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STRANGE-QUARK VECTOR CURRENT PSEUDOSCALAR-MESON TRANSITION FORM FACTORS

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Similarly to the electromagnetic pseudoscalar-meson transition form factors, one can also define strange-quark vector current pseudoscalar-meson transition form factors, contributing only to the behaviour of the isoscalar parts of the former ones. Their explicit form is found by constructing unitary and analytical models of the strange pseudoscalar-meson transition form factors dependent only on ω and ϕ coupling constant ratios as free parameters. Numerical values of these ratios are then determined from the corresponding pseudoscalar-meson transition form factors by employing the ω-ϕ mixing and a special assumption on the coupling of the quark components of vector-meson wave functions to the flavour component of currents under consideration.

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1. Introduction

In the last years, there was an experimental effort [1−3] to confirm non-zero contributions of the sea of strange quark-antiquark pairs to the structure of nucleons, which are built by nonstrange up and down quarks. The results of those experiments were values of the nucleon strange electric and magnetic form factors (FFs), or of their combinations at nonzero values of the four-momentum transfer squared, $t = -Q^2$.

On the other hand, there are various theoretical approaches [4−8] in the framework of which one can predict strange electric and magnetic or strange Dirac and Pauli FFs of nucleons. One of these approaches [8], utilizing the unitary and analytical models of electromagnetic (EM) structure of hadrons [9], appeared to be successful in a description of the scarce experimental information on nucleons and it can be directly extended to the pseudoscalar-meson transition FFs $F_{\gamma\mathrm{P}}(t)$.

The idea is the following. If the unitary and analytical models, with all known properties of the EM pseudoscalar-meson transition FFs, are constructed, $F^{\mathrm{EM}}_{\gamma\mathrm{P}}(t) = f[t; a_\rho, a_\omega, a_\phi]$, where the free parameters $a_\rho = (f_{\rho\gamma\mathrm{P}}/f^{\mathrm{EM}}_\rho)$, $a_\omega = (f_{\omega\gamma\mathrm{P}}/f^{\mathrm{EM}}_\omega)$, $a_\phi = (f_{\phi\gamma\mathrm{P}}/f^{\mathrm{EM}}_\phi)$ are determined by a comparison of the model with all existing data on $|F^{\mathrm{EM}}_{\gamma\mathrm{P}}(t)|$ in space-like and time-like regions simultaneously, and unitary and analytical models of the same inner structure (besides the asymptotic behaviour and normalization) with all known properties of the strange-quark vector current pseudoscalar-meson transition FFs are established, $F^{\mathrm{s}}_{\gamma\mathrm{P}}(t) = g[t; b_\omega, b_\phi]$, with unknown parameters $b_\omega = (f_{\omega\gamma\mathrm{P}}/f^{\mathrm{s}}_\omega)$, $b_\phi = (f_{\phi\gamma\mathrm{P}}/f^{\mathrm{s}}_\phi)$, then the latter parameters are determined from the known a_ω, a_ϕ from the relations [4]

$$\begin{aligned} b_\omega &= -\sqrt{6}\frac{\sin\epsilon}{\sin(\epsilon+\theta_0)}a_\omega, \quad \text{and} \\ b_\phi &= -\sqrt{6}\frac{\cos\epsilon}{\cos(\epsilon+\theta_0)}a_\phi, \end{aligned} \tag{1}$$

where $\epsilon = 3.7^\circ$ is the deviation from the ideal ω-ϕ mixing angle $\theta_0 = 35.3^\circ$.

In the next section, we review briefly the unitary and analytical model of EM pseudoscalar-meson transition FFs. Section 3 is devoted to a prediction of behaviour of strange-quark vector current pseudoscalar-meson transition FFs. In the last section we present discussion and conclusions.

