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TOMOGRAPHY OF QUARK-GLUON PLASMA BY VECTOR MESONS

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The fireball formed in a heavy-ion collision is characterized by the impact parameter vector $\vec{b}$, which is orthogonal to the beam direction and can be determined from the multiplicity and the angular distribution of the reaction products. By appropriate rotations, the $\vec{b}$ vectors of each collision can be oriented into a fixed direction. Using the measured values of the momentum distributions of the vector mesons, an integral equation can be formulated for the unknown emission densities ($E_{\rm M}(\vec{r})$) and for the unknown absorption densities ($\Delta\mu_{\rm M}(\vec{r}, p)$) of the various vector mesons M ($\omega^0, \rho^0, \phi^0, \psi^0, \psi^{0\prime}, \Upsilon$).

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1. Introduction

The most important aim of relativistic heavy -ion experiments is the observation of the quark-gluon plasma. In order to detect the transition into the plasma state, it is desirable to map the density profile of the fireball formed in the collision. This mapping can be performed by various methods [1-3]. The Hanbury-Brown and Twiss effect, observable by the help of pions produced in heavy-ion collision [4], provides the possibility to measure the Fourier transform of the source density. Its interpretation, however, in the case of the CERN and the RHIC data, is rather difficult, since geometry of the fireball practically does not depend on the energy of the collision [5]. Here we investigate an alternative possibility, namely the mapping by tomography [6]. The tomography is a method for producing a picture of a thin layer with the help of penetrating radiation. It is widely applied in medicine and for material testing. We try to apply the tomography for the study of relativistic heavy ion reactions in order to "see" the spot of the quark-gluon plasma formation.

The initial state of the fireball formed in heavy-ion collision can be defined as the overlap of two identical, relativistically contracted pancakes. A collision of two identical relativistic heavy ions produced by a collider can be characterized by the impact parameter vector $\vec{b}$ which is orthogonal to the beam direction. Both the geometry and the orientation of the initial fireball can be characterized by this vector. Its length can be determined by the multiplicity of particles produced in the collision and its direction can be obtained from the angular distribution of the reaction products. The possibility of the application of the idea of the tomography is based on the fact that by the help of an appropriate rotation around the beam direction (z-axis) the impact parameter vector can be oriented into a standard direction (x-axis). In this way the fireball of all particular collision events have the same orientation in the laboratory frame of reference (see Fig. 1).

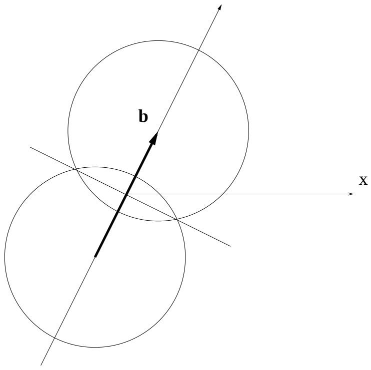

Fig. 1. In an individual collision event the fireball, defined as the overlap of two identical spheres, can be characterised by the impact parameter vector $\vec{b}$. The length and the orientation of $\vec{b}$ can be obtained from the multiplicity and from the angular distribution of the reaction products, respectively.

The neutral vector mesons of type M ($\rho, \omega, \phi, \psi, \psi', \Upsilon$) seem to be the most appropriate for the purpose of tomography [7–9], since they can be observed as well defined peaks above the background due to their large rest mass. Nevertheless, other type mesons may be considered, as well. It was discovered recently by Gyulassy et al. [10] that the absorption of the produced jets may be a sensitive indicator of the quark-gluon plasma formation. In a collision, the number of mesons of type M emitted from the volume element $\mathrm{d}\vec{r}$ with momentum $\vec{p}$ is given by $\mathcal{E}_\mathrm{M}(\vec{r},\vec{p})\,\mathrm{d}\vec{r}\,\mathrm{d}\vec{p}$. During the collision, the system has a large momentum anisotropy. In the momentum distribution of the mesons, this anisotropy is inherently present. We assume that this anisotropy is a global characteristic feature of the collision, i.e. it is independent of $\vec{r}$. Using this assumption, we write the emission density as the product

$$\mathcal{E}_\mathrm{M}(\vec{r},\vec{p}) = E_\mathrm{M}(\vec{r})G_\mathrm{M}(\vec{p})\,. \tag{1}$$

