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AXIAL AND ELECTROMAGNETIC NUCLEON FORM FACTORS IN THE UNPHYSICAL REGION

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We study the annihilation channel in antiproton-nucleon collisions with production of a single charged or neutral pion and a lepton-antilepton pair. These processes offer a unique possibility to study the nucleon electromagnetic form factors in the unphysical region. The differential cross section in an experimental set-up, where the pion is fully detected, is given with explicit dependence on the relevant nucleon form factors.

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1. Motivation

The electromagnetic nucleon form factors are usually measured in the scattering processes such as $\ell + N \to \ell + N$, where we access the space-like region of the form factor, or in the annihilation processes $\bar{N} + N \to \ell^+ + \ell^-$, where we access the time-like region of the form factor. However, in the annihilation processes $\bar{N} + N \to \ell^+ + \ell^-$ it is not possible to measure nucleon electromagnetic form factors in the time-like region under the threshold of the annihilation – so called unphysical region. Also there are no experimental data on the nucleon axial form factor in the time-like region.

In this paper we study the annihilation processes

$$\bar{p}(p_1) + n(p_2) \to \pi^-(q_\pi) + \ell^-(p_-) + \ell^+(p_+) \quad \text{and} \tag{1}$$
$$\bar{p}(p_1) + p(p_2) \to \pi^0(q_\pi) + \ell^-(p_-) + \ell^+(p_+)\,,\ \ell = e, \mu\,,$$

which are the crossed processes of pion electroproduction on a nucleon $e^- + N \to e^- + N + \pi$. They contain the same information on the nucleon form factors in the different kinematical region. These processes are also related to the pion scattering process $\pi + N \to N + \ell^- + \ell^+$ which was studied in Ref. [1]. In that paper it was pointed out, that annihilation processes (1) give possibility of determination of the nucleon electromagnetic form factors in the unphysical region, which is otherwise unreachable by the annihilation process $\bar{p} + p \to e^+ + e^-$. In another paper [2], a general expression for the cross section was derived and numerical estimations were given in the kinematical region near threshold. Now we take into account a larger set of diagrams contributing to the processes and we give a special emphasis to the possibility of extraction of the axial nucleon form factor in the time-like region. The aim of this section is to estimate differential cross section for experimental conditions achievable at the future FAIR facility [3] by using existing models of nucleon structure extended to the time-like region.

2. Formalism

Our approach is based on the Compton-like annihilation Feynman amplitudes and aims to establish the matrix elements of the processes (1). The main uncertainty in this description in terms of Green functions of mesons and nucleons is related to the model-dependent description of hadron form factors (FFs). Possible tree-level Feynman diagrams of the considered processes (1) are shown in Figs. 1 and 2, and they differ in a particle emitting lepton-antilepton pair. In the case of the first process, there are only two possible tree-level diagrams, while spinless neutral particle π^0 can not emit the $\ell\bar{\ell}$ pair.

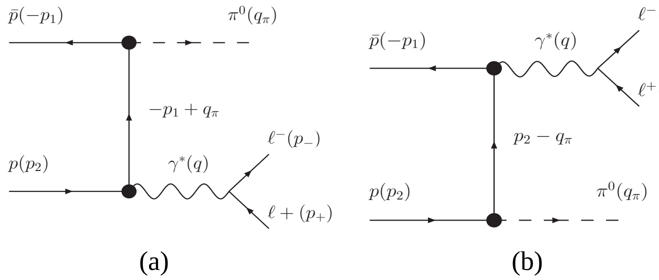

Fig. 1. Feynman diagrams for the process $\bar{p} + p \to \pi^0 + \ell^+ + \ell^-$.

As it has been already discussed in Ref. [2], vertices of pion and nucleons, $\gamma^* NN^*$ and $\gamma^*\pi\pi^*$, contain virtual hadrons, and rigorously speaking, electromagnetic form factors should be modified taking into account off-mass shell effects. However, we will use standard on-mass shell form factors as errors of such approximation are at the level of 3% [4].