2. *EM pseudoscalar-meson transition form factors*

The EM pseudoscalar-meson transition FFs are understood to be functions $F^{\mathrm{EM}}_{\gamma\mathrm{P}}(t)$ describing any $\gamma^* \to \gamma\mathrm{P}$ transition, where P stands for π^0, η or η'. Only recently a progress in the EM pseudoscalar-meson transition FFs has been made [10] thanks to the sophisticated unitary and analytical model of EM structure of hadrons [9] and the appearance of new experimental information, especially in the time-like region [11]. There is a single FF for each $\gamma^* \to \gamma\mathrm{P}$ transition to be defined by a parametrization of the matrix element of the EM current $J^{\mathrm{EM}}_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$

$$\langle P(p)\gamma(k)|J^{\mathrm{EM}}_\mu|0\rangle = \epsilon_{\mu\nu\alpha\beta}p^\nu\epsilon^\alpha k^\beta F^{\mathrm{EM}}_{\gamma\mathrm{P}}(t), \tag{2}$$

where ϵ^α is the polarization vector of the photon γ, $\epsilon_{\mu\nu\alpha\beta}$ appears as only the pseudoscalar-meson belongs to the abnormal spin-parity series. Every $F^{\mathrm{EM}}_{\gamma\mathrm{P}}(t)$ for

$\mathrm{P} = \pi^0,\ \eta,\ \eta'$ in the framework of the unitary and analytical model of the EM structure of hadrons takes the form

$$F_{\gamma \mathrm{P}}^{\mathrm{EM}}(t) = F_{\gamma \mathrm{P}}^{I=0}[V(t)] + F_{\gamma \mathrm{P}}^{I=1}[W(t)] \tag{3}$$

with

$$F_{\gamma \mathrm{P}}^{I=0}[V(t)] = \left(\frac{1-V^2}{1-V_{\mathrm{N}}^2}\right)^2 \left\{\frac{1}{2}F_{\gamma \mathrm{P}}^{\mathrm{EM}}(0)H(\omega') + [L(\omega)-H(\omega')]a_\omega + [L(\phi)-H(\omega')]a_\phi\right\}$$

$$\text{and} \quad F_{\gamma \mathrm{P}}^{I=1}[W(t)] = \left(\frac{1-W^2}{1-W_{\mathrm{N}}^2}\right)^2 \left\{\frac{1}{2}F_{\gamma \mathrm{P}}^{\mathrm{EM}}(0)H(\rho) + [L(\rho)-H(\rho')]a_\rho\right\},$$

where $V(W)$ is the conformal mapping

$$V(t) = \mathrm{i}\frac{\sqrt{q_{\mathrm{in}}^{I=0}+q} - \sqrt{q_{\mathrm{in}}^{I=0}-q}}{\sqrt{q_{\mathrm{in}}^{I=0}+q} + \sqrt{q_{\mathrm{in}}^{I=0}-q}}, \tag{4}$$

$$\text{with} \quad q = \sqrt{(t-t_0)/t_0} \quad \text{and} \quad q_{\mathrm{in}}^{I=0} = \sqrt{(t_{\mathrm{in}}^{I=0}-t_0)/t_0},$$

of the four-sheeted Riemann surface in t-variable into one V-plane (W-plane),

$$F_{\gamma \mathrm{P}}^{\mathrm{EM}}(0) = \frac{2}{\alpha\, m_{\mathrm{P}}}\sqrt{\frac{\Gamma(\mathrm{P}\to\gamma\gamma)}{\pi\, \mathrm{m_P}}}\,, \tag{5}$$

where $t_0 = m_{\pi^0}^2$, $t_{\mathrm{in}}^{I=0}$ and $t_{\mathrm{in}}^{I=1}$ are the effective square-root branch points including in average contributions of all higher important thresholds in both, isoscalar and isovector case, respectively, and

