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EXPONENTIATION FOR THE WILSON LOOP IN THE LIGHT-CONE GAUGE

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Despite the fact that the Mandelstam-Leibbrandt prescription for the light-cone gauge fails to give $C_R C_G$ terms correctly to order g^4, it does give C_R^2 terms which do agree with those in the Feynman gauge. Thus, the exponentiation theorem remains satisfied.

1. Introduction

The set of algebraic non-covariant (i. e. gauge) conditions of the kind $n^\mu A_\mu = \Phi$, where $n_\mu = (n_0, \vec{n})$ is a constant four-vector and Φ is a given field in the adjoint representation of a semisimple Lie algebra, has been widely and increasingly adopted in the treatment of many aspects of quantum non-abelian gauge theories. The appealing features of this class of gauge choices are

(a) the decoupling of the Faddeev-Popov ghost sector;
(b) the freedom of Gribov's ambiguities;
(c) the possibility of a simple »parton-like« interpretation of the theory owing to the apparent decoupling (or even absence) of the unphysical degrees of freedom.

On the other hand, in addition to entailing the break-down of manifest Lorentz covariance, the use of algebraic non-covariant gauges entails several delicate mathematical problems arising from the presence of spurious gauge singularities of the kind $[n \cdot k]^{-1}$, $[n \cdot k]^{-2}$ in the free gauge-field propagator.

If such singularities are defined by means of the Cauchy principal value (CPV) prescription, several pathologies arise in perturbation theory:

(i) the positivity of the sum over the polarization vectors is lost in the axial gauge $n \cdot A = 0$;

(ii) the axial gauge does not reproduce the expected exponential behaviour of the Wilson loop in the large-T limit, either in the temporal[1] or in the space-like[2] case;

(iii) in the light-cone case ($n^2 = 0$), inconsistencies arising in loop calculations[3] are essentially due to pinches between spurious poles and Feynman poles in the evaluation of Feynman integrals.

In order to solve the problem of the light-cone gauge, Mandelstam and Leibd brandt proposed a prescription[4,5] according to which the spurious poles an- the Feynman poles are always located in the same quadrants of the complex k_0 plane. This prescription is given by

$$\frac{1}{n \cdot k} \equiv \frac{1}{n \cdot k + i\omega (n^* \cdot k)} = \frac{n^* \cdot k}{(n \cdot k)(n^* \cdot k) + i\varepsilon}, \tag{1}$$

where $n^*_\mu = (n_0, -\vec{n})$ is usually called the dual (or conjugate) vector of n_μ. The first expression is Mandelstam's proposal, the second one is Leibbrandt's.

In this paper we show in detail that the exponentiation theorem remains satisfied with the Mandelstam-Leibbrandt prescription to order g^4 for the Wilson loop with light-like sides.

2. *Diagrams*

We use the same Wilson loop and notation as in Ref. 6. The diagrams contributing to C_R^2 terms in the light-cone gauge are shown in Figs. 1, 2 and 3.

The contribution of the diagram shown in Fig. 1 is of the form

$$W_1 = -16 g^4 \operatorname{Tr}(t_a t_b t_a t_b) \int d^n p\, d^n k \, \times$$

$$\times \frac{n^*_\mu}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k} \right) n^*_\nu \frac{n^*_\alpha}{p^2} \left(\delta_{\alpha\beta} - \frac{p_\alpha n_\beta + p_\beta n_\alpha}{n \cdot p} \right) n^*_\beta \, \times$$

$$\times \int_0^L dt_1 \int_0^L dt_2\, \Theta(t_2 - t_1) \int_{L-T}^{-T} dt_3 \int_{L-T}^{-T} dt_4 \Theta(t_3 - t_4) \, \times$$

$$\times \exp i [k_0 (t_1 - t_3) - k_3 (x_1 - x_3)] \cdot \exp i [p_0 (t_2 - t_4) - p_3 (x_2 - x_4)]. \tag{2}$$

Here Θ is a step function.