Therefore, the expression for the nucleon electromagnetic current can be written

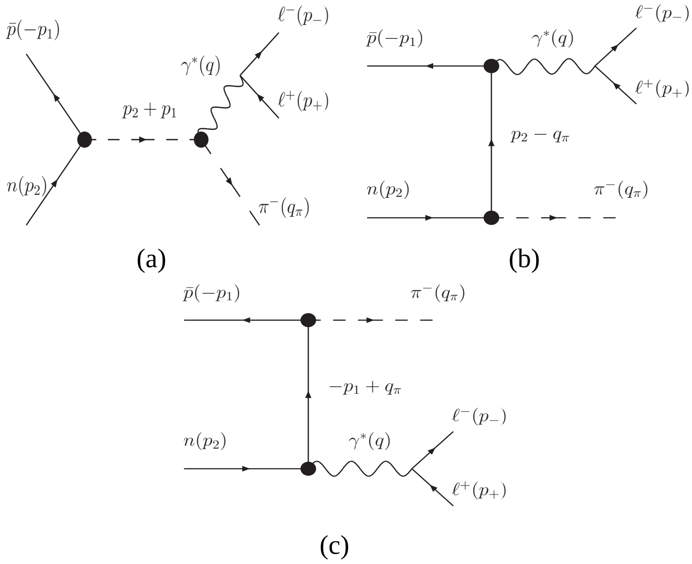

Fig. 2. Feynman diagrams for the process $\bar{p}+n \to \pi^- + \ell^+ + \ell^-$.

as

$$\left\langle N(p') \left| \Gamma_\mu^N(q) \right| N(p) \right\rangle = \bar{u}(p') \left[F_1^N(q^2)\gamma_\mu + \frac{F_2^N(q^2)}{4M}(\hat{q}\gamma_\mu - \gamma_\mu\hat{q}) \right] u(p) \ , \ N = n, p, \tag{2}$$

where M is the nucleon mass, q is the four-momentum of the virtual photon and F_1^N, F_2^N are the Dirac and Pauli form factors of the proton and neutron.

The pion (π^-) electromagnetic form factor $F_\pi(q^2)$ is also introduced in the standard way through the corresponding EM current as

$$J_\mu^\pi = (q_1 + q_2)_\mu F_\pi(q_\pi^2), \tag{3}$$

where q_1, q_2 are momenta of incoming and outgoing charged pion and $q_\pi = q_1 - q_2$. Special attention must be devoted to the pion-nucleon interaction in the vertices πNN, which are parametrized as

$$\bar{v}(p_1)\gamma_5 u(p_2) g_{\pi NN}(s), \quad \text{and} \quad \bar{\mathrm{v}}(\mathrm{p}_1 - \mathrm{q})\gamma_5 \mathrm{u}(\mathrm{p}_2) \mathrm{g}_{\pi\mathrm{NN}}(\mathrm{m}_\pi^2), \tag{4}$$

with $s = (p_1 + p_2)^2$.

The coupling constant $g_{\pi\bar{N}N}$ can be related to the nucleon axial form factor by the Golberger-Treiman relation in the form

$$g(s) = g_{\pi\bar{N}N}(s) = \frac{MG_A(s)}{f_\pi}, \quad G_A(0) = 1.2673 \pm 0.0035, \tag{5}$$

where $g(s)$, $g(m_\pi^2)$ are the pion-nucleon coupling constants for pion off- and on-mass shell, respectively. This assumption can be justified by the fact that f_π is weakly depending on q^2 and it is in agreement with the ChPT expansion at small q^2 [5]. Therefore, measuring the $g_{\pi\bar{N}N}(s)$ coupling constant gives information on the axial and induced pseudoscalar FFs of the nucleon in the chiral limit (neglecting the pion mass).

The matrix element can be expressed in terms of the hadronic H and leptonic J currents

$$\mathcal{M}^i = \frac{4\pi\alpha}{q^2} H^i_\mu J^\mu(q), \quad H^i_\mu = \bar{v}(p_1) O^i_\mu u(p_2), \quad J^\mu(q) = \bar{v}(p_+)\gamma_\mu u(p_-), \tag{6}$$

where the index $i = 0, -$ refers to π^0 and π^- respectively. The cross section for the case of unpolarized particles has a standard form (we imply that the nucleon target is at rest in the laboratory frame)

$$d\sigma^i = \frac{1}{16PM} \sum |\mathcal{M}^i|^2 d\Gamma, \quad P^2 = E^2 - M^2, \tag{7}$$

where E is the energy, P is the modulus of the momentum and $d\Gamma$ is the phase space volume.