$$\begin{aligned}
L(\mathrm{s}) &= \frac{(V_{\mathrm{N}}-V_{\mathrm{s}})(V_{\mathrm{N}}-V_{\mathrm{s}}^*)(V_{\mathrm{N}}-1/V_{\mathrm{s}})(V_{\mathrm{N}}-1/V_{\mathrm{s}}^*)}{(V-V_{\mathrm{s}})(V-V_{\mathrm{s}}^*)(V-1/V_{\mathrm{s}})(V-1/V_{\mathrm{s}}^*)}, \\
& \qquad \mathrm{s} = \omega,\ \phi, \quad V_{\mathrm{N}} = V(t)|_{t=0}, \\
H(\omega') &= \frac{(V_{\mathrm{N}}-V_{\omega'})(V_{\mathrm{N}}-V_{\omega'}^*)(V_{\mathrm{N}}+V_{\omega'})(V_{\mathrm{N}}+V_{\omega'}^*)}{(V-V_{\omega'})(V-V_{\omega'}^*)(V+V_{\omega'})(V+V_{\omega'}^*)}, \\
L(\rho) &= \frac{(W_{\mathrm{N}}-W_\rho)(W_{\mathrm{N}}-W_\rho^*)(W_{\mathrm{N}}-1/W_\rho)(W_{\mathrm{N}}-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)}, \\
& \qquad W_{\mathrm{N}} = W(t)|_{t=0}, \\
H(\rho') &= \frac{(W_{\mathrm{N}}-W_{\rho'})(W_{\mathrm{N}}-W_\rho^*)(W_{\mathrm{N}}+W_{\rho'})(W_{\mathrm{N}}+W_{\rho'}^*)}{(W-W_{\rho'})(W-W_{\rho'}^*)(W+W_{\rho'})(W+W_{\rho'}^*)}.
\end{aligned}$$

If in a comparison of Eq. (3) with existing data masses and width of all vector-mesons under consideration are fixed at the table values, then other free parameters of the model acquire the following values:

$$\pi^{\mathbf{0}}: \quad \chi^2/ndf = 0.79, \quad t_{\rm in}^{I=0} = 0.9714\ \mathrm{GeV}^2, \quad t_{\rm in}^{I=1} = 1.0198\ \mathrm{GeV}^2, \tag{6}$$
$$(f_{\omega\gamma\pi^0}/f_\omega^{\rm EM}) = 0.0120 \pm 0.0002, \quad (f_{\phi\gamma\pi^0}/f_\phi^{\rm EM}) = -0.0002 \pm 0.0001,$$
$$(f_{\rho\gamma\pi^0}/f_\rho^{\rm EM}) = 0.0208 \pm 0.0006,$$

$$\eta: \quad \chi^2/ndf = 1.08, \quad t_{\rm in}^{I=0} = 0.6081\ \mathrm{GeV}^2, \quad t_{\rm in}^{I=1} = 0.6299\ \mathrm{GeV}^2, \tag{7}$$
$$(f_{\omega\gamma\eta}/f_\omega^{\rm EM}) = 0.0201 \pm 0.0020, \quad (f_{\phi\gamma\eta}/f_\phi^{\rm EM}) = -0.0013 \pm 0.0001,$$
$$(f_{\rho\gamma\eta}/f_\rho^{\rm EM}) = 0.0119 \pm 0.0012,$$

$$\eta': \quad \chi^2/ndf = 1.29, \quad t_{\rm in}^{I=0} = 1.0106\ \mathrm{GeV}^2, \quad t_{\rm in}^{I=1} = 0.9578\ \mathrm{GeV}^2, \tag{8}$$
$$(f_{\omega\gamma\eta'}/f_\omega^{\rm EM}) = -0.1049 \pm 0.0011, \quad (f_{\phi\gamma\eta'}/f_\phi^{\rm EM}) = 0.0757 \pm 0.0017,$$
$$(f_{\rho\gamma\eta'}/f_\rho^{\rm EM}) = 0.0859 \pm 0.0009\,.$$

A prediction of the behaviour of the corresponding FFs and their comparison with exiting data are graphically presented in Figs. 1 − 3.

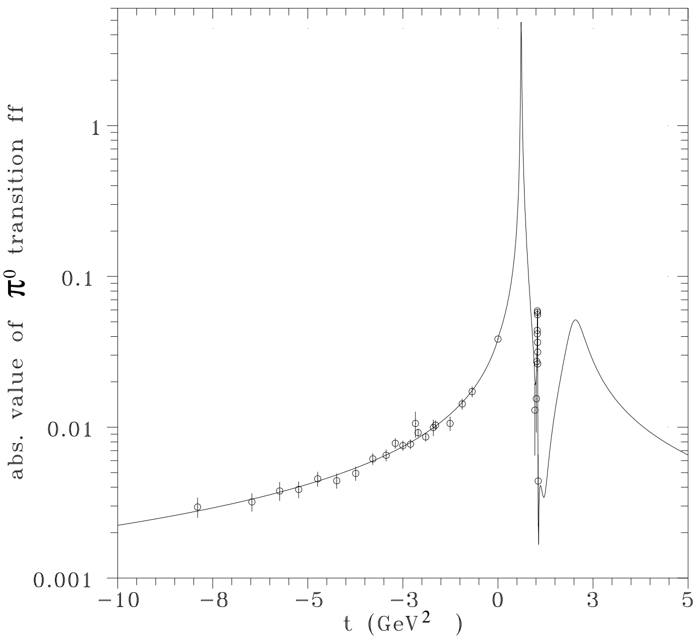

Fig. 1. π^0 transition form factor.

