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BIANCHI TYPE-III ANISOTROPIC UNIVERSES WITH A CLOUD OF STRINGS IN LYRA'S GEOMETRY

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Bianchi type-III string cosmological models are investigated in the presence of string fluid as a source of matter in the framework of Lyra's geometry. It is observed that the models start with singular state and evolve continuously as $t \to \infty$. It is found that, in all cases, the displacement vector $\beta(t)$ is a decreasing function of time analogous to the behaviour of the cosmological constant Λ. The physical and geometrical aspects of the models are also discussed.

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1. Introduction

The string theory plays a significant role in the study of physical situation at the very early stages of formation of the universe. It is generally assumed that after the big-bang, the universe may have undergone a series of phase transitions as its temperature lowered down below some critical temperature as predicted by the grand unified theories [1 − 6]. At the very early stages of evolution of the universe, it is believed that during the phase transition, the symmetry of the universe was broken spontaneously. That could have given rise to topologically-stable defects such as domain walls, strings and monopoles [1]. Of all these three cosmological structures, only cosmic strings have excited the most interesting consequence [7], because they are believed to give rise to density perturbations which lead to formation of galaxies [4, 8]. The cosmic strings can be closed (like loops) or open (like a hair) which move through time and trace out a tube or a sheet, according to whether it is closed or open. The string is free to vibrate and its different vibrational modes present different types of particles carrying the force of gravitation. This is why it is interesting to study the gravitational effect that arises from strings by using Einstein's field equations.

The general relativistic treatment of strings has been initially given by Letelier [9, 10] and Stachel [11]. Letelier [9] obtained the general solution of Einstein's field equations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry. Letelier [10] also obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Banerjee et al. [12] investigated an axially-symmetric Bianchi type-I string dust cosmological model in the presence and absence of magnetic field. Exact solutions of string cosmology for Bianchi type-II, -VI$_0$, -VIII and -IX space-times were studied by Krori et al. [13] and Wang [14, 15]. Bali and Upadhaya [16] presented LRS Bianchi type-I string dust magnetized cosmological models. Singh and Singh [17] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [18, 19] studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times.

Ever since the beginning of the general relativity theory, a multitude of efforts has been devoted to construct alternative theories of gravitation. These modifications, according to their authors' tastes or beliefs, were often conceptually far from Einstein's orthodoxy. One of the most intriguing modifications of general relativity is that proposed by Weyl [20], invented to unify gravitation and electromagnetism by means of fundamental changes in Riemannian geometry. Unfortunatly the Weyl theory suffers from non-integrability of length and is, therefore, physically unacceptable. However, being interesting from the mathematical point of view, it may still have the germs of a future fruitful theory. Later, Lyra [21] suggested a modification of Riemannian geometry by introducing a gauge function which removes the non-integrability condition of the length of a vector under parallel transport. This modified Riemannian geometry is known as Lyra's geometry. Subsequently, Sen et al. [22, 23] proposed a new scalar-tensor theory of gravitation. They constructed an analog of the Einstein's field equation based on Lyra's geometry which in normal gauge may be written as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi G T_{ij} \, ,$$

where ϕ_i is the displacement vector, R_{ij} is the curvature tensor, g_{ij} is the metric tensor and T_{ij} is the energy-momentum tensor.

Halford [24] pointed out that the constant displacement vector field ϕ_i in Lyra's geometry plays the role of a cosmological constant in the normal general relativistic treatment. Halford [25] showed that the scalar-tensor treatment based on Lyra's geometry predicts some effects within observational limits, as in Einstein's theory. Singh and Singh [26 – 28] studied Bianchi type-I, -III, Kantowski-Sachs and a new class of models with a time dependent displacement field in Lyra's geometry. Recently, Bali and Chandani [29, 30] investigated Bianchi type cosmological models based on Lyra's geometry. The cosmological models with variable Hubble parameter and constant deceleration parameter were studied by Berman [31] and Rao et al. [32].

