The $q\bar{q}$ spectrum is studied within a chiral constituent quark model. It provides with a good fit of the available experimental data from light (vector and pseudoscalar) to heavy mesons. The new D states measured at different B-factories are studied. The $0^{++}$ light mesons are analyzed as $q\bar{q}$ pairs or tetraquark structures.

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Keywords: heavy mesons, chiral constituent quark model, tetraquark structures

1. Introduction

The continuously increasing huge amount of data and its simplicity have converted meson spectroscopy into an ideal system to learn about the properties of QCD. Since the Gell-Mann’s conjecture, most of the meson experimental data were classified as $q\bar{q}$ states according to $SU(N)$ irreducible representations. Nevertheless a number of interesting issues remains still open, as for example the understanding of some data recently obtained in the B-factories or the structure of the scalar mesons. The recently measured $D_s$ states are hardly accommodated in a pure $q\bar{q}$ description. They present a much lower mass than the naive prediction of constituent quark models. Besides, the underlying structure of the scalar mesons is still not well established theoretically. In a pure $q\bar{q}$ scheme, the light scalars could be identified with the isoscalars $f_0(600)$ and $f_0(980)$, the isodoublet $\kappa(900)$, and the isovector $a_0(980)$, constituting a $SU(3)$ flavor nonet. However, such identification immediately faces difficulties to explain for example (i) the $f_0(980)$ and $a_0(980)$ mass degeneracy, (ii) why $a_0(980)$ and $f_0(600)$ have such a different mass? and (iii) the similar branching ratios of the $J/\psi \rightarrow f_0(980)\phi$ and $J/\psi \rightarrow f_0(980)\omega$ decays which clearly indicates the existence of strange and nonstrange content in the $f_0(980)$.

The theoretical tools to determine the properties of mesons are based to a large extent on phenomenological models. The study of charmonium and bottomonium
made clear that heavy-quark systems are properly described by nonrelativistic potential models reflecting the dynamics expected from QCD [1]. For heavy quarks and to leading order in \((v/c)^2\), it has been demonstrated that the interaction can be derived from the theory [2]. The light-meson sector has been studied by means of constituent quark models, where quarks are dressed with a phenomenological mass and bound in a nonrelativistic potential [3]. Quite surprisingly, a large number of properties of hadrons have been reproduced in this way [4]. In this work, we present the meson spectra obtained by means of a chiral constituent quark model in a trial to interpret some of the still unclear experimental data in the light scalar and D-meson sectors.

2. SU(3) chiral constituent quark model

Since the origin of the quark model, hadrons have been considered as bound states of constituent (massive) quarks. Nowadays, it is widely recognized that the constituent quark mass appears because of the spontaneous breaking of the original \(SU(3)_L \otimes SU(3)_R\) chiral symmetry at some momentum scale. In this domain, a simple Lagrangian invariant under the chiral transformation can be derived as [5]

\[
L = \bar{\psi} (i \gamma^\mu \partial_\mu - M U^{\gamma_5}) \psi ,
\]

where \(U^{\gamma_5} = \exp(i \pi^a \lambda^a \gamma_5/f_\pi)\) with \(a = 1, \ldots, 8\). \(\pi^a\) denotes the Goldstone pseudoscalar fields \(\pi, K, \) and \(\eta_8, \) and \(M\) is the constituent quark mass. The momentum dependence of the constituent quark mass can be obtained from the theory. It can be parametrized as \(M(q^2) = m_q F(q^2)\) with

\[
F(q^2) = \left[ \frac{\Lambda^2}{\Lambda^2 + q^2} \right]^{1/2} ,
\]

where \(\Lambda\) determines the chiral symmetry breaking scale. \(U^{\gamma_5}\) can be expanded in terms of boson fields as,

\[
U^{\gamma_5} = 1 + \frac{i}{f_\pi} \gamma^5 \lambda^a \pi^a - \frac{1}{2f_\pi} \pi^a \pi^a + \ldots ,
\]