We choose the two light-like vectors to be

$$n = (1, 0, 0, -1),$$

$$n^* = (1, 0, 0, 1), \qquad (3)$$

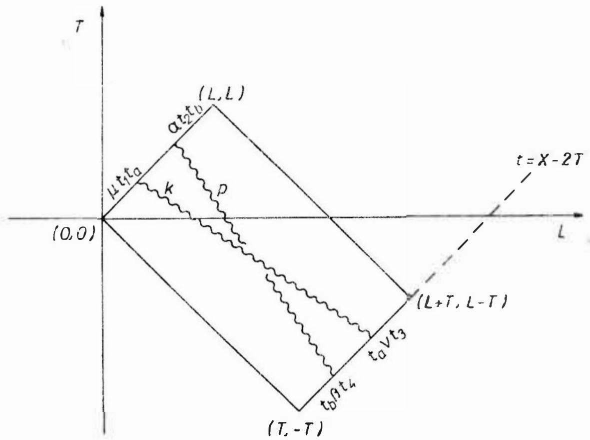


Fig. 1. Graphs contributing to the vacuum expectation value of the Wilson-loop operator at order g^4. The sides of the loop are in the directions of the two null vectors n, n^*. Two gluons are exchanged. We follow the graphs only with the groups factor C_R^2.

and use Mandelstam's variables

$$n \cdot p = p_+,$$

$$n^* \cdot p = p_- . \qquad (4)$$

Thus we obtain

$$W_1 = -16 g^4 \operatorname{Tr}(t_a t_b t_a t_b) \int \mathrm{d}^n p \mathrm{d}^n k \frac{\mathrm{e}^{\mathrm{i}(k+p)_+ T}}{k^2 k_+ p^2 p_+} \times$$

$$\times \left\{ \frac{1}{(k+p)_-} (\mathrm{e}^{\mathrm{i}(k+p)_- L} - 1) - \frac{1}{p_-} (\mathrm{e}^{\mathrm{i}p_- L} - 1) \right\} \times$$

$$\times \left\{ \frac{1}{(k+p)_-} (\mathrm{e}^{-\mathrm{i}(k+p)_- L} - 1) - \frac{1}{k_-} (\mathrm{e}^{-\mathrm{i}k_- L} - 1) \right\}. \qquad (5)$$

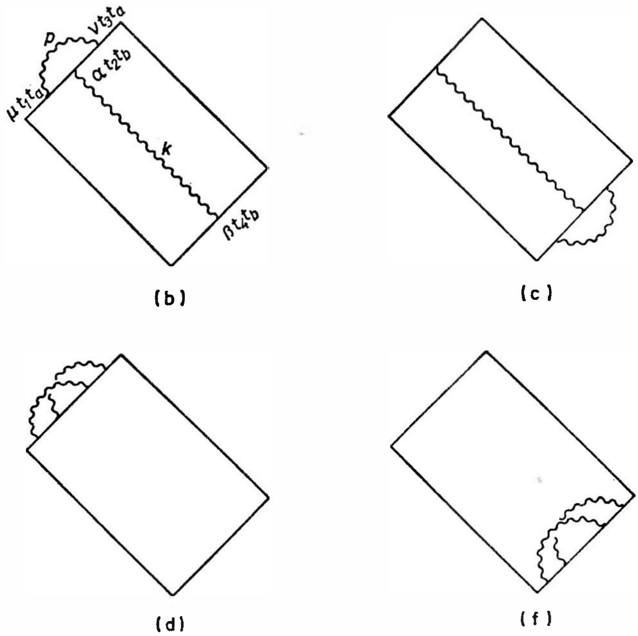


Fig. 2. Same as Fig. 1.

The poles in k_+ and p_+ are to be understood in the sense of Mandelstam's prescription (1). We can soften these poles by adding the diagrams of Fig. 2:

$$W_b + W_c = -16\,\mathrm{Tr}\,(t_a t_b t_a t_b)\, g^4 \int \mathrm{d}^n p\, \mathrm{d}^n k \,\frac{2\cos k_+ T}{k^2 k_+ p^2 p_+} \cdot \frac{1 - \mathrm{e}^{-\mathrm{i}k_- L}}{k_-} \times$$

$$\times \left\{ \frac{1}{(k-p)_-} (\mathrm{e}^{\mathrm{i}k_- L} - 1) - \frac{1}{(p+k)_-} (\mathrm{e}^{\mathrm{i}(p+k)_- L} - 1) + \frac{1}{(p-k)_-} (\mathrm{e}^{\mathrm{i}p_- L} - 1) \right\}, \tag{6}$$

$$W_d + W_f = -16\,\mathrm{Tr}\,(t_a t_b t_a t_b)\, g^4 \int \mathrm{d}^n p\, \mathrm{d}^n k \,\frac{2}{k^2 k_+ p^2 p_+} \times$$

$$\times \left\{ \frac{1}{(p+k)_-^2} (\mathrm{e}^{\mathrm{i}(p+k)_- L} - 1) - \frac{1}{p_- (p-k)_-} (\mathrm{e}^{\mathrm{i}p_- L} - 1) + \right.$$

$$\left. + \frac{1}{k_- (p-k)_-} (\mathrm{e}^{\mathrm{i}k_- L} - 1) \right\}. \tag{7}$$