Recently, Pradhan et al. [33] obtained LRS Bianchi type-II string cosmological models for perfect fluid distributin in general relativity. Yadav et al. [34] investigated string cosmological models in cylindrically-symmetric inhomogeneous space time. Several authors studied Bianchi type-III string cosmological models in different contex in Refs. [35 − 39]. Mahanta and Mukherjee [40] presented some string cosmological models based on Lyra's geometry. In recent years Bali et al. [41 − 43] investigated magnetised string cosmological models in general relativity. Mohanty et al [44, 45] studied higher-dimensional string cosmological models in Lyra's geometry with time dependent displacement field.

Motivated by the situation discussed above, in this paper we investigate Bianchi type-III string cosmological models in Lyra's geometry for the Nambu string, massive string and Takabayasky string. The physical and geometrical aspects of the models are also discussed. It is found that gauge function $\beta(t)$ is large at the beginning and reduces fast during evolution of the universe.

2. *Metric and field equations*

We consider the Bianchi type-III metric of the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(t)\mathrm{d}x^2 - B^2(t)e^{-2ax}\mathrm{d}y^2 - C^2(t)\mathrm{d}z^2, \tag{1}$$

where a is a constant.

The energy momentum tensor of cosmic strings in co-moving cordinate system is given by

$$T^i_j = \rho u_j u^i - \lambda x_j x^i, \tag{2}$$

where u_i and x_i satisfy conditions

$$u^i u_i = -x^i x_i = 1, \qquad u^i x_i = 0\,, \tag{3}$$

and in co-moving coordinate system,

$$T^1_1 = T^2_2 = 0, \quad T^3_3 = \lambda, \quad T^4_4 = \rho, \quad T^i_j = 0 \ \text{ for } \ i \neq j\,. \tag{4}$$

Here, ρ is the rest energy of the cloud of strings with massive particles attached to them. It is given by $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the density of tension that characterizes the strings.

The field equation in the normal gauge for Lyra's manifold, as obtained by Sen, is

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi G T_{ij}\,, \tag{5}$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = (0, 0, 0, \beta(t)) \ , \tag{6}$$

and other symbols have their usual meaning as in Riemannian geometry.

The Einstein's field Eq. (2) for line element (1) lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\frac{3}{4}\beta^2, \tag{7}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\frac{3}{4}\beta^2, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = 8\pi\lambda - \frac{3}{4}\beta^2, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{a^2}{A^2} = 8\pi\rho + \frac{3}{4}\beta^2, \tag{10}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0\,. \tag{11}$$

The quantities with dots overhead refer to their partial derivatives with respect to the time co-ordinate.

On integrating Eq. (11), we have

$$A = \mu B\,, \tag{12}$$

where μ is the constant of integration.

Using Eq. (12), Eqs. (7) – (10) reduces to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\frac{3}{4}\beta^2, \tag{13}$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\psi}{B^2} = 8\pi\lambda - \frac{3}{4}\beta^2, \tag{14}$$

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{B}\dot{C}}{BC} - \frac{\psi}{B^2} = 8\pi\rho + \frac{3}{4}\beta^2, \tag{15}$$

where $\psi = \dfrac{a^2}{\mu^2}$, is constant.

3. Solution of the field equations in some cases

The simplest relation between ρ and λ is the proportionality relation, written as

$$\rho = \alpha\lambda\,, \tag{16}$$

where α is a proportionality constant which gives rise to the following three cases:

(I) For $\alpha = 1$, we get geometric strings or Nambu strings.

(II) For $\alpha = -1$, we get massive strings.

(III) For $\alpha = 1 + \omega, \quad \omega \geq 0$, we get p-strings or Takabayaski strings.

