ISSN 1330–0016
CODEN FIZBE7

# GENERAL RELATIVISTIC DOMAIN WALL IN LYRA GEOMETRY

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Received 24 July 2002; revised manuscript received 24 March 2004
Accepted 29 March 2004 Online 7 February 2005

An exact solution for the thick domain wall in Lyra geometry is found. The energy density decreases on both sides of the wall and space-time is reflection symmetric with respect to the wall. The space-time shows that the thick domain wall will not collapse. It is shown that the gravitational field experienced by a test particle is repulsive.

PACS numbers: 98.80.cq, 04.20.jb UDC 530.12

Keywords: Lyra geometry, thick domain wall, energy density, no collapse, repulsive gravitational field

## 1. Introduction

At the early stages of its evolution, the Universe underwent phase transitions and as a result, several topological defects occured, namely, domain walls, cosmic strings, monopoles and textures [1,2]. The existence of domain wall is associated with the breaking of a discrete symmetry, i.e., the vacuum manifold M consists of several disconnected components. So the homotopy group $\pi_0$(M) is non-trivial ($\pi_0(\mathrm{M}) \neq 1$) [2]. Recently, Pando, Valls-Gabaud and Fang [3] proposed that the topological defects are responsible for the structure formation of our Universe. In general relativity, domain walls are getting special attention due to their peculiar and interesting gravitational effects. In recent years, due to a new scenario of galaxy formation as suggested by Hill, Schram and Fry [4], the study of domain walls of finite thickness has gained renewed cosmological interest. Also, the study of domain walls and space-times associated with them, have received considerable attention due to their application in the structure formation of the Universe [5].

Many authors [6,7] have discussed non-static solutions of the Einstein scalar-field equations for thick domain wall. But these solutions have a peculiar behavior.

In these solutions, the energy scalar is independent of time while the metric tensor depends on both space and time. Subsequently, Wang [8] obtained a class of solutions to the Einstein's equations representing the gravitational collapse of a thick domain wall.

Thick domain walls are characterized by the energy momentum tensors

$$T_{ij} = \rho(g_{ij} + w_i w_j) + p w_i w_j \quad \text{where} \quad w_i w^i = -1 \,, \tag{1}$$

and $\rho$ is the energy density of the wall, $p$ is the pressure in the direction normal to the plane of the wall and $w_i$ is a unit space like vector in the same direction.

In last few decades, there has been a considerable interest in alternative theories of gravitation. The most important among them are the scalar-tensor theories proposed by Lyra [9] and by Brans-Dicke [9]. Lyra [9] proposed a modification Riemannian geometry by introducing a gauge function into the structureless manifold that bears a close resemblance to the Weyl's geometry. In general relativity, Einstein succeeded in geometrising gravitation by identifying the metric tensor with the gravitational potentials.

In the scalar-tensor theory of Brans-Dicke, on the other hand, scalar field remains alien to the geometry. Lyra geometry is more in keeping with the spirit of Einstein's principle of geometrisation, since both the scalar and tensor fields have more or less intrinsic geometrical significance. In the consecutive investigations, Sen [10] and Sen and Dunn [10] proposed a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equation based on Lyra geometry, which in normal gauge may be written as

$$R_{ik} - \frac{1}{2} g_{ik} R + \frac{3}{2} \phi_i \phi_k - \frac{3}{4} g_{ik} \phi_m \phi^m = -8\pi T_{ik} \,, \tag{2}$$

where $\phi_i$ is the displacement vector and other symbols have their usual meaning as in Riemannian geometry.

According to Halford [11], the present theory predicts the same effects within the observational limits, as far as the classical solar system tests are concerned, as well as tests based on the linearised form of field equations. Soleng [11] has pointed out that the constant displacement field in Lyra geometry will either include a creation field and be equal to Hoyle's creation field cosmology, or contain a special vacuum field, which together with the gauge vector term may be considered as a cosmological term.

Subsequent investigations were done by several authors in the scalar tensor-theory and cosmology within the framework of Lyra geometry [12]. Recently, Farook has studied some topological defects within the framework of Lyra geometry [13].

In this work we shall deal with the domain wall, assuming time-dependent displacement vectors based on Lyra geometry in normal gauge, i.e., the displacement vector

$$\phi_i = (\beta(t), 0, 0, 0) \tag{3}$$

and look further whether the domain wall shows any significant properties due to introduction of the gauge field in the Riemannian geometry.

## 2. Field equations and their solutions

In this section, we develop a general relativistic model of a plane-symmetric thick domain wall within the framework of Lyra geometry.

The metric ansatz describing a plane-symmetric space-time is

$$\mathrm{d}s^2 = \mathrm{e}^A(\mathrm{d}t^2 - \mathrm{d}z^2) - \mathrm{e}^C(\mathrm{d}x^2 + \mathrm{d}y^2)\,, \tag{4}$$

where $A = A(z,t)$ and $C = C(z,t)$

The energy-stress components in the co-moving coordinates for the thick domain wall are given by

$$T^t_t = T^x_x = T^y_y = \rho\,, \quad T^z_z = -p \quad \text{and} \quad T^z_t = 0\,. \tag{5}$$

