QUASIELASTIC ELECTRON SCATTERING IN ^{4}He BETWEEN 328 AND 725 MeV

JOSEPH J. BEVELACQUA*

Wisconsin Electric Power Company, 6610 Nuclear Road, Two Rivers, Wisconsin 54241, USA

^{4}He (e, e') cross section data at 328, 365, and 725 MeV have been analyzed using the quasi-free scattering (QFS) model of Lightbody and O'Connell. The QFS model provides an improved description of the shape and magnitude of the cross section data when compared with plane-wave impulse approximation calculations, particularly at higher values of the electron energy loss. Although the nucleon separation energy is about 20 MeV in ^{4}He, the best QFS results are obtained for a 6 MeV value.

1. Introduction

Recent plane-wave impulse approximation (PWIA) studies of the ^{4}He (e, e') reaction in the 32—725 MeV range of incident electron energy fail to accurately describe the data[1]. The discrepancies between calculations and data have been attributed to the failure to properly account for various physical processes, including multinucleon knockout, final-state interactions, meson-exchange currents and possible modification to the nucleon form factors in the nuclear medium[2-7]. Von Reden et al.[1] note that even though some of these effects may be large, a consistent improvement in the theoretical predictions of quasielastic electron scattering has yet to be achieved.

---

*Address all correspondence to the author at 934 Regent Lane, Green Bay, Wisconsin 54311, U.S.A.

The PWIA calculations predict a single broad peak with a decreasing cross section at higher values of electron energy loss. The data are typified by a broad peak, but also contain a rising cross section at higher electron energy loss values. The PWIA model suggests essentially a zero value for the cross section at the higher electron energy loss values. The ability to describe this higher energy region is a major challenge to theoretical models attempting to describe the (e, e') data[1].

Recent theoretical advances by O'Connell et al.[8] and Lightbody and O'Connell[9] have proposed a quasi-free scattering (QFS) model for inelastic electron scattering. This model has successfully described a wide variety of inclusive (e, e') data and its application to the 328—725 MeV data of von Reden et al.[1] is warranted.

The QFS model contains a wealth of physical contributions to the (e, e') cross section. Calculation in the 328—725 MeV energy range will provide additional insight into the capabilities of the QFS model. Moreover, the assessment of von Reden et al.'s data will further quantify the applicability of the QFS and PWIA models to this energy region of the ^{4}He plus electron system.

2. Quasi-free scattering (QFS) model

QFS is a phenomenological model which describes electron scattering cross sections. The model, as formulated by Lightbody et al.[8] and Lightbody and O'Connell[9] assumes largely incoherent electron-nucleon scattering and includes the dominant physical processes that contribute to the cross section. These processes include:

(1) quasielastic (e, e') scattering from a bound nucleon;
(2) scattering from two interacting nucleons;
(3) delta electroproduction;
(4) electroproduction at energies above the delta threshold described by two resonances (one centered at an invariant mass of 1500 MeV and the other at 1700 MeV); and
(5) deep inelastic scattering in the X-scaling regime [$X = Q^2/(2M\omega)$, where M is the nucleon mass and ω is the energy loss].

The QFS mode is constrained to yield a real photon total absorption cross section that approximately reproduces available data. This model must be applied with care to low energy electron scattering below the Fermi momentum, or to energy losses in the range of coherent nuclear excitations[8,9].

QFS considers an incident electron of momentum $\mathbf{k}$ and energy E which scatters from a nucleus of atomic weight A, containing Z protons and N neutrons. If the scattered electron has momentum $\mathbf{k'}$ and energy E', then the three momentum ($\mathbf{q}$) and energy transfer (ω) to the nucleus (nuclear excitation energy) are

$$\mathbf{q} = \mathbf{k} - \mathbf{k'} \tag{1}$$

$$\omega = E - E' \tag{2}$$

Following Lightbody and O'Connell[9], the square of the four-momentum transfer (Q^2) is

$$Q^2 = q^2 - \omega^2. \tag{3}$$

Atomic units are used in the subsequent discussion

$$\hbar = c = 1. \tag{4}$$

The QFS electron scattering cross section is described in terms of a virtual photon flux factor, a virtual photon cross section and a form factor. Cross sections in the nuclear scaling region are taken to be A times the free nucleon scattering cross section. The QFS nucleon electron scattering cross section is a function of the four-momentum transfer, nuclear excitation energy and scattering angle (Θ) with respect to the incident electron direction

