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MIXING ANGLES IN S_{11} AND D_{13} BARYONS

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The nature of the short-range interactions between quarks in baryons is of fundamental importance to our understanding of the nature of nuclear matter in the soft-QCD regime. Models of the structure of the negative-parity excited baryons in the S_{11} and D_{13} partial waves in Nπ scattering differ in their predictions for the amount by which states of different quark spin but the same total spin are mixed. It may be possible to distinguish among these models by an accurate determination of the mixing angles in these states. A study of the dependence on these mixing angles of their decay branching fractions to Nπ, Nη, and ΛK, using a constituent quark model and the $^3P_0$ decay model, allows conclusions to be drawn about the values of the mixing angles.

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1. Introduction

Models of the baryon spectrum use different effective short-range interactions between the quarks, such as one-gluon exchange (OGE) [1, 2], one-boson exchange (OBE) [3], and instanton-induced interactions [4]. Although these models have differing consequences for baryon masses, a comparison with the spectrum extracted from analyses of data alone is not enough to distinguish between them. A study of the wave functions resulting from these different interactions can be more sensitive.

An interesting test case is the mixing caused by short-range interactions in the near-degenerate nucleon ($I = 1/2$) low-lying excited states with $J^P = 1/2^-$ (S_{11} in $N\pi$) and $3/2^-$ (D_{13}). There are two orbital spaces in the three-quark system, and with equal quark masses, these have degenerate $L = 1$ orbital excitations. There are also two possible total quark spins, leading to two pairs of degenerate states. This degeneracy is broken by contact spin-spin interactions present in all of these models, which are scalar operators in the total orbital and quark-spin spaces. In addition, the tensor and spin-orbit interactions which are necessarily associated with these contact interactions cause large mixings of these states. Chizma and Karl [5] show that OGE and OBE effective interactions, adjusted to reproduce the contact splittings of these states, have very different tensor interactions, which cause different mixing angles in these low-lying S_{11} and D_{13} states.

In a quark model, the two light $J^P = 1/2^-$ nucleon resonances $S_{11}(1535)$ and $S_{11}(1650)$ seen in analyses can be described as mixtures of $L^\pi = 1^-$ orbital excitations of the nucleon with total quark spin $S = 1/2$, or $S = 3/2$, combined to total angular momentum and parity $J^P = 1/2^-$, which we can label $|N^2P\,1/2^-\rangle$ and $|N^4P\,1/2^-\rangle$, and similarly for the $J^P = 3/2^-$ states $D_{13}(1520)$ and $D_{13}(1700)$. Typical models of the interactions between light quarks, such as one-gluon exchange or one-boson exchange, have interactions which are simultaneous orbital and spin scalars, and so cannot mix the sub-states with different total quark spin. Examples are the confining interaction and the relativistic kinetic energy, and the contact piece of the short-range interactions, proportional to $\sum_{ij} f(r_{ij})\mathbf{S}_i \cdot \mathbf{S}_j$, and responsible for large splittings in the baryon spectrum such as $\Delta - N$, $\Sigma - \Lambda$, etc.

There are also associated interactions that can mix the sub-states with different quark spins but the same J^P, and since these are essentially isolated two-state systems, this mixing can be represented by two angles, θ_S and θ_D. Examples include the tensor part of the short-range spin-spin interactions, as well as spin-orbit interactions associated with the short-range interactions and those that must arise from Thomas-precession of the quark spins in the confining potential. The amount by which the quark-spin sub-states are mixed by these interactions depends on the model used to describe the short-distance interactions between the quarks. If the mixing angles can be determined from decay data, it may allow us to distinguish between these models. These angles have been determined using a model based on broken $SU(6)_W \otimes O(3)$ symmetry for baryon decay strong vertices [6], which was extended to include pion photo-production in Ref. [7], and in a recent analysis of eta photo-production data [8]. Here an independent determination of these angles based on the relativized constituent quark model of baryon decays [9] is described.