The global momentum distribution $G_\mathrm{M}(\vec{p})$ can be obtained from the average momentum distribution of the emitted mesons of type M. The meson produced at $\vec{r}$ and emitted with momentum $\vec{p}$ may reach the surface of the fireball if it is not absorbed. The absorption can be taken into account in the following way. The number of mesons at $\vec{r}\,'$ emitted at $\vec{r}$ in the direction $\vec{p}$ is denoted by $N_{\mathrm{M}}(\vec{r}\,', \vec{r}, \vec{p})\ \mathrm{d}\vec{r}\ \mathrm{d}\vec{p}$. This number is decreasing because of the absorption

$$\mathrm{d}N_{\mathrm{M}}(\vec{r}\,', \vec{r}, \vec{p}) = -N_{\mathrm{M}}(\vec{r}\,', \vec{r}, \vec{p})\ \mu(\vec{r}\,', p)\ \mathrm{d}l' \,, \tag{2}$$

where $p = |\vec{p}|$ and the absorption coefficient is denoted by μ. The line element $\mathrm{d}l'$ is taken along the straight line defined by

$$\vec{r}\,' = \vec{r} + l'\vec{p}_0, \tag{3}$$

where the unit vector $\vec{p}_0$ is defined by $\vec{p}_0 = \vec{p}/p$. The number of mesons of type M starting at $\vec{r}$ and arriving to the surface of the fireball at $\vec{R}(\vec{r}, \vec{p}_0)$ is given by

$$N_{\mathrm{M}}(\vec{R}, \vec{r}, \vec{p}) = N_{\mathrm{M}}(\vec{r}, \vec{r}, \vec{p})\ \exp\left(- \int\limits_{\vec{r}}^{\vec{R}(\vec{r},\vec{p}_0)} \mathrm{d}l' \mu_{\mathrm{M}}(\vec{r}\,', p) \right), \tag{4}$$

where $\vec{R}$ is defined by $\vec{R} = \vec{r} + L\vec{p}_0$, and the length $L(\vec{r}, \vec{p}_0)$ is the distance between the points $\vec{r}$ and $\vec{R}$. The quantity $N_{\mathrm{M}}(\vec{r}, \vec{r}, \vec{p})$ can be expressed as follows

$$N_{\mathrm{M}}(\vec{r}, \vec{r}, \vec{p}) = E_{\mathrm{M}}(\vec{r})\ G_{\mathrm{M}}(\vec{p}). \tag{5}$$

Now we define the dimensionless, “normalized”, momentum distribution, $\mathcal{N}_{\mathrm{M}}(\vec{p}, b)$,

$$\mathcal{N}_{\mathrm{M}}(\vec{p}, b) = G_{\mathrm{M}}^{-1}(\vec{p}) \int\limits_{V(b)} \mathrm{d}\vec{r} N_{\mathrm{M}}(\vec{R}, \vec{r}, \vec{p}), \tag{6}$$

where the integral is taken over the volume $V(b)$ of the fireball. It has to be noted that both factors in this definition can be obtained from measured data. The integral gives the momentum distribution of the mesons of type M reaching the surface of the fireball, i.e. reaching the detector. It is expected that the value of the “normalized” momentum distribution $\mathcal{N}_{\mathrm{M}}(\vec{p}, \vec{b})$ would be unity if there was no absorption. In other words its momentum dependence is produced by the meson absorption. The normalized momentum distribution can be expressed in the following form

$$\mathcal{N}_{\mathrm{M}}(\vec{p}, \vec{b}) = \int\limits_{V(b)} \mathrm{d}\vec{r}\ E_M(\vec{r})\ \exp\left(- \int\limits_{\vec{r}}^{\vec{R}(\vec{r},\vec{p}_0)} \mathrm{d}l' \mu_{\mathrm{M}}(\vec{r}\,', p) \right). \tag{7}$$

