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GEOMETRICAL HIGGS MASS AND BOSON STARS FROM COMPLEX SCALAR FIELD BRANS-DICKE GRAVITY

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We discuss Higgs gauge symmetry breaking and mass stability, Brans-Dicke scalar gravity, dark energy and dark matter within the framework of non-minimal coupling, where the observable cosmological constant was shown to be the sum of the vacuum (Λ_{vac}) and the induced term $\Lambda_{ind} = -3m^2/4$ with m being the ultra-light mass ($\approx$ the Hubble parameter) implemented in the theory from supergravities arguments. We show that the dynamical Higgs mass and the generated electron mass are geometrical, depending on Λ_{vac} and m, on condition that the scalar curvature and the non-minimal coupling parameter are of opposite signs. In order to take into account the dark matter and dark energy, we have generalized the theory by adding a decreasing exponential complex potential to the Higgs-like complex scalar one. We show that the whole clusters, galaxies physical scenarios and boson stars may depend also on the sign of the scalar curvature and on the non-minimal coupling constant.

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1. Introduction

It is well-known that the universe is accelerating with time, spatially approaching de-Sitter regime and is filled with strange dark matter and dark energy responsible of its accelerated expansion. From the theoretical point of view, one can interpret the dark energy either by vacuum energy (cosmological constant), or by weakly interacting massive particles (WIMPs), or as a slowly-varying scalar field energy. Of course, some other alternative theories have been suggested to explain the cosmological observations and their corresponding problems, in particular the cosmological constant problem, the dark matter and the rotating spiral galaxies problem (string cosmology, network of topological defects, extra-dimensions, brane world, extended supergravity models, quintessence scalar fields, complex scalar field, dark fluid, etc) [1 − 7]. All these phenomenological and conceptual models and problems need, in fact, to be further clarified. From the observational point of view, there is no direct evidence for their existence causing some kind of difficulties to actual dark cosmological models. In recent years, a lot of articles have been devoted to the investigation of the cosmological models with the non-minimal coupling between gravity and inflaton complex scalar field and to their important connection with inflationary cosmology. We have shown that in the case of non-minimal coupling between the scalar curvature and the density of the complex scalar field such as $L = -(\xi/2)\sqrt{-g}\, R\phi\phi^*$ (R is the scalar curvature or the Ricci scalar) and for a particular scalar complex potential field $V(\phi\phi^*) = (3m^2/4)(\omega\phi^2\phi^{*2} - 1)$ (ω is a tiny parameter), inspired from supergravity inflation theories, ultra-light masses m are implemented naturally in the Einstein field equations (EFE), leading to a cosmological constant Λ in accord with observations and contributing to the dark energy problem [8]. The metric tensor of the spacetime is treated as a background and the Ricci scalar in the non-minimal coupling term, regarded as an external parameter, was found to be $R = 4\Lambda - 3m^2 = 4\bar{\Lambda}$ where $\bar{\Lambda} = \Lambda - (3m^2/4)$ is the effective cosmological constant. That is to say, there is another induced contribution to the vacuum cosmological constant with $\Lambda_{\text{induced}} = -(3m^2/4)$. These tiny ultra-light masses have the desirable feature for the description of the accelerated universe and contribute to the dark energy and matter problem. They occur in supergravity quintessence model with the de-Sitter minimum and have important additional features in astrophysics, cosmology, the early universe and the standard electroweak model [9, 10]. It is worth-mentioning that within the framework of classical general relativity, the presence of a positive cosmological constant implies the existence of a minimal mass and of a minimal density in nature [11]. Adopting this result, we explore some of the consequences concerning the presence of these ultra-light bosons in the field equations. Ignoring all gauge coupling, the Klein-Gordon equation (KGE) with no mass term associated with this choice reads

$$\Box\phi - \frac{R}{6}\phi - \frac{\partial V}{\partial \phi^*} = 0\,. \tag{1}$$

