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LETTER TO THE EDITOR

KALUZA-KLEIN FRW STIFF FLUID COSMOLOGICAL MODEL IN LYRA MANIFOLD

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In this paper we made an attempt to construct a five-dimensional FRW stiff fluid cosmological model in Lyra manifold. This model reduces to curvature-free ($k = 0$) expanding stiff fluid cosmological model. Some physical and geometrical properties of the model are discussed.

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In view of Kaluza-Klein theories [1 − 4], the study of higher-dimensional cosmological models acquired much significance. An interesting possibility known as the cosmological dimensional reduction process is based on the idea that, at the very early stage, all dimensions in the universe are comparable. Later, the scale of the extra dimensions becomes so small as to be unobservable by experiencing contraction. This process was first proposed by Chodos and Detweiler [5] who showed that, in the framework of pure gravitational theory of Kaluza-Klein, the extra dimension contracts to a very small scale, while the other spatial dimensions expand isotropically. Guth [6] and Alvarez and Gavela [7] observed that during the contraction process, extra dimensions produce large amount of entropy. Weyl [8] proposed a modification of Riemannian manifold in order to geometrize the whole of gravitation and electromagnetism. His theory is found to be physically unsatisfactory. Later, further modification of Riemannian geometry proposed by Lyra [9] bears a close resemblance to Weyl's geometry. Sen [10] showed that the static model with finite density in Lyra manifold is similar to the static model in Einstein theory. Subsequently, Halford [11] studied cosmological theory within the framework of Lyra's geometry and pointed out that the vector field ϕ_i in Lyra's geometry plays a similar role of cosmological constant Λ in general theory of relativity. Further, he suggested that energy conservation law does not hold in the cosmological theory based on Lyra's geometry and he developed a theory within the framework of Lyra's geometry which gives rise to non-static perfect fluid models. Halford [12] showed that the scalar-tensor theory of gravitation in Lyra manifold predicts the same effects, within observational limits, as in the Einstein theory. Beesham [13] constructed four-dimensional FRW cosmological models in Lyra manifold. Assuming the energy density of the universe equal to its critical value, he showed that the models have $k = -1$ geometry. Singh and Singh[14] gave a general review of the work done in Lyra's geometry. Further, they constructed cosmological models with both constant and time-dependent displacement field and have shown that some of these models solve the singularity, entropy and horizon problems. Singh and Desikan [15] obtained the exact solutions for four-dimensional FRW cosmological models in Lyra geometry with constant deceleration parameter. Further, they examined the behavior of the displacement field β and the energy density ρ for perfect fluid distribution. However, they found that the expressions for β^2 and ρ are not valid for empty universe and the stiff matter distribution. Pradhan et al. [16] obtained exact solutions of the field equations constituting an isotropic homogeneous universe with a bulk viscous fluid in the cosmological theory based on Lyra geometry. Various authors [17−25] constructed various five-dimensional cosmological models in Lyra manifold. Recently Yadav[26] studied various aspects of Lyra's geometry.

The Einstein's field equations based on Lyra's manifold, as proposed by Sen [10] and Sen and Dunn [27] in normal gauge, may be written as

$$R_{ik} - \frac{1}{2}g_{ik}R + \frac{3}{2}\phi_i\phi_k - \frac{3}{4}g_{ik}\phi_m\phi^m = -\chi T_{ik}\,, \tag{1}$$

where ϕ_i is the displacement vector and other symbols have their usual meanings in the Riemannian geometry. Here we consider the five dimensional FRW metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right) + A^2\mathrm{d}m^2, \tag{2}$$

where R and A are functions of the cosmic time t only, and k characterizes the spatial curvature. The fifth co-ordinate m is assumed to be space like.

The energy momentum tensor for perfect fluid distribution is taken as

$$T_{ik} = (p+\rho)u_iu_k - pg_{ik}\,, \tag{3}$$

together with the co-moving co-ordinates

$$g^{ik}u_i u_k = -1\,, \tag{4}$$

where p and ρ are isotropic pressure and energy density of the cosmic fluid distribution, respectively, and u_i is the velocity five-vector of the fluid which has components $(1,0,0,0,0)$. The displacement vector ϕ_h is defined as

$$\phi_h = (\beta(t),0,0,0,0)\,. \tag{5}$$

The field Eqs. (1) together with Eqs. (3), (4) and (5) for the metric (2) yield the following explicit equations

$$2\frac{R''}{R} + \left(\frac{R'}{R}\right)^2 + 2\frac{R'A'}{RA} + \frac{A''}{A} + \frac{3}{4}\beta^2 + \frac{k}{R^2} = -\chi\rho\,, \tag{6}$$

$$-3\left(\frac{R'}{R}\right)^2 - 3\frac{R'A'}{RA} + \frac{3}{4}\beta^2 - \frac{3k}{R^2} = -\chi\rho\,, \tag{7}$$

and

$$\frac{3R''}{R} + 3\left(\frac{R'}{R}\right)^2 + \frac{3}{4}\beta^2 + \frac{3k}{R^2} = -\chi P\,, \tag{8}$$

where prime denotes differentiation with respect to the cosmic time t.