$$\sigma(Q^2, \omega, \Theta) = G_\nu \, \sigma_\gamma(\omega) \, (1 + eR_x) \, F_x^2(Q^2) \tag{5}$$

where σ_γ is defined in Eq. (6), G_ν is the virtual photon flux and $F_x(Q^2)$ is a form factor which is chosen to be a sum-of-exponential fit to the actual ^{1}H (e, e') form factor below 5 GeV electron energy[9]. Lightbody and O'Connell[9] find that this parametrization is better at low Q^2 than one based solely on X scaling. The virtual photon polarization (Θ) is a measure of the ratio of longitudinal to transverse photon flux in the effective photon beam, and R_x is the ratio of longitudinal to transverse cross sections.

The real photon cross section is parametrized using the following form

$$\sigma_\gamma(\omega) = \left[\sigma_0 + \frac{\sigma_1}{(\omega - \omega_\pi)}\right] \times \left[1 - \exp\left\{-\frac{(\omega - \omega_\pi)^2}{2G_x^2}\right\}\right] \tag{6}$$

where $\sigma_0 = 100$ μb and $\sigma_1 = 54$ mb MeV (1b $= 10^{-28}$ m^2). The threshold for the photon cross section ω_π is taken to be at the free nucleon pion electroproduction threshold

$$\omega_\pi = Q^2/(2M) + m_\pi + m_\pi^2/(2M) \tag{7}$$

where M is the nucleon mass.

The width parameter $G_x = 650$ MeV was determined by a fit to free nucleon photodata[9].

In addition to the nuclear effects noted above, the QFS model also includes radiative effects, and treats both nuclear and electromagnetic radiative effects in a consistent manner. The QFS model provides a description of the continuum radiation process which distorts the pure nuclear processes.

QFS does not treat the coherent radiation effects from the elastic scattering process, nor does it treat any two-step effects. The model's treatment of the ra-

diation tail is within the framework of the »peaking approximation«, in which photon emission is confined to the direction of the initial or scattered electron[9].

The cross section used by the QFS model for the »peaking approximation« radiation tail is

$$d\sigma = \left[\sigma\left(E, \Theta, \omega'\right) \frac{a}{\pi} \frac{d\omega'}{\omega - \omega'} \times \left(\log\frac{Q_1^2}{m_e^2} - 1 \right) \left\{ \frac{(E - \omega')^2 + (E - \omega)^2}{2\left(E - \omega'\right)^2} \right\} + \right.$$

$$+ \sigma\left(E - \omega + \omega', \Theta, \omega'\right) \frac{a}{\pi} \frac{d\omega'}{\omega - \omega'} \left(\log\frac{Q_2^2}{m_e^2} - 1 \right) \times$$

$$\left. \times \left\{ \frac{E^2 + (E - \omega + \omega')^2}{2E^2} \right\} \right] \left(1 + \bar{d}_2\right) \left(\frac{\omega - \omega'}{\bar{E}}\right)^{\bar{d}_1} \tag{8}$$

where

$$\bar{d}_1 = \frac{2a}{\pi} \left(\log\frac{\overline{Q}^2}{m_e^2} - 1 \right) \tag{9}$$

$$\bar{d}_2 = \frac{2a}{\pi} \left(\frac{13}{12} \left\{ \log\frac{\overline{Q}^2}{m_e^2} - 1 \right\} - \frac{17}{36} - \frac{1}{2} \left\{ \frac{\pi^2}{6} - L_2 \left[\cos^2(\Theta/2)\right] \right\} \right) \tag{10}$$

$$Q_1^2 = 4E\left(E - \omega'\right) \sin^2\left(\Theta/2\right) \tag{11}$$

$$Q_2^2 = 4\left(E - \omega + \omega'\right)\left(E - \omega\right) \sin^2\left(\Theta/2\right) \tag{12}$$

$$\overline{Q}^2 = \left(Q_1^2 Q_2^2\right)^{1/2} \tag{13}$$

$$\overline{E} = \left(E - \omega\right)\left(E - F'\right)]^{1/2} \tag{14}$$

and L_2 is Spence's integral ($n = 2$) and m_e is the electron mass.