For the studied processes, it can be expressed as

$$d\sigma^i = \frac{\alpha^2}{6s\pi r} \frac{\beta(q^2 + 2\mu^2)}{(q^2)^2} \mathcal{D}^i \frac{d^3 q_\pi}{2\pi E_\pi}, \tag{8}$$

where μ is the lepton mass

$$\beta = \sqrt{1 - (4\mu^2/q^2)}, \quad s = (q_\pi + q)^2 = 2M(M + E), \quad r = \sqrt{1 - (4M^2/s)} \tag{9}$$

and

$$\mathcal{D}^i = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{1}{4} Tr(\hat{p}_1 - M) O^i_\mu (\hat{p}_2 + M)(O^i_\nu)^*, \; i = 0, -. \tag{10}$$

Using Feynman rules, we can write (see Figs. 1 and 2)

$$O^-_\mu = \Gamma^p_\mu(q) \frac{\hat{p}_1 - \hat{q} - M}{(p_1 - q)^2 - M^2} \gamma_5 g(m^2_\pi)$$
$$- \gamma_5 \frac{\hat{p}_2 - \hat{q} + M}{(p_2 - q)^2 - M^2} \Gamma^n_\mu(q) g(m^2_\pi) + \frac{(2q_\pi + q)_\mu}{s - m^2_\pi} g(s) F_\pi(q^2) \gamma_5 \tag{11}$$

$$O^0_\mu = \Gamma^p_\mu(q) \frac{\hat{p}_1 - \hat{q} - M}{(p_1 - q)^2 - M^2} \gamma_5 g(m^2_\pi) - \gamma_5 \frac{\hat{p}_2 - \hat{q} + M}{(p_2 - q)^2 - M^2} \Gamma^p_\mu(q) g(m^2_\pi). \tag{12}$$

Note that the hadronic current $\mathcal{J}^0_\mu = \bar{v}(p_1) O^0_\mu u(p_2)$ is conserved $J^0_\mu q^\mu = 0$, but $J^-_\mu = \bar{v}(p_1) O^-_\mu u(p_2)$ is not conserved,

$$q_\mu \mathcal{J}^-_\mu = \left[(-F^p_1(q^2) + F^n_1(q^2)) g(m^2_\pi) + g(s) F_\pi(q^2) \right] \bar{v}(p_1)\gamma_5 u(p_2) = \mathcal{C} \bar{v}(p_1)\gamma_5 u(p_2). \tag{13}$$

Therefore, to provide the gauge invariance, it is necessary to add to O_μ^- a contact term with the appropriate structure (13).

The explicit expression for $\mathcal{D}^0$ (corresponding to the process $p+\bar{p} \to \ell^+ + \ell^- + \pi^0$) can be written as

$$\mathcal{D}^0 = |f_{2p}|^2 \left[\frac{E-M}{M} - \frac{1}{2}\left(1-\frac{q^2}{4M^2}\right)\frac{(1-X)^2}{X}\right] + |f_{1p}-f_{2p}|^2 \frac{(X+1)^2}{X}, \quad (14)$$

and the expression for $\mathcal{D}^-$, which corresponds to the process $n+\bar{p} \to \ell^+ + \ell^- + \pi^-$, is

$$\mathcal{D}^- = \frac{1}{4}\left[\sum_i C_{i,i}|f_i|^2 + 2\sum_{j,k;j<k} C_{j,k} Re(f_j f_k^*) + \frac{2|\mathcal{C}|^2 s}{q^2}\right], \quad i,j,k = 1p, 2p, 1n, 2n, a, \quad (15)$$

where q^2-dependent terms, which contain FFs, are

$$f_a(s) = F_\pi(q^2) G_{\pi N\bar{N}}(s), \ f_{iN}(q^2) = g(m_\pi^2) F_i^N(q^2), \ i = 1, 2, \ N = n, p,$$

