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LETTER TO THE EDITOR

A LATE-TIME COSMOLOGICAL MODEL WITH SLOWLY GROWING GRAVITATIONAL COUPLING CONSTANT AND DECAYING VACUUM ENERGY DENSITY

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A simple cosmology with slowly growing Newton's coupling constant motivated from Brans-Dicke coupling theory and decaying vacuum energy density in universe with flat spatial curvature is considered. Despite the simplicity of the model, it exhibits interesting late-time dynamics consequences in agreement with recent observations.

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One of the most remarkable discoveries of our time is the accelerated expansion of our universe. This fact is supported and based on CMBR dataset of the Three-Year WMAP observations and results obtained using combined WMAP data and data from Supernova Legacy Survey of type SNeIa and galaxy clustering. These recent observations suggest that in the standard paradigm, the cosmos is dominated by a mysterious form of dark energy (DE) with large negative pressure and consequently with negative equation of state parameter (EoSP) $w = -1.06^{+0.13}_{-0.08}$ [1 − 7]. The recent observations on baryon oscillations support strongly the DE hypothesis [8]. Further, these observations suggest that the universe has experienced at least two stages of accelerated expansion: the early universe inflationary stage and the present acceleration of the cosmos. The DE problem represents today one of the most intriguing problems in modern physics. There are strong reasons to believe that answering this question will have much to do with the possibility to understand the very early physics. Models of phenomenological DE are abundant ranging from alternative higher gravity theories [9] to the ΛCDM model [10], quintessence with a very shallow many-forms inverse power-law potential with EoSP $-1 \leq w \leq 0$ [11], K-essence [12], viscous fluid [13], Chaplygin gas [14, 15], generalized Chaplygin gas model [16, 17], Brans-Dicke (BD) theory [18], decaying Higgs fields [19], dilaton field [20], unstable tachyon field [21, 22] etc. It is worth-mentioning that, using the concept of tracking fields, the coincidence and the fine-tuning problems can be solved. Despite the number of efforts, there is no consistent theory which may explain the late-time accelerated expansion of the universe and solve the singularity cosmology at the origin of time, in particular within the context of Friedmann-Robertson-Walker isotropic homogeneous cosmological model.

Many scientists believe that the resolution of the Big Bang initial singularity problem will come from modifications of the Einstein general relativity (EGR) due to quantum gravity at the Planck scale, e.g. time-variations of the Newton's constant as predicted by some alternative theories of gravity [23] and a number of modern cosmological models. Many theoretical field theories, such as cosmology with extra dimensions, string theories, and scalar-tensor quintessence models [24], have been proposed and discussed through the literature in which the gravitational coupling parameter is time-dependent. At the moment, the question over the constancy of the gravitational coupling constant has been revitalized by recent astronomical observations of distant high-red-shift type Ia supernovae and galaxy clustering. Since then, many attempts have been made to find astrophysical signs due to the possible time-variation of the gravitational coupling constant [25].

In this letter, we investigate whether a slow-time dependence of the gravitational coupling constant can affect the result. There are several phenomenological models which predict a time-dependence of the gravitational constant. The well-recognized ones are generally based on a Brans-Dicke coupling theory and are parameterized as $G = G_0\big(1+A(t-t_0)\big)^\beta$, where G_0 is the value of the gravitational constant at the present epoch, i.e. $G = G_0$ for $t = t_0$, β is a real parameter, and $|A| \equiv |\dot{G}_0/G_0|$ [26]. The *present* (late-time) constraints on $\dot{G} \equiv \mathrm{d}G/\mathrm{d}t$ can be expressed as follows: $|\dot{G}/G|_p \approx |\beta| t_p^{-1} = |\beta| H_p/m < \delta\ 10^{-10}\ \mathrm{yr}^{-1}$, i.e. $|\beta| < m\delta$, where δ is a real parameter constrained from observations. Here we have assumed that $H_p t_p = m$, $H_p = 10^{-10}\ \mathrm{yr}^{-1}$ is the present Hubble length and m is constrained by observations [27]. It was recently shown using the density dependence of the nuclear symmetry energy, constrained by recent terrestrial laboratory data on isospin diffusion in heavy-ion reactions at intermediate energies, and the size of neutron skin in ^{208}Pb, within the gravitochemical heating formalism developed by Jofré et al., that an upper limit for $|\dot{G}/G|_p \approx (4.5 - 21) \times 10^{-12}\ \mathrm{yr}^{-1}$ and, therefore, one expects that $|\beta|$ is a small parameter [28].