In (7) we symmetrize p_μ, $k_\mu \leftrightarrow -p_\mu$, $-k_\mu$ and denote

$$C = -16 g^4 \operatorname{Tr} (t_a t_b t_a t_b). \tag{8}$$

In this way we obtain

$$W_d + W_f = C \int \mathrm{d}^n p \mathrm{d}^n k \frac{2}{k^2 k_+ p^2 p_+} \left\{ \frac{1}{(p+k)_-^2} (\cos (p+k)_- L - 1) - \right.$$

$$\left. - \frac{1}{p_- (p-k)_-} (\cos p_- L - 1) + \frac{1}{k_- (p-k)_-} (\cos k_- L - 1) \right\}. \tag{9}$$

After changing $k_\mu \to -k_\mu$ in the term with $1/(p+k)_-$, we can express Eq. (6) in the form

$$W_b + W_c = C \int \mathrm{d}^n p \mathrm{d}^n k \frac{\cos k_+ T - 1}{k^2 k_+ p^2 p_+} \cdot \frac{1}{k_-} \cdot \frac{1}{(k-p)_-} \times$$

$$\times 4 \{\cos k_- L - 1 + \cos (p-k)_- L - \cos p_- L\} +$$

$$+ 4C \int \mathrm{d}^n p \mathrm{d}^n k \frac{1}{\cancel{k^2} k_+ p^2 p_+} \cdot \frac{1}{k_- (k-p)_-} \times$$

$$\times \{\cos k_- L - 1 + \cos (p-k)_- L - \cos p_- L\}. \tag{10}$$

We denote the second expression in (10) by R and add it to $W_d + W_f$. Thus we obtain

$$W_d + W_f + R = C \int \mathrm{d}^n p \mathrm{d}^n k \frac{2}{k^2 k_+ p^2 p_+} \left\{ \frac{1}{(p+k)_-^2} (\cos (p+k)_- L - 1) - \right.$$

$$- \frac{1}{p_- (p-k)_-} (\cos p_- L - 1) - \frac{1}{k_- (p-k)_-} (\cos k_- L - 1) +$$

$$\left. + \frac{2}{k_- (k-p)_-} (\cos (p-k)_- L - \cos p_- L) \right\}. \tag{11}$$

We obtain all the denominators in the form $1/(p+k)_-$ by changing $k_\mu \to -k_\mu$ where appropriate. After making the exchange $p_\mu \leftrightarrow k_\mu$ and some algebraic manipulations, we obtain

$$W_d + W_f + R = C \int \mathrm{d}^n p \mathrm{d}^n k \frac{2}{k^2 k_+ p^2 p_+} \left\{ \frac{1}{(p+k)_-^2} (\cos (p+k)_- L - 1) - \right.$$

$$- \frac{1}{p_- (k+p)_-} (\cos p_- L - 1 + \cos (p+k)_- L - \cos k_- L) +$$

$$+ \frac{1}{k_- (p+k)_-} (\cos k_- L - 1) - \frac{1}{p_- (k+p)_-} (\cos (p+k)_- L - \cos k_- L) \Big\}. \quad (12)$$

The first two lines are ready to be added to Eq. (5), whereas the last line, which we denote by $\mathscr{L}$, still requires some manipulations:

$$\mathscr{L} = C \int \mathrm{d}^n p \mathrm{d}^n k \frac{2}{k^2 k_+ p^2 p_+} \Big\{ \frac{1}{k_- (p+k)_-} (\cos k_- L - 1) -$$

$$- \frac{1}{p_- (k+p)_-} (\cos (p+k)_- L - \cos k_- L) \Big\}. \quad (13)$$

Symmetrizing Eq. (13) under $p_\mu \leftrightarrow k_\mu$ and writing it back in terms of the exponentials (while making the change $p_\mu \to -p_\mu$, $k_\mu \to -k_\mu$ where necessary), we obtain the form

$$\mathscr{L} = -C \int \mathrm{d}^n p \mathrm{d}^n k \frac{1}{k^2 k_+ p^2 p_+} \cdot \frac{1}{k_- p_-} (\mathrm{e}^{\mathrm{i} p_- L} - 1)(\mathrm{e}^{-\mathrm{i} k_- L} - 1). \quad (14)$$