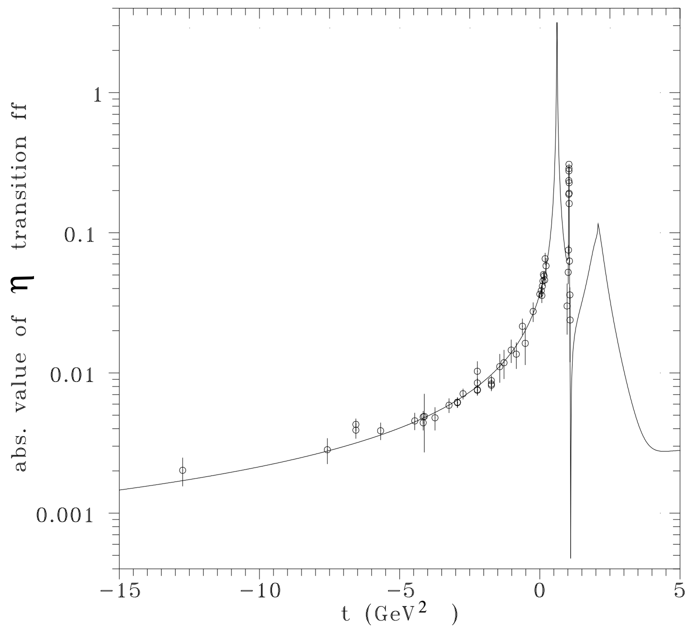

Fig. 2. η transition form factor.

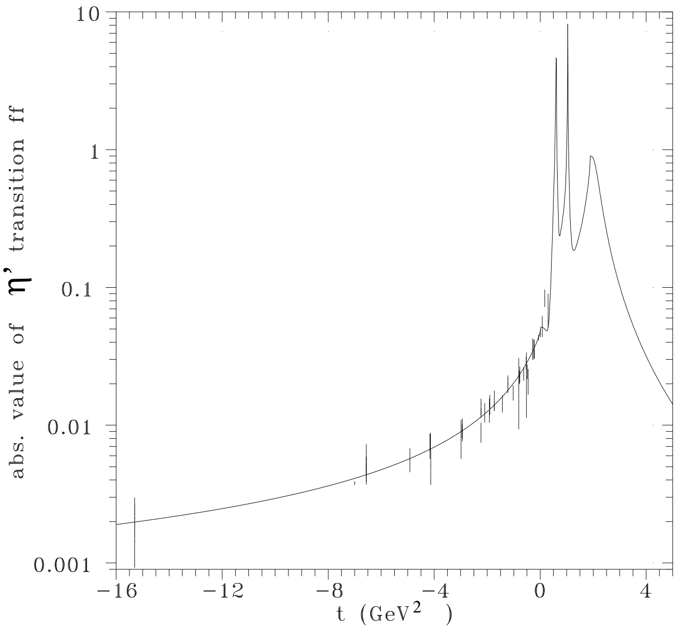

Fig. 3. η' transition form factor.

3. *Strange pseudoscalar-meson transition form factors*

The strange-quark vector current pseudoscalar-meson transition FFs, $F^{\rm s}_{\gamma\rm P}(t)$, can be defined in analogy to Eq. (2) by the parametrization

$$\langle P(p)\gamma(k)|J^{\rm s}_\mu 0\rangle = \epsilon_{\mu\nu\alpha\beta}p^\nu\epsilon^\alpha k^\beta F^{\rm s}_{\gamma\rm P}(t)\,, \tag{9}$$

where $J^{\rm s}_\mu = \bar{s}\gamma_\mu s$ is the strange-quark vector current.

