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QCD CONTRIBUTIONS TO THE FROISSART BOUND FOR THE TOTAL CROSS-SECTION

A. ACHILLI[a], A. GRAU[b], G. PANCHERI[c] and Y. N. SRIVASTAVA[a]

[a]*INFN and Physics Department University of Perugia, I-06123 Perugia, Italy*
E-mail-addresses: achilli@fisica.unipg.it, Yogendra.Srivastava@pg.infn.it

[b]*Departamento de Fisica Teórica y del Cosmos, Universidad de Granada, 18071 Granada, Spain E-mail-address: igrau@ugr.es*

[c]*INFN Frascati National Laboratories, I-00044 Frascati, Italy*
E-mail-address: pancheri@lnf.infn.it

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We discuss the effect of infrared soft gluons on the asymptotic behaviour of the total cross-section. We use a singular but integrable expression for the strong coupling constant in the infrared limit and relate its behaviour to the satisfying of the condition of the Froissart bound, giving a specific phenomenological example.

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1. *Introduction*

The understanding of the energy behaviour of total cross-sections in hadron and photon scattering is still an unresolved challenge. In our opinion, the major problem can be summarized as follows. The simplest and the most successful parametrization of the total cross-section for proton-proton and proton-antiproton scattering follows from the optical theorem and describes the initial fall through the exchange of Regge trajectories, i.e. $s^{\alpha_R(0)-1}$ with $\alpha_R(0) < 1$, and the rise as due to Pomeron exchange, i.e. $s^{\alpha_P(0)-1}$ with $\alpha_P(0) > 1$. Although successful, this parametrization violates the Froissart bound on total cross-sections, established long time ago, namely that $\sigma^{\rm tot} \leq \log^2 s$ as $s \to \infty$. Other parametrizations have then been proposed, which are based on purely phenomenological grounds, but as such do not really give an insight into the mechanisms of the rise of total cross-sections. In another vein, there are the eikonal models which use the impact parameter distribution of protons, often derived from the Fourier trasform of the proton form factor, and basic scattering cross-sections. Among them is the eikonal mini-jet model in which the term which drives the rise is a perturbatively calculated QCD jet cross-section. Our proposal is to use this eikonal mini-jet model with an impact parameter factor derived from the Fourier transform of the initial transverse momentum distribution of valence quarks in the proton, calculated through soft gluon resummation. This distribution is energy-dependent and produces a saturation mechanism which quenches the mini-jet rise in such a way that one restores the Froissart bound. In the following, we shall first describe the soft gluon transverse momentum distribution, then the eikonal mini-jet model and its application to the proton cross-sections. We shall then show how this model in the very large asymptotic regime, and with a particular ansatz for the zero momentum modes of the emitted soft gluons, satisfies the Froissart bound.

2. *The soft gluon transverse momentum distribution*

Soft gluon emission from the initial quarks and gluons introduces an acollinearity between the initial partons. Such acollinearity is energy dependent, and can be responsible for reducing the fast rise of the mini-jet cross-section [1]. We have described this effect in a number of papers. Here we first discuss in detail the main characteristics of the soft gluon resummed transverse momentum distribution.

The emission of an infinite number of low-momentum massless quanta results in a four-momentum distribution

$$\mathrm{d}^4P(K) = \frac{\mathrm{d}^4K}{(2\pi)^4}\int \mathrm{d}^4x e^{\mathrm{i}\,K\cdot x} e^{-h(x)}\,, \tag{1}$$

with

$$h(x) = \int \mathrm{d}^3\bar{n}(k)[1-e^{-\mathrm{i}\,k\cdot x}]\,, \tag{2}$$

where $\mathrm{d}^3\bar{n}(k)$ is the average number of soft quanta, gluons or photons, emitted by a color or electrically charged source. Upon integrating over the energy and longitudinal momentum variable, one obtains the well known transverse momentum distribution

$$\mathrm{d}^2P(K_\perp) = \frac{\mathrm{d}^2\mathbf{K}_\perp}{(2\pi)^2}\int \mathrm{d}^2\mathbf{b} e^{\mathrm{i}\,\mathbf{K}_\perp\cdot\mathbf{b}} e^{-h(b)}\,, \tag{3}$$

with

$$h(b) = \int \mathrm{d}^3\bar{n}(k)\big[1-e^{-\mathrm{i}\,\mathbf{k_t}\cdot\mathbf{b}}\big]\,. \tag{4}$$

The above transverse momentum distribution does not admit a closed-form solution, unlike the energy distribution in QED, which easily integrates to a power law function. This has led to various strategies to evaluate the integral in Eq. (3), none of which is however fully satisfying, for reasons which we shall try to illustrate.