In fact, a possible mass term for the scalar field and the cosmological constant are embodied in our quartic complex potential $V(\phi\phi^*)$ and our scalar curvature $R = 4\Lambda - 3m^2$. By substituting into Eq. (1), we obtain

$$\Box\phi - M^2\phi = 0\,, \tag{2}$$

where $M^2 = \tilde{M}^2 + 2h\phi\phi^*$, $\tilde{M}^2 = 2\Lambda/3 - m^2/2 = \xi R$ and $h = 3\omega m^2/4 > 0$. From Eqs. (1) and (2), if we treat the Ricci scalar as a parameter, the potential takes the following form $\tilde{V}(\phi\phi^*) = (2\Lambda - (3m^2/2))\xi\phi\phi^* + (h/2)(\phi\phi^*)^2$. If we assume that the Higgs's complex scalar is chargeless, then the Higgs scalar can only be interpreted as gravitational. Such a coupling could play a major role in gravitational interaction transfer [12]. For the particular case $4\Lambda = 9m^2$ and $\omega = 0$, we have $R = 6m^2 > 0$, and as a result we can obtain $(\Box - R/6)\phi = 0$. The Higgs field is then supplanted by a massless KGE conformally coupled to the scalar curvature with a tiny real rest mass. Consequently, for $h = \Lambda = 0$, the effective cosmological constant is negative, spontaneous symmetry breaking (SSB) still occurs and is controlled by the presence of the term m^2. For a homogeneous isotropic curvature Ricci positive scalar, that is $4\Lambda < 3m^2$, there exists a non-trivial minimum distributed on the circle $|\phi| = \sqrt{-\xi R/(2h)} := \mathrm{v}\sqrt{2}$ and we have a gauge SSB. In other words, the presence of ultra-light particles is responsible for inducing the SSB provided that $4\Lambda < 3m^2$. A very small cosmological constant is, therefore, equivalent to the almost flatness of the cosmological spacetime and consequently the SSB do not need to be altered.

In order to see how the model accommodates dark energy, instead of complex scalar field, we use alternative field variables by assuming $\varphi(x) = \phi(x)\exp(\mathrm{i}\theta(x))$ and more precisely $\varphi(t) = \phi(t)\exp(\mathrm{i}\theta(t))$. We write the Higgs potential in our theory on its usual form as $V(\varphi) = (1/2)\mu^2\varphi^2 + (\lambda/4)\varphi^4$, where $\mu^2 = (4\Lambda - 3m^2)\xi$ and $\lambda = 3\omega m^2/2 > 0$ and we refer to the flat FRW universe strongly favoured by cosmological observations and where, from the theoretical point of view, the inflaton is distributed homogeneously. In addition, we assumed that it is free of matter. Remember that recent observations suggest that up to 90% of our observed universe is empty. The resulting, pressure and density for the inflaton scalar field are [13–16]

$$p=\frac{1}{2}\dot{\varphi}^2-\frac{1}{2}(4\Lambda-3m^2)\xi\varphi^2-\frac{3}{8}\omega m^2\varphi^4-\xi\left[4H\varphi\dot{\varphi}+2\dot{\varphi}^2+2\varphi\ddot{\varphi}+(2\dot{H}+3H^2)\varphi^2\right] \quad (3)$$

$$\rho = \frac{1}{2}\dot{\varphi}^2 + 3\xi H\varphi(H\varphi + 2\dot{\varphi}) + \frac{1}{2}(4\Lambda - 3m^2)\xi\varphi^2 + \frac{3}{8}\omega m^2\varphi^4 \quad (4)$$

