions is shown in Fig 2. Both at 4.5 AGeV and 200 AGeV, the degree of anisotropy is different at different scales. At 4.5 AGeV, the fluctuation pattern is self similar in bins $2 \le M \le 30$, $2 \le M \le 15$ and $15 \le M \le 30$, while it is self-affine in bins $10 \le M \le 20$ and $12 \le M \le 25$. High values of the Hurst exponent suggest that the anisotropy is not strong in this case.

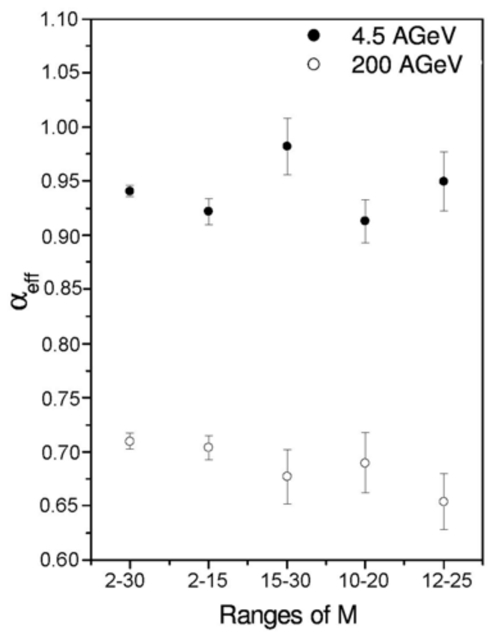


Fig. 2. Plot of $\alpha_{\rm eff}$ vs. ranges of M for both interactions.

However, in the case of ^{32}S-AgBr interactions at 200 AGeV, the fluctuation pattern is self-similar in bins $15 \le M \le 30$ and $12 \le M \le 25$, while it is self-affine in bins $2 \le M \le 30$, $2 \le M \le 15$ and $10 \le M \le 20$. Anisotropy in η and ϕ directions is pronounced for the bin $2 \le M \le 15$, while for $2 \le M \le 30$ and $10 \le M \le 20$ the anisotropy is not very strong in η and ϕ directions. Thus, we find that the fluctuation pattern is scale-dependent both at the relativistic and ultra-relativistic energies and the scale dependence w.r.t. degree of anisotropy is more pronounced at ultra-relativistic energies.

For ^{24}Mg-AgBr interactions at 4.5 AGeV, when a given range of values of M, $2 \leq M \leq 30$ is divided into sub-intervals, then the values of $\alpha_{\rm eff}$ varies significantly for the bins $2 \leq M \leq 15$, $15 \leq M \leq 30$ and $10 \leq M \leq 20$, while the change is not so prominent for the bin range $12 \leq M \leq 25$. A non-monotonic behaviour of the fluctuation strength is revealed in this case. However at 200 AGeV, a significant change from the values of $\alpha_{\rm eff}$ $2 \leq M \leq 30$ is noticed for the bins $15 \leq M \leq 30$ and $12 \leq M \leq 25$. The change is not pronounced for bins $2 \leq M \leq 15$ and $10 \leq M \leq 20$. At 200 AGeV, the fluctuation strength is found to be more in those bins where self-affinity is observed. But unlike in the case of ^{24}Mg-AgBr interactions at 4.5 AGeV, $\alpha_{\rm eff}$ decreases when a given interval is spilt into subintervals and no non-monotonic behaviour is revealed. From the values of $\alpha_{\rm eff}$ provided in Table 2 as well as in Fig. 2, we observe that the effective fluctuation strength as well as its scale dependence are more pronounced at 4.5 AGeV. Thus we can summarize as follows:

a) The fluctuation pattern of pions is scale dependent both at relativistic and ultra-relativistic energies.

b) The scale dependence w.r.t. the degree of anisotropy is more pronounced at ultra-relativistic energies.

c) The scale dependence w.r.t. the fluctuation strength is more pronounced at the lower energy.

TABLE 2. Values of a_2, $\alpha_{\rm eff}$ and $\chi^2/d.o.f.$ for different ranges of M for both interactions.

^{24}Mg-AgBr interaction at 4.5 GeV			
M	α_2	$\alpha_{\rm eff}$	$\chi^2/d.o.f.$
$2 \leq M \leq 30$	0.443±0.005	0.941±0.005	0.224
$2 \leq M \leq 15$	0.425±0.011	0.922±0.012	0.373
$15 \leq M \leq 30$	0.482±0.026	0.982±0.026	0.138
$10 \leq M \leq 20$	0.417±0.018	0.913±0.020	0.067
$12 \leq M \leq 25$	0.451±0.026	0.950±0.027	0.178
^{32}S-AgBr interaction at 200 GeV			
M	α_2	$\alpha_{\rm eff}$	$\chi^2/d.o.f.$
$2 \leq M \leq 30$	0.252±0.005	0.710±0.007	0.419
$2 \leq M \leq 15$	0.248±0.008	0.704±0.011	0.268
$15 \leq M \leq 30$	0.229±0.017	0.677±0.025	0.133
$10 \leq M \leq 20$	0.238±0.019	0.690±0.028	0.234
$12 \leq M \leq 25$	0.214±0.017	0.654±0.026	0.274

Acknowledgements

We thank Prof. K. D. Tolstov of JINR, Dubna and Prof. P. L. Jain, State University of New York at Buffalo, U.S.A. for giving us the exposed emulsion plates. One of our authors Srimonti Dutta (J.R.F) gratefully acknowledges financial support of CSIR (India).

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JESU LI KOLEBANJA PIONA U RELATIVISTIČKIM I ULTRARELATIVISTIČKIM NUKLEARNIM SUDARIMA OVISNA O LJESTVICI?

Proučavamo kolebanje piona nastalih u sudarima ^{24}Mg-AgBr na 4.5 AGeV i ^{32}S-AgBr na 200 AGeV za široko područje energije. Načinili smo dvodimenzijske analize. Zbog anizotropije faznog prostora, primijenili smo Hurstov eksponent H radi postizanja ispravne razdjele faznog prostora. Proveli smo strog račun prilagodbe jednodimenzijskih krivulja zasićenja faktorijalnih momenata. Izračunali smo efektivnu jakost kolebanja $\alpha_{\rm eff}$. Našli smo ovisnost slike kolebanja o ljestvici za relativističke i ultrarelativističke energije.