Indeed, while the importance of this function has been known for a long time, in our opinion such importance has not been fully exploited, and in this note we shall discuss this point and present our approach to the problem.

The simplest case to examine is that of the low-momentum soft quanta in an Abelian gauge theory with a constant but not small coupling constant. It is then possible to make some simple approximations, as in Ref. [2], and obtain a closed form for the transverse momentum distribution, namely

$$\mathrm{d}^2P(K_\perp) = \frac{\beta(2\pi)^{-1}}{\Gamma(1+\beta/2)}\,\frac{\mathrm{d}^2K_\perp}{2E^2A}\left(\frac{K_\perp}{2E\sqrt{A}}\right)^{\frac{\beta}{2}-1}\times\mathcal{K}_{1-\beta/2}\left(\frac{K_\perp}{E\sqrt{A}}\right), \tag{5}$$

where $\mathcal{K}_{1-\beta/2}$ is the modified Bessel function of the third kind, β is obtained by performing the angular integration over $\mathrm{d}^3\bar{n}(k)$ and E is the maximum energy allowed to the single soft quantum emitted.

Clearly, the above procedure fails in the non Abelian case, where the strong coupling constant is large but not constant. This was included [3, 4, 5] in the subsequent QCD version of the transverse momentum distribution, which is presently used in many phenomenological applications. Using the asymptotic freedom expression for α_s, the relative transverse momentum distribution induced by soft gluon emission from a pair of, initially collinear, colliding partons is obtained, at LO, as

$$h(b,E)=\frac{16}{3}\int_\mu^E\frac{\alpha_s(k_t)}{\pi}\frac{\mathrm{d}k_t}{k_t}\ln\frac{2E}{k_t}\approx\frac{32}{33-2N_f}\left\{\ln\left(\frac{2E}{\Lambda}\right)\left[\ln\left(\ln\left(\frac{E}{\Lambda}\right)\right)-\ln\left(\ln\left(\frac{\mu}{\Lambda}\right)\right)\right]-\ln\left(\frac{E}{\mu}\right)\right\}, \tag{6}$$

where the integration only extends down to a scale μ. This expression fails to reproduce the entire range of low-energy transverse momentum effects and an *ad hoc* constant, intrinsic transverse momentum is introduced for phenomenological applications.

The problem we observe in the above use of this distribution is its limited applicability to minimum bias physics and its lack of connection to the question of confinement. Our approach instead is to extend the integral over the soft gluon momentum down to zero modes; to propose an ansatz for the behaviour of α_s in the infrared region, and compare its consequences on measurable quantities such as the total cross-section. In the next section, we shall present one such phenomenological application through the calculation of total cross-sections and in the last section discuss the consequences of our ansatz for the infrared soft gluon limit to the limitations imposed by the Froissart bound on the total cross-section.

3. A model for the total cross-section

Recently, we obtained predictions for the total cross-section [7, 6, 8] at LHC energies using a model based on hard component of scattering responsible for the rise of the total cross-section and soft gluon emission from scattering particles, which softens the rise and gives the impact parameter distribution. In this model, mini-jet events coming from parton-parton high-energy collisions are responsible for the rise of the total cross-section. From perturbative QCD, we have the following expression for parton-parton cross section at high energy,

$$\sigma_{\rm jet}^{AB}(s, p_{t\,\min}) = \int_{p_{t\,\min}}^{\sqrt{s/2}} \mathrm{d}p_t \int_{4p_t^2/s}^{1} \mathrm{d}x_1 \int_{4p_t^2/(x_1 s)}^{1} \mathrm{d}x_2 \times \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{\mathrm{d}\hat{\sigma}_{ij}^{kl}(\hat{s})}{\mathrm{d}p_t}\,, \tag{7}$$