where κ is the gravitational constant and $H = \dot{a}/a$ is the Hubble parameter, $a(t)$ being the scale factor. Clearly, the presence of an effective cosmological constant $\kappa_{\mathrm{eff}} = \kappa(1 - \kappa\xi\varphi^2)$ and the critical values of the scalar field $\varphi_c = \pm(\kappa\xi)^{-1/2}$ for $\xi > 0$. The ultra-light field is usually known as a high-energy field not related to the Standard Model, i.e., $\varphi \approx M_\alpha$ where M_α is some high energy scale around the Planck mass [17]. The effective state equation of the model at one of the following critical values $\varphi = \pm\varphi_c$ will take the following form $w(t) \approx -4\,(1+\Delta)\,/\,(1+4\Delta)$, $\Delta = \xi(4\Lambda - 3m^2)/\left(3\omega m^2\varphi_c^2\right) \equiv \widetilde{\varphi}^2/\varphi_c^2$ where $\widetilde{\varphi}^2 = \xi(4\Lambda - 3m^2)/(3\omega m^2)$ and $H \ll 1$. For a positive (negative) coupling parameter and a positive (negative) scalar curvature, assuming $\Delta \gg 1$ (for $\omega \ll 1$) yields $w \approx -1$ (vacuum energy) and superacceleration always occurs. While for $\Delta \ll 1$ ($R \approx 0$ or $\xi \ll 1$) $w \approx -4$ (phantom field), corresponding to a superluminal case. This shows briefly some of the contributions of the ultra-light masses to the dark-energy problem. If the ultra-light mass field is time-dependent, it will help us certainly to understand more about the cosmological-constant problem as well as the coincidence problem, and, maybe, about neutrino condensation [18].

2. *Symmetry and generalized Brans-Dicke gravity*

In fact, the two fundamental paradigms of modern physics are the Big Bang cosmology and the standard $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ model of the strong and electroweak interactions. Two key ingredients were recently added: old and new inflation on the cosmological side and axion as a pseudo Goldstone boson associated with the spontaneous breakdown of the Peccei-Quinn symmetry in particle physics. Inflation requires the existence of dark matter, and axions have long been candidates for the cold dark matter, even though there is no physical reason why the two notions should be related at all [19]. In fact, cosmology with ultra-light pseudo-Nambu-Goldstone bosons was explored in the literature. It was suggested that with global SSB scale of about 10^{18} GeV and explicit breaking scale to be in a mass range whose upper bound is of about 10^{-3}eV, the field acquires a mass of the order of the Hubble parameter, dominating the energy density of the present Universe [20]. Moreover, it was recently argued that one of the theoretical approaches to solve the gauge hierarchy problem is a gravity-gauge-Higgs unification scenario in which the Higgs is identified as extra-dimensional components of the metric tensor field [20]. In reality, we still ignore where does this come from. A phenomenological explanation was proposed in the framework of inflation-axion scenario which concerns an unusual realization of the Higgs mechanism which converts the latent energy of the vacuum into mass or pseudo-Goldstone bosons [19]. It is worth-mentioning that gravity or gravitational fields may be considered as the Goldstone realization of a spontaneously broken diffeomorphism group in which Goldstone coordinates are considered as a dynamical fluid of reference [21]. A Higgs mechanism for gravity, in which an affine spacetime evolves into a Riemannian one by the condensation of a metric, has been recently constructed. The symmetry-breaking potential in Kirsch approach is identical to that of hybrid inflation, but with the non-inflaton scalar extended to a symmetric second-rank tensor required for the realization of the metric as a Higgs field. The introduction of a scalar Higgs potential and the SSB leads to a non-vanishing value of the vacuum energy which can be interpreted as a direct contribution to the cosmological constant in the Einstein-Hilbert action for the gravitational field.

In return to our scenario, the scalar field is complex (to ensure that the total charge of the field is conserved) and the scalar potential is connected to the scalar curvature and the non-minimal coupling; it will be of interest to study its role in alternative generalized gravity theories, in particular generalized Brans-Dicke gravity. It was argued more recently that a Brans-Dicke Lagrangian can be embedded in the flat spacetime electroweak theory. In other words, the Weinberg-Salam Lagrangian may be converted to a locally conformally flat spacetime into a generally covariant Lagrangian. The modulus of the Higgs field becomes the Brans-Dicke scalar field, the Higgs mass converts into a background scalar Ricci curvature, a gauge invariant vector field can acquire a mass even in the absence of SSB. Near the BPS limit, the cosmological constant of the embedded gravity can be very small and the Planck mass can be very large [22].