$$\mathcal{C}(s) = f_a(s) - f_{1p}(q^2) + f_{1n}(q^2),$$

the quantity X is defined as

$$X = \frac{p_1 q_\pi}{p_2 q_\pi} = (s-q^2)/(2ME_\pi) - 1 \quad (16)$$

and coefficients used in Ref. (15) have the following form

$$\begin{aligned}
C_{1p,1p} &= 4X, \ C_{2p,2p} = \frac{s}{M^2}\left(1+\frac{q^2}{2s}X\right), \ C_{1p,2p} = -3\left(1+X\right), \ C_{a,a} = \frac{2q^2}{s} - 4, \\
C_{1n,1n} &= 4\frac{1}{X}, \ C_{2n,2n} = \frac{s}{M^2}\left(1+\frac{q^2}{2sX}\right), \ C_{1n,2n} = -3\left(1+\frac{1}{X}\right), \\
C_{a,1p} &= 2, \ C_{a,2p} = \left(1-\frac{q^2}{s}\right)(1+X), \ C_{1p,1n} = 4, \ C_{1p,2n} = \left(\frac{1}{X} - 2X - 1\right), \\
C_{2p,2n} &= \left(2+\frac{2}{X} - \frac{q^2}{2M^2} + X\right), \ C_{2p,1n} = \left(X - \frac{2}{X} - 1\right), \\
C_{a,1n} &= -2, \ C_{a,2n} = -\left(1-\frac{q^2}{s}\right)\left(1+\frac{1}{X}\right). \quad (17)
\end{aligned}$$

3. Kinematics

In the laboratory system, useful relations can be derived between the kinematical variables, which characterize the process. The allowed kinematical region, at a fixed incident total energy s, can be illustrated as a function of different useful variables.

One can find the following relation between q^2, the invariant mass of the lepton pair and the pion energy

$$q^2 = (p_1 + p_2 - q_\pi)^2 = 2M^2 + m_\pi^2 + 2M(E - E_\pi) - 2p_1 q_\pi = s + m_\pi^2 - 2E_\pi M - 2p_1 q_\pi, \tag{18}$$

with

$$2p_1 q_\pi = 2E_\pi E - 2\sqrt{E_\pi^2 - m_\pi^2} P \cos\theta_\pi, \tag{19}$$

where $\theta_\pi = \widehat{\boldsymbol{p}_1 \boldsymbol{q}_\pi}$ is the angle between the antiproton and the pion momenta (in the laboratory frame).

The limit $-1 \leq \cos\theta_\pi \leq 1$ translates into a maximal and a minimal value for the pion energy and permits to calculate the allowed kinematical region for differet values of the beam energy E.

For fixed values of the lepton pair invariant mass, the pion energy can take values in the region

$$\frac{E_\pi^{\min}}{M} = \frac{s - q^2}{s(1+r)} \leq \frac{E_\pi}{M} \leq \frac{s - q^2}{s(1-r)} = \frac{E_\pi^{\max}}{M}, \tag{20}$$

neglecting the pion mass.

The phase-space volume of the produced pion can be written (neglecting terms $\simeq m_\pi^2/m_N^2$) in three (equivalent) forms

$$\frac{d^3 q_i}{2\pi E_\pi} = dq^2 \delta[q^2 - 2E_\pi(E + M - P\cos\theta_\pi)] E_\pi dE_\pi d\cos\theta_\pi$$

$$= E_\pi dE_\pi d\cos\theta_\pi\,, \tag{21a}$$

$$= M\frac{dq^2}{sr} dE_\pi\,, \tag{21b}$$

$$= \frac{q^2 M^2 dq^2 d\cos\theta_\pi}{s^2(1 - r\cos\theta_\pi)^2}\,. \tag{21c}$$

4. *Axial and EM form factors*

In order to estimate cross sections of the considered processes we use existing models of nucleon and pion structure. The nucleon EM form factors in the time-like region can be described by the simple pQCD inspired model with smooth behavior, which does not show any discontinuities and can be considered an ‘average’ expectation, and for a comparison we use VMD-based model [6, 7] of the nucleon EM form factors, with characteristic resonance behavior in the unphysical region. Both models have been well discussed in Ref. [8].

Data on the nucleon axial FFs in the TL region do not exist, and they suffer in the SL region from a model-dependent derivation. In the SL region, the nucleon axial FF, $G_A(q^2)$, for the transition $W^*+p \to n$ (W^* is the virtual W-boson), can be described by the simple dipole formula [9]. A simple analytical continuation of this prescription to the TL region presents a pole in the unphysical region. Therefore, we used a 'mirror' parametrization from the SL region

$$FF^{(TL)}(|q^2|) = FF^{(SL)}(|q^2|).$$

Such a prescription is, in principle, valid only at very large q^2, since it obeys the asymptotic analytical properties of FFs [10].

Again, for a comparison, we use the VMD-inspired model of the nucleon axial form factor discussed in Ref. [11].