The cross section is obtained by integrating Eq. (8) over nuclear excitations less than the energy loss (ω) from $\omega' = 0$ to $\omega' = \omega - \Delta E$, and ΔE is chosen to be 10 MeV[9].

This integrated cross section describes the combination of radiation of hard photons and nuclear excitation by all electrons appearing in the energy range of interest. The QFS model treats contributions from electrons which suffer energy loss by a combination of soft photon emission and nuclear excitation in the interval $\omega' = \omega - \Delta E$ to ω by using the radiative correction

$$\sigma = \sigma\left(E, \Theta, \omega\right) \left(1 + d_2\right) \left(\frac{\Delta E}{[E\left(E - \omega\right)]^{1/2}}\right)^{d_1} \tag{15}$$

where

$$d_1 = \frac{2\alpha}{\pi}\left(\log\frac{Q^2}{m_e^2} - 1\right)$$ (16)

$$d_2 = \frac{2\alpha}{\pi}\left(\frac{13}{12}\left\{\log\frac{Q^2}{m_e^2} - 1\right\} - \frac{17}{36} - \frac{1}{2}\left\{\frac{\pi^2}{6} - L_0\left[\cos^2(\Theta/2)\right]\right\}\right).$$ (17)

Since no electrons suffer zero radiation loss, the total radiative cross section is the sum of the hard and soft photon contributions.

The reader should note that radiative effects are generally unfolded from experimental electron scattering data prior to comparison with theory. Radiatively corrected data at lower energy loss, as well as lower momentum transfer, are required for a proper treatment of radiative corrections. This information is also available from the QFS model.

The aforementioned discussion only provides a brief summary of the quasi-free scattering model. For a more complete description of the QFS model and its application, the reader is referred to O'Connell et al.[8] and Lightbody and O'Connell[9].

3. Plane-wave impulse approximation model

The ^{4}He (e, e') data of von Reden et al.[1] have been compared to PWIA calculations[10]. These calculations sum over all possible (e, e' N) states using the PWIA in the final state. The agreement between data and PWIA calculations is best at high momentum transfers which is expected for a PWIA model.

Von Reden et al. consider three sets of incident electron energies (E) and scattering angles (Θ): a) 365 MeV and 60°, b) 328 MeV and 134.5° and c) 725 MeV and 60°. As noted by von Reden et al.[1], PWIA calculations for the lowest (Θ, E) values, Set a, describe the data poorly. Final state interactions were not taken into account in the PWIA model but these effects were important contributors to a proper description of ^{3}H and ^{3}He data[11,12].

Von Reden et al.[1] note that even at high momentum transfers, the low excitation energy sides of the quasielastic peaks are notably enhanced compared to the PWIA calculations. The PWIA calculations also omit the effects of meson exchange currents and real pion production. Therefore, the high excitation energy sides of the peaks are not expected to be reproduced by the PWIA calculations. Since these effects are at least partially included in the QFS model, improved results are expected especially in the tail region at higher values of electron energy loss.

4. Nucleon separation energy

The nucleon separation energy is one of the key parameters involved in the QFS model[8]. O'Connell et al.[8] have shown that the high energy ^{4}He (e, e') data is best represented by a nucleon separation energy of 6 MeV. This value is much

smaller than the known 16 MeV single-nucleon separation energy for nuclear matter and the known single-nucleon separation energy for ^{4}He at roughly 20 MeV[13]. However, the 6 MeV value is consistent with nucleon separation energy measurements[14]. There is evidence in the Kuplennikov et al.[14] data that indicates the nucleon separation energy is not constant and has a minimum value near $|q| = 2k_F$.

As noted by O'Connell et al.[8], the q^2 dependence of the nucleon separation energy may be an artifact of the oversimplified Fermi gas model for single nucleon knockout. However, it may also indicate that another process, such as meson exchange current effects in two-nucleon knockout, is competing with single nucleon knockout and altering the cross section.

In view of the inconsistency between the best fit ^{4}He nucleon separation energy and the measured value of 20 MeV[13], we will investigate the impact of this separation energy on the ^{4}He (e, e') cross section.

