# A MICROSCOPIC MODEL FOR CALCULATING THE INITIAL NUMBER OF EXCITONS IN NUCLEUS-NUCLEUS COLLISIONS

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Received 2 July 1991

UDC 539.17

Original scientific paper

A microscopic model for estimating $n_0$, the initial exciton number used in calculating pre-equilibrium emission from heavy-ion reactions, is presented. The model follows the evolution of the geometrical and phase spaces during the process of fusioning of the target and the projectile. A good agreement is found between the calculated values of $n_0$ and the empirical values extracted from fits to nucleon spectra; empirical trends of physical quantities involving $n_0$ are reproduced. An approximate expression for calculating $n_0$ as a function of the mass and energy of the colliding system is given.

## *1. Introduction*

Among the variety of models used to calculate the nonequilibrium nucleon emission from nucleus-nucleus collisions at intermediate energies, those based on the master equation approach (see e. g. Ref. 1) assume that the equilibration process starts from some simple configuration, characterized by an initial number of degrees of freedom, $n_0$. Although in view of the complexity of the equilibration of a multinucleon system, $n_0$ should be a source term[2,3] rather than a single number, the latter choice has been surprisingly effective in describing the observed spectra with remarkable accuracy[4]. Thus in the Boltzmann master equation approach one has[2,5]

$$N(U) \cdot \Delta U \sim \frac{U^{n_0-1} - (U-\Delta U)^{n_0-1}}{(E^*)^{n_0-1}} \tag{1}$$

for the number of nucleons $N(U)$ emitted in the energy interval $\Delta U$ around the residual excitation energy $U$. $E^*$ is the total excitation energy of the composite system.

Eq. (1) has its partner also in the exciton and the hybrid models[1,6] where

$$N(U) \sim \frac{1}{(E^*)^{n_0-1}} \frac{\mathrm{d}U^{n_0-1}}{\mathrm{d}U} = (n_0-1)\, E^* \left(\frac{U}{E^*}\right)^{n_0-2}. \tag{2}$$

Eq. (2) is the limit of Eq. (1) for $\Delta U \to 0$.

The value of $n_0$ for a given collision is a phenomenological value extracted from appropriate data. A simple way to obtain it is to apply Eq. (2) in its logarithmic form[6]

$$\ln\,[N(U)] \sim (n_0 - 2) \ln U \tag{3}$$

to the high-energy part of nucleon spectra from central collisions. A complete master equation treatment of these spectra yields, ideally, the same value; in practice, the difference between the two is unessential[4]. In this paper we present a simple microscopic model for calculating $n_0$ and extend the basic ideas published recently in Ref. 7. The model is based on momentum space considerations with a few straightforward assumptions and it does not introduce any adjustable parameters.

## 2. *The model*

The colliding system is represented in the momentum space by two spheres of radii $P_F$, whose distance is $P_P + P_T$. $P_F$ is the Fermi momentum and $P_P$ and $P_T$ the respective per nucleon momenta at the touching point of the projectile and the target, respectively (Fig. 1). Clearly,

$$P_F = \sqrt{2m_0E_F}, \quad P_P = \frac{1}{A_P}\sqrt{2A_Pm_0E_P}, \quad P_T = \frac{1}{A_T}\sqrt{2A_Tm_0E_T}, \tag{4}$$

with $m_0$ the nucleon mass, $E_F$ the Fermi energy, and the energies

$$E_P = \frac{A_T}{A_P + A_T}\, E_0, \quad E_T = \frac{A_P}{A_P + A_T}\, E_0. \tag{5}$$

Here, $E_0$ is the center-of-mass energy of the collision, diminished by the corresponding Coulomb barrier $V_C$,

$$E_0 = E_{CM} - V_C. \tag{6}$$

The newly created composite system is represented by a third sphere of radius $P_F$, whose center coincides with the center of mass of the colliding system. The part of this sphere fed by both colliding nuclei is illustrated by the doubly shaded area in Fig. 1. This part has all the levels filled and hence it does not contribute