We expect naturally the appearance of electromagnetic components, in particular the electron charge in the theory described. Assuming for simplicity a $U\,(1)$ symmetry and the required special Higgs gauge transformation

$$\widetilde{\phi}(x) \to \tilde{\phi}'(x) = \eta(x) + \mathrm{v}/\sqrt{2},$$

$$A_\mu \to B_\mu = A_\mu + e^{-1}\nabla_\mu \zeta(x),$$

$$D_\mu = \nabla_\mu + \mathrm{i}eA_\mu \to D'_\mu = \nabla_\mu + \mathrm{i}eB_\mu,$$

where $\widetilde{\phi}(x)$ is the scalar field associated to the standard model, the action of the theory writes as [12].

$$S=\frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\Bigg[w\nabla_\mu\left(\eta+\sqrt{-\frac{\xi(4\Lambda-3m^2)}{2h}}\right)\nabla^\mu\left(\eta+\sqrt{-\frac{\xi(4\Lambda-3m^2)}{2h}}\right)+2\eta^2\xi(4\Lambda-3m^2)$$
$$-2\sqrt{-2\xi(4\Lambda-3m^2)\lambda}\,\eta^3-\lambda\eta^4+we^2B_\mu B^\mu\left(\eta+\sqrt{-\frac{2\xi}{h}(4\Lambda-3m^2)}\right)\eta$$
$$-w\frac{\xi(4\Lambda-3m^2)}{2h}e^2B_\mu B^\mu+\frac{\xi^2(4\Lambda-3m^2)^2}{4\lambda}\Bigg]+\int \mathrm{d}^4x\sqrt{-g}\,L_{\mathrm{matter}}\,. \tag{5}$$

The dynamical mass of the gravitational Higgs scalar field η is therefore

$$m_\eta = \sqrt{\frac{\xi(3m^2-4\Lambda)}{w}}\,, \tag{6}$$

related to the cosmological constant, the ultra-light masses, the non-minimal coupling constant ξ and the Brans-Dicke parameter w, with $3m^2 > 4\Lambda$. Assuming that the fermions couple to gravity via the standard Higgs-lepton coupling (HLC)

$$G_e\cdot\left[e_R\phi^+\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L+\begin{pmatrix}\bar{\nu}_e & \bar{e}\end{pmatrix}_L\phi e_R\right], \tag{7}$$

the SSB requires the following electron mass

$$m_e = G_e\sqrt{\frac{\xi(3m^2-4\Lambda)}{2h}} \tag{8}$$

or in the following square form

$$m_e^2=\frac{3\xi m^2G_e^2}{2h}-\frac{4\xi\Lambda G_e^2}{2h}\equiv(m_{\mathrm{eff}}^2-m_\Lambda^2)G_e^2=m_{\mathrm{eff}}^2G_e^2\left(1-\frac{4\Lambda}{3m^2}\right)\equiv M_{\mathrm{eff}}^2\left(1-\frac{4\Lambda}{3m^2}\right), \tag{9}$$

where $m_{\mathrm{eff}}^2 = 3\xi m^2G_e^2/2h$ is the HLC effective ultra-light mass, $m_\Lambda^2 \equiv 2\xi\Lambda G_e^2/h$ is the HLC effective vacuum energy mass and $M_{\mathrm{eff}}^2 \equiv m_{\mathrm{eff}}^2G_e^2$, whereas the effective modified cosmological constant is given by

$$\Lambda_{\mathrm{eff}} = \frac{\xi^2(3m^2-4\Lambda)^2}{64\lambda\pi}\,. \tag{10}$$