5. Results and discussion — QFS model

Quasielastic scattering is one of the major (e, e') reaction mechanisms at intermediate energies. A broad peak or cross section maximum is observed in the (e, e') double differential cross section spectrum at an excitation energy of approximately $Q^2/2M$. The location of the peak suggests that electrons are elastically scattered from core nucleons and the shape of the peak can then be interpreted as a reflection of the nucleon momentum distribution[1].

Calculations of the radiatively corrected double-differential cross sections for inclusive electron scattering are summarized in Tables 1, 2 and 3. These tables summarize the calculated ^{4}He (e, e') cross sections at 365, 328 and 725 MeV with radiative corrections for nucleon separation energies of 6, 11 and 20 MeV. As noted earlier, the physical content of the QFS model contains the essential elements that contribute to the cross section in the tail or high energy region beyond the location of the cross section peak. Therefore, it is expected that the QFS results will provide a better representation of the data that the corresponding PWIA results.

a) 365 MeV, 60ˢ data (Set a)

Table 1 summarizes QPS, PWIA and measured ^{4}He (e, e') data for 365 MeV electrons. The PWIA cross section reaches its maximum at about 80 MeV energy loss, about 15 MeV beyond the measured peak. PWIA calculations underestimate the measured cross sections below the 65 MeV peak location, and overestimate the cross section between the peak and about 130 MeV energy loss. Beyond 130 MeV, the PWIA model underestimates the cross section.

The QFS cross section below 130 MeV is similar to the predictions of the PWIA model. Both models overestimate the magnitude of the cross section peak, and predict that the peak location lies beyond its experimental location.

The peak QFS cross sections occur at 68, 73 and 82 MeV energy loss for nucleon separation energies of 6, 11 and 20 MeV, respectively. The peak locations further

TABLE 1.

Electron energy loss (MeV)	Cross section (nb/MeV-sr); $1b = 10^{-28} \, m^2$				
		QFS Model			
	PWIA Model[b]	Nucleon separation energy (MeV)			Experiment[a]
		6	11	20	
40	0.4	5.0	3.8	2.1	3.6
60	7.4	9.7	8.9	6.8	8.7
70	10.3	10.2	10.3	9.3	8.8
80	10.7	9.0	9.9	10.6	7.8
100	7.9	4.2	5.3	7.8	5.2
120	4.0	1.7	2.1	3.2	3.4
150	0.9	1.2	1.3	1.4	1.5
170	0.3	1.5	1.5	1.5	1.2
190	0.2	1.7	1.7	1.7	1.4
210	0.1	1.9	1.9	1.9	1.5
230	<0.1	2.1	2.1	2.0	2.0
240	<0.1	2.2	2.2	2.1	2.7
260	<0.1	2.4	2.4	2.4	4.0

[a] Values scaled from Fig. 4, Ref. 1.
[b] Calculational values extrapolated from Fig. 4, Ref. 1.
^{4}He (e, e′) cross section at 365 MeV incident electron energy (Set a).

support Kuplennikov et al.[14] in suggesting the 6 MeV nucleon separation energy is the most appropriate nucleon separation energy value for QFS calculations.

The most dramatic difference between the QFS and PWIA models occurs at higher electron energy loss values. Beyond 130 MeV energy loss, the QFS model provides a more realistic representation of the data. The differences between PWIA and QFS predictions are most dramatic beyond 200 MeV. At this energy loss, the PWIA model underestimates the measured cross section by a factor of 10, while the QFS model reproduces the data within a factor of 1.4. This is attributed to the inclusion of physical mechanisms (See Section 2) not explicitly included in the PWIA model.

b) 328 MeV, 134.5° data (Set b)

The PWIA representation of the 328 MeV data is similar to its description at 365 MeV. The data below the peak are underpredicted more severely than in the previous case, and the peak cross section occurs at about 140 MeV which is about 5 MeV beyond the experimental peak location. In addition, the PWIA model severely underpredicts the measured cross sections beyond about 200 MeV. The PWIA, QFS and data for Set b are summarized in Table 2.

Results of the QFS model below 200 MeV are similar to the PWIA cross section predictions. The QFS model also underestimates the magnitude of the peak cross section. The values of the cross section at the 135 MeV peak are 0.45, 0.68 and 0.70 nb/MeV-sr for the QFS (6 MeV), PWIA and experiment, respectively. The location of the QFS peak cross section is at 136, 141 and 150 MeV for 6, 11 and 20 MeV values of the nucleon separation energy, respectively. The best

135

TABLE 2.