Since the isospin of the strange quark s is zero, then the strange-quark vector current pseudoscalar-meson transition FFs $F^{\rm s}_{\gamma\rm P}(t)$ can contribute only to the isoscalar parts of $F^{\rm EM}_{\gamma\rm P}(t)$, and from this directly follows that $F^{\rm s}_{\gamma\rm P}(t)$ are saturated (unlike $F^{\rm EM}_{\gamma\rm P}(t)$) only by isoscalar vector-mesons. However, since the total strangeness of P and γ is zero, then their normalizations take the form

$$F^{\rm s}_{\gamma\rm P}(0) = 0. \tag{10}$$

The asymptotic behaviour of the strange pseudoscalar-meson transition FFs is

$$F^{\rm s}_{\gamma\rm P}(t)|_{|t|\to\infty} \sim t^{-3} \tag{11}$$

as there are another two $\bar{\rm s}$s quarks contributing to the structure of P.

Analytical properties of $F^{\rm s}_{\gamma\rm P}(t)$ are identical with analytical properties of $F^{I=0}_{\gamma\rm P}(t)$.

Taking into account all above mentioned properties in a construction of the unitary and analytical models of $F^{\rm s}_{\gamma\rm P}(t)$, we start with the corresponding VMD parametrization

$$\widetilde{F}^{\rm s}_{\gamma\rm P}(t) = \sum_{{\rm i}=\omega,\phi,\omega'} \frac{m_{\rm i}^2}{m_{\rm i}^2 - t}(f_{{\rm i}\gamma\rm P}/f^{\rm s}_{\rm i})\,, \tag{12}$$

where $f^{\rm s}_{\rm i}$ is a coupling of the strangeness current to vector meson ${\rm i} = \omega, \phi, \omega'$ and we use the FF denotation $\widetilde{F}^{\rm s}_{\gamma\rm P}(t)$ as it has still the VMD asymptotic behaviour.

Requirement of the normalization (10) leads to the expression

$$\widetilde{F}^{\rm s}_{\gamma\rm P}(t) \;=\; \left[\frac{m^2_\omega}{m^2_\omega - t} - \frac{m^2_{\omega'}}{m^2_{\omega'} - t}\right] b_\omega + \left[\frac{m^2_\phi}{m^2_\phi - t} - \frac{m^2_{\omega'}}{m^2_{\omega'} - t}\right] b_\phi. \tag{13}$$

Then in analogy to Eq. (3), the unitary and analytical model of $\widetilde{F}^{\rm s}_{\gamma\rm P}(t)$ takes the form

$$
\begin{aligned}
\widetilde{F}^{\rm s}_{\gamma P}(t) \;\; = \;\; & \left(\frac{1-V^2}{1-V_{\rm N}^2}\right)^2 \Bigg\{ \Bigg[\frac{(V_{\rm N}-V_\omega)(V_{\rm N}-V^*_\omega)(V_{\rm N}-1/V_\omega)(V_{\rm N}-1/V^*_\omega)}{(V-V_\omega)(V-V^*_\omega)(V-1/V_\omega)(V-1/V^*_\omega)} \\
& - \frac{(V_{\rm N}-V_{\omega'})(V_{\rm N}-V^*_{\omega'})(V_{\rm N}+V_{\omega'})(V_{\rm N}+V^*_{\omega'})}{(V-V_{\omega'})(V-V^*_{\omega'})(V+V_{\omega'})(V+V^*_{\omega'})} \Bigg] b_\omega \\
& + \Bigg[\frac{(V_{\rm N}-V_\phi)(V_{\rm N}-V^*_\phi)(V_{\rm N}+V_\phi)(V_{\rm N}+V^*_\phi)}{(V-V_\phi)(V-V^*_\phi)(V+V_\phi)(V+V^*_\phi)} \\
& - \frac{(V_{\rm N}-V_{\omega'})(V_{\rm N}-V^*_{\omega'})(V_{\rm N}+V_{\omega'})(V_{\rm N}+V^*_{\omega'})}{(V-V_{\omega'})(V-V^*_{\omega'})(V^*+V_{\omega'})(V+V^*_{\omega'})} \Bigg] b_\phi \Bigg\} , \qquad (14)
\end{aligned}
$$

but still with the VMD asymptotics. However, taking into account a change of the exponent in the asymptotic term