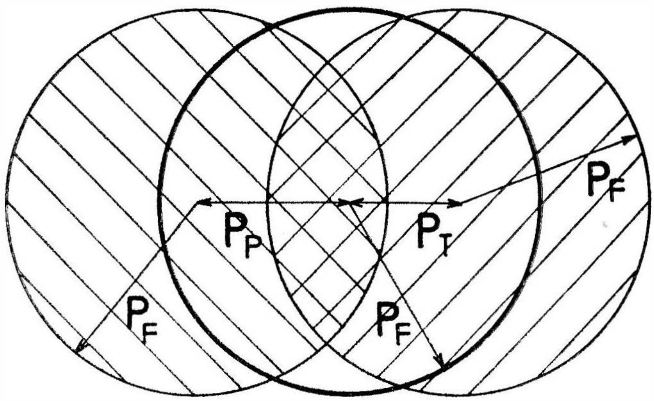


Fig. 1. The colliding system in momentum space. The heavy-line circle represents the final composite system.

particle-hole pairs to the composite system. In all the remaining phase space within the spheres, particles and holes will be created by the collision. Shown schematically, holes from the projectile ($h_P$) will all stem from the region inside the composite system, where this sphere does not overlap with the projectile one (shaded area in Fig. 2 a). Correspondingly, holes from the target ($h_T$) will stem from the region of no overlap between the target and the composite spheres (shaded area in Fig. 2b). Note that the polar regions, fed neither by the projectile nor by the target, are counted twice. On the other hand, nucleons outside the composite sphere do not produce holes. Consequently, all these nucleons should be considered as particles ($p_P + p_T$, shaded areas in Fig. 2c). From simple geometry it follows that the number of holes, $h_0$, equals that of particles, $p_0$. Finally, $n_0 = p_0 + h_0 = (p_P + p_T) + (h_P + h_T)$. Thus the number of particles and holes in the model is calculated as the geometric overlap of parts of three spheres of equal radii in momentum space.

Let us assume that in the heavier partner (say, the target) we can split the nucleons into two groups, the participants and the spectators (the terms are borrowed from relativistic heavy-ion physics). Then the number of participants from the target equals to the number of participants from the lighter partner (say, the projectile). In this case we have

$$p_0 = (1 - f_P)\, A_P + (1 - f_T)\, A_P \tag{7}$$

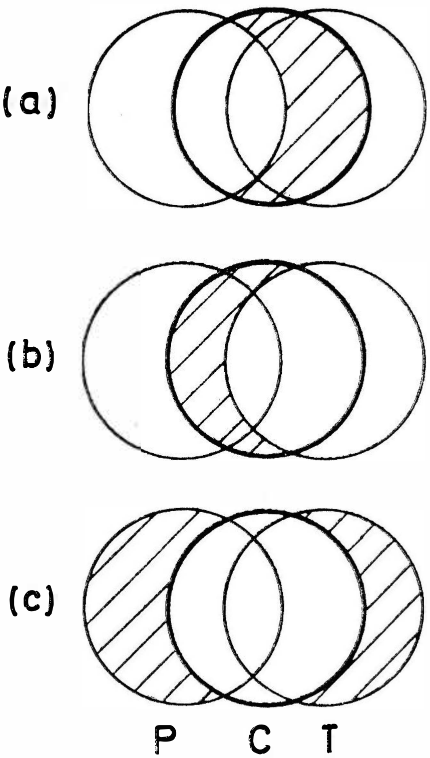


Fig. 2. Overlapping volumes for calculating the number of holes stemming from the projectile (a) and the target (b), and the number of particles (c). The spheres depicted are the same in all three parts a, b, c, and also identical to those in Fig. 1.

($f_P$ and $f_T$ are, respectively, the overlaps of the projectile and the composite spheres and of the target and the composite spheres; $0 \leq f_P, f_T \leq 1$). A similar expression holds for holes, thus limiting $n_0 \leq 4A_P$.

