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POSSIBLE CPT VIOLATION FROM PLANCK SCALE EFFECTS

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At present there is good agreement between the neutrino mass-squared difference determined from the solar neutrino data and the anti-neutrino mass-squared difference determined from the KamLAND reactor anti-neutrino experiment. However, the central values of the two cases differ from each other by about 10^{-5} eV^2. An improvement in the accuracy of both the solar neutrino experiments and reactor anti-neutrino experiments could establish the existence of a non-zero difference between neutrino and anti-neutrino mass-squared differences and provide a signal for CPT violation. In this paper, we show how such a difference can arise through the CPT violating neutrino mass terms from Planck scale physics.

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1. Introduction

The KamLAND experiment recently detected the distortion due to oscillations in the anti-neutrino spectrum from the reactors and determined the corresponding mass-square difference, $\overline{\Delta}_{21}$, to a great precision [1]. At present there is good agreement between $\overline{\Delta}_{21}$ and the mass-squared difference of the neutrinos, Δ_{21}, determined from the analysis of the solar neutrino data [2]. However, the best-fit values of the two Δs differ from each other by 10^{-5} eV^2. Together, solar and KamLAND data impose the constraint $|\Delta_{21} - \overline{\Delta}_{21}| \leq 1.1 \times 10^{-4}$ eV^2 [3]. Future reactor experiments, located at a distance of about 70 Km from the source so that the oscillation minimum coincides with the spectral maximum, are expected to improve the precision of $\overline{\Delta}_{21}$ even further [4]. Similarly future solar neutrino experiments, such as LENS [5] and other experiments [6], are expected to improve the accuracy of Δ_{21}. These future experiments may indeed show that there is a non-zero difference between Δ_{21} and $\overline{\Delta}_{21}$, thus establishing a signal for CPT violation in the neutrino sector [7, 8].

If Δ_{21} and $\overline{\Delta}_{21}$ are indeed found to be different, a natural question to ask is: How does this CPT violation arise? In this letter, we assume that CPT violation in neutrino sector could arise due to the Planck scale effects. We parametrize these effects in terms of Planck scale CPT violating neutrino mass terms and calculate the difference between Δ_{21} and $\overline{\Delta}_{21}$ arising due to these terms.

We assume that neutrino masses mainly arise due to the grand unified theory (GUT) dynamics via see-saw mechanism [9] and these masses are CPT conserving. We further assume that the CPT violation arises only at the Planck scale and parametrize it by the effective neutrino mass term

$$\mathcal{M}_{\rm CPT} = \frac{v^2}{2M_{\rm Pl}} \lambda_{\alpha\beta} \,, \tag{1}$$

where α and β are flavour indices. The mass term for anti-neutrinos will have the opposite sign [10]. Since these are effective masses arising from Planck scale effects, they are suppressed by $1/M_{\rm Pl}$. Since these are assumed to be the masses at the low energy scale, the electroweak vacuum expectation value (VEV) $v = 174$ GeV is used to make these terms have dimension of mass. In Eq. (1), the term λ is a 3×3 matrix in flavour space whose elements are of the order 1. We assume that the Planck scale interaction is "flavour blind", i.e. the elements of the matrix λ are independent of α, β indices. In this case, the contribution to the neutrino mass matrix is of the form

$$\mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{2}$$

where the scale μ is

$$\mu = v^2/2M_{\rm Pl} \simeq 10^{-6} \text{ eV}. \tag{3}$$

In our calculations, we take Eq. (2) as a perturbation to the main part of the neutrino mass matrix, that is generated by the GUT dynamics.

2. Calculation

The theoretical framework, in which the Planck scale mass terms are treated as perturbation to the GUT scale neutrino masses, is developed in Ref. [11]. Here we briefly recapitulate some salient features of this framework. In the discussion below, the labels α and β refer to flavour eigenstates and the labels i and j refer to mass eigenstates. The unperturbed ($0^{\rm th}$-order) neutrino mass matrix $\mathcal{M}$ is diagonalized by a unitary matrix U to yield the matrix M, whose eigenvalues are real and non-negative. As stated before, $\mathcal{M}$ is generated by grand unified dynamics and is related to U and M through the relation

$$\mathcal{M} = U^* \, M \, U^\dagger, \text{where } U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}, \end{pmatrix}. \tag{4}$$

We adopt the usual parameterization [12]: $U_{e2}/U_{e1} = \tan\theta_{12}$, $U_{\mu 3}/U_{\tau 3} = \tan\theta_{23}$ and $|U_{e3}| = \sin\theta_{13}$. We include all possible phases in the definition of the neutrino mixing matrix,

$$U = \mathrm{diag}(\mathrm{e}^{if_i}) \; R(\theta_{23}) \; \Pi \; R(\theta_{13}) \; \Pi^* \; R(\theta_{12}) \; \mathrm{diag}(\mathrm{e}^{ia_i}). \tag{5}$$