In summary, we can write

$$W_1 + W_b + W_c + W_d + W_f = C \int \mathrm{d}^n p \mathrm{d}^n k \frac{\mathrm{e}^{\mathrm{i}(p+k)_+ T} - 1}{k^2 k_+ p^2 p_+} \times$$

$$\times \Big\{ \frac{2}{(k+p)_-^2} (1 - \cos (k+p)_- L - \frac{2}{p_- (k+p)_-} (1 - \cos (k +$$

$$+ p)_- L + \cos k_- L - \cos p_- L) + \frac{1}{p_- k_-} (\mathrm{e}^{\mathrm{i} p_- L} - 1)(\mathrm{e}^{-\mathrm{i} k_- L} - 1) \Big\} +$$

$$+ C \int \mathrm{d}^n p \mathrm{d}^n k \frac{\cos k_+ T - 1}{k^2 k_+ p^2 p_+} \cdot \frac{1}{k_- (k-p)_-} \times$$

$$\times 4 \{ \cos k_- L - 1 + \cos (p-k)_- L - \cos p_- L \}.$$

The second part in Eq. (15) vanishes because of the identity (15)

$$\int \mathrm{d} p_+ \frac{1}{(p^2 + \mathrm{i}\varepsilon)(p_+ + \mathrm{i}\omega p_-)} \equiv 0. \quad (16)$$

The diagram in Fig. 3 reads

$$W_2 = -g^4 \,\mathrm{Tr}\,(t_b t_a t_a t_b) \int \mathrm{d}^n p \mathrm{d}^n k \frac{n^*_\mu}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k} \right) n^*_\nu \times$$

$$\times \frac{n^*_\alpha}{p^2} \left(\delta_{\alpha\beta} - \frac{p_\alpha n_\beta + p_\beta n_\alpha}{n \cdot p} \right) n^*_\beta \times$$

$$\times \int_0^L \mathrm{d}t_1 \int_0^L \mathrm{d}t_2 \, \Theta(t_1 - t_2) \int_{L-T}^{-T} \mathrm{d}t_3 \int_{L-T}^{-T} \mathrm{d}t_4 \Theta(t_3 - t_4) \times$$

$$\times \exp \mathrm{i} \, [k_0 (t_1 - t_3) - k_3 (x_1 - x_3)] \cdot \exp \mathrm{i} \, [p_0 (t_2 - t_4) - p_3 (x_2 - x_4)]. \quad (17)$$

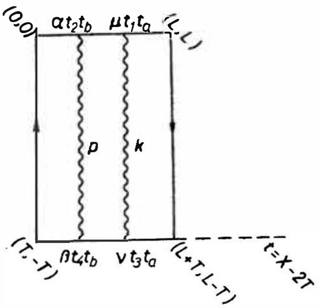


Fig. 3. Same as Fig. 1.

After integrating over $\mathrm{d}t_1$, $\mathrm{d}t_2$, $\mathrm{d}t_3$ and $\mathrm{d}t_4$ and symmetrizing $k_\mu \to -k_\mu$, $p_\mu \to -p_\mu$, we obtain

$$W_2 = -g^4 \,\mathrm{Tr}\,(t_a t_a t_b t_b) \cdot 32 \int \mathrm{d}^n p \mathrm{d}^n k \frac{\cos (p+k)_+ T}{k^2 k_+ p^2 p_+} \times$$

$$\times \left\{ - \frac{1}{(p+k)^2_-} [1 - \cos (p+k)_- L] + \frac{2}{p_- (p+k)_-} (1 - \cos p_- L) \right\}. \quad (18)$$

3. *Evaluation*

We start by evaluating the diagram W_2. After making a shift of integration variables:

$$p + k = p'$$

$$k = k', \qquad p = p' - k' \quad (19)$$

and using partial fractions

$$\frac{1}{n\cdot k\, n\cdot(p-k)} = \frac{1}{n\cdot p}\left[\frac{1}{n\cdot k} + \frac{1}{n\cdot(p-k)}\right], \tag{20}$$

we obtain

$$W_2 = -32g^4\,\mathrm{Tr}\,(t_a t_a t_b t_b)\int \mathrm{d}^n p\mathrm{d}^n k \frac{\cos p_+ T}{k^2 n\cdot k\,(p-k)^2\, n\cdot p}\times$$

$$\times\left\{-\frac{1}{p_-^2}(1-\cos p_- L) + \frac{2}{p_-(p-k)_-}[1-\cos(p-k)_- L]\right\} -$$

$$-32g^4\,\mathrm{Tr}\,(t_a t_a t_b t_b)\int \mathrm{d}^n p\mathrm{d}^n k \frac{\cos p_+ T}{k^2 (p-k)^2\, n\cdot p\, n\cdot(p-k)}\times$$

$$\times\left\{-\frac{1}{p_-^2}(1-\cos p_- L) + \frac{2}{p_-(p-k)_-}[1-\cos(p-k)_- L]\right\}. \tag{21}$$