Concerning the pion FF, a reasonable description exists in the kinematical region of interest here. For a recent discussion see Ref. [12]. For the sake of simplicity, we use here a ρ meson saturated monopole-like parametrization, which takes a Breit Wigner form in the TL region

$$F_\pi(q^2) = \frac{m_\rho^2}{m_\rho^2 - q^2 - i m_\rho \Gamma_\rho}\,. \tag{22}$$

5. Results

The differential and integrated cross sections were calculated for several values of the antiproton energy and different choices of FFs described above.

The differential cross sections, Eq. (8), as a function of E_π and q^2, Eq. (3), are shown in Figs. 3 and 4 for the processes $\bar{p}+p \to \pi^0+\ell^++\ell^-$ and $\bar{p}+n \to \pi^-+\ell^++\ell^-$ and at E= 7 GeV^2. As one can see from the figures, the differential cross sections

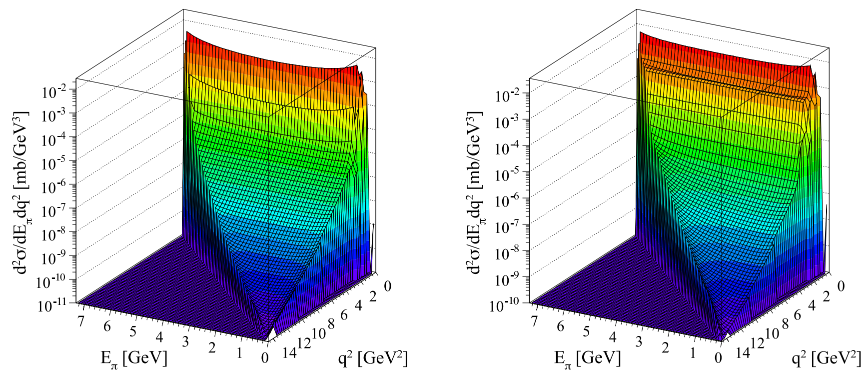

Fig. 3. Left: Double differential cross section for the process $\bar{p}+p \to \pi^0+\ell^++\ell^-$ as a function of q^2 and E_π, using FFs from [6] for nucleon and [11] for axial FF. Right: Same quantity as in the left plot for the process $\bar{p}+n \to \pi^-+\ell^++\ell^-$.

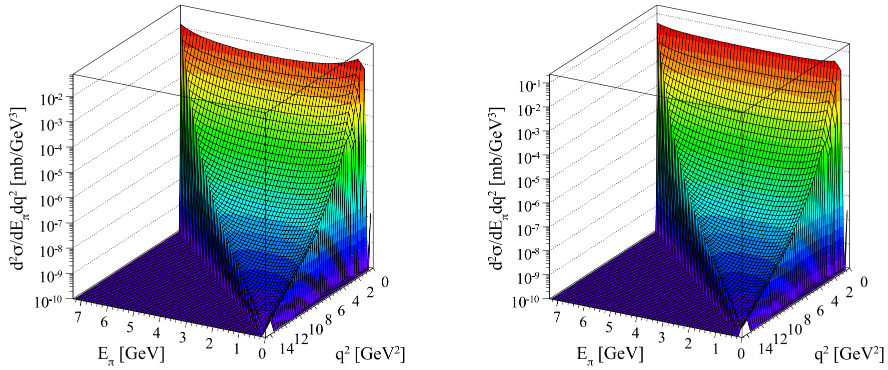

Fig. 4. Left: Double-differential cross section for the process $\bar{p}+p \to \pi^0+\ell^+ +\ell^-$ as a function of q^2 and E_π, using pQCD inspired nucleon FFs and dipole axial FFs. Right: Same quantity as in the left plot for the process $\bar{p}+n \to \pi^- +\ell^+ +\ell^-$.

are large and measurable in a wide range of the considered variables. It is reasonable to assume that the region up to $q^2 = 7$ GeV^2, at least, will be accessible by the experiments at FAIR.

The discontinuities in the small q^2 regions are smoothed out by the steps chosen to histogram the variables. However, depending on the resolution and the reconstruction efficiency, it will be experimentally possible to identify the meson and nucleon resonances.