The field equation (2) for the metric (3) reduces to

$$\frac{1}{4}\mathrm{e}^{-A}[-4C'' - 3(C')^2 + 2A'C'] + \frac{1}{4}\mathrm{e}^{-A}[C^{*2} + 2A^*C^*] + \frac{3}{4}\beta^2\mathrm{e}^{-A} = 8\pi\rho\,, \tag{6}$$

$$\frac{1}{4}\mathrm{e}^{-A}[4C^{**} + 3C^{*2} - 2A^*C^*] + \frac{1}{4}\mathrm{e}^{-A}[-(C')^2 - 2A'C'] + \frac{3}{4}\beta^2\mathrm{e}^{-A} = -8\pi p\,, \tag{7}$$

$$\frac{1}{4}\mathrm{e}^{-A}[-2A'' - 2C'' - (C')^2] + \frac{1}{4}\mathrm{e}^{-A}[2C^{**} + 2A^{**} + C^{*2}] - \frac{3}{4}\beta^2\mathrm{e}^{-A} = 8\pi\rho \quad (8)\,,$$

$$\frac{1}{2}[-C^{*\prime} + C^*(A' - C') + C'A^*] = 0\,. \tag{9}$$

[‘*’ and ‘′’ are differentiations with respect to $t$ and $z$, respectively. ]

To solve the field equations, we shall assume the separable form of the metric coefficients as follows

$$A = A_1(z) + A_2(t) \quad \text{and} \quad C = C_1(z) + C_2(t)\,. \tag{10}$$

From Eq.(9), we get

$$(C_1' - A_1')/C_1' = A_2^*/C_2^* = 1 - m\,, \tag{11}$$

where $(1-m)$ is the separation constant, This implies

$$A_1 = mC_1 \quad \text{and} \quad A_2 = (1-m)C_2\,. \tag{12}$$

For the reason of economy of space, we skip the details of the intermediate steps and write down the final results as

$$C_2 = \ln[\cosh(kt)]\,, \tag{13}$$

and

$$C_1 = \ln[\cosh(kz)]\,, \tag{14}$$

where $k^2$ is the separation constant.

Correspondingly, the energy density and pressure of the wall are given by

$$8\pi\rho = 8\pi p = -\frac{1}{4}(2m+1)\mathrm{sech}^3(kz)\,\mathrm{sech}(kt)\,. \tag{15}$$

The displacement vector is given by

$$\frac{3}{4}\beta^2 = \frac{1}{4}k^2[(3-2m)\mathrm{sech}^2(kt)]\,. \tag{16}$$

## 3. Properties of the solutions

For domain-wall solution, the physical requirement $\rho > 0$ would be satisfied provided $2m + 1 < 0$, i.e., $m < -1/2$.

It is interesting to note that when $m = -1/2$, $\rho$ and $p$ vanish, resulting in an empty space-time given by the metric

$$\mathrm{d}s^2 = \mathrm{sech}^{1/2}(kz)\,\cosh^{3/2}(kt)\,[\mathrm{d}t^2 - \mathrm{d}z^2] - \cosh(kz)\,\cosh(kt)\,[\mathrm{d}x^2 + \mathrm{d}y^2]\,. \tag{17}$$

One can see that the space-time of the solution is reflection-symmetric with respective to the wall. For a thick domain wall, it is desirable that the pressure and density decrease on both sides of the wall away from the symmetry plane, and fall of to zero as $z \to \pm\infty$.

We also see that $\rho$ and $p$ have a single maximum at $z = 0$ and tend to zero as $z \to \pm\infty$.

Thus, the solutions given in (12) – (14) represent a single domain wall with its center located at z = 0.

Here the energy density and pressure of the wall are space and time dependent and the space-time has no particle horizon, as one can show from Eqs. (12) – (14) (which is contrary to the case studied in Refs. [6] and [7], but similar to Wang's wall [8]). However, in contrast to the Wang's wall, our domain wall in Lyra geometry never collapses.

Another aspect of the domain wall is the effect on test particle in its gravitational field. Let us consider an observer with the four-velocity given by

$$V_i = \cosh^{m/2}(kz)\,\cosh^{(1-m)/2}(kt)\;\delta_i^t\,.$$

Then we obtain the acceleration vector $A^i$ as

$$A^i = V^i\, X_k\, V^k = \frac{1}{2}(mk)\ \tanh(kz)\,\mathrm{sech}^m(kz)\ \mathrm{sech}^{(1-m)}(kt)\ \delta^i_z\,. \qquad (18)$$

Since the condition of positive energy implies $m < -1/2$, so $A^z$ is negative. It follows that an observer who wishes to remain stationary with respect to the wall must accelerate towards the wall. In other words the wall exhibits a repulsive nature to the observer.

The repulsion of the wall is attributed to the fact that the pressure of the wall in the perpendicular direction is negative, which is similar to the case study in Refs. [6] and [7], but contrary to the Wang's domain wall.

Thus, our thick domain in Lyra geometry exhibits peculiar features: some properties are similar and some are contrary to The Wang's domain wall, and at the same time, some properties are similar and some are contrary to the Goetz and Mukherji's domain wall. It seems that in Lyra geometry, Weyl's concept of gauge, which is essentially a metrical concept, is modified by the introduction of a gauge function in the structureless manifold.

We note that the displacement vector will not exist after infinite time. For future work, it will be interesting to study different properties of other topological defects within the framework of Lyra geometry.

*Acknowledgements*

We are thankful to Prof. S Chakraborty and Dr. S. Chatterji for helpful discussion. F.R. wishes to thank IUCAA for providing the research facilities.

We are also grateful to the referee for pointing out the errors of the solutions in the earlier version.

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## OPĆI RELATIVISTIČKI GRANIČNI ZID U LYRINOJ GEOMETRIJI

Našli smo točno rješenje za debeo granični zid u Lyrinoj geometriji. Pad gustoće energije na obje strane zidnog vremena-prostora je simetričan u odnosu na zid. Vrijeme-prostor pokazuju da se zid ne urušava. Gravitacijsko polje koje osjeća ispitna čestica je odbojno.