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ANISOTROPIC VISCOUS FLUID COSMOLOGICAL MODELS WITH HEAT FLOW IN SAEZ-BALLESTER THEORY OF GRAVITATION

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We discuss the field equations in Saez-Ballester scalar-tensor theory of gravitation for a Bianchi type-V model filled with viscous fluid together with heat flow. We obtain two classes of cosmological solutions by applying a special law of variation of Hubble's parameter which yields a constant value of deceleration parameter. One class of solutions corresponds to a model of universe which evolves from a big-bang type singularity at $t = 0$ and expands with power-law expansion. The other class of solutions represents an universe expanding exponentially having singularity in the infinite past. The physical and kinematical features of the models are discussed. We observe that the models of the universe in two types of cosmologies are compatible with the results of recent observations.

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1. Introduction

The Friedmann-Robertson-Walker (FRW) models are only globally acceptable perfect fluid space-time which are spatially homogeneous and isotropic. The adequacy of isotropic cosmological models for describing the present state of the universe is no basis for expecting that they are equally suitable for describing the early stages of evolution of the universe. At the early stages of the evolution of the universe, when radiation in the form of photons as well as neutrino decoupled, the matter behaved like a viscous fluid. Since viscosity counteracts the gravitational

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collapse, a different picture of the initial stage of the universe may appear due to dissipative processes caused by viscosity.

Misner [1,2] studied the effect of viscosity on the evolution in the cosmological models and has suggested that the strong dissipation due to the neutrino viscosity may considerably reduce the anisotropy of the black-body radiation. Murphy [3] obtained an exact cosmological model of zero-curvature of FRW type in the presence of bulk viscosity alone which exhibits an interesting feature that the big-bang singularity appears in the infinite past. Roy and Tiwari [4] presented some plane symmetric solutions to Einsteins's field equations representing inhomogeneous cosmological models with viscous fluid and constant bulk viscosity. Szydlowski and Heller [5] constructed models of the universe filled with interacting matter and radiation including bulk viscosity dissipation. Mohanty and Pradhan [6] obtained a class of exact non-static solution in a closed elliptic Robertson-walker space-time filled with viscous fluid in the presence of attractive scalar fields.

Belinski and Khalatnikov [7], while investigating a Bianchi type I cosmological model with the influence of viscosity, have found the important property that near the initial singularity the gravitational field creates matter. Banerjee et al. [8] obtained some Bianchi type I solutions in the case of stiff matter by using the assumption that shear viscosity coefficient are power-law functions of the energy density. Huang [9] presented exact solution for a Bianchi type I cosmological model with viscosity without using shear viscosity. Goener and Kowaleski [10] developed a method for obtaining irrotational anisotropic viscous fluid solutions of Bianchi type I with barotropic equation of state. Banerjee and Sanyal [11] presented an irrotational Bianchi type V model under the influence of both shear and bulk viscosity together with heat flow. Coley [12], Coley and Hoogan [13], while generalizing the work of Coley and Tupper [14], studied diagonal Bianchi type V imperfect fluid models with both viscosity and heat conduction with and without the cosmological term. Recently, Singh and Chaubey [15] investigated the evolution of Bianchi type V model with viscous fluid and cosmological constant.

In this paper, we obtain two classes of exact solutions of the field equations in the scalar-tensor theory of Saez and Ballester [16] for a Bianchi type V space-time filled with a viscous fluid with both bulk and shear viscosities together with heat flow. In order to find solutions of the field equations, we apply a special law of variation of Hubble's parameter as proposed by Berman [17]. One class of solutions represents model of the universe with power-law expansion which evolves from a big-bang singularity at $t = 0$. The other class of solutions corresponds to an exponentially expanding universe having singularity in the infinite past. The physical and kinematical behavior of models are discussed in two types of cosmologies.