Such a situation, however, is an oversimplification of reality. The initial exciton number is modified when the geometrical overlap of the two colliding nuclei is taken into account. In the geometrical space the colliding system is represented by the projectile and target spheres of radii $R_P$ and $R_T$, respectively. In the present calculation the overlap volume of the two colliding nuclei (projectile and target) is calculated at each moment of the collision. The time when two colliding nuclei touch is set as zero time; we stop the calculations at the time when the whole lighter partner has entered into the heavier one. The whole process of gradual fusioning of the target and projectile is followed in steps of $\Delta t = 2.1 \cdot 10^{-23}$ s; this time interval corresponds to the average time between two subsequent nucleon-nucleon collisions in nuclear matter[2]. We also assume that the relative velocity of the collision partners is not significantly affected by friction and that the energy of the composite system increases gradually as the nucleons from the lighter partner, which we always assume to be the projectile, enter the target. Taking into account the conservation of the total number of particles, energy, and momentum we obtain the following relation for the dependence of the available excitation energy on the collision time $t$:

$$E_0(t) = E_0 \frac{A_T + A_P}{A_T + \Delta A(t)} \frac{\Delta A(t)}{A_P}, \tag{8}$$

where $\Delta A(t)$ is the number of nucleons from the lighter partner enveloped into the heavier partner at a given time $t$. At $t = 0$, $\Delta A(0) = 0$, hence $E_0(0) = 0$, while at a certain $t = t_{max}$, when $\Delta A(t) = A_P$, $E_0(t_{max}) = E_0$, with $E_0$ given by Eq. (6). Obviously, since $E_0(t)$ dependens on $t$, all kinematical variables in the phase space (except $P_F$) will also depend on $t$. Therefore, we extend Eqs. (4) and (5) with the following set of equations:

$$P_P(t) = P_P \sqrt{\frac{E_0(t)}{E_0}}, \quad P_T(t) = P_T \sqrt{\frac{E_0(t)}{E_0}} \tag{9}$$

$$E_P(t) = E_P \frac{E_0(t)}{E_0}, \quad E_T(t) = E_T \frac{E_0(t)}{E_0}, \tag{10}$$

where $P_P$, $P_T$, $E_P$ and $E_T$ are defined in Eqs. (4) and (5). Figs. 1 and 2 refer to the case when $t = t_{max}$ and $E_0(t_{max}) = E_0$; at $t = 0$ all three spheres in the phase space coincide, since $E_0(0) = 0$.

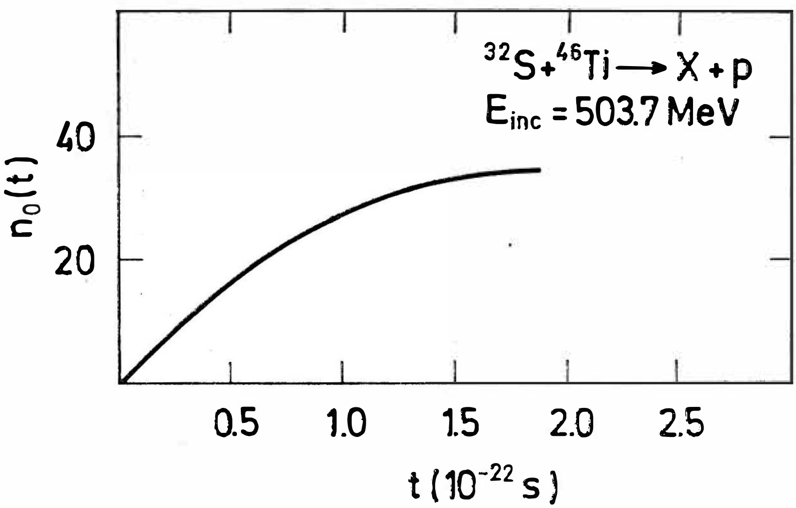


Fig. 3. Evolution of the calculated value of $n_0(t)$ with the collision time $t$.

The calculation of $n_0(t)$ according to the model outlined above is straightforward. Obviously, Eq. (7) is now modified to

$$p_0 = (1 - f_P)[A_P - \Delta A(t)] + (1 - f_T)[A_T + \Delta A(t)]. \tag{11}$$

Fig. 3 shows the dependence of the so calculated values $n_0(t)$ on the collision time $t$ for $^{32}S + {}^{46}Ti$ at 503.7 MeV. Obviously, we take as $n_0$ the maximal (saturation) value of $n_0(t)$.

## 3. *Comparison with empirical values*

Historically, the first estimate of $n_0$ was to set $n_0 = A_P$[2]. The underlying idea was an extension from light ion data, based on the complete break-up of the projectile. With the progress of the analysis, however, some energy dependence emerged. The value of $n_0/A_P$ was found to increase with the available per nucleon energy, $E_0/A_P$. This dependence was observed for a wide range of reactions at energies up to about 20 MeV/nucleon[8]. More recently, a linear dependence of $E^*/n_0$ on the energy brought in by the collision, $(E_{lab} - V_C)/A_P$, has been observed above 10 MeV per nucleon[9].