In the second expression of Eq. (21) we make a change of variables:

$$p = p', \qquad p - k = k',$$

$$k = p' - k'. \tag{22}$$

In this way we have to evaluate

$$W_2 = -64g^4\,\mathrm{Tr}\,(t_a t_a t_b t_b)\int \mathrm{d}^n p\mathrm{d}^n k \frac{\cos p_+ T}{k^2 (p-k)^2\, n\cdot p\, n\cdot k}\times$$

$$\times\left\{-\frac{1}{p_-^2}(1-\cos p_- L) + \frac{1}{p_-(p-k)_-}[1-\cos(p-k)_- L] +\right.$$

$$\left.+\frac{1}{p_- k_-}(1-\cos k_- L)\right\}. \tag{23}$$

We use the integrals listed in Ref. 6:

$$\int \mathrm{d}^n p\mathrm{d}^n k = \frac{1}{4}\int \mathrm{d}p_+\mathrm{d}p_-\mathrm{d}^{2-\varepsilon} P \int \mathrm{d}k_+\mathrm{d}k_-\mathrm{d}^{2-\varepsilon} K, \tag{24}$$

$$B = \int \mathrm{d}^n k \frac{1}{k^2 (p-k)^2\, n\cdot k} = -\mathrm{i}\pi^{2-\varepsilon/2} p_- \Gamma\left(1+\frac{\varepsilon}{2}\right)\times$$

$$\times\int_0^1 \mathrm{d}x\,(1-x)^{-\varepsilon/2}\int_0^1 \mathrm{d}y\,(1-y+xy)^{-1-\varepsilon/2}(P^2 y - p_+ p_- - \mathrm{i}\eta)^{-1-\varepsilon/2}, \tag{25}$$

where $k_- = p_- x$,

$$\int \mathrm{d}^{2-\varepsilon} P \, (P^2 y - p_+ p_- - \mathrm{i}\eta)^{-1-\varepsilon/2} = = \pi^{1-\varepsilon/2} \frac{\Gamma(\varepsilon)}{\Gamma\left(1+\frac{\varepsilon}{2}\right)} \, y^{\varepsilon/2-1} \, (-p_+ p_- - \mathrm{i}\eta)^{-\varepsilon}. \tag{26}$$

Therefore,

$$W_2 = -16 g^4 \operatorname{Tr}(t_a t_a t_b t_b)(-\mathrm{i}) \, \pi^{3-\varepsilon} \, \Gamma(\varepsilon) \int \mathrm{d}p_+ \mathrm{d}p_- \times \times \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \, (-p_+ p_- - \mathrm{i}\eta)^{-\varepsilon} y^{\varepsilon/2-1} (1-x)^{-\varepsilon/2} (1 - y + xy)^{-1-\varepsilon/2} \times \times \frac{\cos p_+ T}{p_+ + \mathrm{i}\omega p_-} \left\{ -\frac{1}{p_-}(1 - \cos p_- L) + \frac{1}{p_- x}(1 - \cos p_- x L) + + \frac{1}{p_- (1-x)} [1 - \cos p_- (1-x) L] \right\}. \tag{27}$$

First, let us perform the integration over $\mathrm{d}p_+ \mathrm{d}p_-$:

$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}p_+ \mathrm{d}p_- \, \frac{\cos p_+ T}{p_+ + \mathrm{i}\omega p_-} (-p_+ p_- - \mathrm{i}\eta)^{-\varepsilon} \frac{1}{p_-} \times \times \left\{ -(1 - \cos p_- L) + \frac{1}{x}(1 - \cos p_- x L) + \frac{1}{1-x} [1 - \cos p_- L (1-x)] \right\} = = \int_{-\infty}^{\infty} \mathrm{d}p_+ \left\{ \int_0^{\infty} \mathrm{d}p_- \, \frac{\cos p_+ T}{p_+ + \mathrm{i}\omega p_-} (-p_+ p_- - \mathrm{i}\eta)^{-\varepsilon} \frac{1}{p_-} \times \right. \times \left[-(1 - \cos p_- L) + \frac{1}{x}(1 - \cos p_- x L) + \frac{1}{1-x}(1 - \cos p_- L (1-x)) \right] - - \int_0^{\infty} \mathrm{d}p_- \, \frac{\cos p_+ T}{p_+ - \mathrm{i}\omega p_-} (p_+ p_- - \mathrm{i}\eta)^{-\varepsilon} \frac{1}{p_-} \times \left. \times \left[-(1 - \cos p_- L) + \frac{1}{x}(1 - \cos p_- x L) + \frac{1}{1-x}(1 - \cos p_- L (1-x)) \right] \right\}. \tag{28}$$