2. Field equations and general expressions

We consider the spatially homogeneous Bianchi type V model in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(t)\mathrm{d}x^2 - e^{2mx}\left[(B^2(t)\mathrm{d}y^2 + C^2(t)\mathrm{d}z^2\right], \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the cosmic scale factors and m is a constant.

The energy-momentum tensor T_{ij} in a viscous fluid with heat conduction is given by

$$T_{ij} = (\rho + \bar{p})\, u_i u_j - \bar{p} g_{ij} + \eta \delta_j^k \left(u_{i;k} + u_{k;i} - u_i u^k u_{j;k} - u_j u^k u_{i;k} \right) + h_i u_j + h_j u_i \,, \quad (2)$$

where ρ is the energy density and $\bar{p}$ the effective pressure given by

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta \right) u^k_{;k}. \quad (3)$$

Here u^i is the 4-velocity of the fluid, ξ and η are coefficients of bulk and shear viscosity, and h_i is the heat flow vector orthogonal to u^i. If we assume that the heat flow is in the x-direction only, then $h_i = (h_1, 0, 0, 0)$, h_1 being a function of time.

The field equations in Saez-Ballester scalar-tensor theory of gravitation are

$$R_{ij} - \frac{1}{2} R g_{ij} - \omega \phi^r (\phi_{;i} \phi_{;j} - \frac{1}{2} g_{ij} \phi_{;k} \phi^{;k}) = -T_{ij}. \quad (4)$$

The scalar field ϕ satisfies the equation

$$2\phi^r \phi^{;k}_{;k} + r\phi^{r-1} \phi_{;k} \phi^{;k} = 0, \quad (5)$$

where r is an arbitrary constant and ω is a dimensionless coupling constant. A semicolon denotes covariant derivative. We have taken $8\pi G = c = 1$ in Eq. (4). The energy-momentum tensor T_{ij} satisfies the conservation equation

$$T^{ij}_{;j} u_i = 0. \quad (6)$$

For the metric (1), the field equations (2) – (4), in comoving coordinates, yield

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -\left(\bar{p} - 2\eta \frac{\dot{A}}{A} \right) + \frac{1}{2}\omega \phi^r \dot{\phi}^2 \,, \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -\left(\bar{p} - 2\eta \frac{\dot{B}}{B} \right) + \frac{1}{2}\omega \phi^r \dot{\phi}^2 \,, \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -\left(\bar{p} - 2\eta \frac{\dot{C}}{C} \right) + \frac{1}{2}\omega \phi^r \dot{\phi}^2 \,, \quad (9)$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = \rho - \frac{1}{2}\omega \phi^r \dot{\phi}^2 \,, \quad (10)$$

$$m\left(2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=h_1\,, \tag{11}$$

where an over dot denotes differentiation with respect to time t. The conservation equation (6) gives

$$\dot{\rho}+(\rho+\bar{p})\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-2\eta\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2\right]=\frac{2m}{A^2}h_1. \tag{12}$$

The scalars of expansion and shear are calculated as

$$\theta=u^i_{;i}=\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{13}$$

$$2\sigma^2=\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2\right]-\frac{\theta^2}{3}. \tag{14}$$

For the metric (1), the average scale factor R is defined by

$$R=(ABC)^{1/3}\,. \tag{15}$$

The spatial volume V is given by

$$V=R^3=ABC. \tag{16}$$

The generalized mean Hubble's parameter H is defined by

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{17}$$

where $H_1=\dot{A}/A$, $H_2=\dot{B}/B$ and $H_3=\dot{C}/C$ are the directional Hubble's parameters in the directions of x, y and z, respectively. The deceleration parameter q is given by

$$q=-\frac{R\ddot{R}}{\dot{R}^2}. \tag{18}$$

Subtracting Eqs. (7) and (8), Eqs. (8) and (9), and Eqs. (7) and (9), and integrating the resulting equations, we obtain the quadrature solutions for the average scale factors A, B and C as follows:

$$A(t)=l_1R\exp\left[X_1\int\frac{\exp\left(-2\int\eta\mathrm{d}t\right)}{R^3}\mathrm{d}t\right], \tag{19}$$