Using the model described above, we calculated the values of $n_0$ for a number of nucleus-nucleus collisions and compared them to the values extracted from the data using a Boltzmann master equation analysis, where we have selected data from central collisions only[9]. This comparison is shown in Table 1. The agreement with the extracted values is rather good.

TABLE 1.

| Proj. | $^{16}O$; 403 MeV | | $^{32}S$; 504 MeV | | $^{32}S$; 679 MeV | | $^{58}Ni$;876 MeV | |
|---|---|---|---|---|---|---|---|---|
| Target | fit | calc. | fit | calc. | fit | calc. | fit | calc. |
| $^{27}Al$ | 16 | 23.0 | 23 | 29.5 | 23 | 34.7 | 26 | 28.4 |
| $^{46}Ti$ | 19 | 23.0 | 28 | 34.5 | 28 | 40.9 | 35 | 47.4 |
| $^{60}Ni$ | 19 | 23.0 | 29 | 34.2 | 29 | 40.8 | | 59.3 |
| $^{120}Sn$ | 21 | 23.1 | 35 | 33.8 | 35 | 40.7 | 46 | 58.0 |
| $^{124}Sn$ | | 23.2 | 35 | 33.9 | 35 | 40.8 | 46 | 58.2 |
| $^{197}Au$ | 22 | 22.7 | 37 | 32.7 | 37 | 40.1 | 61 | 56.3 |

Extracted (fit)[9] and calculated values of $n_0$ for $^{16}O$, $^{32}S$ and $^{58}Ni$ projectiles on selected targets.

In our calculations, we have used $E_F = 30$ MeV, and for the Coulomb barrier we have used $R = 1.2A^{1/3}$ fm. However, a reasonable variation of the value of the Fermi energy does not destroy the agreement: varying $E_F$ by $\pm 15\%$ changes $n_0$ by only about 10%.

Finally, Fig. 4 displays the dependence of $E^*/n_0 = (E_{CM} + Q_{fus})/n_0$ on $e' \equiv$ $\equiv (E_{CM} - V_C)/A_P$ observed earlier in reactions induced by $^{16}O$, $^{32}S$ and $^{58}Ni$ projectiles on various targets[9], drawn together with results obtained from our model. On the top (open symbols) are the presently calculated values of $E^*/n_0$; the corresponding empirical (extracted) values are shown by the corresponding full symbols (bottom). The calculated values reproduce the empirical values rather

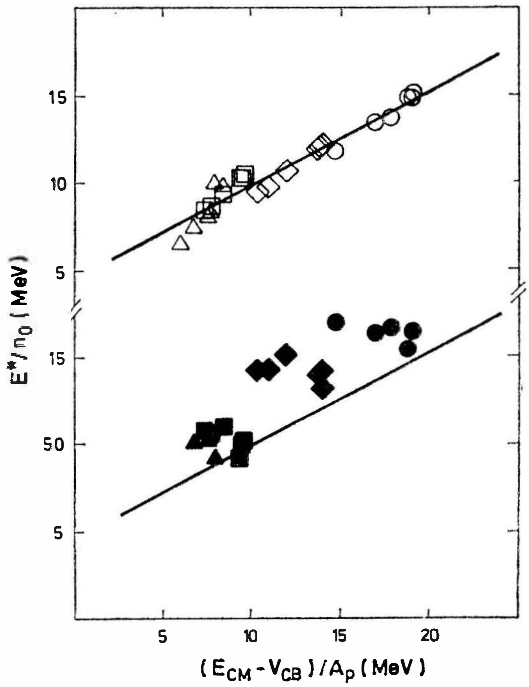


Fig. 4. Dependence of $E^*/n_0$ on the per nucleon energy above the Coulomb barrier for systems from Table 1. For each projectile (and incident energy) the extracted (fit) values of $n_0$ are shown as full symbols (bottom) and the corresponding calculated values as empty symbols (top). Symbols denote: circles for $^{16}$O, squares and diamonds for $^{32}$S and triangles for $^{58}$Ni projectiles. Lines: The average calculated trend of $E^*/n_0$, Eq. (12).