In addition, changing the integration over $\mathrm{d}p_+$ into the positive range of integration, we obtain

$$I = 2 \int_0^{\infty} \int_0^{\infty} \mathrm{d}p_+ \mathrm{d}p_- \cos p_+ T \left[\frac{e^{\mathrm{i}\pi\varepsilon}}{p_+ + \mathrm{i}\omega p_-} - \frac{1}{p_+ - \mathrm{i}\omega p_-} \right] \times \times p_+^{-\varepsilon} p_-^{-1-\varepsilon} \left\{ -(1 - \cos p_- L) + \frac{1}{x}(1 - \cos p_- x L) + + \frac{1}{1-x} [1 - \cos p_- L (1-x)] \right\}. \tag{29}$$

The evaluation of the integral over $\mathrm{d}p_+$ is shown in detail in Appendix B. Using it, we have

$$I = 2\,(\mathrm{e}^{\mathrm{i}\varepsilon\pi} - 1)\,(TL)^{\varepsilon} \cos^2 \frac{\varepsilon\pi}{2}\,\Gamma^2(-\varepsilon)\,[1 - x^{\varepsilon-1} - (1-x)^{\varepsilon-1}]. \tag{30}$$

Therefore,

$$W_2 = -16 g^4\, \mathrm{Tr}\,(t_a t_a t_b t_b)\,(-\mathrm{i})\,\pi^{3-\varepsilon}\,\Gamma(\varepsilon) \cdot 2\,(\mathrm{e}^{\mathrm{i}\pi\varepsilon} - 1)\,(TL)^{\varepsilon} \cos^2 \frac{\varepsilon\pi}{2}\,\Gamma^2(-\varepsilon) \times$$
$$\times \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, y^{-1+\varepsilon/2}\,(1-x)^{-\varepsilon/2}\,(1-y+xy)^{-1-\varepsilon/2}\,[1 - x^{\varepsilon-1} - (1-x)^{\varepsilon-1}]. \tag{31}$$

Second, there are two more integrations to perform:

$$\int_0^1 \mathrm{d}y\, \frac{y^{\varepsilon/2-1}}{[1-(1-x)\,y]^{1+\varepsilon/2}} = \frac{2}{\varepsilon}\, x^{-\varepsilon/2}, \tag{32}$$

$$\int_0^1 \mathrm{d}x\, x^{-\varepsilon/2}\,(1-x)^{-\varepsilon/2}\,[1 - x^{\varepsilon-1} - (1-x)^{\varepsilon-1}] = \frac{\Gamma(1-\varepsilon/2)\,\Gamma(1-\varepsilon/2)}{\Gamma(2-\varepsilon)} -$$
$$- 2\,\frac{\Gamma(\varepsilon/2)\,\Gamma(1-\varepsilon/2)}{\Gamma(1)}. \tag{33}$$

The final result is

$$W_2 = 64 g^4\, \mathrm{Tr}\,(t_a t_a t_b t_b)\,\pi^{3-\varepsilon}\,\frac{1}{\varepsilon}\,\Gamma(\varepsilon) \cdot \mathrm{i}\,(\mathrm{e}^{\mathrm{i}\pi\varepsilon} - 1)\,\Gamma^2(-\varepsilon)\,\cos^2 \frac{\varepsilon\pi}{2} \times$$
$$\times \left[\frac{\Gamma(1-\varepsilon/2)\,\Gamma(1-\varepsilon/2)}{\Gamma(2-\varepsilon)} - 2\Gamma\left(\frac{\varepsilon}{2}\right)\Gamma\left(1-\frac{\varepsilon}{2}\right)\right] \cdot (TL)^{\varepsilon}. \tag{34}$$

The leading divergence ε^{-4} becomes

$$W_2 = 2^8 g^4 C_R^2 \pi^{4-\varepsilon}\,(TL)^{\varepsilon} \cdot \frac{1}{\varepsilon^4}. \tag{35}$$

This expression is to be multiplied by the factor $(2\pi)^{-8}$ coming from the two-loop phase-space.