2. *Mixing angles*

Within a model it is possible to turn off the interactions which mix states of definite total orbital and total quark spin angular momentum. In the relativized model [2], these are the tensor and spin-orbit interactions associated with short range interactions between the quarks arising from exchange of a gluon, and the spin-orbit interactions arising from precession of the quark spins in the confining potential. Although expanded in a large harmonic-oscillator basis, with these interactions turned off the unmixed model eigenstates associated with the light S$_{11}$ and D$_{13}$ states have $L = 1$, and $S = 1/2$ or $3/2$. The mixed states

$$\begin{aligned} [S_{11}(\theta_S)]_1 &= \cos\theta_S\,|N^2P\,1/2^-\rangle - \sin\theta_S\,|N^4P\,1/2^-\rangle, \\ [S_{11}(\theta_S)]_2 &= \sin\theta_S\,|N^2P\,1/2^-\rangle + \cos\theta_S\,|N^4P\,1/2^-\rangle, \end{aligned} \tag{1}$$

and

$$\begin{aligned} [D_{13}(\theta_D)]_1 &= \cos\theta_D\,|N^2P\,3/2^-\rangle - \sin\theta_D\,|N^4P\,3/2^-\rangle, \\ [D_{13}(\theta_D)]_2 &= \sin\theta_D\,|N^2P\,3/2^-\rangle + \cos\theta_D\,|N^4P\,3/2^-\rangle, \end{aligned} \tag{2}$$

will correspond to the wave functions of Ref. [2] for particular values of θ_S and θ_D. Here a ^{3}P$_0$ pair creation model [9] is used to calculate the amplitudes for strong decays into Nπ, Nη and ΛK final states of the states in Eqs. (1) and (2) for *arbitrary* values of θ_S and θ_D. In this model hadrons decay by the creation of a quark pair with the quantum numbers of the vacuum. The emitted mesons are allowed to have structure, the initial baryon wave functions are given by Eqs. (1) and (2), and the final baryon wave functions are determined by the structure model of Ref. [2].

If this model of the structure and decays of the unmixed states is capable of describing the decay amplitudes of the physical states, then there should be unique angles θ_S and θ_D, within errors, where the calculated amplitudes agree with those extracted from analyses of data. Note that for each state and for each decay channel, this comparison allows an independent determination of the mixing angle. In what follows there are, in principle, five such determinations of θ_S, from the decays of each of the two states in Eq. (1) to πN and ηN, and from the decay of the upper state to KΛ, and similarly for θ_D. It is important to note that the angles extracted in this way do not depend on any particular model of the effects which may cause the mixing, but do depend on the model of the structure of the unmixed states and of the decays.

3. *Comparison to amplitudes extracted from data*

Given the importance of consistency between the mixing angles from the decay channels Nπ, and Nη, the calculated amplitudes should be compared to a set of amplitudes extracted from a single analysis of data, wherever possible. This is chosen here to be the multi-channel, unitary analysis Ref. [10] (Pitt-ANL). The decay amplitudes to ΛK shown in Table 1 are calculated from the Particle Data Group [11] estimates of $(\Gamma_{\mathrm{N}\pi}\Gamma_{\Lambda\mathrm{K}})^{1/2}/\Gamma$ and the Pitt-ANL values for the total width Γ and Nπ branching fraction, and are significantly less certain. The Pitt-ANL analysis extracts the mass, width, and branching fractions to the various open final state channels of each resonance. In addition, the relative signs of the amplitudes to go from a reference channel (usually Nπ) to a given decay channel through

TABLE 1. Central value (uncertainty) of widths, branching fractions, and partial decay amplitudes to $N\pi$ *and* $N\eta$ *(Ref. [10]) and* ΛK *(Refs. [10] and [11]) extracted from data for light* S_{11} *and* D_{13} *baryons.*