Equations (6) – (8) contain five unknowns, viz. R, A, β, P and ρ. In order to obtain explicit exact solutions, we assume that $\sigma \propto \theta$ which leads to

$$A = R^n \qquad (n \neq 0) \tag{9}$$

and the equation of state, i.e.

$$p = \rho\,. \tag{10}$$

Equation (10) represents a stiff matter or Zel'dovich fluid. The models with $p = \rho$ are important in the description of very early stages of the universe.

Using Eqs. (9) and (10) in the field equations (6) – (8), we obtained

$$(n+2)\frac{R'}{R} + \frac{R''}{R'} = 0\,, \tag{11}$$

which yields

$$R = [(n+2)(bt+c)]^{1/(n+3)}\,, \tag{12}$$

where $b\,(\neq 0)$ and c are arbitrary constants of integration. Using Eq. (12) in Eq. (9), we get

$$A = [(n+2)(bt+c)]^{n/(n+3)}\,. \tag{13}$$

Now using Eqs. (12) and (13) in the field Eq. (6), we obtained

$$\chi p = \frac{3b^2(n+1)}{[(n+3)(bt+c)]^2} + \frac{k}{[(n+3)(bt+c)]^2} - \frac{3}{4}\beta^2 . \tag{14}$$

But Eq. (14) satisfies the field Eqs. (7) and (8) for $k = 0$ only. Hence we get

$$\chi\rho\,(=\chi p) = \frac{3b^2(n+1)}{[(n+3)(bt+c)]^2} - \frac{3}{4}\beta^2 . \tag{15}$$

Thus geometry of the curvature-free ($k = 0$) five-dimensional FRW stiff fluid cosmological model is described by the metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + [(n+3)(bt+c)]^{2/(n+3)}(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2) + [(n+3)(bt+c)]^{2n/(n+3)}\mathrm{d}m^2 . \tag{16}$$

At the initial epoch $t = 0$, the metric (16) becomes flat. Further, as time increases, the scale factor R expands indefinitely while the other one, namely A, gradually decreases for $n \in (-3, 0)$. Thus the extra dimension becomes insignificant as time proceeds after the creation and we are left with the real four-dimensional world. From Eq. (15) we observed that, as time t tends to infinity, $\chi\rho = -\frac{3}{4}\beta^2$. Therefore, we can say that the universe is guided by the gauge function β. This type of dependence of ρ and β in the case of stiff fluid distribution has already been shown by Singh and Singh [28, 29] and Singh and Sri Ram [30].

The scalar of expansion (θ) is obtained as

$$\theta = \frac{b}{bt+c} .$$

At the initial epoch $t = 0$, θ is finite and $\theta \to 0$ when $t \to \infty$. Hence there is finite expansion in the model.

The shear scalar is obtained as

$$\sigma^2 = \frac{1}{2}\left[\frac{4}{9} + 3\left\{\frac{b}{bt+c} + \frac{1}{3}\right\}^2 + \left\{\frac{nb}{(n+3)(bt+c)} + \frac{1}{3}\right\}^2\right] .$$

Since $\lim_{t\to\infty} \sigma^2/\theta^2 \neq 0$, the universe remains anisotropic throughout the evolution.

The spatial volume (V) of the universe is

$$V = [(n+3)(bt+c)]^2 r^2 \sin^2\theta .$$

The volume of the universe increases with time.

The deceleration parameter (q) is

$$q = -\frac{1}{2} .$$

which indicates that the model (16) represents an accelerating universe.

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KALUZA-KLEINOV FRW KOZMOLOŠKI MODEL S KRUTOM TEKUĆINOM U LYRINOJ MNOGOSTRUKOSTI

Pokušali smo ustrojiti pet-dimenzijski Friedmann-Robertson-Walker kozmološki model s krutom tekućinom u Lyrinoj mnogostrukosti. Taj se model svodi na kozmološki model bez zakrivljenosti ($k = 0$) s krutom tekućinom i širenjem. Raspravljaju se neka fizička i geometrijska svojstva modela.