Therefore, the electron mass depends also on the ultra-light masses and the cosmological constant and the scalar curvature. One may then suggest that a small cosmological constant may be generated by ordinary electromagnetic vacuum energy and, consequently, this could have important consequences on dark energy and dark matter in the Universe [23] and in high energy physics [24]. In other words, the electron mass is a geometrical mass and could not stable in an evolving Universe. If SSB occurs at cosmological levels, some additional terms can be naturally introduced into the theory. In addition, SSB leads of the scalar field non-minimally coupled to gravity and generates a massive gauge boson with ultra-light mass with a critical temperature far above the Universe temperature [10]. If these particles were proved to condensate into a degenerate Bose-Einstein gas (BEG), then we have a possible candidate to the missing dark matter [25]. After relaxation of the symmetry breaking, the BEG can become not only ultra-light but relativistic. For $\Lambda = 0$, $(m_e/m) \propto G_e$ and this is an interesting result. If in contrast $m^2 = 0$, one needs to have $\Lambda < 0$ for positive ξ, or $\Lambda > 0$ for negative ξ. The SSB occurs for negative curvature scalar or depends on the sign of the cosmological constant and the non-minimal coupling constant in case the ultra-light masses are zero. In other words, both the Higgs mass and the electron mass depend on the spacetime geometry. For a time-decaying cosmological constant or ultra-light masses, we find also a time-decaying electron mass and the whole standard particle theory is unstable. In most of the models described in the literature $\Lambda, m^2 \propto t^{-2}$, where t is the time scale [26 − 39]. Mass stability may occur then if we assume the conjecture $G_e^2\Lambda$ and $G_e^2 m^2$ are constants. This time of increasing of G_e may be interpreted as due to a beautiful physical balance between expansion and gravity. That is, since gravity increases, expansion must accelerates in time. A variable Newton coupling constant is widely discussed in literature [40]. Within the framework of inflation cosmology, the time-dependence of the gravitational constant causes the effective cosmological constant in the de Sitter phase to decrease with time. This in the long run causes the expansion rate of the universe to drop below the rate of formation of true vacuum bubbles, allowing the phase transition from SSB to complete itself. This success obviously provides good motivation for more detailed studies of aspects of BD-Higgs theories of gravity.

In this way, the Higgs mechanism can be considered as time-independent and consequently stable in time. The complex potential was shown to have important features and implications in the physics of the early universe, gravitation and the spontaneous symmetry breaking, but does not take into account of the dark matter and energy problem [10]. A previous study has shown that, in order to take into account the dark matter (as a possible explanation of the additional attractive gravitational effects in galaxies and clusters) and the dark energy (as a possible explanation of the repulsive effect at large cosmological scales), one needs to add a decreasing exponential potential to the quadratic one [3]. Being interested at first in clusters with typical density around 10^{-25} kgm^{-3}, we may generalize our complex scalar potential to the following form

$$V(\widetilde{\phi}\widetilde{\phi}^*) = \left(2\Lambda - \frac{3m^2}{2}\right)\xi\widetilde{\phi}\widetilde{\phi}^* + \left(\frac{h}{2}\right)(\widetilde{\phi}\widetilde{\phi}^*)^2 + \alpha\exp(-\beta\widetilde{\phi}\widetilde{\phi}^*) \tag{11}$$

α and β may be chosen adequately so that the quadratic term is dominant until a late time of the structure formation, and the third exponential part of the potential becomes non-negligible only later leading to a quintessence behavior [3]. Thus we let $\phi^*(x,t) = (\sigma(r)/\sqrt{2})\exp(-\mathrm{i}\omega t)$, that is, we assume that the complex potential has a conserved charge. At large cluster scale, the potential reads

$$V(\sigma) = \alpha - \frac{1}{2}m_{\rm eff}^2\sigma^2 + \frac{h}{4}\sigma^4 \equiv \frac{h}{4}(\sigma^2 - \mathrm{v}^2)^2 \tag{12}$$

where

$$m_{\rm eff}^2 = \left(\frac{3m^2\xi}{2} - 2\Lambda\xi\right) \equiv m'^2 - m^2 \tag{13}$$

with $m^2 = 2\Lambda\xi$ and $2m'^2 = 3m^2\xi$. $V(\sigma)$ has a minimum at $\sigma = \pm\mathrm{v}$ and a maximum at $\sigma = 0$ with curvature $V'' = -m_{\rm eff}^2$. The major difference between our approach and those present in the literature is that the effective mass depends on the cosmological constant, the ultra-light masses and the coupling constant. Comparing with Arbey's approach, $\alpha = 9m^4\xi^2/8h$ and $\beta = 4h/(3m^2\xi)$ depends on the ultra-light masses rather than on the cosmological constant [3]. The KGE associated with a static and isotropic spherical symmetric metric

$$\mathrm{d}\tau^2 = e^{2u(r)}\mathrm{d}t^2 - e^{2\mathrm{v(r)}}(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2) \tag{14}$$

gives the effective density of the scalar field as

$$\rho_{\rm eff} = \left(2\omega^2 - 2\Lambda\xi + \frac{3m^2\xi}{2}\right)\sigma^2 - \frac{h}{4}\sigma^4 - \alpha\,. \tag{15}$$