The phase δ appearing in $\Pi = \mathrm{diag}(\mathrm{e}^{i\delta/2}, 1, \mathrm{e}^{-i\delta/2})$ is the one that enters oscillation probabilities and leads to CP violation in neutrino oscillations. a_i are the so called Majorana phases and the matrix with these phases has the form $\mathrm{diag}(\mathrm{e}^{ia_1}, \mathrm{e}^{ia_2}, 1)$. The diagonal phase matrix on the left is given by $\mathrm{diag}(\mathrm{e}^{if_1}, \mathrm{e}^{if_2}, \mathrm{e}^{if_3})$, which are usually absorbed into the definition of the respective charged lepton field. It is possible to rotate away the phases f_i, if the mass matrix (4) *is the complete mass matrix*. However, since we are going to add another contribution to this mass matrix, the phases f_i of the zeroth order mass matrix have an impact on the complete mass matrix and thus must be retained. By the same token, the Majorana phases which are usually redundant for oscillations have a dynamical role to play now.

Planck scale effects will add other contributions to the mass matrix. The additional term has the form given in Eq. (2) and with its inclusion the neutrino mass matrix becomes

$$\mathcal{M} \to \mathcal{M}' = \mathcal{M} + \mu \, \lambda, \tag{6}$$

It is possible that the CPT violating mass terms may be arise not at Planck scale but at some scale M_X below M_{Pl} which is well above the GUT scale (by an order of magnitude or so). In such a case, the perturbation parameter $\mu = v^2/2M_X$, rather than $v^2/2M_{\mathrm{Pl}}$.

We now define the hermitian matrix $\mathcal{M}'^\dagger \mathcal{M}'$ and find its eigenvalues and eigenvectors. The differences of pairs of eigenvalues give us the modified mass-squared differences and the eigenvectors give us the modified neutrino mixing matrix. To the first order in the small parameter μ, we have

$$\mathcal{M}'^\dagger \mathcal{M}' = \mathcal{M}^\dagger \mathcal{M} + \mu \lambda^\dagger \mathcal{M} + \mathcal{M}^\dagger \mu \lambda. \tag{7}$$

This matrix is diagonalized by a new unitary mixing matrix U'. We denote this diagonal matrix, correct to the first order in μ, to be M'^2. Using Eq. (4), we can rewrite $\mathcal{M}$ in the above equation in terms of the diagonal matrix M. Converting $\mathcal{M}'$ also into its diagonal form M', we can rewrite the above equation as

$$U' M'^2 U'^\dagger = U(M^2 + m^\dagger M + Mm)U^\dagger \quad \text{with} \quad m = \mu \; U^t \; \lambda \; U. \tag{8}$$

From the above equation, it follows that the mixing matrix U', correct to the first order in μ, is related to zeroth order mixing matrix U by

$$U' = U\ (1 + i\delta\theta), \tag{9}$$

where $\delta\theta$ is a hermitian matrix proportional to μ.

Substituting the expression for U' from Eq. (9) in Eq. (8), we obtain

$$M^2 + m^\dagger M + Mm = M'^2 + [i\delta\theta, M'^2]. \tag{10}$$

Because M'^2 is diagonal, the diagonal terms of $[i\delta\theta, M'^2]$ in the above equation are zero. Therefore, to the first order in μ, the mass squared differences $\Delta M_{ij}^2 = M_i^2 - M_j^2$ get modified as

$$\Delta M_{ij}^{'2} = \Delta M_{ij}^2 + 2\ (M_i \mathrm{Re}[m_{ii}] - M_j \mathrm{Re}[m_{jj}]), \tag{11}$$

where there is no summation over the repeated indices. The above equation shows the correction for neutrino mass-squared difference. Because the Planck scale corrections are assumed to be CPT violating, the correction for anti-neutrino mass-squared difference will have the opposite sign.

3. *Results*

Note from Eq. (11) that the correction term depends crucially on the type of neutrino mass spectrum. For a hierarchial or inverse hierachial spectrum, the correction is negligible. Hence, we consider a degenerate neutrino spectrum and take the common neutrino mass to be 2 eV, which is the upper limit from the tritium beta decay experiment [13].