State	Γ (MeV)	$Br(N\pi)$ (%)	$\sqrt{Br(N\pi)\Gamma}$ (MeV$^{1/2}$)	$Br(N\eta)$ (%)	$\sqrt{Br(N\eta)\Gamma}$ (MeV$^{1/2}$)	$\sqrt{Br(\Lambda K)\Gamma}$ (MeV$^{1/2}$)
$S_{11}(1535)$	127(19)	35(4)	6.7(0.6)	55(5)	8.4(0.7)	
$S_{11}(1650)$	225(40)	73(2)	12.8(1.2)	-2(1)	-2.1(0.6)	-3.9(1.2)
$D_{13}(1520)$	118(4)	61(2)	8.5(0.2)	0(1)	0.0(1.1)	
$D_{13}(1700)$	178(133)	4(1)	2.7(1.1)	7(1)	3.5(1.3)	-0.7(3.3)

each resonance are determined, and are shown in Table 1 by giving a sign to the branching fraction.

These values are shown as shaded horizontal bands, along with the calculated model decay amplitudes plotted versus the mixing angles θ_S or θ_D, in Figs. 1, 2 and 3. The calculated amplitudes shown are the square roots of the partial widths for decay of a given resonance into the final states $N\pi$, $N\eta$, and ΛK, multiplied by the calculated sign of the $N\pi$ amplitude for this state at the mixing angle θ^p preferred by the data [12]. Note that this implies an $N\pi$ amplitude which is always positive at the extracted mixing angle, necessary since the sign of the $N\pi$ amplitude is

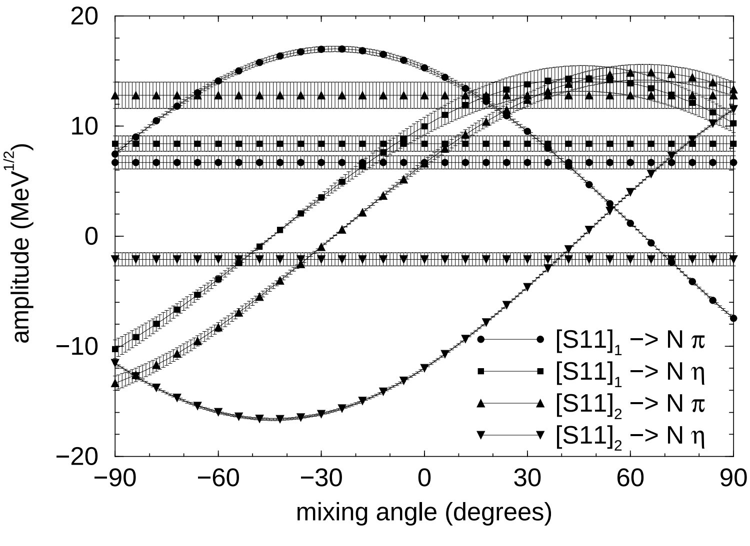

Fig. 1. Calculated amplitudes for the two S_{11} *states from Eq. (1) to decay to* $N\pi$ *and* $N\eta$*, plotted as a function of mixing angle* θ_S *(curved bands), and the amplitudes from Ref. [10] listed in Table 1 (horizontal bands).*