$$B(t) = l_2 R \exp\left[X_2 \int \frac{\exp\left(-2\int \eta \mathrm{d}t\right)}{R^3}\mathrm{d}t\right], \tag{20}$$

$$C(t) = l_3 R \exp\left[X_3 \int \frac{\exp\left(-2\int \eta \mathrm{d}t\right)}{R^3}\mathrm{d}t\right], \tag{21}$$

where constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0 \qquad \text{and} \qquad l_1 l_2 l_3 = 1. \tag{22}$$

For the metric (1), Eq. (5) provides

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{r}{2\phi}\dot{\phi}^2 = 0\,, \tag{23}$$

which, on integration yields

$$\phi = \left[\frac{h(r+2)}{2}\int \frac{\mathrm{d}t}{R^3}\right]^{2/(r+2)}. \tag{24}$$

From Eqs. (7) – (10), we obtain the expressions for energy density and effective pressure as given by

$$\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2\,, \tag{25}$$

$$\bar{p} = H^2\left(2q-1\right) - \sigma^2 + \frac{m^2}{A^2} + \frac{2}{3}\eta\theta + \frac{1}{2}\omega\phi^r\dot{\phi}^2. \tag{26}$$

Clearly, we can find the solutions of the Eqs. (19) – (21) for the scale factors A, B, C if the shear viscosity coefficient η and R are known. Regarding η, we make the physically valid assumption that the shear is proportional to the expansion, $\eta \propto \theta$, i.e.

$$\eta = \eta_0\theta\,, \tag{27}$$

where η_0 is a positive constant.

In the next section, we find two explicit forms of the average scale factor R by applying a special law of variation for Hubble's parameter. Using the explicit forms of R, we find the solutions of Eqs. (19) – (21) in two type of cosmologies.

3. *Variation law for Hubble's parameter*

In order to obtain explicit solutions of the Eqs. (19) – (21), we assume that the mean Hubble's parameter H is related to the average scale factor R by the relation

$$H = lR^{-n}\,, \tag{28}$$

where l and n are non-negative constants. Such type of relation has already been used by Berman [17], Berman and Gomide [18] for solving field equations in FRW cosmologies, Singh and Kumar [19], Kumar and Singh [20], for Bianchi type I space-times and Singh et al. [21], Shri Ram et al. [22] for Bianchi type V models in different physical contexts. This relation yields a constant value of the deceleration parameter. The positive value of q corresponds to standard decelerating universe, whereas the negative value indicates inflation.

Substituting Eq. (28) in Eq. (17), we obtain

$$\dot{R} = lR^{-n+1}\,, \tag{29}$$

which, on differentiation, gives

$$\ddot{R} = -l^2(n-1)R^{-2n+1}. \tag{30}$$

From Eqs. (18), (29) and (30), we find that

$$q = n - 1. \tag{31}$$

Thus, the variation law Eq. (28) of Hubble's parameter gives a constant value of deceleration parameter. For $n > 1$, we have a decelerating model and for $0 \leq n < 1$, we have an accelerating model of the universe.

Integration of Eq. (29) provides

$$R = (nlt + k)^{1/n}\,, \quad n \neq 0\,, \tag{32}$$

$$R = c\exp(lt)\,, \quad n = 0\,, \tag{33}$$

where k and c are integration constants. Without loss of any generality, we can take $k = 0$. Then Eq. (32) becomes

$$R = (nlt)^{1/n}\,. \tag{34}$$

We now use Eqs. (33) and (34) to solve Eqs. (19) – (21) for scale factors A, B and C in two types of cosmology.