If the cosmological term is constant from the Planck time to the present epoch, there is the famous *cosmological constant problem*: the theoretical estimation of the quantum vacuum energy (the Casimir-like energy density for the whole universe) gives an enormous large value, whereas the experimentally estimated value is 120 orders smaller! Such a significantly huge value of the vacuum energy density represents a big problem in itself, independently of the problem of the accelerated expansion of the universe. There are dozens of candidates for the solution of this problem. Unfortunately, most of the approaches found in the literature shift the problem rather than solve it. To solve this problem, it is thus natural to consider that the cosmological term decreases from the large value at the early epoch to the present value. Observing that the cosmological constant has dimension of inverse length square, several ansatzes have been proposed in which the cosmological constant decays with time [29]. One interesting ansatz introduced in the literature is $\Lambda = 3\alpha(\dot{a}^2/a^2)$, where $a(t)$ is the scale factor of the FRW metric and α is a real constant assumed to be positive [30]. It is worth-mentioning that a non-zero lambda term helps to reconcile inflation with observation. Moreover, we make another assumption that is common in modern cosmology: the matter density decreases like $\rho = \epsilon a^n$, n is a real parameter and $\epsilon = 3/(8\pi G_0)$ for convenience, e.g. $n = 3$ for non-relativistic matter, $n = 4$ for radiation and $n = 0$ for the cosmological constant. Here we assume that n is unknown.

Let us assume a spatially-flat, isotropic and homogeneous FRW space-time described by the usual metric $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \sum_{i=1}^{3} (\mathrm{d}x^i)^2$, where $a(t)$ is the scale factor of the universe. The Friedmann equation $\dot{a}^2/a^2 = 8\pi G\rho/3 + \Lambda/3$ is written using the previous ansatzs as

$$(1-\alpha)\frac{\dot{a}^2}{a^2} = \big(1 + A(t-t_0)\big)^{\beta} a^{-n}. \tag{1}$$

The conservation equations for the stress energy-momentum tensor $T_{\mu\nu}$, i.e. $\nabla_\nu T^{\mu\nu} = 0$ take the usual forms $\dot{\rho} + 3H(\rho + p) = 0$ and $\dot{\Lambda} + 8\pi\dot{G}\rho = 0$. p and ρ are, respectively, the pressure and density of the perfect fluid. For a complete determinacy of the system, we will adopt the equation of state $p = w\rho$, where w is the EoSP, i.e. $\dot{\rho} + 3H(w+1)\rho = 0$. Although the proper way is to look for a field phenomenological framework for the variation of the gravitational coupling constant and the cosmological constant, nevertheless the present approach could be considered as a limiting case of some covariant theory yet to be discovered. Brans-Dicke scalar theory is one prototype of these theories but as it is well-known, this theory allows only a decreasing gravitational constant with the cosmological time. The solution of Eq. (1) is easily deduced and looks like

$$a(t) = \left\{ a_0^{n/2} + \frac{n}{A(\beta+2)\sqrt{1-\alpha}} \left[\big(1 + A(t-t_0)\big)^{(\beta/2)+1} - 1 \right] \right\}^{2/n}, \tag{2}$$