On the boundaries on which the gravitational potential is

$$\Phi_0 = \frac{1}{2}\left(1 - \frac{(4\Lambda - 3m^2)\xi}{2\omega^2}\right), \tag{16}$$

the scalar potential is in a Bose condensate state. As Φ_0 is too small, $2\omega^2 \approx (4\Lambda - 3m^2)\xi$ which is positive as long as $4\Lambda \gtrless 3m^2$ and $\xi \gtrless 0$. Far away from the center of the clusters, the effective density behaves as $\rho_{\rm eff} \approx -9m^4\xi^2/4h$, which is proportional to Λ, yielding a repulsing behaviour outside clusters as long as $\Lambda > 0$. At galactic scale, one can neglect h and the potential reads [5]

$$V(\sigma) = \frac{1}{2}\left(2\Lambda - \frac{3m^2}{2}\right)\xi\sigma^2 + \alpha\exp\left(-\frac{\beta\sigma^2}{2}\right). \tag{17}$$

The Klein-Gordon and Einstein equations give

$$\rho_{\rm eff} \approx V\left(2\Lambda - \frac{3m^2}{2}\right)\xi\sigma^2\,, \tag{18}$$

which is positive as long as $4\Lambda \gtrless 3m^2$ and $\xi \gtrless 0$. The size and the density of the halos are fixed by the value of $\omega \approx \left([2\Lambda - (3m^2/\xi)^{1/2}]\right)$. In time-decreasing cosmological constant models, where Λ and m^2 tend to zero at late times, $\omega \to 0$ and the whole scenario changes, and some effects occur inside a specific galaxy unless the coupling constant increases with time with the same rate.

3. Boson stars with Higgs-gauge symmetry

Having addressed the BD gravity with generalized complex scalar potential (11) in this paper, it will be of interest to explore finally the boson stars composed of bosons condensed into their ground state [41] which in fact may exist in nature [42] and may contribute to the dark matter problem [43]. For non-interacting massive scalar field, Colpi et al. find the mass of the boson star in the Schwarzschild metric to be of the order of $M \approx M_P^2/m$, where m is the mass of the scalar field and M_P is the Planck's mass. Despite this interesting result, this mass does not contribute to the dark matter problem, e.g., for $m \approx 1000$ MeV, $M \approx 10^{-19} M_\odot$, where $M_\odot$ is one solar mass. For self-interacting complex scalar field, the authors find $M \approx M_P^3/m^2$, e.g., for $m \approx m_{proton}$, $M \approx M_\odot$ and such mass may contribute to the dark matter problem [44]. Our aim now is to re-analyze the work done by Gunderson and Jensen in the context of BD gravity theory with complex scalar field assumed to be related to the standard model with complex scalar potential

$$V(\widetilde{\phi}\widetilde{\phi}^*) = \left(2\Lambda - \frac{3m^2}{2}\right)\widetilde{\xi}\widetilde{\phi}\widetilde{\phi}^* + \left(\frac{h}{2}\right)(\widetilde{\phi}\widetilde{\phi}^*)^2. \tag{19}$$

$\widetilde{\xi}$ is a new parameter in the theory. The action of the theory looks like