3.1. Case I: Geometric string or Nambu string ($\rho = \lambda$)

In this case, substracting Eq. (15) from (14), we have

$$\frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC} = -\frac{3}{4}\beta^2, \tag{17}$$

Equating Eqs. (7) and (17), we get

$$\frac{\ddot{C}}{C} + \frac{2\dot{B}\dot{C}}{BC} = 0\,. \tag{18}$$

The field equations (7) − (11) represent five independent equations containing the six unknowns A, B, C, ρ, λ and β. For a complete determinacy of the system, we need one extra condition. Referring to Throne [46], current observations suggest that the Hubble expansion of the universe is isotropic today to within $\simeq 30$ percent [47, 48]. Hence, one can adopt the assumption that shear scalar is proportional to the scalar of expansion, i.e. $\sigma \propto \theta$, which leads to

$$B = C^n. \tag{19}$$

Using Eq. (19) in (18), we have

$$\frac{\ddot{C}}{C} + 2n\frac{\dot{C}^2}{C^2} = 0\,, \tag{20}$$

which on intregation yields

$$C = (k_1 t + k_2)^{\frac{1}{(2n+1)}}\,. \tag{21}$$

Hence, from Eq. (19), we get

$$B = (k_1 t + k_2)^{\frac{n}{(2n+1)}}\,, \tag{22}$$

and

$$A = \mu(k_1 t + k_2)^{\frac{n}{(2n+1)}}. \tag{23}$$

Thus the metric (1) reduces to the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mu^2(k_1 t + k_2)^{\frac{2n}{(2n+1)}} \mathrm{d}x^2 - (k_1 t + k_2)^{\frac{2n}{(2n+1)}} \mathrm{d}y^2 + (k_1 t + k_2)^{\frac{2}{(2n+1)}} \mathrm{d}z^2. \tag{24}$$

This represents Nambu string cosmological model in Lyra's geometry.

Physical behaviour of the model

Using Eqs. (21) and (22) in Eq. (17), we get

$$\beta = \frac{2k_1\sqrt{n(n+1)}}{\sqrt{3}(2n+1)(k_1 t + k_2)}, \tag{25}$$

Using Eqs. (21), (22) and (25) in Eqs. (14) and (15), we have

$$8\pi\lambda = 8\pi\rho = -\frac{\psi}{(k_1 t + k_2)^{\frac{2n}{(2n+1)}}}. \tag{26}$$

For a physically viable cosmological model, we choose $\psi < 0$ what implies $\rho > 0$ and $\lambda > 0$. In this case also we see that $\rho = \lambda$, i.e., matter behaves as a cloud of geometric strings.

The proper volume is given by

$$V^3 = \sqrt{-g} = \mu(k_1 t + k_2)e^{-ax}. \tag{27}$$

When $t \to -\dfrac{k_2}{k_1}, V^3 \to 0, \beta \to \infty$. Thus the model has a singularity at $t = -\dfrac{k_2}{k_1}$. The gauge function $\beta(t)$ is large at the beginning and reduces fast during its evolution, as shown in Fig. 1.

Halford [24] has described that the constant displacement field vector ϕ_i plays the role in Lyra's geometry as the cosmological constant Λ in general relativity [24, 25]. From Eq. (25), we observe that the displacement vector β is a decreasing function of time and it approaches a small positive value. It is large at the beginning and decreases fast with the evolution of the universe. Figure 1 clearly shows the behaviour of β as a decreasing function of time.

The cosmological parameter expansion scalar θ, shear scalar σ^2 and deceleration parameter q are given by the following expresions

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{28}$$

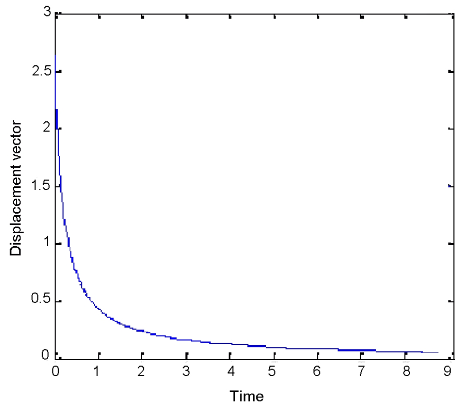

Fig. 1. Plot of the displacement vector β vs. time for the model (24) with parameters $k_1 = 1$, $k_2 = 0.2$, $n = 2$.