3.1. Cosmological model with power-law expansion ($n \neq 0$)

Using the power-law solution to the average scale factor R, given in Eq. (34), we obtain

$$H = (nt)^{-1}\,, \tag{35}$$

$$\theta = 3\,(nt)^{-1}\,, \tag{36}$$

$$\eta = 3\eta_0 (nt)^{-1}. \tag{37}$$

Substituting the value of R and η in Eqs. (19) – (21) and integrating, we obtain

$$A(t) = l_1\,(nlt)^{1/n} \exp\left[\frac{X_1}{l(n-3-6\eta_0)}\,(nlt)^{(n-3-6\eta_0)/n}\right], \tag{38}$$

$$B(t) = l_2\,(nlt)^{1/n} \exp\left[\frac{X_2}{l(n-3-6\eta_0)}\,(nlt)^{(n-3-6\eta_0)/n}\right], \tag{39}$$

$$C(t) = l_3\,(nlt)^{1/n} \exp\left[\frac{X_3}{l(n-3-6\eta_0)}\,(nlt)^{(n-3-6\eta_0)/n}\right], \tag{40}$$

provided $n \neq (3+6\eta_0)$. The directional Hubble's parameters H_1,H_2,H_3 are obtained as

$$H_1 = (nt)^{-1} + X_1\,(nlt)^{-(3+6\eta_0)/n}\,, \tag{41}$$

$$H_2 = (nt)^{-1} + X_2\,(nlt)^{-(3+6\eta_0)/n}\,, \tag{42}$$

$$H_3 = (nt)^{-1} + X_3\,(nlt)^{-(3+6\eta_0)/n}\,. \tag{43}$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\left(X_1^2 + X_2^2 + X_3^2\right)(nlt)^{-(6+12\eta_0)/n}\,. \tag{44}$$

Eq. (24), on integration, yields

$$\phi = \left[\frac{h(r+2)}{2l(n-3)}\right]^{2/(r+2)} (nlt)^{2(n-3)/\{(n(r+2)\}}\,, \quad n \neq 3\,. \tag{45}$$

Using Eqs. (38) – (40) in Eq. (11), we find that

$$h_1 = 3mX_1\,(nlt)^{-(3+6\eta_0)/n}\,. \tag{46}$$

With these solutions, the conservation equation (12) is identically satisfied.

From Eqs. (25) and (26), the energy density and the effective pressure are obtained as

$$\begin{aligned}\rho &= 3(nt)^{-2} - \frac{1}{2}\left(X_1^2 + X_2^2 + X_3^2\right)(nlt)^{-(6+12\eta_0)/n} \\ &\quad - \frac{3m^2}{{l_1}^2}(nlt)^{-2/n}\exp\left[\frac{2X_1}{l(3+6\eta_0-n)}(nlt)^{(n-3-6\eta_0)/n}\right] \\ &\quad + \frac{1}{2}\omega h^2 (nlt)^{-6/n} \qquad (47)\end{aligned}$$

$$\begin{aligned}\bar{p} &= (2n+6\eta_0-3)(nt)^{-2} - \frac{1}{2}\left(X_1^2 + X_2^2 + X_3^2\right)(nlt)^{-(6+12\eta_0)/n} \\ &\quad + \frac{m^2}{{l_1}^2}(nlt)^{-2/n}\exp\left[\frac{2X_1}{l(3+6\eta_0-n)}(nlt)^{(n-3-6\eta_0)/n}\right] \\ &\quad + \frac{1}{2}h^2\omega(nlt)^{-6/n} \qquad (48)\end{aligned}$$

provided $n \neq (3+6\eta_0)$.

If we assume that the energy density ρ and pressure p satisfy the barotropic equation of state $p = \gamma\rho$, $0 \leq \gamma \leq 1$, then, from Eq. (3), we find that

$$\begin{aligned}\xi &= \frac{(3\gamma-2n+3)l}{3}(nlt)^{-1} + \frac{\left({X_1}^2 + {X_2}^2 + {X_3}^2\right)(1-\gamma)}{6l}(nlt)^{(n-6-12\eta_0)/n} \\ &\quad - \frac{m^2(3\gamma+1)}{3{l_1}^2 l}(nlt)^{(n-2)/n}\exp\left[\frac{2X_1}{l(3+6\eta_0-n)}(nlt)^{(n-3-6\eta_0)/n}\right] \\ &\quad + \frac{\omega(\gamma-1)h^2}{6l}(nlt)^{(n-6)/n}. \qquad (49)\end{aligned}$$

The coefficients of bulk and shear viscosities are time-dependent.