where the first term generates the constituent quark mass and the second gives rise to the pseudoscalar one-boson exchange between quarks. The main contribution of the third term comes from the exchange of two-pseudoscalar mesons which is usually simulated by means of scalar-exchange potentials. From the nonrelativistic reduction of the Lagrangian, one can generate in the static approximation the quark-meson exchange potentials. Among them, the one-pion, one-kaon and one-eta exchanges contribute with central and tensor interactions, while the scalar one-sigma exchange gives central and spin-orbit terms. Explicit expressions of these potentials can be found elsewhere [6]. The parameters of the Goldstone boson...
exchange interaction are fixed assuming that $SU(3)$ flavor-symmetry is exact, only broken by the different mass of the strange quark. In the heavy-quark sector, chiral symmetry is explicitly broken and therefore these interactions will not appear.

For higher momentum transfer, quarks still interact through gluon exchanges. Following de Rújula et al. [7], the one-gluon-exchange (OGE) interaction is taken as a standard color Fermi-Breit potential. In order to obtain a unified description of light, strange and heavy mesons, a running strong coupling constant has to be used [4]. The perturbative expression for $\alpha_s(Q^2)$ diverges when $Q \to \Lambda_{QCD}$ and, therefore, the coupling constant has to be frozen at low energies. We parametrize this behavior by means of an effective scale-dependent strong coupling constant [8]

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln \left( (\mu^2 + \mu_0^2)/\Lambda_0^2 \right)}$$

where $\mu$ is the reduced mass of the $q\bar{q}$ system and $\alpha_0$, $\mu_0$ and $\Lambda_0$ are fitted parameters [6]. This equation gives rise to $\alpha_s \sim 0.54$ for light quarks, a value consistent with the one used in the study of the nonstrange hadron phenomenology [9], and it also has an appropriate high $Q^2$ behavior, $\alpha_s \sim 0.127$ at the $Z_0$ mass [10]. The $\delta$ function appearing in the OGE has to be regularized in order to avoid an unbound spectrum from below. For a coulombic system, the size scales with the reduced mass, what suggests the use of a flavor-dependent regularization $r_0(\mu) = \hat{r}_0/\mu$ [6]. Moreover, the Schrödinger equation cannot be solved numerically for potentials containing $1/r^3$ terms. This is why the non-central terms of the OGE are regularized with a monopole form factor [6].

The other nonperturbative property of QCD is confinement. Lattice QCD studies show that $q\bar{q}$ systems are well reproduced at short distances by a linear potential that is screened at large distances due to pair creation [11]. We have simulated this behavior by means of a screened potential of the form $V_{\text{CON}}(r_{ij}) = -a_c(1 - e^{-\mu r_{ij}})(\chi_i \chi_j)$ [6]. One important question is the covariance property of confinement what determines the sign and strength of the spin-orbit Thomas precession term. While the spin-orbit splittings in heavy-quark systems suggest a scalar confining potential [12], Ref. [13] showed that the Dirac structure of confinement is of vector nature in the heavy quark limit of QCD. On the other hand, a significant mixture of vector-scalar confinement has been used to explain the decay widths of P-wave D mesons [14]. Therefore, we consider the spin-orbit contribution of the confining interaction as an arbitrary combination of scalar and vector terms $V_{\text{CON}}^{SO}(r_{ij}) = (1 - a_s)V_V^{SO}(r_{ij}) + a_s V_S^{SO}(r_{ij})$, where $a_s$ is the mixing parameter.