$$
\left(\frac{1-V^2}{1-V_{\rm N}^2}\right)^2 \to \left(\frac{1-V^2}{1-V_{\rm N}^2}\right)^{2n} , \quad n = 1,2,3,\ldots \qquad (15)
$$

leading to the change of the asymptotic behaviour

$$
|_{|t|\to\infty} \sim t^{-1} \to |_{|t|\to\infty} \sim t^{-n} \qquad (16)
$$

of any unitary and analytical FF, one can multiply both sides of Eq. (14) by the factor $\left(\frac{1-V^2}{1-V_{\rm N}^2}\right)^4$ and redefine the FF

$$
F^{\rm s}_{\gamma {\rm P}}(t) = \widetilde{F}^{\rm s}_{\gamma {\rm P}}(t) \left(\frac{1-V^2}{1-V_{\rm N}^2}\right)^4 , \qquad (17)
$$

in order to achieve the unitary and analytical model of $F^{\rm s}_{\gamma {\rm P}}(t)$ with the required asymptotic behaviour (11) and dependent only on the unknowns b_ω and b_ϕ, to be determined by the relations (1) from the values of a_ω, a_ϕ given by Eqs. (6) – (8).

Now taking into account the numerical values (6) – (8) and utilizing relations (1), one gets

$$
\begin{aligned}
\pi_{\bf 0}: &\quad (f_{\omega\gamma\pi_0}/f^{\rm s}_\omega) = +0.0062, \quad (f_{\phi\gamma\pi_0}/f^{\rm s}_\phi) = +0.0006, \\
\eta: &\quad (f_{\omega\gamma\eta}/f^{\rm s}_\omega) = -0.0050, \quad (f_{\phi\gamma\eta}/f^{\rm s}_\phi) = +0.0041, \\
\eta': &\quad (f_{\omega\gamma\eta'}/f^{\rm s}_\omega) = +0.0263, \quad (f_{\phi\gamma\eta'}/f^{\rm s}_\phi) = -0.2386 \qquad (18)
\end{aligned}
$$

Predictions of behaviour of the corresponding strange pseudoscalar-meson transition FFs are graphically presented in Figs. 4 – 6.

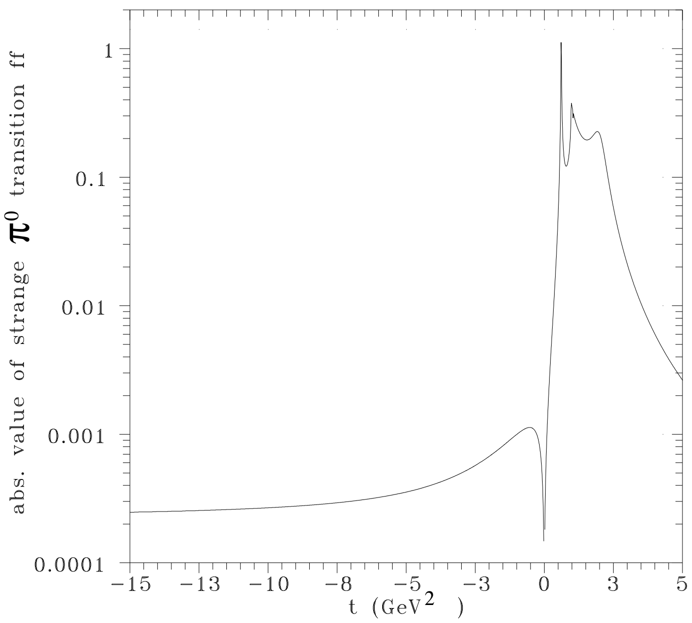

Fig. 4. Strange π^0 transition form factor.