where $a_0 = a(t = t_0)$ and $\alpha < 1$. Equation (2) may be approximated for very large times by $a(t) \propto t^q$, $q = (2+\beta)/n$, and therefore the cosmological constant decays like $\Lambda = \alpha(2+\beta)^2/(n^2 t^2) = 3\alpha H^2$, while the matter density decays as $\rho \propto t^{-2-\beta}$. It is noteworthy that the parameter q is to completely dissimilar from the deceleration parameter $\bar{q} \equiv -\ddot{a}a/a^2$ which is related to the Hubble parameter $H(t) \equiv \dot{a}/a$ by $\bar{q} \equiv -\ddot{a}/H\dot{a} = -(q-1)/q$. For $q > 1$, $\bar{q} < 0$ and this match an accelerated expanding universe for the present epoch. The continuity equations give in their turn $(2+\beta)(3(w+1)-n) = 0$ and $3\beta n^2 = 2\alpha(2+\beta)^2$ with $\beta \neq -2$. For $\beta > 0$, we find $\alpha > 0$, and therefore the cosmological constant is positive. Accordingly, we find $n = 3(w+1)$, $w \neq -1$. As $n > 0$, we get $w > -1$, and consequently the universe in our framework is free from phantom energy. The vacuum energy density is given by $\rho_v(t) = \Lambda/(8\pi G) \approx \alpha(2+\beta)^2/(8\pi G_0 n^2 t^{2+\beta})$ and the density parameter of the universe is given by $\Omega_m = \rho/\rho_c \approx (1-\alpha)n^2/4$, $(\alpha < 1)$, where $\rho_c = 3H^2/(8\pi G)$ is the critical energy density of the universe. The density parameter due to the vacuum contribution is given by $\Omega_\Lambda = \Lambda/(3H^2) = \alpha$, and therefore we shall define $\Omega_{\mathrm{Total}} = \Omega_\Lambda + \Omega_m \approx \alpha + (1-\alpha)n^2/4$. This particular class of solution, with constant deceleration factor, sheds light on the cosmological constant problems, leading to a vacuum parameter $\Omega_\Lambda = \Lambda/(3H^2) = \alpha$, and to a constant ratio between the vacuum and matter energy densities. Due to the constant ratio between the energy densities of matter and vacuum, a possible solution for the cosmic coincidence problem, that is the approximated coincidence presently observed between the matter density and critical density, may be realized. However, this ansatz presents some problems. The mainly severe of them is the existence of a constant deceleration factor, i.e. the universe expansion is uniform, whereas a large structure formation requires a phase of decelerated phase. For this main reason, one should take into account this ansatz applicable merely in the limit of late-times, restricting in this way the predictive power of the model. As suggested from various observations concerning clusters [30−32] the allowed range of the density parameter is $0.2 \leq \Omega_m \leq 0.4$, therefore $0.6 \leq \Omega_\Lambda \leq 0.8$, $0.7 \leq \beta \leq 5.2$ and $1.8H_0^2 \leq \Lambda \leq 2.4H_0^2$, while $n = 2$ and $w = -1/3$. The above mentioned observations restrict the dimensionless age parameter $H_p t_p$ (which is 2/3 in the standard flat FRW model) to the interval $1.35 \leq H_p t_p = m \leq 3.65$, which should be compared with the rather conservative bounds $0.6 \leq H_p t_p \leq 1.4$, adopted in Ref. [33]. The model, therefore predicts that the minimum age of the universe is $1.35 H_p^{-1}$.

As we expect β to be a tiny parameter, we choose $\beta \approx 0.7$, for which $G(t) \propto t^{0.7}$, $\rho \propto t^{-2.7}$, $\Lambda \propto t^{-2}$ and $a(t) \propto t^{1.35}$. The universe at this stage is accelerated in time, dominated by dark energy with EoSP $w = -1/3$, a decaying lambda, a decaying matter density and a slowly increasing gravitational coupling constant. For these ranges of β, the present day variation of G, $(\dot{G}/G)_p \approx 0.5 H_p$ per year is in agreement with recent astronomical data [34]. Using the previous numerical data, the rate of particle creation/annihilation defined by $N \equiv a_p^{-3}\big(\mathrm{d}\rho a^3/\mathrm{d}t\big)_p$ is $N = 1.35\rho_p H_p$ less than that of the steady state model $\big(3\rho_p H_p\big)$ [35]. It is noteworthy that for $\alpha = 0$, i.e. cosmology free from the cosmological constant, $\beta = 0$, $n = 2$, $w = -1/3$, and therefore $a(t) \propto t$, $\rho(t) \propto t^{-2}$ and $G = G_0$. The universe, for this particular case, is uniformly expanding in time and is dominated by dark energy. However, an inflationary solution is obtained for $\rho = \epsilon = constant$, i.e. $n = 0$, from which one obtains easily using Eq. (1) the solution $a(t) = a_0 \exp\big[(1+t)^{1+\beta/2} - 1\big]/\big[(1+\beta/2)\sqrt{1-\alpha}\,\big]$. This is a modified inflationary scenario in a sense $G(t) \propto t^\beta$ and $\Lambda(t) = 3\alpha H^2$, in contrast to the standard inflationary paradigm where G is constant, $\Lambda(t) = 3H^2$ and $\rho = \epsilon = 0$.