If one would know the emission density $E_{\mathrm{M}}(\vec{r})$ and the density of the absorption $\mu_{\mathrm{M}}(\vec{r}\,', p)$, then the number of mesons arriving to the detector could be computed. As a matter of fact, one does not know these functions. Instead of them, it is the distribution of the mesons arriving to a detector what is known, since it is measured. Thus the last equation can be considered an integral equation for the unknown emission density $E_{\mathrm{M}}(\vec{r})$ and the unknown absorption density $\mu_{\mathrm{M}}(\vec{r}, p)$. This can be the basis of the tomography of the fireball by vector mesons. It is the analogue of the tomography applied in medicine. In the case of the medical emission tomography, the analogue of the mesons are the gamma rays emitted by previously incorporated radionuclides. The impact parameter vector $\vec{b}$ corresponds to a vector fixed to the body and the momentum vector $\vec{p}$ corresponds to the momentum of the gamma quantum selected by the collimator of the gamma detector. The definition of the fireball is given by

$$\left(x - \frac{b}{2}\right)^2 + y^2 + \frac{z^2}{1 - v^2/c^2} < R^2,$$

$$\left(x + \frac{b}{2}\right)^2 + y^2 + \frac{z^2}{1 - v^2/c^2} < R^2.$$

The fireball has the reflection symmetries:

$$\begin{aligned} x &\longrightarrow -x, \\ y &\longrightarrow -y, \\ z &\longrightarrow -z. \end{aligned}$$

These symmetries are not realised necessarily in every particular events. They must, however, show up in the various densities averaged over a great number of events. The two pancakes contracted in the z-direction at the moment of the collision seem to be a narrow layer when viewed from the laboratory system of coordinates. Later on, the extension of the fireball is increasing in the z-direction. We are unable to follow the time development, therefore we restrict our study to a rather narrow layer situated at $z = 0$ and we are going to make a pictures of this layer. The actual picture of this layer will be a time average taken on the whole history of a collision event.

2. *Iterative solution of the integral equation*

At a fixed value of M and b, the equation for $\mathcal{N}_M(\vec{p}, b)$, can be rewritten in the following form

$$\mathcal{N}_{\mathrm{M}}(\vec{p}, b) = \int\limits_{V(b)} \mathrm{d}\vec{r}\, E_{\mathrm{M}}(\vec{r})\ \exp[-\bar{\mu}_{\mathrm{M}}(p) L(\vec{r}, \vec{p}_0)]\ \exp\left(-\int\limits_{\vec{r}}^{\vec{R}(\vec{r},\vec{p}_0)} \mathrm{d}l' \Delta\mu_{\mathrm{M}}(\vec{r}\,', p)\right), \quad (8)$$

where the average value of the absorption coefficient having no $\vec{r}$ dependence is denoted by $\bar{\mu}_{\mathrm{M}}(p)$, while $\Delta\mu_{\mathrm{M}}$ is defined as $\Delta\mu_{\mathrm{M}} = \mu_{\mathrm{M}} - \bar{\mu}_{\mathrm{M}}$. The length $L(\vec{r}, \vec{p}_0)$

is obtained by solving the following equation

$$\left(x + L\sin\theta\cos\phi + \frac{b}{2}\right)^2 + (y + L\sin\theta\sin\phi)^2 + \frac{(z + L\cos\theta)^2}{1 - v^2/c^2} = R_0^2 . \qquad (9)$$

Here θ and ϕ are the polar angles of the vector $\vec{p}_0$. The integral equation given above can not be transformed into an exact system of linear equations. An iterative procedure, however, can be constructed in such a way that in every iterative step a linear system of equations must be solved. We expand the second exponential factor in Taylor series. The zero-order approximation $E_{\rm M}^{[0]}(\vec{r})$ is calculated by solving the equation in which only the zero-order term of the Taylor expansion is kept

$$\mathcal{N}_{\rm M}(\vec{p}, b) = \int\limits_{V(b)} {\rm d}\vec{r}\ \exp[-\bar{\mu}_{\rm M}(p)\ L(\vec{r}, \vec{p}_0)] E_{\rm M}^{[0]}(\vec{r})\ . \qquad (10)$$