From the definition of the matrix m in Eq. (8), we find

$$\begin{aligned} m_{11} &= \mu e^{i2a_1}\left(U_{e1}e^{if_1} + U_{\mu 1}e^{if_2} + U_{\tau 1}e^{if_3}\right)^2, \\ m_{22} &= \mu e^{i2a_2}\left(U_{e2}e^{if_1} + U_{\mu 2}e^{if_2} + U_{\tau 2}e^{if_3}\right)^2. \end{aligned} \tag{12}$$

The contribution of the terms in the Planck scale correction, $\mathrm{Re}(m_{22}) - \mathrm{Re}(m_{11})$, can be additive or subtractive depending on the values of the phases a_1 and a_2. Similarly, the magnitudes of $\mathrm{Re}(m_{22})$ and $\mathrm{Re}(m_{11})$ are functions of the phases f_i. Thus we try to find a combination of these phases which can give rise to a significant difference between the neutrino and anti-neutrino mass-squared difference. In our calculations, we used $\theta_{12} = 34°$, $\theta_{13} = 10°$, $\theta_{23} = 45°$ and $\delta_{cp} = 0$. We define the percentage correction P to be ratio of the difference between the corrected neutrino and anti-neutrino mass-squared difference to the unperturbed mass squared difference among the first and the second mass eigenstates as

$$P = 100 \times (\Delta M_{21}'^2 - \overline{\Delta} M_{21}'^2)/\Delta M_{21}^2 \tag{13}$$

In Fig. 1, we plot our results as contours of constant P in the $a_1 - a_2$ plane, for $f_i = 0$. We note that the maximum difference allowed is about $\pm 5\%$ only. This is because the magnitudes of m_{22} and m_{11} remain relatively small in the limit $f_i = 0$. In Fig. 2, we plot contours of constant P in the $f_1 - f_2$ plane with $f_3 = 0 = a_1 = a_2$. When one phase is large and the other is small, there is a possibility that all terms in m_{22} and m_{11} add up to give a large value for their difference and hence a large value for P. We note that a much larger change, varying from 30% to – 20% is

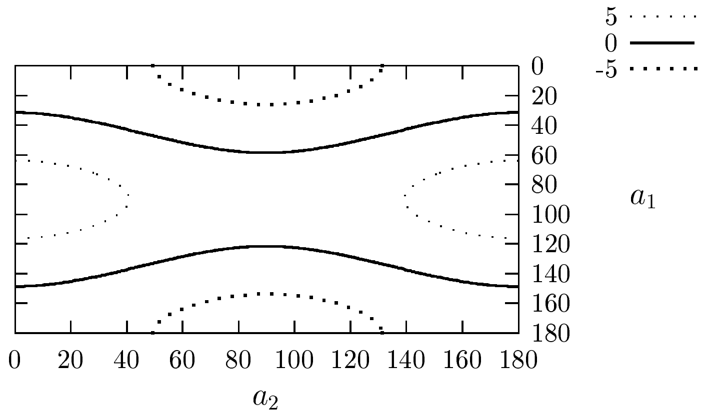

Fig. 1. P as a function of the Majorana phases a_1 and a_2.

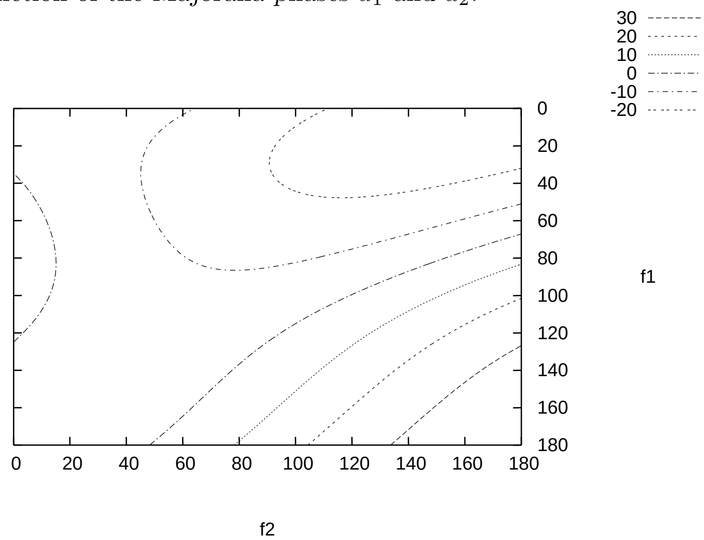

Fig. 2. P as a function of the phases f_1 and f_2.

possible. In Figs. 3 and 4, we plot contours of constant P in the $f_1 - f_3$ plane and in the $f_2 - f_3$ plane. In generating these plots, we set all other phases to zero, as in the case of Fig. 2.