well. The upper full line in Fig. 4, shows the best-fit line through the values calculated from the model. This line is given by

$$\frac{E^*}{n_0} = 4.6 + 0.54 \cdot e' \tag{12}$$

and is reproduced also on the lower side of the figure. On the other hand, the empirical trend observed in Ref. 9 could be represented by

$$\frac{E^*}{n_0} = 6.8 + 0.54 \cdot e' \tag{13}$$

(the difference of this equation and Eq. (2) of Ref. 9 stems from the use of c. m. energies instead of the laboratory ones). The comparison between Eqs. (12) and (13) shows that the empirical dependence of $E^*/n_0$ on $e'$ from Ref. 9 and the one obtained in this work are essentially identical, except for a difference in the absolute values of $E^*/n_0$.

Another representation of our model calculations would be the dependence of $n_0/A_P$ on the available per nucleon energy $e'$. An empirical expression for this dependence, obtained from fitting the calculated $n_0/A_P$ values for several colliding systems is

$$n_0/A_P = 0.09 + \left(0.38 - 0.08 \cdot \frac{A_T - A_P}{A_T + A_P}\right) \cdot \sqrt{e'} \tag{14}$$

with $E_{CM}$ and $V_C$ in MeV shows that these values vary with, essentially, the momentum corresponding to the available per nucleon energy.

## 4. *Discussion*

The reliability of the two approximate expressions in calculating $n_0$ for specific cases asks for some comments. A small uncertainity in the absolute term in Eq. (12) would introduce a large error in $E^*/n_0$, especially for small values of $n_0$ (which occur predominantly at small per-nucleon incident energies $e'$). Hence, Eq. (12) would be preferred at higher energies, whereas Eq. (14) is more suitable at moderate energies. The energy, where one has to »shift« from one expression to the other is somewhat arbitrary, but $e'$ about 15 MeV/$A$ might serve as a rough estimate for the change of representation.

An interesting verification of our model would be its application to the low-energy reactions induced by nucleons and $\alpha$-particles, a traditional domain of the pre-equilibrium model. Therein, an extensive set of analyses has been performed; the usual prescription is $n_0 = 1$ or $n_0 = 3$ for nucleon-induced reactions (these two initial exciton numbers are, in fact, indistinguishable), and $n_0 = 4$ to 6 for $\alpha$-induced reactions. Clearly, direct application of our model to the reactions induced by nucleons is not feasible. However, the approximate expressions (12) and (14) can be used for this purpose; the latter of these equations is more suitable because it is geared to lower energies. Here, Eq. (14) yields $n_0 \approx 1.2$ for 14 MeV nucleon-induced reactions and only raising the incident energy up to 100 MeV increases the value of $n_0$ to 3. For $\alpha$-particles of 20 MeV per nucleon above the Coulomb barrier ($E \simeq 80$ MeV, an energy higher than that in majority of $\alpha$-induced reactions analysed within the pre-equilibrium model), Eq. (14) yields $n_0 \approx 5$, in agreement with the data obtained from systematics.

In our calculations, we assume that the Fermi momentum $P_F$ is the same for all the three nuclei (target, projectile and the composite system). However, detailed analyses give individual values of $P_F$ which are smaller for »compact« nuclei like $^{12}C$ and larger for the other ones, the differences being up to 20%. Moreover, different values are reported for neutrons and protons[10]. Though in a detailed study one should take this effect into account, we did not feel the necessity to incorporate it into the calculation at its present stage.

The models treated in this paper refer to calculations of angle-integrated energy spectra. To calculate angular distributions, one has to have at least some knowledge on the momenta involved, especially, on the momentum of the system which emits particles. In this system the main portion of angular information has its origins in the early stages of nuclear reaction and we can use the conditions valid at the beginning of the reaction as a zero-order approximation. Thus, the momentum introduced to the composite system by the projectile is

$$\mathscr{P}_P = P_P \cdot A_P \tag{15}$$

(here, $P_P$ is the per nucleon momentum). Assuming that this momentum is carried (after fusioning) by the projectile-created excitons, we have

$$\mathscr{P}_P = (p_P + h_P) \cdot \mathscr{P}_{aver}. \tag{16}$$

Thus, the average per-exciton momentum due to the projectile is

$$\mathscr{P}_{aver, proj} = \frac{P_P \cdot A_P}{p_P + h_P}, \tag{17}$$

and similarly for the target part. All this is in the c. m. system. In order to transform to the laboratory system, one adds the center-of-mass momentum in the laboratory frame, $P_{CM}$. This gives us the tool for expressing the direction-dependent quantities of the pre-equilibrium decay in heavy-ion reactions.