For the evaluation of Eq. (15), we refer to Ref. 6. The leading ε^{-4} contribution is

$$W_1 + W_b + W_c + W_d + W_f = 2^8 \left(C_R^2 - \frac{1}{2}\, C_R C_G\right)(g\pi)^4\,(TL)^{\varepsilon}\,\frac{1}{\varepsilon^4}\,(2\pi)^{-8}. \tag{36}$$

The sum of Eqs. (35) and (36) gives the coefficient of the term C_R^2 which is identical to the contributions from the two-gluon graphs in the Feynman gauge and thus satisfies the exponentiation theorem.

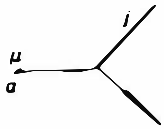
4. *Conclusion*

As a gauge-invariant object, the Wilson loop represents an ideal test laboratory for different gauges and regularization prescriptions. Usually, lowest-order calculations to g^2 in perturbation theory are in agreement with the Feynman gauge. However, higher-order calculations to g^4 in the coupling constant give surprising results. We have found that the Mandelstam-Leibbrandt prescription for the light-cone gauge is not in agreement with the Feynman gauge for $C_R C_G$ terms[6]. However, C_R^2 terms do satisfy the exponentiation theorem.

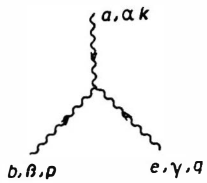
Appendix A

The Feynman rules:

$$\mathrm{i}g\gamma_\mu (t_a)_{ij},$$



$$-\mathrm{i}gf_{abc}\,[\delta_{\beta\gamma}(q-p)_\alpha + \delta_{\gamma\alpha}(k-q)_\beta + \delta_{\alpha\beta}(p-k)_\gamma],$$



$$\frac{\delta_{ab}}{k^2 + \mathrm{i}\eta}\left\{\delta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k}\right\}.$$

Additional factors:

$(2\pi)^4\,\mathrm{i}$ for each vertex,

$\frac{1}{(2\pi)^4\,\mathrm{i}}$ for each propagator.

Appendix B:

Evaluation of the integrals:

$$I = \int_0^\infty \mathrm{d}p_+ p_+^{-\varepsilon} \cos p_+ T \left[\frac{e^{i\pi\varepsilon}}{p_+ + i\omega p_-} - \frac{1}{p_+ - i\omega p_-}\right], \tag{B1}$$

$$I = \int_0^\infty \mathrm{d}p_+ p_+^{-1-\varepsilon} [\cos p_+ T - 1] (e^{i\pi\varepsilon} - 1) +$$

$$+ \int_0^\infty \mathrm{d}p_+ p_+^{-\varepsilon} \left[\frac{e^{i\pi\varepsilon}}{p_+ + i\omega p_-} - \frac{1}{p_+ - i\omega p_-}\right]. \tag{B2}$$

We know the first integral in (B2); its contribution is

$$(e^{i\pi\varepsilon} - 1)\, T^\varepsilon\, \Gamma(\varepsilon) \cos \frac{\varepsilon\pi}{2}.$$

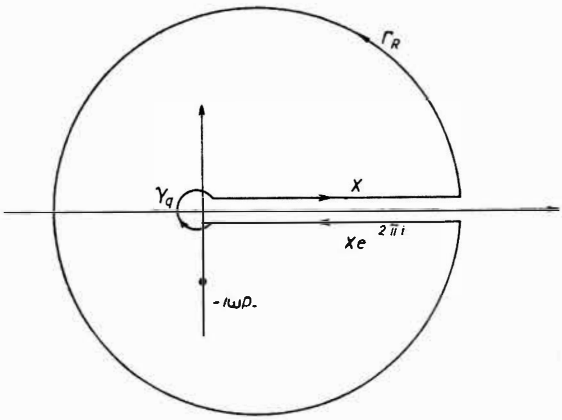


Fig. 4. The path in the complex p_+ plane.