### 3. Results

Let us briefly discuss the parameters of our model. Most of the parameters of the Goldstone boson fields are taken from the NN sector. The eta and kaon cutoff masses are related with the sigma and pion one as explained in Ref. [15]: $\Lambda_{[u(d)s]} \approx \Lambda(ud) + m_s$ where $m_s$ is the strange quark current mass. The confinement parameters $a_c$ and $\mu_c$ are fitted to reproduce the energy difference between the $\rho$
meson and its first radial excitation and the $J/\psi$ and the $\psi(2S)$. The parameters involved in the OGE are obtained from a global fit to the hyperfine splittings well established in the Particle Data Group (PDG) [16]. Finally, one has to fix the relative strength of the scalar and vector confinement. Using a pure scalar confining potential, one obtains 1363 MeV and 1142 MeV for the $a_1(1260)$ and $a_2(1320)$, respectively, in complete disagreement with the order and magnitude of the experimental data. Introducing a mixture of vector confinement, $\alpha_s = 0.777$, the experimental order is recovered, being now the masses 1198 MeV and 1322 MeV, respectively, both within the experimental error bars. This value also allows to obtain a good agreement with the experimental data in the $c\overline{c}$ and $b\overline{b}$ systems.

In Fig. 1 we show results for the light pseudoscalar and vector mesons and for heavy mesons. The agreement with experimental data is remarkable. Let us emphasize that with only 11 parameters more than 110 states are described [6].

![Meson spectra](image)

**Fig. 1.** Meson spectra (a) light pseudoscalar and vector mesons (b) $b\overline{b}$ mesons.

### 3.1. The $\eta_c(2S)$ and $D_{sJ}^*$ states

Recently Belle and BaBar collaborations have reported new measurements that immediately prompted different interpretations. The Belle collaboration has reported the following values for the mass of the $\eta_c(2S)$

$$
M[\eta_c(2S)] = \begin{cases} 
3654 \pm 6 \pm 8 \text{ MeV Ref. [17]}, \\
3622 \pm 6 \pm 6 \text{ MeV Ref. [18]}, \\
3630 \pm 8 \text{ MeV Ref. [19]}, 
\end{cases}
$$
that are significantly larger than most predictions of constituent quark models and the previous experimental value of the PDG: \( M[\eta_c(2S)] = 3594 \pm 5 \) MeV. It has been pointed out that these values cannot be easily explained in the framework of constituent quark models because the resulting 2S hyperfine splitting (HFS) would be smaller than the predicted for the 1S ones. In fact the predicted ratio of the 2S to 1S HFS in charmonium is

\[
R = \frac{\Delta M_{2S}^{\text{HFS}}}{\Delta M_{1S}^{\text{HFS}}} = \begin{cases} 
0.84 \text{ Ref. [20]}, \\
0.67 \text{ Ref. [21]}, \\
0.60 \text{ Ref. [22]},
\end{cases}
\]

whereas the experimental one is \( R = 0.273 \) if \( M[\eta_c(2S)] = 3654 \) MeV, \( R = 0.547 \) for 3622 MeV and \( R = 0.479 \) for 3630 MeV. In view of these differences, some authors have claimed for a quark-gluon coupling constant depending on the radial excitation.

Our result is \( M[\eta_c(2S)] = 3627 \) MeV, within the error bar of the last two Belle measurements, the ones obtained with higher statistics. Moreover, the ratio 2S to 1S HFS is found to be 0.537, in perfect agreement with the experimental data. The reason for this agreement can be found in the shape of the confining potential that also influences the HFS, the linear confinement being not enough flexible to accommodate both excitations [23].

The experimental measurement reported by BaBar is a narrow state near 2317 MeV known as \( D^*_S(2317) \) [24]. This state has been confirmed by CLEO [25] together with another possible resonance around 2460 MeV. Both experiments interpret these resonances as \( J^P = 0^+ \) and \( 1^+ \) states. This discovery has triggered a series of articles [26] either supporting this interpretation or presenting alternative hypothesis.

The most striking aspect of these two resonances is that their masses are much lower than expected. The mass difference between the PDG mass value \( D_1(2420) \) and the lowest value of \( D_0^* \) reported by the Belle collaboration is about 135 MeV. Using these data and the PDG mass value for the \( D_S(2536) \), one would expect the \( D_0^* \) mass to be around 2400 MeV, almost 100 MeV greater than the measured values. Our results are shown in Table 1 (next page). They agree reasonably well with the values of the PDG for both for the D’s and D_s’s states, but fail to reproduce the two D_s states reported by Belle and CLEO. Our results agree with the results by FOCUS [28] for the D_0 state but are far from the result of Belle for the same state.