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\theta^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC}\right], \tag{29}$$

$$q = -\frac{V\ddot{V}}{\dot{V}^2}. \tag{30}$$

Hence

$$\theta = \frac{k_1}{(k_1 t + k_2)}, \tag{31}$$

$$\sigma^2 = \frac{1}{3}\frac{(3n^2+1)}{(2n+1)}\left[\frac{k_1^2}{(k_1 t + k_2)}\right], \tag{32}$$

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3}\frac{(3n^2+1)}{(2n+1)} = constant. \tag{33}$$

Thus the model does not approch isotropy.

The deceleration parameter

$$q = 0. \tag{34}$$

The rate of expansion H_x, H_y and H_z in directions of x, y and z, respectively, are given by

$$H_x = \frac{\dot{A}}{A} = \frac{n}{(2n+1)(k_1 t + k_2)},$$

$$H_y = \frac{\dot{B}}{B} = \frac{n}{(2n+1)(k_1 t + k_2)},$$

$$H_z = \frac{\dot{C}}{C} = \frac{1}{(2n+1)(k_1 t + k_2)} \,.$$

Hence, the Hubble parameter

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{k_1}{3(k_1 t + k_2)} \,. \tag{35}$$

This model has a singularity at $t = -\dfrac{k_2}{k_1}$. As $t \to \infty$, the shear dies out and the expansion stops. Thus the gauge function $\beta(t)$ is large at the beginning but decays continuously during the evolution.

3.2. Case II: Massive string ($\rho + \lambda = 0$)

Adding Eqs. (14) and (15), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{C}}{BC} - \frac{\psi}{B^2} = 0 \,, \tag{36}$$

Here again, assuming the same relation as taken in the case of Nambu strings between the metric cofficients, the field Eqs. (13) − (15) admit the solution given by

$$A = l_1(t + k_3) \,, \tag{37}$$

$$B = l_2(t + k_3) \,, \tag{38}$$

$$C = l_2^m (t + k_3)^m , \tag{39}$$

where $l_1 = \mu l_2, l_2 = \sqrt{\dfrac{a^2}{\mu^2(m+1)}}$ and $m = \dfrac{1}{n}$.

Thus the metric (1) can be written in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - l_1^2 (t + k_3)^2 \mathrm{d}x^2 - l_2^2 (t + k_3)^2 e^{-2ax} \mathrm{d}y^2 - l_2^{2m} (t + k_3)^{2m} \mathrm{d}z^2 . \tag{40}$$

Physical behaviour of the model

Using Eqs. (38) and (39) in Eq. (17), we get

$$\beta = \frac{2\sqrt{(1-m)(t+k_3)^m - 1}}{\sqrt{3}(t + k_3)} \,. \tag{41}$$

Using Eqs. (38),(39) and (41) in Eq. (15), we get

$$\rho = \left(1 - \frac{\psi}{l_2^2}\right) \frac{1}{8\pi(t+k_3)} \,. \tag{42}$$

For $\alpha = -1$, we have $\rho = -\lambda$. Since for physically viable cosmological models $\rho > 0$, this implies that $\lambda < 0$. A negative string tension density λ indicates that string phase of the universe should disappear and the universe be filled with an anisotropic fluid of particles.

The proper volume is given by

$$V^3 = \sqrt{-g} = l_1 l_2^{m+1} (t+k_3)^{m+2} e^{-ax}, \tag{43}$$

When $t \to -k_3, V^3 \to 0$ and $\beta \to \infty$. Thus the model has a singularity at $t = -k_3$. The gauge function $\beta(t)$ is large at the beginning and reduces fast during the evolution.

The variation of the displacement vector β with respect to time t for the model (40) is given in Table 1. From the table, it is clear that initially the displacement vector β is very small and aproaches to zero at cosmic time $t = 3.8$. After that,

TABLE 1. Variation of the displacement vector β with respect to the cosmic time t for the model (40).