For this model, the spatial volume V tends to zero as $t \to 0$. The energy density, pressure, scalar expansion and shear scalar all assume infinite values at this epoch. Heat function is also infinite at $t = 0$. Thus, the model evolves from a big-bang type singularity at $t = 0$ and eventually expands with power-law expansion. The scalar function ϕ, viscosity coefficients ζ and η are also infinite at $t = 0$. As the cosmic time increases, the volume and all scale factors will increase, but the expansion scalar decreases. The physical and kinematical quantities σ^2, H_1,H_2, H_3, H, ρ and p are decreasing functions, as t increases. As $t \to \infty$, the spatial volume becomes infinite and other physical and kinematical quantities tend to zero. The heat function dies out for large time. Also σ^2/θ tends to zero as $t \to \infty$ provided $n < 6(1+2\eta_0)$. Thus, the model gives essentially empty universe for large time.

3.2. *Cosmological model with exponential expansion* ($n = 0$)

Using the exponential form of R given in Eq. (33), we obtain

$$H = l\,, \tag{50}$$

$$\theta = 3l\,. \tag{51}$$

The shear viscosity coefficient η and volume scalar V are given by

$$\eta = 3l\eta_0\,, \tag{52}$$

$$V = c^3 \exp(3lt). \tag{53}$$

Integrating Eqs. (19) – (21), we obtain the solutions for the metric functions as

$$A(t) = l_1 c \exp\left[lt - \frac{X_1}{3lc^3(1+2\eta_0)} \exp\{-3l(1+2\eta_0)t\}\right], \tag{54}$$

$$B(t) = l_2 c \exp\left[lt - \frac{X_2}{3lc^3(1+2\eta_0)} \exp\{-3l(1+2\eta_0)t\}\right], \tag{55}$$

$$C(t) = l_3 c \exp\left[lt - \frac{X_3}{3lc^3(1+2\eta_0)} \exp\{-3l(1+2\eta_0)t\}\right]. \tag{56}$$

From Eq. (24), the scalar function ϕ has the solution

$$\phi(t) = \left[\frac{h(r+2)}{6l}\right]^{2/(r+2)} \exp\left(-\frac{6lt}{r+2}\right). \tag{57}$$

The directional Hubble's parameters H_1, H_2 and H_3 have the values

$$H_1 = l + \frac{X_1}{c^3} \exp\{-3l(1+2\eta_0)t\}\ , \tag{58}$$

$$H_2 = l + \frac{X_2}{c^3} \exp\{-3l(1+2\eta_0)t\}\ , \tag{59}$$

$$H_3 = l + \frac{X_3}{c^3} \exp\{-3l(1+2\eta_0)t\}\ . \tag{60}$$

The shear scalar σ has the value given by

$$\sigma^2 = \frac{\left(X_1^2 + X_2^2 + X_3^2\right)}{2c^6} \exp\{-6l(1+2\eta_0)t\}\,. \tag{61}$$

The heat function h_1 is given by

$$h_1 = \frac{3mX_1}{c^3} \exp\{-3l(1+2\eta_0)t\}\,. \tag{62}$$

The expressions for the energy density and effective pressure are obtained as

$$\begin{aligned}\rho &= 3l^2 - \frac{\left(X_1^2+X_2^2+X_3^2\right)}{2c^6}\exp\{-6l(1+2\eta_0)t\} \\ &\quad -\frac{3m^2}{l_1^2c^2}\exp\left[\frac{2X_1}{3c^3l(1+2\eta_0)}\exp\{-3l(1+2\eta_0)t\}-2lt\right] \\ &\quad +\frac{\omega h^2}{2}\exp(-6lt)\end{aligned} \tag{63}$$