4. The scalar sector: \( q\bar{q} \) study

It is still not clear which are the members of the \( 0^{++} \) nonet corresponding to \( L = S = 1 \) \( q\bar{q} \) multiplets. There are too many \( 0^{++} \) mesons observed below 2 GeV to be explained as \( q\bar{q} \) states. Two isovectors \( IJ^{PC} = 10^{++} \): \( a_0(980) \) and \( a_0(1450) \), five isoscalars \( IJ^{PC} = 00^{++} \): \( f_0(600), f_0(980), f_0(1370), f_0(1500) \) and \( f_0(1710) \), and
TABLE 1. Masses of D and D_s mesons in MeV compared to all known experimental data [27].

<table>
<thead>
<tr>
<th></th>
<th>M^*(0^+)</th>
<th>M(1^+)</th>
<th>M(1^+)</th>
<th>M^*(2^+)</th>
<th>M(0−) (2S)</th>
<th>M^+(1−) (2S)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D mesons</strong></td>
<td></td>
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<tr>
<td>PDG (0)</td>
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<tr>
<td>PDG (±)</td>
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<tr>
<td>FOCUS (0)</td>
<td>~2420</td>
<td></td>
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<tr>
<td>FOCUS (±)</td>
<td>~2420</td>
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<td></td>
<td></td>
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<tr>
<td>Belle (0)</td>
<td></td>
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<tr>
<td></td>
<td>2290±30</td>
<td>2400±36</td>
<td>2424±2</td>
<td>2461±4</td>
<td>2637±7</td>
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<tr>
<td></td>
<td></td>
<td>2461±51</td>
<td></td>
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<tr>
<td>Our result</td>
<td>2437.9</td>
<td>2496.0</td>
<td>2495.7</td>
<td>2500.5</td>
<td>2641.7</td>
<td>2700.1</td>
</tr>
<tr>
<td><strong>D_s mesons</strong></td>
<td></td>
<td></td>
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<tr>
<td>PDG (±)</td>
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<tr>
<td>FOCUS (±)</td>
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<tr>
<td>BaBar</td>
<td>2317</td>
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<tr>
<td>CLEO</td>
<td>2317</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Our result</td>
<td>2470.5</td>
<td>2565.5</td>
<td>2549.7</td>
<td>2584.4</td>
<td>2698.7</td>
<td>2764.3</td>
</tr>
</tbody>
</table>

Three IJ^{PC} = \frac{1}{2}0^{++}: K_0^*(1430), K_0^*(1940) and recently κ(900) have been reported in the PDG. The naive quark model predicts the existence of one isovector, two isoscalars and two I = 1/2 states.

Our results are shown in Table 2. Using this table one can proceed to assign physical states to 0^{++} nonet members. We observe that there are no nonstrange states with I = 0 and mass close to 1 either 1.5 GeV, which would correspond to the f_0(980) and the f_0(1500). The same occurs in the strange sector around 0.8 GeV, we do not find a qq partner for the κ(900).

Let us discuss each state separately. With respect to the isovector states, there appears a candidate for the a_0(980), the \(^3P_0\) member of the lowest \(^3P_J\) isovector multiplet. The other candidate, the a_0(1450), is predicted to be the scalar member of a \(^3P_J\) excited isovector multiplet. This reinforces the predictions of the naive quark model, where the LS force makes lighter the J = 0 states with respect to the J = 2. The assignment of the a_0(1450) as the scalar member of the lowest \(^3P_J\) multiplet would contradict this idea, because the a_2(1312) is well established as a qq pair. The same behavior is evident in the c\overline{c} and the b\overline{b} spectra, making impossible to describe the a_0(1450) as a member of the lowest \(^3P_J\) isovector multiplet without
TABLE 2. Light-scalar meson masses in MeV.