S.N.	Cosmic time	Displ. vector	S.N.	Cosmic time	Displ. vector
1	0	3.11×10^{-16}	16	50	0.03668
2	3.8	0	17	100	0.02301
3	3.9	0.03139	18	200	0.01421
4	4.0	0.04320	19	300	0.01065
5	5.0	0.08314	20	500	0.00731
6	6.0	0.09255	21	1000	0.004443
7	6.9	0.09375	22	2000	0.002668
8	7.0	0.09274	23	5000	0.001354
9	8.0	0.09253	24	10000	0.0008083
10	9.0	0.09021	25	15000	0.0005975
11	10.0	0.08743	26	20000	0.0004821
12	15.0	0.07402	27	25000	0.0004113
13	20.0	0.06384	28	50000	0.0002431
14	30.0	0.05055	29	100000	0.0001456
15	40.0	0.04231	30	200000	0.0000861

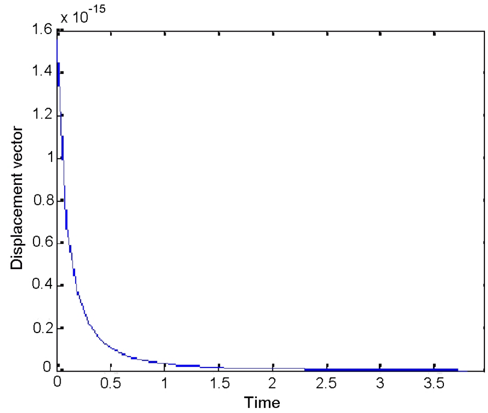

Fig. 2. Plot of the displacement vector β vs. time for the model (40) with parameters $k_2 = 0.2$, and $m = 0.5$.

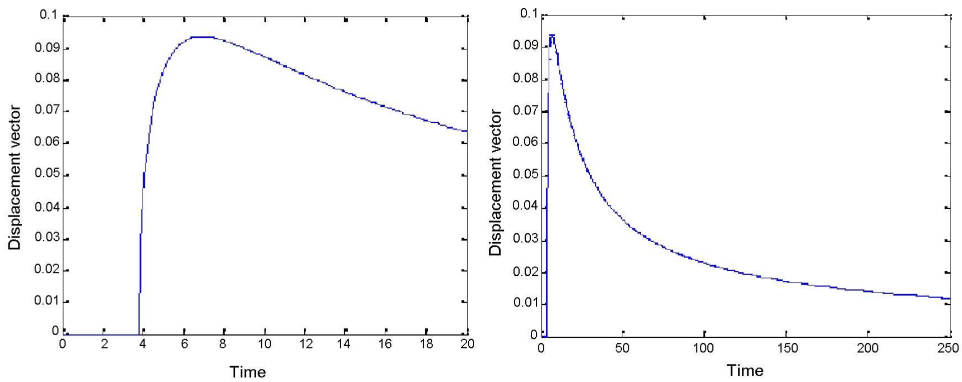

Fig. 3. a) (left) Plot of the displacement vector β vs. time for the model (40) with parameters $k_2 = 0.2$, and $m = 0.5$. b) the same on comacted scale.

its value increases suddenly and a peak is found as shown in Fig. 3a for the model (40). At late time, β decreases and approaches to a small positive value as shown in Fig. 3b. Thus, we observe that β is a decreasing function of time and it plays the same role in Lyra's geometry as the cosmological constant Λ in general relativity [24, 25]. This behaviour is clearly shown in Fig. 2 and Fig. 3

Other physical quantities, the expansion scalar θ and shear scalar σ^2 are given by the following expresions

$$\theta = \frac{m+2}{(t+k_3)}\,, \tag{44}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left[\frac{(m+2)}{(t+k_3)}\right]^2\,, \tag{45}$$

$$\frac{\sigma^2}{\theta^2} = \frac{1}{6} = constant\,. \tag{46}$$

Thus the model does not approch isotropy. The deceleration parameter

$$q = -\frac{(m+2)}{(m+1)} = constant\,. \tag{47}$$

The value of the deceleration parameter q is found to be constant and negative what is supported by the current observations of SN Ia and CMBR, favouring an accelerating model ($q < 0$).