which depends on the minimum transverse momentum allowed to the scattered partons in the final state, $p_{t\,\min}$, $\approx 1-2$ GeV. This parameter represents the energy scale which distinguishes hard processes from the soft ones. Evaluating the mini-jet cross-sections using different LO parametrizations available for the partonic density functions $f_{i|A}$ [9], we obtain a growth of $\sigma_{\rm jet}$ with energy as a (small) power of s. Soft-gluon emission changes the parton collinearity and contributes in softening the rise of the total cross-section. The average number of soft-gluon emissions increases with energy, and more emissions means more acollinearity between colliding partons and less energy available for mini-jet production. These processes are best studied through an impact parameter (b-)distribution. The overlap function which describes the b-distribution of the incident partons in our model is

$$A_{BN}(b,s) = N \int \mathrm{d}^2\mathbf{K}_\perp \; e^{-\mathrm{i}\mathbf{K}_\perp \cdot \mathbf{b}} \frac{\mathrm{d}^2 P(\mathbf{K}_\perp)}{\mathrm{d}^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{\max})}}{\int \mathrm{d}^2\mathbf{b}\; e^{-h(b,q_{\max})}}\,, \tag{8}$$

with

$$h(b, q_{\max}) = \frac{16}{3} \int_0^{q_{\max}} \frac{\alpha_s(k_t^2)}{\pi} \frac{\mathrm{d}k_t}{k_t} \log \frac{2q_{\max}}{k_t} \left[1 - J_0(k_t b)\right], \tag{9}$$

similar to the one obtained in the first section, but for the fact that the integral extends down to zero momentum and $J_0(k_t b)$ needs to be retained for infrared finiteness. For α_s we use a phenomenological expression, which coincides with the usual QCD limit for large k_t, but is singular for $k_t \to 0$, namely

$$\alpha_s(k_t^2) = \frac{12\pi}{33 - 2N_f} \frac{p}{\ln\left[1 + p(k_t/\Lambda)^{2p}\right]}\,, \tag{10}$$

with $p < 1$ for the integral over k_t to exist and $p \geq 1/2$ for the correct analyticity in the momentum transfer variable.

The parameter $q_{\max}$ is linked by kinematics to the maximum transverse momentum allowed to single soft-gluon emission in a given hard collision, and for this purpose, we use an expression averaged over all parton densities [8],

$$q_{\max}(s) = \sqrt{\frac{s}{2}} \frac{\sum_{i,j} \int (\mathrm{d}x_1/x_1) \int (\mathrm{d}x_2/x_2) \int_{z_{\min}}^{1} \mathrm{d}z f_i(x_1) f_j(x_2) \sqrt{x_1 x_2}(1-z)}{\sum_{i,j} \int (\mathrm{d}x_1/x_1) \int (\mathrm{d}x_2/x_2) \int_{z_{\min}}^{1} \mathrm{d}z f_i(x_1) f_j(x_2)}, \qquad (11)$$

with $z_{\min} = 4p_{t\,\min}^2/(sx_1x_2)$.

The use of eikonal representation allows to consider multiple scattering. Neglecting the real part of the eikonal function, the expression for the total cross section is given by

$$\sigma_{\text{tot}} = 2 \int \mathrm{d}^2\mathbf{b} \left[1 - e^{-n(b,s)/2}\right], \qquad (12)$$

with $n(b,s) = n_{\text{soft}}(b,s) + n_{\text{hard}}(b,s) = n_{\text{soft}}(b,s) + A_{BN}(b,s)\sigma_{\text{jet}}(s,p_{t\,\min})$, denoting the average number of partonic collisions during the scattering. The hard term is