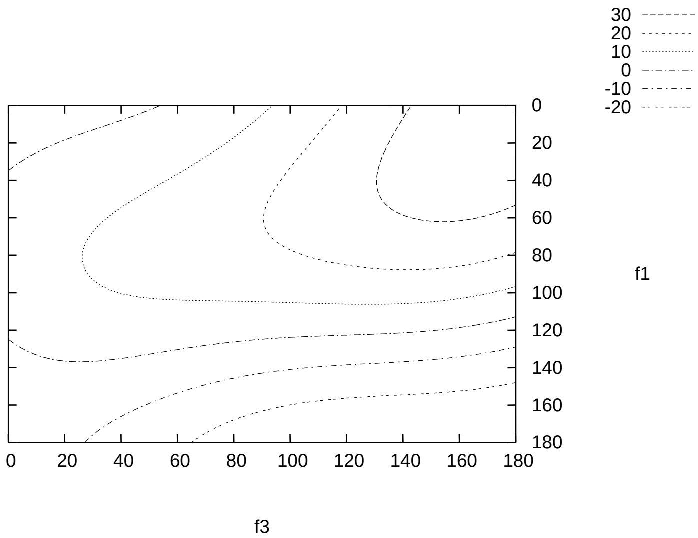

Fig. 3. P as a function of the phases f_1 and f_3.

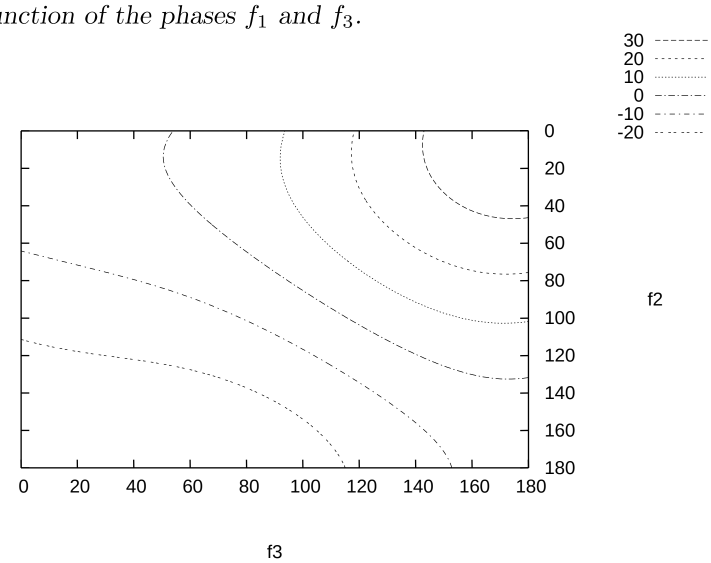

Fig. 4. P as a function of the phases f_2 and f_3.

The effect scales directly as the common neutrino mass. So if one takes the WMAP [14] bound on the common neutrino mass, which is 0.23 eV, then there is a negligible effect. However, if there are new flavour blind interactions between the GUT and Planck scales, then a low common mass is compensated by a lower scale of these new interactions and we still can get appreciable effects. In fact, one can even say that if the gap between Δ_{21} and $\overline{\Delta}_{21}$ persists and future data indicate small neutrino mass, this can be an indication of new CPT violating flavour blind interactions below the Planck scale.

4. Conclusions

Both the solar and reactor data are well accounted for by invoking neutrino oscillations. The fit to the solar data gives a large region for the neutrino mass squared difference in the two flavour parameter space. The fit to the reactor data, however, gives a very strongly constrained anti-neutrino mass squared difference. The best fits of the two mass squared differences are appreciably different from each other. Further improvement in KamLAND systematics and future solar neutrino data may remove this discrepancy. However, if this mismatch between the best fits persists, then the CPT breaking in the neutrino sector will be established. We have demonstrated that flavour blind CPT violating neutrino masses from Planck scale physics can nicely accomodate this effect. This effect is crucially dependent on the neutrino mass spectrum and gives rise to an observable difference between Δ_{21} and $\overline{\Delta}_{21}$ only for a degenerate neutrino mass spectrum with $m_\nu \simeq 2$ eV, which is the largest allowed value from tritium beta decay data. The low value of the common mass implied by the WMAP bound leads to a negligible difference between Δ_{21} and $\overline{\Delta}_{21}$. This can however be compensated for by considering a slightly lower scale for the flavour blind CPT violating mass terms rather than the usual Planck scale.

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MOGUĆE KRŠENJE CPT ZBOG UČINAKA NA PLANCKOVOJ LJESTVICI

Sada je sklad između razlika kvadrata neutrinskih masa određenih mjerenjima s neutrinima sa Sunca i mjerenjima na KamLAND reaktoru s antineutrinima dobar. Međutim, utvrđena je mala razlika središnjih vrijednosti od oko 10^{-5} eV^2. Poboljšanje točnosti kako u mjerenjima s neutrinima sa Sunca, tako i s reaktorskim antineutrinima moglo bi pokazati postojanje stvarnog neslaganja neutrinske i antineutrinske razlike kvadrata masa, što bi ukazalo na kršenje CPT simetrije. U ovom se radu pokazuje kako takav nesklad može nastati zbog članova u neutrinskim masama koji krše CPT simetriju a posljedica su fizike na Planckovoj ljestvici.