The first-order approximation, $E_{\rm M}^{[1]}(\vec{r})$ and $\Delta\mu_{\rm M}^{[1]}(\vec{r}, p)$, is obtained by the solution of the equation in which only the first order term of the Taylor expansion is kept

$$\begin{aligned} \mathcal{N}_{\rm M}(\vec{p}, b) \quad &= \quad \int\limits_{V(b)} {\rm d}\vec{r}\ \exp[-\bar{\mu}_{\rm M}(p)\ L(\vec{r}, \vec{p}_0)] \Bigg[E_{\rm M}^{[1]}(\vec{r}) - E_{\rm M}^{[0]}(\vec{r}) \\ &\quad \times \int\limits_{\vec{r}}^{\vec{R}(\vec{r}, \vec{p}_0)} {\rm d}l'\ \Delta\mu_{\rm M}^{[1]}(\vec{r}\,', p) \Bigg] . \end{aligned} \qquad (11)$$

This iterative procedure can be continued until the required precision is attained

$$\begin{aligned} \mathcal{N}_{\rm M}(\vec{p}, b) \quad &= \quad \int\limits_{V(b)} {\rm d}\vec{r}\ \exp[-\bar{\mu}_{\rm M}(p)\ L(\vec{r}, \vec{p}_0)] \Bigg[E_{\rm M}^{[n]}(\vec{r}) - E_{M}^{[n-1]}(\vec{r}) \sum_{k=0}^{(n-1)} \frac{1}{(k+1)!} \\ &\quad \times \Bigg(- \int\limits_{\vec{r}}^{\vec{R}(\vec{r}, \vec{p}_0)} {\rm d}l'\ \Delta\mu_{M}^{[n-1]}(\vec{r}\,', p) \Bigg)^k \int\limits_{\vec{r}}^{\vec{R}(\vec{r}, \vec{p}_0)} {\rm d}l'\ \Delta\mu_{\rm M}^{[n]}(\vec{r}\,', p) \Bigg] . \end{aligned} \qquad (12)$$

Since the Taylor expansion of an exponential function is always convergent, we may assume that this iterative procedure is also convergent.

The values of the emission density, $E_{\rm M}(\vec{r})$, and the absorption coefficient, $\Delta\mu_{\rm M}(\vec{r}, p)$, are defined on a lattice, taking into account the symmetry discussed above. The integral equation given above, at each order of the iteration, can be approximated by the following system of linear equations

$$\begin{aligned} \mathcal{N}_{\rm M}(\vec{p}^{\,i}, b) \quad &= \quad \sum_{k=1}^{N} [K_{\rm M}(\vec{p}^{\,i}, \vec{r}^{\,k}, b) E_{\rm M}^{[n]}(\vec{r}^{\,k}) + H_{\rm M}(\vec{p}^{\,i}, \vec{r}^{\,k}, b) \Delta\mu_{\rm M}^{[n]}(\vec{r}^{\,k}, p)] \\ (i \quad &= \quad 1, 2, ..., 2N). \end{aligned} \qquad (13)$$

3. Summary

The accuracy and the performance of this tomographic method can be and will be checked by comparison of various assumed density profiles with their tomographic reconstruction. For this purpose, a computer code was written which solves Eq. (8) by iteration. Up till now, it was demonstrated by the help of this computer code that the iterative procedure advocated above really converges. So the hope is justified that using the measured momentum distribution of a vector meson of type M, at the fixed value of the impact parameter b, one can obtain maps both for the emission density $E_{\rm M}(\vec{r})$ and for the absorption density $\Delta\mu_{\rm M}(\vec{r})$. The reliability of these maps can be increased by repeating the analysis for different values of the momentum p. From the analysis of different vector mesons of type M, independent maps can be obtained. Finally, it is necessary to emphasize that even in the case of a perfect agreement among the independently obtained maps, the result is a "map", nothing more. The interpretation of the map requires additional informations.

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TOMOGRAFIJA KVARK-GLUONSKE PLAZME VEKTORSKIM MEZONIMA

Značajka vatrene lopte stvorene u sudaru teških iona je vektor parametra sudara $\vec{b}$, koji je okomit na smjer snopa i može se odrediti iz višestrukosti i kutne raspodjele produkata reakcije. Na osnovi mjerenih impulsnih raspodjela vektorskih mezona $(\omega^0, \rho^0, \phi^0, \psi^0, \psi^{0'}, \Upsilon)$ može se izvesti integralna jednadžba za emisijske i apsorpcijske gustoće, $E_{\rm M}(\vec{r})$ odnosno $\Delta\mu_{\rm M}(\vec{r}, p)$.