Electron energy loss (MeV)	PWIA Model[b]	Cross section (nb/MeV-sr); $1b = 10^{-28} m^2$			Experiment[a]
		QFS Model			
		Nucleon separation energy (MeV)			
		6	11	20	
70	<0.02	0.08	0.06	0.04	0.09
80	0.02	0.13	0.10	0.07	0.17
88	0.07	0.18	0.15	0.11	0.23
103	0.20	0.29	0.26	0.20	0.36
110	0.33	0.34	0.31	0.25	0.44
120	0.49	0.40	0.38	0.33	0.62
135	0.68	0.45	0.45	0.44	0.70
150	0.60	0.42	0.45	0.48	0.55
165	0.39	0.33	0.37	0.44	0.39
178	0.26	0.26	0.29	0.36	0.37
190	0.15	0.21	0.23	0.28	0.27
210	0.06	0.27	0.27	0.29	0.23
226	0.03	0.32	0.33	0.33	0.37
236	0.02	0.35	0.36	0.36	0.49

[a]Values scaled from Fig. 4, Ref. 1.
[b]Calculational values extrapolated from Fig. 4, Ref. 1.
^{4}He (e, e') cross section at 328 MeV incident electron energy (Set b).

peak location and representation of the data is provided by a nucleon separation energy of 6 MeV.

The most significant differences between the data and PWIA calculations occur beyond 200 MeV where the experimental cross section magnitude is underpredicted by as much as a factor of 25. The QFS model provides a more reasonable description of the data in the tail region, but still underpredicts the data by a factor of 1.4.

c) 725 MeV, 60° *data (Set c)*

Table 3 summarizes the 725 MeV ^{4}He (e, e') data and calculations utilizing the QFS and PWIA methodology. The PWIA methodology provides a reasonable representation of the data below 280 MeV energy loss, but the energy region below the peak is somewhat underpredicted. Beyond 280 MeV the cross section rises while the PWIA model decreases. The discrepancy in the cross section magnitude between the PWIA model and data is greater than a factor of 10 beyond 370 MeV energy loss.

An improved representation of the 725 MeV data is provided by the QFS model, particularly at higher values of electron energy loss. Beyond 370 MeV energy loss, the QFS model underpredicts the measured cross section by less than a factor of 1.4. The most significant shortcoming of the QFS model is that it underpredicts the cross section magnitude in the peak region. The peak location is predicted to lie at 218, 223 and 232 MeV energy loss for nucleon separation energy values of 6, 11 and 20 MeV, respectively. The 6 MeV value provides the best representation of the peak location.

TABLE 3.

Electron energy loss (MeV)	PWIA Model[b]	QFS Model			Experiment[a]
		Nucleon separation energy (MeV)			
		6	11	20	
107	<0.01	0.03	0.02	0.01	0.04
153	0.12	0.22	0.19	0.15	0.25
172	0.32	0.36	0.33	0.27	0.49
200	0.71	0.56	0.54	0.49	0.82
225	0.84	0.60	0.61	0.62	0.89
243	0.76	0.53	0.56	0.62	0.74
260	0.61	0.42	0.46	0.53	0.68
304	0.18	0.33	0.34	0.38	0.38
358	0.04	0.43	0.43	0.43	0.37
370	0.03	0.47	0.47	0.47	0.50
400	0.02	0.58	0.58	0.58	0.64
450	c)	0.67	0.67	0.67	0.72
480	c)	0.65	0.65	0.65	0.88

Cross section (nb/MeV-sr); $1b = 10^{-28} m^2$

[a] Values scaled from Fig. 4, Ref. 1.
[b] Calculational values extrapolated from Fig. 4, Ref. 1.
[c] Not provided in Fig. 4, Ref. 1.
^{4}He (e, e') cross section at 725 MeV incident energy (Set c).

For Sets a, b and c, QFS model cross section results improve in both shape and magnitude as the nucleon separation energy decreases. The cross section reproduces the data most closely with a 6 MeV nucleon separation energy value. The results of Tables 1, 2 and 3 continue to support O'Connell et al.[8] in suggesting that the QFS model favours nucleon separation energy values which are considerably less than the measured nucleon separation energy in ^{4}He.

