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DETERMINATION OF GENERALIZED ENTROPY IN HEAVY-ION COLLISIONS

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Assuming uniformly distributed nucleons in the configuration space at the time of freeze-out, we propose a way for a unique determination of the cell-size in momentum space. Then, using the nucleon occupation probability function in momentum space, we discuss the possibility for the determination of generalized entropy in heavy-ion reaction systems at freeze-out.

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1. Introduction

Nuclear matter at densities of a few times the ground-state density ($\rho \leq 3\rho_0$) and at temperatures well below the so-called Hagedorn temperature [1] ($T_H \approx 140$ MeV, i.e. the pion mass) is a system of strongly interacting hadrons whose properties are still poorly understood. One of the goals of the relativistic nuclear physics is to study the behaviour of hot and dense nuclear matter in terms of different physical quantities related to the hadronic equation of state, such as density, temperature, specific heat, compressibility, entropy and others. Indeed, investigations of heavy-ion collisions offer an experimental method of probing nuclei far from the low temperature and density state. The relativistic heavy-ion experiments studying the break-up of nuclei have greatly contributed to the understanding of particle production in heavy-ion collisions [2–5]. The conditions for particle production have been examined theoretically in Refs. [6], [7] and [8].

It is generally assumed that entropy is a quantity which remains almost unchanged throughout the final decompression-expansion stages of the interaction [9]. Thus, a measurement of baryonic entropy in the final state yields information on the state of hot and dense nuclear matter (a "memory effect"). The entropy in central-collision reactions of heavy nuclei stems from several sources, but most of the entropy is produced by shock waves. Additionally, entropy increases during expansion. The entropy information on nuclear matter produced in heavy-ion collisions is not easily accessible. In a number of experimental papers, the value of entropy has been determined for various nuclear systems and projectile energies using several (static) models for the formation of a hot nucleus and different global observables [10–12]. These observables have been compared with those obtained using different models. Obviously, such a determination of entropy is model-dependent.

In this work, we propose to study the amount of entropy generated at the time of freeze-out without recurring to simulation models. For this purpose, we use a general approach for the determination of entropy in non-interacting fermionic systems.

Section 2 introduces the idea for the determination of generalized entropy. The estimation of entropy at freeze-out is discussed in Sect. 3 and the cell-size definition for central events in Sect. 4. A summary is given in Sect. 5.

2. *Generalized entropy*

An approach to the determination of entropy in fermionic systems was introduced by Boltzmann providing a statistical basis of thermodynamics. The entropy in a microcanonical ensemble defined by Boltzmann is proportional just to the logarithm of the number of possible microscopic states in phase space at a given excitation energy,

$$S = k \ln N(E) = -k \ln P(E), \tag{1}$$

where k is the Boltzmann constant and $P(E) = N^{-1}(E)$ is the probability assumed to have the same value for each of the microstates in an equilibrated system.

For a non-equilibrated system of independent fermions, the time-dependent generalized entropy $S(t)$ can be defined as

$$S(t) = -k\frac{g}{h^3}\iint \{f(t,\boldsymbol{r},\boldsymbol{p})\ln f(t,\boldsymbol{r},\boldsymbol{p}) + [1-f(t,\boldsymbol{r},\boldsymbol{p})]\ln[1-f(t,\boldsymbol{r},\boldsymbol{p})]\}\ \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{p}, \tag{2}$$

where g is the spin-isospin degeneracy factor ($g = 4$ in our case), h is the Planck constant and $f(t,\boldsymbol{r},\boldsymbol{p})$ is the single-particle occupation probability in phase space. The explicit dependence on time of the generalized entropy and of the distribution function can be omitted since our study is restricted to the freeze-out conditions

$$S = -k\frac{g}{h^3}\iint \{f(\boldsymbol{r},\boldsymbol{p})\ln f(\boldsymbol{r},\boldsymbol{p}) + [1-f(\boldsymbol{r},\boldsymbol{p})]\ln[1-f(\boldsymbol{r},\boldsymbol{p})]\}\ \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{p}. \tag{3}$$

This entropy definition leads to the usual thermodynamic entropy in the case of global thermodynamic equilibrium.

It should be stressed that the generalized entropy is well defined only if the cell-size determination is unique [13]. Thus, according to the Heisenberg uncertainty principle, we divide a 6-dimensional single-particle phase space into a set of "small" cubic cells of the size

$$\Delta\Omega = (\Delta x)^3 (\Delta p_x)^3 = h^3. \tag{4}$$

In this way, the generalized entropy at the time of freeze-out defined by (3) becomes ($k = 1$)

$$S = -g \sum_{\boldsymbol{r}} \sum_{\boldsymbol{p}} \left\{ f(\boldsymbol{r},\boldsymbol{p}) \ln f(\boldsymbol{r},\boldsymbol{p}) + [1 - f(\boldsymbol{r},\boldsymbol{p})] \ln[1 - f(\boldsymbol{r},\boldsymbol{p})] \right\}. \tag{5}$$