## 5. *Conclusions*

To summarize, a microscopic model to calculate $n_0$, the value of the initial number of degrees of freedom that share the energy in a nucleus-nucleus collision (the initial exciton number) has been developed. These values were so far extracted from master-equation and plot analyses of nucleon spectra. The calculated values of $n_0$ reproduce those extracted from the data rather well; they also reproduce two main trends observed for $n_0$, namely the parabolic increase of $n_0/A_P$ with the available energy $(E_{CM} - V_C)/A_P$ and the linear dependence of $E^*/n_0$ (the share of the excitation energy per initial degree of freedom) on the same quantity.

### Acknowledgement

This work was supported in part by the US-Yugoslav collaboration project JF 939 DOE/IRB. Two of the authors (N. C. and J. J. G.) acknowledge the hospitality of the Joint Institute for Heavy Ion Research, Oak Ridge, TN, where this work was initiated.

### References

1) M. Blann, Ann. Rev. Nucl. Sci. **25** (1975) 123;

2) M. Blann, Proc. Int'l School Nucl. Phys., Predeal, 1974, ed. by A. Ciocanel, p. 249 (Romanian Acad. of Sci., Bucharest, 1976);
M. Blann, Phys. Rev. **C23** (1981) 205;

3) T. Otsuka and K. Harada, Phys. Lett. **121B** (1983) 106;
K. Niita, Z. Phys. **A316** (1984) 309;

4) E. Holub, M. Korolija and N. Cindro, Z. Phys. **A314** (1983) 347;

5) G. D. Harp, J. M. Miller and B. J. Berne, Phys. Rev. **165** (1968) 1166;

6) J. J. Griffin, Phys. Rev. Lett. **17** (1966) 478 and Phys. Lett. **24B** (1967) 5;

7) N. Cindro, M. Korolija, E. Běták and J. J. Griffin, Phys. Rev. Lett. **66** (1991) 868;

8) M. Korolija, E. Holub, N. Cindro and D. Hilscher, Proc. Int. Conf. on Nucl. Physics, Florence, 1983, Contributed Papers, p. 589 (Compositori, Bologna);
E. Betak, in *Collective Motion and Giant Resonances*, Proc. Int. 15th Summer School on Nucl. Physics, Mikolajki, 1983, Ed. Z. Wilhelmi et al., p. 132 (Warsaw University);
E. Běták, Czech. J. Phys. **B34** (1984) 850;

9) M. Korolija, N. Cindro and R. Čaplar, Phys. Rev. Lett. **60** (1988) 193;

10) E. J. Moniz, I. Sick, R. R. Whitney, J. R. Ficenec, R. D. Kephart and W. P. Trower, Phys. Rev. Lett. **26** (1971) 445;
R. J. Vojtech, R. Butsch, V. M. Datar, M. G. Herman, R. L. McGrath, P. Paul and M. Thoennessen, Phys. Rev. **C40** (1989) R244.

# MIKROSKOPSKI MODEL ZA RAČUNANJE POČETNOG BROJA EKSCITONA U SUDARIMA ATOMSKIH JEZGARA

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UDK 539.17

Originalni znanstveni rad

Prikazan je model pomoću kojeg je moguće odrediti $n_0$, početni broj ekscitona na koji se dijeli energija sudara atomskih jezgara. Taj je parametar potreban za računanje predravnotežne emisije čestica iz teškoionskih reakcija Prikazani model slijedi evoluciju geometrijskog faznog prostora tijekom procesa fuzije jezgre mete i projektila. Dobro slaganje opaženo je između računatih vrijednosti parametara $n_0$ i onih, dobivenih na empirički način analizom nukleonskih spektara iz teškoionskih reakcija. Model također uspješno reproducira trendove fizikalnih veličina povezanih s $n_0$. U članku je dan izraz za računanje približne vrijednosti parametra $n_0$ u ovisnosti o masi i energiji sudarnog sustava.