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left(-\phi_{\mathrm{BD}}R - wg^{\mu\nu}\frac{\partial_\mu\phi_{\mathrm{BD}}\partial_\nu\phi_{\mathrm{BD}}}{\phi_{\mathrm{BD}}}\right) \tag{20}$$

$$+\int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^*_{\mathrm{BD}}\partial_\nu\phi_{\mathrm{BD}} - V(\widetilde{\phi}\widetilde{\phi}^*)\right),$$

and the corresponding equations of motion are

$$G_{\mu\nu} = \frac{8\pi}{\phi_{\mathrm{BD}}}T_{\mu\nu} + \frac{w}{\phi^2_{\mathrm{BD}}}\left(\partial_\mu\phi_{\mathrm{BD}}\partial_\nu\phi_{\mathrm{BD}} - \frac{1}{2}g_{\mu\nu}\partial_\lambda\phi_{\mathrm{BD}}\partial^\lambda\phi_{\mathrm{BD}}\right) + \frac{1}{\phi_{\mathrm{BD}}}(\phi_{\mathrm{BD};\mu;\nu} - g_{\mu\nu}\Box\phi_{\mathrm{BD}}) \tag{21}$$

$$\Box\phi_{\mathrm{BD}} = \frac{8\pi}{2w+3}T\,, \tag{22}$$

where $T = g^{\mu\nu}T_{\mu\nu}$ and

$$T_{\mu\nu} = \frac{1}{2}\partial_\mu\widetilde{\phi}^*\partial_\nu\widetilde{\phi} + \partial_\mu\widetilde{\phi}\partial_\nu\widetilde{\phi}^* - \frac{1}{2}g_{\mu\nu}(g^{\rho\eta}\partial_\rho\widetilde{\phi}^*\partial_\eta\widetilde{\phi} + V(\widetilde{\phi}\widetilde{\phi}^*)) \tag{23}$$

We adapt the well-known Schwarzschild metric

$$\mathrm{d}\tau^2 = \left(1 - \frac{2\bar{M}(r)}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{2\bar{M}(r)}{r}\right)^{-1}\mathrm{d}r^2 - (r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2)\,, \tag{24}$$

$\bar{M}(r)$ is the amount of the star's mass within the radius r, and demanding a spherically symmetric matter field $\widetilde{\phi}$ having the form $\widetilde{\phi}(\boldsymbol{r},t) = \widetilde{\varphi}(r)\exp(-\mathrm{i}\omega_G t)$, where

ω_G is the ground state boson's energy. Following the arguments of Gunderson and Jensen by finding the equations of motion or the boson star and imposing on the system realistic boundary conditions, we find

$$M_{\max} \approx 0.087 h^{1/2} \frac{M_P^3}{(4\Lambda - 3m^2)\widetilde{\xi}} \equiv 0.087 h^{1/2} \frac{M_P^3}{\widetilde{\xi} R} . \tag{25}$$

This mass is positive for $R > 0$ and $\widetilde{\xi} > 0$, or $R < 0$ and $\widetilde{\xi} < 0$. For $R = 0$, $M_{\max} \gg 1$ and for $R \ll 1$, $M_{\max} \ll 1$. For $m \approx 0$,

$$M_{\max} \approx 0.02 h^{1/2} \frac{M_P^3}{\widetilde{\xi}\Lambda} , \tag{26}$$

and thus depends on the tiny cosmological constant and may contribute to the dark matter problem. Assuming again $U(1)$ symmetry and the previous Higgs gauge transformation, we may write Eq. (25) using Eq. (8) as follows

$$\left| M_{\max}^{Ge} \right| \approx 0.0435 h^{-1/2} G_e^2 \frac{M_P^3}{m_e^2} , \tag{27}$$

which is of interest as it represents the maximum mass associated to the electron mass, and using equation (6),

$$\left| M_{\max}^{GH} \right| \approx 0.087 h^{1/2} \frac{M_P^3}{w m_{GH}^2} , \tag{28}$$

which is the maximum mass associated to the Higgs mass, and which tends to zero for $w \to \infty$, unless $w = \zeta h^{1/2}$, where ζ is a real parameter, that is

$$\left| M_{\max}^{GH} \right| \approx 0.087 \frac{M_P^3}{\zeta m_{GH}^2} , \tag{29}$$

of the order of the Chandrasekhar mass if $\zeta \approx 0.04$. In Eq. (29), m_{GH} is the gravitational Higgs mass associated to the theory. In this special case, for the large coupling effect $h^{1/2}/ \gg 1$ and tiny BD parameter $w \ll 1$, the maximum mass may contribute to the dark matter problem through SSB. An important contribution may occur for weak coupling effect $h^{1/2} \ll 1$ and big BD parameter $w \gg 1$. This is to say that tiny massive bosons stars could have formed in the early universe even in the presence of strong coupling effect and fairly large BD parameter. Moreover, from Eq. (27), a star may be composed of both fermions and bosons without interactions [45]. For $w = 6$,