The differential cross section as a function of q^2 can be obtained after integrating over the pion energy, Eq. (8), with the help of Eq. (3),

$$\frac{d\sigma^i}{dq^2} = \frac{\alpha^2}{6s\pi r}\frac{\beta(q^2+2\mu^2)}{(q^2)^2}\frac{M}{sr}\int_{E_\pi^{\min}}^{E_\pi^{\max}} \mathcal{D}^i dE_\pi, \tag{23}$$

where the integration of the hadronic terms (14, 15) can be done analytically by using the following integrals

$$\int_{E_\pi^{min}}^{E_\pi^{max}} \frac{dE_\pi}{M} = \frac{r(s-q^2)}{2M^2} = rb\ , \quad b = \frac{s-q^2}{2M^2} \tag{24}$$

$$\int_{E_\pi^{min}}^{E_\pi^{max}} \frac{dE_\pi}{M} X = \int_{E_\pi^{min}}^{E_\pi^{max}} \frac{dE_\pi}{M}\frac{1}{X} = \frac{s-q^2}{2M^2}\left[\ln\frac{1+r}{1-r} - r\right] = b(\ell - r), \tag{25}$$

where r is given in (9) and $\ell = \ln[(1+r)/(1-r)]$.

The result of the calculation is shown in Fig. 5. For charged-pion production, the presence of the axial FF is the reason of a larger cross section as compared to

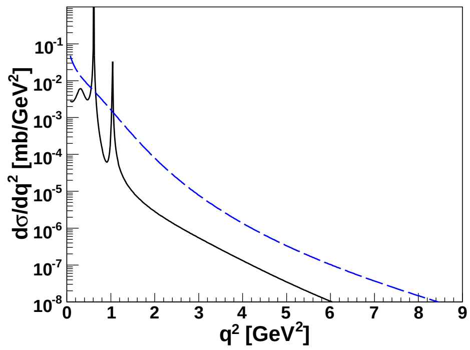

Fig. 5. Left: Differential cross section for the process $\bar{p} + p \to \pi^0 + \ell^+ + \ell^-$ *as a function of* q^2*, with FFs from [6] for nucleon and [11] for axial FF (solid line) and with FFs from pQCD inspired nucleon FFs and dipole axial FFs (dashed line). Right: Same quantity as left, for the process* $\bar{p} + n \to \pi^- + \ell^+ + \ell^-$.

the neutral pion case. For both processes, again, the present calculation gives an integrated cross section of the order of several μb in the unphysical region, for both choices of FFs.

The q^2 dependence is driven by the choice of FFs. In the case of pQCD-like FFs, the behavior is smooth and similar for proton and neutron. In the case of FFs from Ref. [6], the resonant behavior due to ρ, ω and ϕ poles appears in the figures.

6. Conclusion and perspectives

The differential cross section for the processes $\bar{p} + n \to \pi^- + \ell^- + \ell^+$ and $\bar{p} + p \to \pi^0 + \ell^- + \ell^+$ has been calculated in the kinematical range which will be accessible in near future at FAIR. The main interest of these processes is related to the possibility of measuring nucleon electromagnetic and axial FFs in the time-like and in the unphysical regions. As previously pointed out [1, 2], varying the momentum of the emitted pion allows to scan the q^2 region of interest, keeping the beam energy fixed.

The detailed measurement of the double-differential cross section, as a function of q^2 and E_π, allows in principle to extract all nucleon FFs which are involved in the considered processes. A precise simulation of different processes involving the production of a pion will be necessary with a study of the best kinematical conditions in order to minimize background contribution. In particular, the process $\bar{p} + p \to \pi^0 + \pi^0$ has been identified as a potential source of background in the e^+e^--spectrum due to its Dalitz decay $\pi^0 \to e^+e^-\gamma$.

The assumption about the validity of a generalized form of the Golberger-Treiman relation, which allows to relate the pseudoscalar $g_{\pi NN}$ coupling constant to the axial nucleon FF, can be experimentally verified in the case of a small invariant mass of the lepton pair. We do not consider processes involving vector mesons,

Δ resonances and higher excited nucleon states, estimating that their contributions does not exceed 10%.

Acknowledgements

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AKSIJALNI I ELEKTROMAGNETSKI NUKLEONSKE STRUKTURNE FUNKCIJE U NEFIZIČKOM PODRUČJU

Proučavamo kanal poništavanja u sudarima antiproton–nukleon s tvorbom jednog nabijenog ili neutralnog piona i para lepton-antilepton. Ti procesi pružaju jedinstvenu mogućnost za proučavanje nukleonskih elektromagnetskih strukturnih funkcija u nefizičkom području. Dajemo diferencijalni udarni presjek u mjernoj postavi u kojoj je pion cjelovito detektiran uz izričitu ovisnost o dotičnim nukleonskim strukturnim funkcijama.