Let us now determine the variation rate of the gravitational coupling constant at the present epoch dominated by dark energy. The evolution law $G(t) \propto t^{0.7}$ leads to the present relative variation rate $|\dot{G}/G|_p = 0.5 H_0$, which is in good agreement with recent astronomical observations [28]. Another interesting neoclassical test concerns the horizon at time t_0, the proper distance travelled by light and emitted at $t = t_1 \to 0$, which is given by [36] $\mathrm{d}(z) = a(t) \int\limits_{t_1 \to 0}^{t_{\max}} \mathrm{d}t/a(t) \approx H_0^{-1} z$, where $z = 1 - \dot{a}/a$ is the redshift. As long as z increases, the distance increases and there is no event horizon, i.e. the causal communication between two observers exists. Other kinetic tests may be derived including the luminosity distance-redshift d_L *(the ratio of the* *detected energy flux and the apparent luminosity)*, the angular distance-redshift d_A *(the measure of how large objects appear to be)* and the look back time-redshift $t_1(z = 0 - t_{\max}(z)$ *(the difference between the age of the universe at the present time and the age of the universe when photons was emitted at redshift z)*. Following Ref. [36], it is easy to prove that at larger time: $d_L \approx H_0^{-1}[z + (1 - q)z^2/2 + \ldots]$, $d_A(z) = d_L(z)/(1 + z)^2 \approx zH_0^{-1}$ and finally $H_0(t_1 - t_{\max}) \approx z - (1 + q/2)z^2 + \ldots$, which are in better agreement with recent available data than the results obtained in Refs. [36] and [29]. Here $q \equiv -\ddot{a}a/a^2$ is the deceleration parameter. These relations, in particular the angular distance-redshift and the luminosity distance-redshift are difficult to test observationally for two main reasons: first, objects of fixed size like spherical galaxies do not possess sharp edges used in practice for measuring angular size and, consequently, one has rather to measure isophotal diameters, while objects with well-defined linear dimensions such as double radio sources, are highly dynamical and as a result their intrinsic size is unknown. Therefore, these neoclassical tests are in fact difficult to use in practice [37, 38].

In summary, we have analyzed in this short communication the effect of the assumed phenomenological growing law for the gravitational coupling constant $G = G_0\big(1 + A(t - t_0)\big)^{0.7}$ on the late-time dynamics of the universe. We have found that the universe is spatially flat and undergoing a late-time accelerated expansion. This acceleration has been attributed to a dark energy component with negative pressure which can induce repulsive gravity. The matter density decays like $\rho \propto a^{-2}$ in contrast to the standard FRW model. Regardless of the simple analysis we made about the observational limits for the age and matter density parameters, a supplementary comprehensive analysis of the whole set of current observational data is still in order. A careful study of the whole scenario constitutes the subject of a forthcoming publication.

References

[1] A. G. Riess et al., Astron. J. **116** (1998) 1009.

[2] S. Perlmutter et al., Astrophys. J. **517** (1999) 565.

[3] B. R. Schmidt et al., Astrophys. J. **507** (1998) 46.

[4] P. J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D **59** (1999) 123504.

[5] M. Persic, P. Salucci and F. Stel, Mon. Not. Roy. Astron. Soc. **281** (1996) 27.

[6] M. Carmeli and T. Kuzmenko, Int. J. Theor. Phys. **41** (2002) 131.

[7] S. Behar and M. Carmeli, Int. J. Theor. Phys. **39** (2002) 1375;

[8] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[9] A. G. Riess et al., Astrophys. J. **607** (2004) 665.