<table>
<thead>
<tr>
<th>State ((n^{2F+1,2S+1}L_J))</th>
<th>Meson</th>
<th>Our result</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1^{3,3}P_0</td>
<td>a_0(980)</td>
<td>983.5</td>
<td>984.7 ± 1.2</td>
</tr>
<tr>
<td>2^{3,3}P_0</td>
<td>a_0(1450)</td>
<td>1586.3</td>
<td>1474 ± 19</td>
</tr>
<tr>
<td>1^{1,3}P_0</td>
<td>f_0(600)</td>
<td>402.7</td>
<td>400–1200</td>
</tr>
<tr>
<td>1^{1,3}P_0</td>
<td>f_0(1370)</td>
<td>1341.7</td>
<td>1200–1500</td>
</tr>
<tr>
<td>2^{1,3}P_0</td>
<td>f_0(1370)</td>
<td>1391.2</td>
<td>1200–1500</td>
</tr>
<tr>
<td>2^{1,3}P_0</td>
<td>f_0(1710)</td>
<td>1751.8</td>
<td>1713 ± 6</td>
</tr>
<tr>
<td>3^{1,3}P_0</td>
<td>f_0(2020)</td>
<td>1893.8</td>
<td>1992 ± 16</td>
</tr>
<tr>
<td>3^{1,3}P_0</td>
<td>f_0(2200)</td>
<td>2212.2</td>
<td>2197 ± 11</td>
</tr>
<tr>
<td>1^{2,3}P_0</td>
<td>K_0^*(1430)</td>
<td>1213.5</td>
<td>1412 ± 6</td>
</tr>
<tr>
<td>2^{2,3}P_0</td>
<td>K_0^*(1950)</td>
<td>1768.5</td>
<td>1945 ± 30</td>
</tr>
</tbody>
</table>

spoiling the description of heavy-quark multiplets. However, in spite of the correct description of the mass of the \(a_0(980)\), the model predicts a pure light-quark content, what seems to contradict some experimental decays. The \(a_0(1450)\) is predicted to be also a pure light-quark structure obtaining a mass somewhat higher than the experiment.

In the case of the isoscalar states, one finds a candidate for the \(f_0(600)\) with a mass of 402 MeV, in the lower limit of the experimental error bar and with a strangeness content around 8%. The \(f_0(980)\) and \(f_0(1500)\) cannot be found for any combination of the parameters of the model. It seems that a different structure rather than a naive \(q\bar{q}\) pair is needed to describe these states. The \(f_0(1500)\) is a clear candidate for the lightest glueball [29] and our results support this assumption. Concerning the \(f_0(1370)\) (which may actually correspond to two different states [30]), we obtain two almost degenerate states around this energy, the lower one with a predominantly nonstrange content, and the other with a high \(s\bar{s}\) content. Finally, a state corresponding to the \(f_0(1710)\) is obtained.

Concerning the \(I = 1/2\) sector, as a consequence of the larger mass of the strange quark as compared to the light ones, our model always predicts a mass for the lowest 0^{++} state 200 MeV greater than the \(a_0(980)\) mass. Therefore, with \(a_0(980)\) being the member of the lowest isovector scalar multiplet, the \(\kappa(900)\) cannot be explained as a \(q\bar{q}\) pair. We find a candidate for the \(K_0^*(1430)\) although with a smaller mass.