The distribution function $f(\boldsymbol{r},\boldsymbol{p})$ depends on six variables and it would not be practical to make a brute-force division into cells in all six dimensions. Since experimental data permit to deduce the occupation probability only in the momentum space, we should reduce the distribution function dependence to a set of three momentum variables. To do this, we start from the spatial density of nucleons, which is related to the occupation probability $f(\boldsymbol{r},\boldsymbol{p})$ by

$$\rho = \frac{g}{h^3} \int f(\boldsymbol{r},\boldsymbol{p})\, \mathrm{d}\boldsymbol{p}. \tag{6}$$

Assuming the momentum space divided into small cubic cells of the size $(\Delta p_x)^3$, we have for the spatial density

$$\rho = \frac{(\Delta p_x)^3}{h^3} g \sum_{\boldsymbol{p}} f(\boldsymbol{r},\boldsymbol{p}). \tag{7}$$

The sum of the occupation-probability function in (7) over the whole momentum space times the spin-isospin factor g gives the total number of nucleons in the colliding system

$$g \sum_{\boldsymbol{p}} f(\boldsymbol{r},\boldsymbol{p}) = A, \tag{8}$$

and, thus, for spatial density we obtain

$$\rho = \frac{(\Delta p_x)^3}{h^3} A. \tag{9}$$

Using the above expression and the expression (4), we obtain the size of the cells $(\Delta x)^3$ in the configuration space

$$(\Delta x)^3 = h^3 \frac{1}{(\Delta p_x)^3} = \frac{A}{\rho}. \tag{10}$$

Obviously, the size of the cells in the configuration space is equal to the total volume of the colliding system

$$(\Delta x)^3 = V. \tag{11}$$

3. *Time of freeze-out*

It is very important to gain a rough qualitative picture of the phase-space distribution of particles. At the beginning of the reaction, the distributions in momentum and configuration spaces are uncorrelated. After the violent initial phase of the collision with a rapid increase of the entropy, density and local excitation energy, the system begins to expand and cool. Freeze-out characterizes the situation where the particles are far enough from each other, so that they are not colliding any longer. If this happens in all space during a narrow time interval, it is possible to define a freeze-out time.

Depending on the speed of the expansion, freeze-out will "freeze" the velocities of nucleons and clusters in different conditions on the way towards the equilibrium. Thus, we can suppose that nucleons are uniformly distributed in the configuration space at the time of freeze-out and, thus, the occupation probability function depends only on the momentum coordinates. Hence, the entropy may be expressed by

$$S = -g \sum_{\boldsymbol{r}} \sum_{\boldsymbol{p}} \left\{ f(\boldsymbol{p}) \ln f(\boldsymbol{p}) + [1 - f(\boldsymbol{p})] \ln[1 - f(\boldsymbol{p})] \right\}. \tag{12}$$

From the expression (11), we see that there is only one cell in configuration space. Thus, the generalized entropy can be expressed as a sum over momentum space only,

$$S = -g \sum_{\boldsymbol{p}} \left\{ f(\boldsymbol{p}) \ln f(\boldsymbol{p}) + [1 - f(\boldsymbol{p})] \ln[1 - f(\boldsymbol{p})] \right\}. \tag{13}$$

From Eq. (9), it is easy to calculate the size of cubic cells in momentum space as a function of the freeze-out density ρ,

$$(\Delta p_x)^3 = h^3 \frac{\rho}{A}. \tag{14}$$

For example, for the Au + Au reaction system ($A = 394$) at the assumed freeze-out density $\rho = 0.3\rho_0$, one obtains an approximative value $\Delta p_x \approx 60$ MeV/c for the size of cells in momentum space.

4. *Topology of central events*

The generalized entropy in Eq. (13) is expressed by the three-dimensional sum in momentum space. Since central events of reaction systems have two-dimensional topology, the generalized entropy of these events can be expressed by the two-dimensional sum in momentum space (transversal and longitudinal). For central events, we can assume cylindrical symmetry and, thus, the occupation probability function $f(p_\perp, p_\|)$ is a function of transverse and longitudinal components of the momentum. Hence, the generalized entropy is given by

$$S = -g \sum_{p_\perp} \sum_{p_\|} \left\{ f(p_\perp, p_\|) \ln f(p_\perp, p_\|) + [1 - f(p_\perp, p_\|)] \ln[1 - f(p_\perp, p_\|)] \right\}. \quad (15)$$

In the above sum, the size of the transversal $(\Delta p_\perp)_{(i)} = (\Delta P_\perp)_{(i)} - (\Delta P_\perp)_{(i-1)}$ and longitudinal $(\Delta p_\|)_{(i)}$ components of the ith cell (see Fig. 4) can be expressed by the following relation

$$\pi(\Delta P_\perp)^2_{(i)} (\Delta p_\|)_{(i)} = i(\Delta p_x)^3. \quad (16)$$

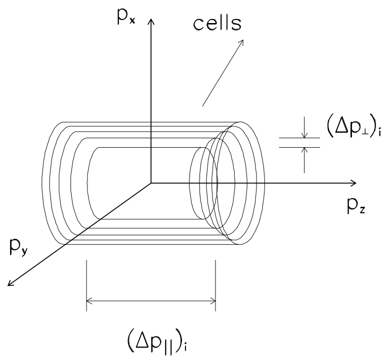

Fig. 1. Topology of a central event with the sizes of the transversal $(\Delta p_\perp)_{(i)}$ and longitudinal $(\Delta p_\|)_{(i)}$ components of the cell.