[10] J. S. Alcaniz, Phys. Rev. D **69** (2004) 083521.

[11] P. de Bernardis, Nature **377** (2000) 600.

[12] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75** (2003) 559.

[13] Ph. Brax and J. Martin, Phys. Lett. B **468** (1999) 45.

[14] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. D **63** (2001) 103510.

[15] J. C. Fabris, S. V. B. Concalves and R. de Sa Ribeiro, Gen. Rel. Grav. **38** (2006) 495.

[16] A, Yu. Kamenshchik, V. Moschella and V. Pasquier, Phys. Lett. B **511** (2001) 265.

[17] J. C. Fabris, S. V. B. Concalves and P. E. de Souza, Gen. Rel. Grav. **34** (2002) 53.

[18] N. Bilić, G. B. Tupper and R. D. Viollier, Phys. Lett. B **535** (2002) 17.

[19] N. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D **66** (2002) 043507.

[20] A. B. Batista, J. C. Fabris and R. de Sa Ribeiro, Gen. Rel. Grav. **33** (2001) 1237.

[21] A. A. Sen, S. Sen and S. Sethi, Phys. Rev. D **63** (2001) 10750.

[22] S. Sen and T. R. Seshadri, Int. J. Mod. Phys. D **12** (2003) 445; T. Chiba, Phys. Rev. D **60** (1999) 083508.

[23] see K. Koyama, Gen. Rel. Grav. **40** (2008) 421.

[24] see R. A. El-Nabulsi, Mod. Phys. Lett. A **23**, 6 (2008) 401 and references therein.

[25] P. G. Krastev and B.-A. Li, Phys. Rev. C **76** (2007) 055804.

[26] L. Amendola, S. Corasaniti and F. Occhionero, astro-ph/9907222.

[27] L. Amendola, Phys. Rev. D **60** (1999) 043501.

[28] P. Jofré, A. Reisenegger and R. Fernandez, Phys. Rev. Lett. **97** (2006) 131102.

[29] see A. Pradhan, G. S. Khadekar and D. Srivastava, Astrophys. Space Sci. **305** (2006) 415 and references therein; see also G. P. Singh, S. Kotambkar and A. Pradhan, Fizika B **15**, 1 (2006) 23.

[30] A. S. Al-Rawaf and M. O. Taha, Gen. Rel. Grav. **28** (1996) 935; A. S. Al-Rawaf, Mod. Phys. Lett. A **13** (1998) 429; J. M. Overdin and F. I. Cooperstock, Phys. Rev. D **58** (1998) 043506; A. I. Arbab, JCAP05 008; Class. Quantum Grav. **20** (2003) 93; A. I. Montenegro and S. Carneiro, gr-qc/0609059.

[31] T. Futamase and S. Yoshida, gr-qc/0011083.

[32] For a recent review see, e.g., N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science **284** (1999) 1481.

[33] J. A. S. Lim and J. M. F. Maia, Phys. Rev. D **49** (1994) 5597.

[34] E. V. Pitjeva, Astron. Lett. **31** (2005) 340.

[35] J. Matyjasek, Phys. Rev. D **51** (1995) 4154.

[36] G. P. Singh, S. Kotambkar, D. Srivastava and A. Pradhan, Rom. J. Phys. **53**, 3-4 (2008) 607.

[37] J. V. Cunha, J. A. S. Lima and N. Pires, Phys. Rev. D **74** (2006) 023532.

[38] M. Novello and H. Heitzmann, Gen. Rel. Grav. **16** (1984) 535; M. Novello and J. M. Salim, Phys. Rev. D **20** (1979) 377.

KASNODOBNI KOZMOLOŠKI MODEL SA SPORORASTUĆOM GRAVITACIJSKOM KONSTANTOM I OPADAJUĆOM GUSTOĆOM VAKUUMSKE ENERGIJE

Razmatra se jednostavna kozmologija s rastućom Newtonovom konstantom potaknuta Brans-Dickeovom teorijom vezanja i opadajućom gustoćom vakuumske energije u ravnom svemiru. Unatoč jednostavnosti modela, pokazuju se zanimljivi ishodi kasnodobne dinamike, u skladu s nedavnim opažanjima.