In conclusion, our results indicate that the light scalar sector cannot be described in a pure \(q\bar{q}\) scheme and more complicated structures or mixing with multiquark states seem to be needed. Our conclusion concerning the \(f_0(980)\) and the \(\kappa(900)\) agrees with Ref. [31] using the extended Nambu-Jona-Lasinio model in an improved ladder approximation of the Bethe-Salpeter equation. This seems to indicate that relativistic corrections would not improve the situation and the conclusions would remain model independent.
5. The scalar sector: tetraquark study

In the naive quark model, a $q\bar{q}$ positive-parity state requires a unit of orbital angular momentum. Apparently, this takes an energy around 1 GeV since similar meson states ($1^{++}$ and $2^{++}$) lie above 1.2 GeV. However a $q^2\bar{q}^2$ structure, suggested twenty years ago by Jaffe [32], can couple to $0^{++}$ without orbital excitation and therefore could be a serious candidate to explain the structure of some light scalar mesons.

In this section we study tetraquark bound states, focusing our attention to those states with the quantum numbers of the scalar mesons. We have solved the Schrödinger equation using a variational method where the spatial trial wave function is a linear combination of Gaussians. The technical details are given in Ref. [33]. Let us only mention that neglecting the exchange terms in the variational wave function is fully justified for the heavy-light tetraquarks, but it may induce some corrections in the present study. Due to the presence of the kaon-exchange, there is a mixture among different configurations with the same isospin. In particular, in the isoscalar sector, the configurations: $[(qq)(\bar{q}\bar{q})]$, $[(qs)(\bar{q}\bar{s})]$, and $[(ss)(\bar{s}\bar{s})]$ are mixed. The same happens in the isovector case for the configurations: $[(qq)(\bar{q}\bar{q})]$, and $[(qs)(\bar{q}\bar{s})]$, and in the $I = 1/2$ case for the configurations: $[(qq)(\bar{q}\bar{s})]$, and $[(qs)(\bar{s}\bar{s})]$. In all cases q stands for a u or d quark.

![Fig. 2. Tetraquark masses for the scalar sector.](image)

The results are shown in Fig. 2. We present the lowest states for the three isospin sectors. The $\times$’s show the results obtained within the model described in Sect. 2.
As one can see, there appear two states, in the isoscalar and isovector sectors, with almost the same mass, although too heavy to be identified with the $f_0(980)$ and $a_0(980)$. In the $I = 1/2$ sector, there appears a candidate to be identified with the $\kappa(900)$. It has recently been argued about the possible importance of three-body color forces arising from the confining interaction for those systems containing at least three quarks [34]. We have performed a calculation including a three-body color confining term as the one reported in Ref. [34]. The strength of this interaction has been chosen to reproduce the mass of the $f_0(980)$. The results are denoted by *’s in Fig. 2. As one can see, the degeneracy between the isoscalar and isovector states remains, while the lowest states of the isoscalar and $I = 1/2$ sectors are almost not affected. From these results one could infer that the scalar sector needs the presence of tetraquark structures to be understood.

A similar calculation to the present work was performed in Ref. [35] describing the $a_0(980)$ and the $f_0(980)$ in terms of K-K molecules. Let us note that the color structure of a tetraquark is more involved than a meson-meson molecule, being possible to obtain a color singlet not only through $1_c \otimes 1_c$ (a meson-meson molecule) but also through $8_c \otimes 8_c$ (colored-qq states) [36]. Our results indicate a more involved structure for the $a_0(980)$ and the $f_0(980)$. These states present an important colored-qq component (31% of the total wave function) besides the meson-meson structure considered in Ref. [35].

6. Conclusion

We have obtained candidates for all light scalar mesons reported in the literature. Our results suggest that there would be some states, as it is the case of the $a_0(980)$ and $f_0(600)$ that would be a mixture of a $q\bar{q}$ and tetraquark structure, but it definitively assigns a tetraquark structure to the $f_0(980)$ and the $\kappa(900)$.

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References


STRUKTURA SKALARNIH I D MEZONA

Proučavamo spektar q q pretpostavljajući kiralni model sastavnih kvarkova. Dobiva se dobro slaganje s eksperimentalnim podacima od lakih (vektorskih i pseudoskalarnih) do teških mezona. Proučavamo i nova stanja D izmjerena u više B-tvornica. Lake mezone 0++ analiziramo kao parove q q ili kao tetrakvarkove.