6. Conclusions

QFS model results lead to an improved description of the ^{4}He (e, e') cross section data of von Reden et al. The QFS model includes the higher order processes essential to properly describing the cross section tail region at higher values of energy loss. The major differences between the PWIA and QFS models occur in the tail region where the QFS model provides an improved representation of the data. The model also supports a nucleon separation energy of 6 MeV in contrast to the measured 20 MeV value.

Acknowledgements

References

1) K. F. von Reden, C. Alcorn, S. A. Dytman, B. Lowry B. P. Quinn, D. H. Beck, A. M. Bernstein, K. I. Blomqvist, G. Dodson K. A. Dow, J. Flanz G. Retzlaff, C. P. Sargent, W. Turchinetz, M. Farkhondeh, J. S. McCarthy, T. S. Ueng and R. R. Whitney, Phys. Rev. **C41** (1990) 1084;

2) J. M. Laget, Can. J. Phys. **62** (1984) 1046;

3) Z. E. Meziani, P. Barreau, M. Bernheim, J. Morgenstern, S. Turck-Chieze, R. Altemus, J. McCarthy, L. J. Orphanos, R. R. Whitney, G. P. Capitani, E. Desanctis, S. Frullani and F. Garibaldi, Phys. Rev. Lett. **52** (1984) 2130;

4) C. C. Blatchley, J. J. LeRose, O. E. Pruet, P. D. Zimmerman, C. F. Williamson and M. Deady, Phys. Rev. **C34** (1986) 1243;

5) M. Deady, C. F. Williamson, P. D. Zimmerman, R. Altemus and R. R. Whitney, Phys. Rev. **C33** (1986) 1897;

6) S. A. Dytman, A. M. Bernstein, K. I. Blomqvist, T. J. Pavel, B. P. Quinn, R. Altemus, J. S. McCarthy G. H. Mechtel, T. S. Ueng and R. R. Whitney, Phys. Rev. **C38** (1988) 800;

7) B. P. Quinn, A. M. Bernstein, K. I. Blomqvist, S. A. Dytman, T. J. Pavel, R. M. Altemus, J. S. McCarthy, G. Mechtel, R. R. Whitney, T. S. Ueng, H. Arenhövel, W. Leidemann and J. M. Laget, Phys. Rev. **C37** (1988) 1609;

8) J. S. O'Connell, W. R. Dodge, J. W. Lightbody Jr., X. K. Maruyama, J. O. Adler, K. Hansen, B. Schroder, A. M. Bernstein, K. I. Blomqvist, B. H. Cottman, J. J. Comuzzi, R. A. Miskimen, B. P. Quinn, J. H. Koch and N. Ohtsuka, Phys. Rev. **C35** (1987) 1063;

9) J. W. Lightbody, Jr. and J. S. O'Connell, Computers in Physics **1** (1988) 57;

10) C. Ciofi degli Atti, Nucl. Phys. **A463** (1987) 127c;

11) C. Ciofi degli Atti, E. Pace and G. Salme, Phys. Lett. **127B** (1983) 303;

12) R. Schiavilla and V. R. Pandharipande, Phys. Rev. **C36** (1987) 2221;

13) S. Fiarman and W. E. Meyerhof, Nucl. Phys. **A206** (1973) 1;

14) E. L. Kuplennikov, S. I. Nagornyi and E. V. Inopin, Sov. J. Nucl. Phys. **41** (1985) 9.

KVAZIELASTIČNO RASPRŠENJE ELEKTRONA NA ^{4}He IZMEĐU 328 I 725 MeV

JOSEPH J. BEVELACQUA

Wisconsin Electric Power Company, 6610 Nuclear Road, Two Rivers, Wisconsin 54241, USA

Podaci za udarni presjek ^{4}He na energijama od 328, 365 i 725 MeV analizirani su koristeći model kvazi-slobodnog raspršenja Lightbodyja i O'Connella. Model omogućava poboljšan opis oblika i iznosa udarnog presjeka u odnosu na rezultate dobivene u aproksimaciji impulsa ravnih valova. To naročito vrijedi za veće gubitke energije elektrona. Iako je energija separacije oko 20 MeV u ^{4}He, najbolji rezultati dobiveni su za 6 MeV-a.