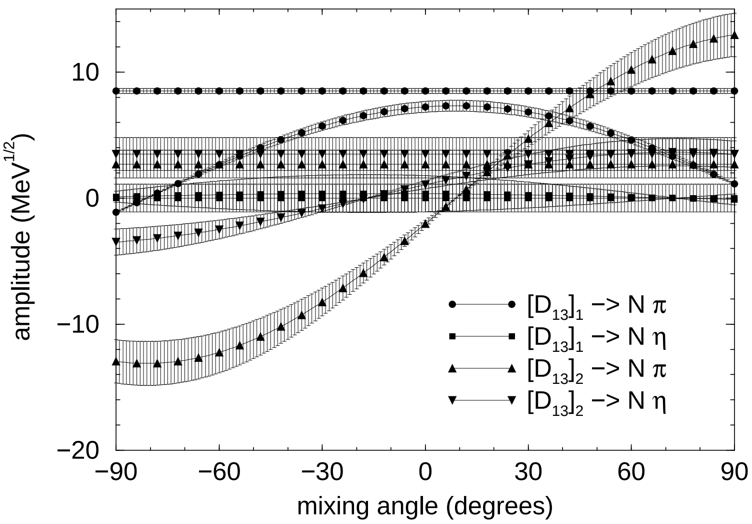

Fig. 2. Calculated amplitudes for the two D_{13} *states from Eq. (2) to decay to* $\mathrm{N}\pi$ *and* $\mathrm{N}\eta$*, plotted as a function of mixing angle* θ_D *(curved bands), and the amplitudes from Ref. [10] listed in Table 1 (horizontal bands).*

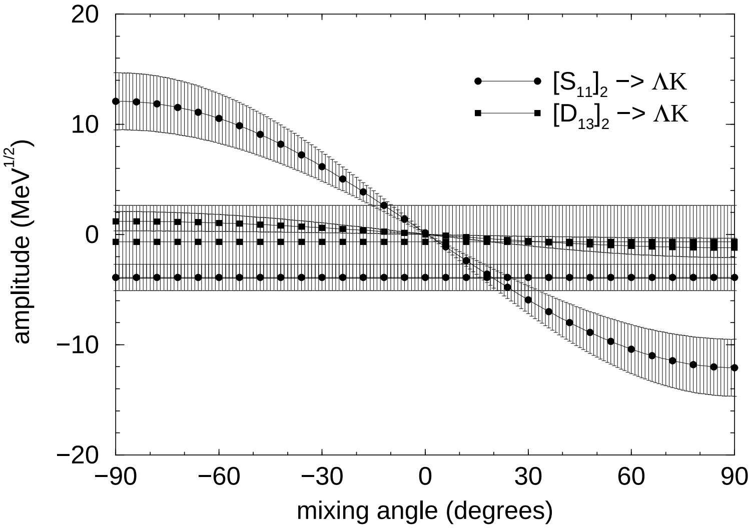

Fig. 3. Calculated amplitudes for the $[\mathrm{S}_{11}]_2$ *and* $[\mathrm{D}_{13}]_2$ *states from Eqs. (1) and (2) to decay to* $\Lambda\mathrm{K}$*, plotted as a function of the mixing angles* θ_S *and* θ_D*, respectively (curved bands), and the amplitudes from Refs. [10] and [11] listed in Table 1.*

unobservable in $\mathrm{N}\pi$ elastic scattering, and that the $\mathrm{N}\eta$ amplitudes plotted are

$$\frac{A_{\mathrm{X}\to\mathrm{N}\pi}(\theta^p)}{|A_{\mathrm{X}\to\mathrm{N}\pi}(\theta^p)|}A_{\mathrm{X}\to\mathrm{N}\eta}(\theta), \tag{3}$$

and similarly for ΛK, which compare directly to the partial amplitudes with signs extracted from the data and described above. Only calculated quantities bilinear in the wave function of the initial (X) and final (N) baryon wave functions, such as those in Eq. (3), are independent of sign conventions and so can be compared to physical observables.

In Figs. 1 − 3, the error bars (central values) of the calculated values of the decay amplitudes are found by using the PDG quoted range ("our estimate" value) of the masses of these states in the calculation of the decay momentum and phase space. The results for mixing angles are given in Table 2. Because of the small (or zero) branching fractions for $D_{13}(1520) \to N\eta$ and $D_{13}(1700) \to \Lambda K$, no mixing angle information can be extracted from the comparison with the calculated amplitudes for these cases. There is no value of θ_D for which the calculated amplitude for $[D_{13}]_2 \to N\eta$ agrees within errors with the amplitude for $D_{13}(1700) \to N\eta$, although they are within roughly 2σ at $\theta_D \simeq 9°$.