It is interesting to calculate the rates of expansion H_i along the directions of x, y and z axes. They are given by the following expresions

$$H_x = \frac{\dot{A}}{A} = \frac{1}{(t+k_3)}\,,$$

$$H_y = \frac{\dot{B}}{B} = \frac{1}{(t+k_3)}\,,$$

$$H_z = \frac{\dot{C}}{C} = \frac{m}{(t+k_3)}\,.$$

The Hubble parameter is given by

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{m+2}{3(t+k_3)}\,. \tag{48}$$

This model has singularity at $t = -k_3$. As $t \to \infty$, the shear dies out and the expansion stops. Thus the gauge function $\beta(t)$ is large at the beginning, but reduces fast during the evolution.

3.3. Case III: p-string or Takabayaski string $\left[\rho = (1+\omega)\lambda\right]$

In this case, $\rho = (1+\omega)\lambda$, hence from Eqs. (14) and (15) we have

$$2(1+\omega)\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - 2\frac{\dot{B}\dot{C}}{BC} - \frac{\omega\psi}{B^2} = 0\,. \tag{49}$$

The differential equation (48) gives similar results as in the case II. Thus, the physical behaviour of the Takabayaski string model is same as that obtained in the case II.

4. Concluding remarks

We studied cosmological models generated by a cloud of strings with particles attached to them in the framework of Lyra's geometry. In the case I, we have a model of the universe that evolves from a pure geometric string-dominated era with or without a remnant of strings. In the case II, we have $\rho + \lambda = 0$, what implies that $\rho = -\lambda$. Since $\rho = \rho_p + \lambda$, we have in this case $\frac{\rho_p}{|\lambda|} = 2$. According to the Refs. [10] and [13], when $\frac{\rho_p}{|\lambda|} > 1$, in the process of evolution, the universe is dominated by massive strings, and when $\frac{\rho_p}{|\lambda|} < 1$, the universe is dominated by strings. Thus in our model, throughout the whole process of evolution, the universe is dominated by massive strings. Since $\frac{\sigma}{\theta} = constant$ in all cases, the models do not approach isotropy for large value of t. The models start with a singular state and evolve continuously as $t \to \infty$. We obtained string-dominated smooth universe with shear. As $t \to \infty$, the shear dies out and the expansion stops. It is observed that the displacement vector $\beta(t)$ coincides with the nature of the cosmological constant Λ. In the recent time, Λ-term has attracted theoreticians and observers for many reasons. The nontrivial role of the vacuum in the early universe generates a Λ-term that leads to the inflationary phase. Observationally, this term provides an additional parameter to accommodate conflicting data on the values of the Hubble constant, the deceleration parameter, the density parameter and the age of the universe. In recent past, there has been an upsurge of interest in the scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology [49, 50]. Therefore, the study of cosmological models in Lyra's geometry may be relevant for inflationary models. There seems a good possibility of Lyra's geometry to provide a theoretical foundation for the relativistic gravitation, astrophysics and cosmology. However, the importance of Lyra's geometry for astrophysical bodies is still an open question. In fact, it needs a fair trial by experiment.

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ANIZOTROPNI SVEMIRI BIANCHIJEVE III VRSTE S OBLAKOM STRUNA U LYRINOJ GEOMETRIJI

Istražujemo Bianchijeve kozmološke modele III vrste u prisustvu tekućine struna kao izvora tvari u okviru Lyrine geometrije. Nalazimo da prema modelima svemir počinje sa singularnim stanjem i razvija se postepeno kako $t \to \infty$. U svim slučajevima smo našli da je vektor posmika $\beta(t)$ opadajuća funkcija vremena, poput svojstva kozmološke stalnice Λ. Raspravljamo fizičke i geometrijske značajke modela.