$$\begin{aligned}\bar{p} &= l^2\,(6\eta_0-3) - \frac{\left(X_1^2+X_2^2+X_3^2\right)}{2c^6}\exp\{-6l(1+2\eta_0)t\} \\ &\quad +\frac{m^2}{c^2l_1^2}\exp\left[\frac{2X_1}{3c^3l(1+2\eta_0)}\exp\{-3l(1+2\eta_0)t\}-2lt\right] \\ &\quad +\frac{\omega h^2}{2}\exp(-6lt).\end{aligned} \tag{64}$$

By a straightforward calculation it can easily be seen that the conservation Eq. (12) is identically satisfied.

If the energy density ρ and pressure p satisfy the gamma law of equation of state $p = \gamma\rho$, $0 \le \gamma \le 1$, then the bulk viscosity coefficient ξ has the value given by

$$\begin{aligned}\xi &= (\gamma+1)l + \frac{\left(X_1^2+X_2^2+X_3^2\right)(1-\gamma)}{6lc^6}\exp\{-6l(1+2\eta_0)t\} \\ &\quad -\frac{m^2(3\gamma+1)}{3l{l_1}^2c^2}\exp\left[\frac{2X_1}{3c^3l(1+2\eta_0)}\exp\{-3l(1+2\eta_0)t\}-2lt\right] \\ &\quad +\frac{(\gamma-1)}{6l}\omega h^2\exp(-6lt).\end{aligned} \tag{65}$$

Thus, in this model of the universe, bulk viscosity coefficient ξ is function of time and shear viscosity coefficient is constant.

The spatial volume V tends to zero, and energy density and pressure become infinite as t tends to $-\infty$. This means that the model has a singularity in the infinite past. All physical and kinematical quantities are well behaved for $-\infty < t < \infty$. At $t \to \infty$, the spatial volume becomes infinite, and energy density and pressure tend to constant values. Also $\sigma^2/\theta \to 0$ as $t \to \infty$. Therefore the model becomes isotropic for large t.

4. Conclusions

We have presented two classes of exact solutions to the field equations in the framework of Saez-Ballester scalar-tensor theory for a Bianchi type V model in the presence of a viscous fluid together with heat flow. We have applied a special law of variation for Hubble's parameter to generate models of the universe in two type of cosmologies. One class of models represents a singular universe which evolves from a big-bang singularity at $t = 0$ and expands with expansion rate of power-law type. The other class of models with negative deceleration parameter corresponds to exponentially expanding model having singularity in the infinite past. The evolution of the universe in such a scenario is consistent with the present observation predicting an accelerated expansion. We have also discussed the physical and kinematical behavior of the models of the universe in two types of cosmologies. These models could give appropriate description of the universe at its early stages of evolution.

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NEIZOTROPNI KOZMOLOŠKI MODELI S VISKOZNOM TEKUĆINOM I TOKOM TOPLINE U SAEZ-BALLESTEROVOJ TEORIJI GRAVITACIJE

Razmatramo jednadžbe polja u Saez-Ballesterovoj skalarno-tenzorskoj teoriji gravitacije za Bianchijev model tipa V s viskoznom tekućinom i tokom topline. Postigli smo dvije vrste kozmoloških rješenja primjenom posebne relacije za promjene Hubbleovog parametra koje vode na stalnu vrijednost parametra usporavanja. Jedna vrsta rješenja odgovara modelu svemira koji se razvija iz singularnosti tipa velikog praska u $t = 0$ i širi se prema potencijalnom zakonu. Druga vrsta rješenja predstavlja svemir koji se širi eksponencijalno a singularnost mu je u beskonačnoj prošlosti. Raspravljaju se fizičke i kinematičke osebine oba modela. Nalazimo da su ova dva modela svemira u skladu s nedavnim opažanjima.