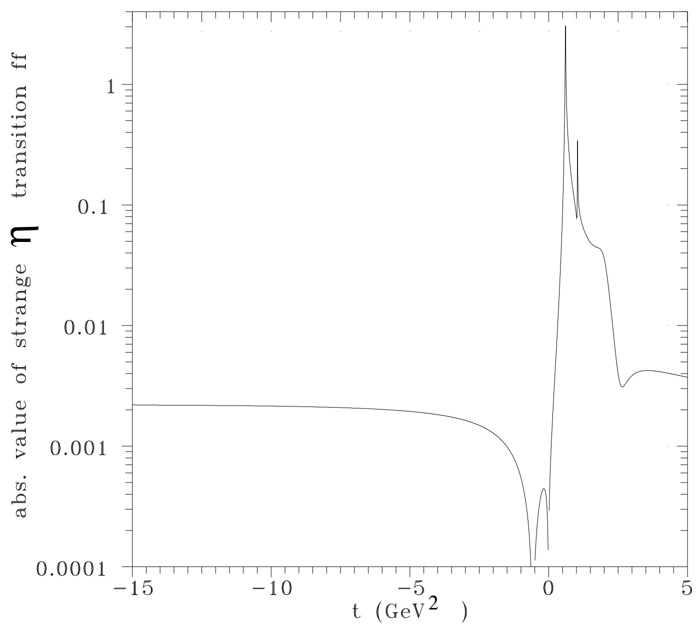

Fig. 5. Strange η transition form factor.

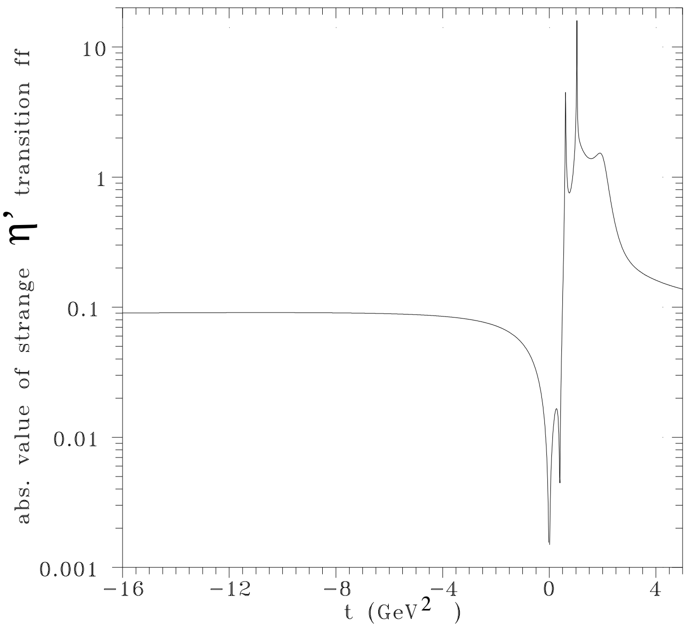

Fig. 6. Strange η' transition form factor.

4. Discussion and conclusions

The study of strange-quark vector current nucleon FF behaviour, which is interesting in relation to the experimental effort to confirm non-zero contributions of the sea of strange quark-antiquark pairs to the nucleon structure, is extended to pseudoscalar-meson transition FFs. An explicit form of strange-quark vector current of pseudoscalar-meson transition FFs is found by constructing unitary and analytical models dependent only on the ω and ϕ coupling constant ratios as the only unknown parameters. Their numerical values are determined from the corresponding coupling constant ratios of the EM pseudoscalar-meson transition FFs by employing the ω-ϕ mixing and a special assumption on the coupling of the quark components of vector-meson wave functions to flavour components of quark-current under consideration.

However, we still don't know how to measure the strange pseudoscalar-meson transition FFs.

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FAKTORI OBLIKA PSEUDOSKALARNIH MEZONA OD VEKTORSKE STRUJE STRANIH KVARKOVA

Poput elektromagnetskih prijelaznih faktora oblika pseudoskalarnih mezona, mogu se definirati i prijelazni faktori oblika pseudoskalarnih mezona od vektorske struje stranih kvarkova, koji doprinose samo svojstvima izoskalarnih dijelova prvih. Njihova se eksplicitna formulacija nalazi razvijanjem unitarnih i analitičkih modela za strane prijelazne faktore oblika pseudoskalarnih mezona koji ovise samo o omjerima konstanti vezanja ω and ϕ mezona kao slobodnim parametrima. Iznosi tih omjera se određuju iz odnosnih prijelaznih faktora oblika pseudoskalarnih mezona upotrebom posebnog miješanja ω-ϕ i posebnom pretpostavkom o vezanju kvarkovskih komponenata valnih funkcija vektorskih mezona na okusnu komponentu studiranih struja.