The size of the transversal and longitudinal components of the cell are related to each other by the asymmetry parameter Y, which is defined as a ratio of average transversal and longitudinal components of momentum distributions

$$Y = \frac{< p_\perp^2 >}{< p_\|^2 >}. \quad (17)$$

Using the asymmetry parameter Y, which can also be written as

$$Y = \frac{(\Delta P_\perp)^2_{(i)}}{(\Delta p_\|)^2_{(i)}}, \quad (18)$$

the longitudinal component of the ith cell can be obtained from the expression (16)

$$\begin{aligned} \pi[Y(\Delta p_\|)^2_{(i)}](\Delta p_\|)_{(i)} &= i(\Delta p_x)^3, \\ \pi Y(\Delta p_\|)^3_{(i)} &= i(\Delta p_x)^3. \end{aligned} \quad (19)$$

Inserting $(\Delta p_x)^3$ from (14) into the above expression, we obtain for the longitudinal $(\Delta p_\parallel)_{(i)}$ component of the ith cell

$$\begin{aligned} \pi Y (\Delta p_\parallel)^3_{(i)} &= h^3 \frac{\rho}{A} i, \\ (\Delta p_\parallel)_{(i)} &= h \sqrt[3]{\frac{\rho}{\pi Y A} i}. \end{aligned} \tag{20}$$

The size of the transverzal component, $(\Delta p_\perp)_{(i)} = (\Delta P_\perp)_{(i)} - (\Delta P_\perp)_{(i-1)}$, of the ith cell can be obtained by combining expressions (18) and (20) as follows

$$\begin{aligned} (\Delta p_\perp)_{(i)} &= (\Delta P_\perp)_{(i)} - (\Delta P_\perp)_{(i-1)} \\ &= \sqrt{Y}[(\Delta p_\parallel)_{(i)} - (\Delta p_\parallel)_{(i-1)}] \\ &= \sqrt{Y} h \sqrt[3]{\frac{\rho}{\pi Y A}} (\sqrt[3]{i} - \sqrt[3]{i-1}) \\ &= h \sqrt[3]{\frac{\rho \sqrt{Y}}{\pi A}} (\sqrt[3]{i} - \sqrt[3]{i-1}). \end{aligned} \tag{21}$$

Finally, the values of $(\Delta p_\parallel)_{(i)}$ and $(\Delta p_\perp)_{(i)}$ are the following

$$\begin{aligned} (\Delta p_\parallel)_{(i)} &= h \sqrt[3]{\frac{\rho}{\pi Y A} i}, \\ (\Delta p_\perp)_{(i)} &= h \sqrt[3]{\frac{\rho \sqrt{Y}}{\pi A}} (\sqrt[3]{i} - \sqrt[3]{i-1}), \qquad i = 1, 2, 3, \dots. \end{aligned} \tag{22}$$

The occupation probability function $f(p_\perp, p_\parallel)$ and the asymmetry parameter Y can be obtained from experimental data, and from these quantities the value of the generalized entropy at the assumed freeze-out can be derived.

5. *Summary*

The understanding of the origin of the entropy content of reaction systems is one of the problems in nuclear physics which is still open. Entropy production requires deviations from thermodynamic equilibrium, which are assumed to be realized at very high temperatures.

We have proposed a generalized approach to the determination of single-particle entropy in central nucleus-nucleus collisions. The approach is based on a unique determination of the cell size in momentum space for a chosen freeze-out density. Assuming a cylindrical topology of central collisions at freeze-out, it is possible to determine the entropy from experimental heavy-ion data. The present approach, however, has inherent uncertainties. The principal uncertainty stems from the freeze-out density assumption, and we have not taken into account that the particle freeze-out is a continuous rather than a sudden process. Also, this approach gives the single-particle entropy, while, in general, one should consider the N-body entropy, for which the distribution in the full $2N$-dimensional phase space should be involved.

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ODREĐIVANJE POOPĆENE ENTROPIJE U SUDARIMA TEŠKIH IONA

Pretpostavljajući jednoliku raspodjelu nukleona u konfiguracijskom prostoru u trenutku zamrznuća, predlažemo pristup jedinstvenog određivanja veličine ćelija u impulsnom prostoru. Nadalje, primjenom funkcije vjerojatnosti zaposjednuća u impulsnom prostoru, razmatramo mogućnost određivanja poopćene entropije u teškoionskim reakcijskim sustavima u trenutku zamrznuća.