The second integral is performed over the path shown in Fig. 4:

$$\int_C \frac{\mathrm{d}z}{z^\varepsilon (z + i\omega p_-)} = \int_0^\infty \frac{\mathrm{d}x}{x^\varepsilon (x + i\omega p_-)} + \int_{\Gamma_R} + \int_{\gamma_\varrho} - \int_0^\infty \frac{\mathrm{d}x}{x^\varepsilon e^{2\pi i\varepsilon} (x + i\omega p_-)} =$$

$$= 2\pi i \operatorname{res} \left(\frac{1}{z^\varepsilon (z + i\omega p_-)}\right) z = -i\omega p_-, \tag{B3}$$

$$\left| \int_{\Gamma_R} \frac{dz}{z^\varepsilon (z + i\omega p_-)} \right| \leq \int_0^{2\pi} \frac{|Rie^{i\varphi}| \, d\varphi}{R^\varepsilon |e^{i\varepsilon\varphi}| \, |Re^{i\varphi} + i\omega p_-|} \leq \int_0^{2\pi} R^{-\varepsilon} d\varphi \underset{\substack{R \to \infty \\ \varepsilon > 0}}{\to} 0, \tag{B4}$$

$$\left| \int_{\gamma_\varrho} \frac{dz}{z^\varepsilon (z + i\omega p_-)} \right| = \left| \int_0^{2\pi} \frac{i\varrho \, e^{i\varphi} \, d\varphi}{\varrho^\varepsilon + e^{i\varepsilon\varphi}(\varrho e^{i\varphi} + i\omega p_-)} \right| \leq \int_0^{2\pi} \frac{\varrho^{1-\varepsilon} \, d\varphi}{(\varrho^2 + \omega^2 p_-^2)^{1/2}} \underset{\substack{\varrho \to 0 \\ \omega p_- \neq 0}}{\to} 0, \tag{B5}$$

$$\operatorname{res} f(z) \, |_{z = -i\omega p_-} = (-i\omega p_-)^{-\varepsilon}.$$

For $z^{-\varepsilon}$, we choose $e^{-\varepsilon(\ln|z| + i \arg z)}$, $0 < \arg z < 2\pi$. Thus

$$\operatorname{res} f(z) = \exp[-\varepsilon(\ln|\omega p_-| + 3\pi i/2)] = (\omega p_-)^{-\varepsilon} e^{-3\pi i\varepsilon/2}. \tag{B6}$$

We obtain

$$\int_0^\infty \frac{dx}{x^\varepsilon (x + i\omega p_-)} = \frac{2\pi i \, e^{-3\pi i\varepsilon/2}}{1 - e^{-2\pi i\varepsilon}} (\omega p_-)^{-\varepsilon}. \tag{B7}$$

In the same way,

$$\int_0^\infty \frac{dx}{x^\varepsilon (x - i\omega p_-)} = \frac{2\pi i \, e^{-i\pi\varepsilon/2}}{1 - e^{-2\pi i\varepsilon}} (\omega p_-)^{-\varepsilon}. \tag{B8}$$

Therefore,

$$\int_0^\infty dp_+ p_+^{-\varepsilon} \left\{ \frac{e^{i\pi\varepsilon}}{p_+ + i\omega p_-} - \frac{1}{p_+ - i\omega p_-} \right\} =$$

$$= \frac{2\pi i}{1 - e^{-2\pi i\varepsilon}} (\omega p_-)^{-\varepsilon} \cdot (e^{-i\pi\varepsilon/2} - e^{-i\pi\varepsilon/2}) \equiv 0. \tag{B9}$$

However, if we had had

$$\int_0^\infty dp_+ p_+^{-\varepsilon} \left\{ \frac{1}{p_+ + i\omega p_-} - \frac{1}{p_+ - i\omega p_-} \right\} =$$

$$= \frac{2\pi i}{1 - e^{-2\pi i\varepsilon}} (\omega p_-)^{-\varepsilon} (e^{-3\pi i\varepsilon/2} - e^{-i\pi\varepsilon/2}) =$$

$$= \frac{\pi}{\sin \varepsilon\pi} (\omega p_-)^{-\varepsilon} (e^{-i\pi\varepsilon/2} - e^{i\pi\varepsilon/2}) \tag{B10}$$

and if we had let $\varepsilon \to 0$ before ω, we should have obtained $(\omega p_-)^{-\varepsilon} \to 1$ and the well-known result

$$\int_0^\infty dp_+ \left\{ \frac{1}{p_+ + i\omega p_-} - \frac{1}{p_+ - i\omega p_-} \right\} = -i\pi. \tag{B11}$$

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WILSONOVA PETLJA I EKSPONENTNI TEOREM U UVJETU SVJETLOSNOG KONUSA

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Premda Mandelstam-Leibbrandtova preskripcija za uvjet svjetlosnog konusa ne daje $C_R C_G$ članove korektne do reda g^4, članovi proporcionalni C_R^2 slažu se s onima u Feynmanovom uvjetu. Tako je eksponentni teorem zadovoljen.