TABLE 2. Mixing angles (degrees) extracted from Figs 1, 2 and 3.

Angle	$[\,]_1 \to N\pi$	$[\,]_1 \to N\eta$	$[\,]_2 \to N\pi$	$[\,]_2 \to N\eta$	$[\,]_2 \to \Lambda K$
θ_S	41 ± 3	-8 ± 7	33^{+22}_{-13}	39 ± 2	20^{+20}_{-10}
θ_D	$\simeq 9$		21^{+9}_{-6}	> 11	

Four of the five determinations of θ_S are consistent within errors, with a weighted average of $38 \pm 4°$, while the mixing angle required to fit the measured Nη decay branch of the $[S_{11}]_1$ state is clearly incompatible with the other values. This implies that it is not possible to understand the Nη decay of the state $S_{11}(1535)$ in the constituent quark model described above and the $^3P_0$ pair-creation decay model. Its proximity to the Nη threshold, and its very strong (given the small phase space available) decay branch to Nη have led to alternative descriptions of this state [13].

The situation is less clear for the mixing in the D_{13} states, in part because of small branches to Nη for both states, and a small Nπ branch for $D_{13}(1700)$ state, which is found to decay predominantly to Nππ in the D-wave quasi-two-body channel Δπ in the Pitt-ANL analysis [10]. A comparison with calculated decay amplitudes for this channel has been made, but does not constrain θ_D because of a large uncertainty in the branching fraction extracted from data. The most reliable information comes from the Nπ decay of $D_{13}(1520)$, which favors a mixing angle of roughly 10°. With this in mind, it would appear that this analysis points to $\theta_D \simeq 10 - 15°$, with an uncertainty of at least 5°.

Additional research is necessary to compare the mixing angles extracted here to those of Refs. [6] and [7], based on broken $SU(6)_W \otimes O(3)$ symmetry, and from an analysis of new eta photo-production data in Ref. [8], as these mixing angles depend on sign conventions for the wave functions.

4. *Conclusion*

Angles for the mixing of low-lying negative-parity excited nucleon states of different quark spin, but the same total spin, have been extracted from a comparison between amplitudes calculated using a model of baryon structure and strong decays, and those extracted from scattering data. These are $\theta_S = +38 \pm 4$ degrees for the two S_{11} states, and $\theta_D \simeq +10-15$ degrees, with a significantly larger uncertainty on the latter value. This comparison also shows that it is not possible to understand the amplitude for the decay $S_{11}(1535) \to N\eta$ using the model for structure and strong decays described here.

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[12] An equivalent approach would be to multiply each amplitude by the calculated sign of the $N\pi$ amplitude, independently for each mixing angle. This would lead to curves for the amplitudes with derivatives discontinuous at the mixing angles where the $N\pi$ amplitude changes sign, but the mixing angles implied by the analyses would be unchanged.

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KUTOVI MIJEŠANJA U BARIONIMA S_{11} I D_{13}

Priroda kratkodosežnih međudjelovanja među kvarkovima u barionima ima osnovnu važnost za razumijevanje značajki nuklearne tvari u uvjetima mekog QCD. Modeli strukture uzbuđenih bariona negativne parnosti s parcijalnim valovima S_{11} i D_{13} u raspršenju $N\pi$ razlikuju se prema svojim predviđanjima ovisno o miješaju stanja s istim ukupnim spinom ali različitih kvarkovskih spinova. Možda će se ti modeli moći razlikovati točnim određivanjem kutova miješanja u tim stanjima. Proučavanje ovisnosti kutova miješanja preko omjera grananja u raspadima $N\pi$, $N\eta$ i ΛK, primjenom modela sastavnih kvarkova i modela raspada $^3P_0$, omogućuje izvođenje zaključaka o iznosima kutova miješanja.