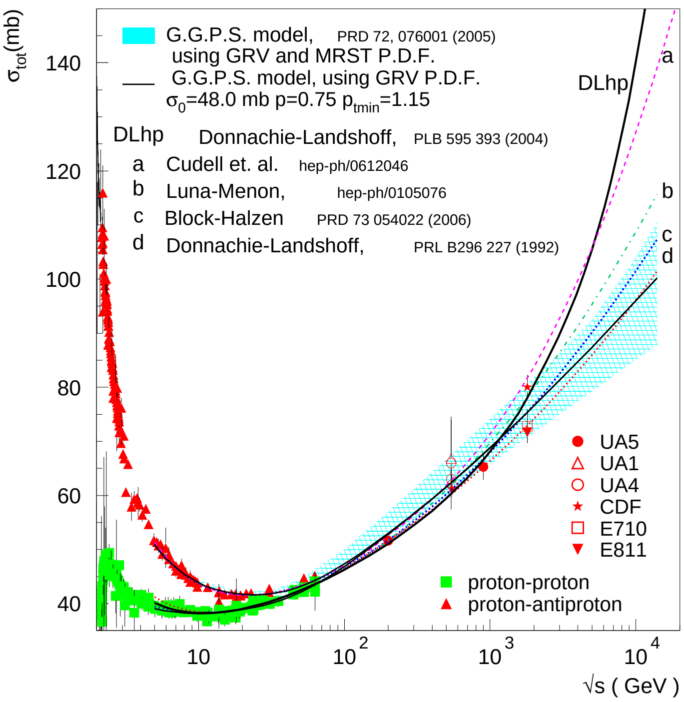

Fig. 1. Total cross-section obtained from our model [8] using different PDF's [9], compared with data [10] and with other models [11].

evaluated using the expressions (7) and (8) for $\sigma_{\rm jet}$ and A_{BN}, while the soft one has a phenomenological parametrization,

$$n_{\rm soft}(b,s) = A_{BN}^{\rm soft}(b,s)\sigma_0\left(1+\epsilon\frac{2}{\sqrt{s}}\right), \tag{13}$$

where $\sigma_0 = constant$ is a parameter necessary to reproduce the right normalization of the total cross-section, while $\epsilon = 0,1$ refer to pp and $p\bar{p}$ scattering, while $A_{BN}^{\rm soft}$ is parametrized as in Ref. [7].

Figure 1 from Ref. [6] shows the range of values of the total cross-section obtained with this model using a set of phenomenological values for the parameters $p_{t\,\rm min}$, σ_0 and p, and varying the parton densities.

4. Soft gluons in the infrared limit and total cross-section

At very large energies, using the eikonal representation, the total cross-section in our model [7] reads

$$\sigma_T(s) \approx 2\pi\int_0^\infty {\rm d}b^2\left[1-{\rm e}^{-n_{\rm hard}(b,s)/2}\right], \tag{14}$$

where $n_{\rm hard}(b,s) = \sigma_{\rm jet}(s)A_{\rm hard}(b,s)$. We consider the asymptotic expression for $\sigma_{\rm jet}$ at high energies, which grows as a power of s: $\sigma_{\rm jet}(s) \approx \sigma_1(s/s_0)^{\varepsilon}$. $A_{\rm hard}(b,s)$ is obtained through soft gluon resummation as

$$A_{\rm hard}(b,s) \propto e^{-h(b,s)}, \tag{15}$$

where $h(b,s)$ is given by Eq. (4). If we take now the limit $k_t \to 0$, where ${\rm d}^3n(k) \propto \alpha_s(k_t^2)$ and $\alpha_s(k_t) \approx (\Lambda/k_t)^{2p}$, with $p < 1$, the integral (4) gives $h(b,s) \propto (b\bar{\Lambda})^{2p}$. This implies $A_{\rm hard}(b) \propto e^{-(b\bar{\Lambda})^{2p}}$ and

$$n_{\rm hard} = 2C(s)e^{-(b\bar{\Lambda})^{2p}}, \tag{16}$$

where $C(s) = A_0(s/s_0)^{\varepsilon}\sigma_1$. The resulting expression for σ_T is

$$\sigma_T(s) \approx 2\pi\int_0^\infty db^2\left[1-e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}\right] \tag{17}$$

from which we derive

$$\sigma_T \to [\varepsilon\ln(s)]^{(1/p)} \tag{18}$$

The result shows explicitly that soft gluon resummation can restore the asymptotic growth of σ_T to lie within the Froissart bound. (Recall that $1/2\ \leq p < 1$.)

5. Conclusions

We have shown how soft gluon resummation acts as a saturation mechanism in the rise of the total cross-section. This effect, embedded into the eikonal formalism with QCD mini-jets to drive the rise, leads to asymptotic cross-sections which satisfy the Froissart bound.

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DOPRINOSI QCD-a FROISSARTOVOJ GRANICI UKUPNOG UDARNOG PRESJEKA

Raspravljamo učinak infracrvenih mekih gluona na asimptotsko ponašanje ukupnog udarnog presjeka. Primjenjujemo singularan ali integrabilan izraz za konstantu jakog vezanja u infracrvenoj granici i usklađujemo njeno ponašanje s Froissartovom granicom, dajući poseban fenomenološki primjer.