$$\left| M_{\max}^{GH} \right| \approx 0.0145 h^{1/2} \frac{M_P^3}{m_{GH}^2} , \tag{30}$$

less than the one derived by Gunderson and Jensen, and for $w = 500$ as derived from the current observation [46]

$$\left| M_{\max}^{GH} \right| \approx 1.74 \times 10^{-4} h^{1/2} \frac{M_P^3}{m_{GH}^2} , \tag{31}$$

which may contribute to the dark matter problem for the strong coupling effect. Note that the maximum mass of the bosons star may increase in time if the cosmological constant decreases like $\Lambda \propto t^{-2}$ and thus we may have important contributions to the dark matter problem. In the so-called hyperextended models of inflation [47], the BD parameter may vary in time. From current observations, one might expect, since the Universe expands, that w was lower in the past and thus increases in time. As a result from Eq. (28), in order that the maximum mass contributes to the dark matter problem dominated by the strong coupling effect, m_{GH}^2 must decreases in time in a way the the product $wm_{GH}^2 \propto const$. It is interesting to have a decaying Higgs-gravity mass with the cosmological time in the Standard Model [48]. One should also mention that as a result, all the other parameters, e.g., Yukawa couplings, vacuum expectation value of the Higgs field may have a time dependence.

4. Conclusions

As a generalization of BD gravity with complex scalar field, we showed that the dynamical Higgs mass and the generated electron mass are geometrical, depending on Λ_{vac} and m on condition that the scalar curvature and the non-minimal coupling parameter are of opposite signs. It has been found that dark matter could take the form of the ultra-light masses whose effective Compton wavelength is of the same order as of the galaxy core. Smaller scale structure arises due to the fact that dark matter cannot be concentrated on smaller scales. It is very difficult to understand the ultra-light masses in particle physics and, certainly, this is one of the major problems underlying the quintessence models. The whole model described in this letter is simplistic. It is natural to couple the complex scalar field to the fields of the standard model which will lead to new interaction via a fifth forth type and would be another source of time variation for the parameters of the Standard Model [49]. We have also shown that it is possible for bosons stars to contribute to the dark matter problem even for weak coupling effect but large BD parameter. The complex scalar field and its associated geometrical Higgs complex scalar potential could thus have impact on the standard cosmology and the particle physics and be at the origin of a time variations of the couplings and masses of the standard models (big bang and electroweak Standard Model). Further details and consequences are in progress.

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GEOMETRIJSKA HIGGSOVA MASA I BOZONSKE ZVIJEZDE IZ KOMPLEKSNOG POLJA BRANS-DICKEOVE GRAVITACIJE

Raspravljamo lomljenje Higgsove mjerne simetrije i stabilnost mase, Brans-Dickeovu skalarnu gravitaciju, tamnu energiju i materiju, u okviru neminimalnog vezanja, gdje je pokazano da je kozmološka konstanta zbroj vakuumskog (Λ_{vac}) i induciranog člana $\Lambda_{ind} = -3m^2/4$, gdje je m ultra lagana masa ($\approx$ Hubbleov parametar), ugrađena u teoriju na osnovi super-gravitacije. Pokazujemo da su dinamička Higgsova masa i generirana elektronska masa geometrijske, i ovise o Λ_{vac} i m, uz uvjet da su skalarna zakrivljenost i neminimalni parametar vezanja suprotnog predznaka. Radi uključivanja tamne energije i tamne mase, poopćili smo teoriju uvođenjem opadajućeg eksponencijalnog kompleksnog potencijala Higgsovom kompleksnom skalarnom potencijalu. Pokazujemo da cijele nakupine, fizički scenariji galaksija i bozonske zvijezde možda ovise o predznaku skalarne zakrivljenosti i o neminimalnoj konstanti vezanja.