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Maja Cindrić, PhD, Assistant Professor
University of Zadar
Department of Teacher and Preschool Teacher Education
mcindric@unizd.hr
https://orcid.org/009-0005-9405-787X

Josip Cindrić, lecturer
University of Zadar
Department of Teacher and Preschool Teacher Education
jcindric@unizd.hr
https://orcid.org/0009-0002-5073-5438

LEGACY OF FIRST ARITHMETIC TEXTBOOKS IN CROATIA ON TODAY'S MATHEMATICS TEACHING

Abstract: *The first books in Croatia that teach the reader the basics of arithmetic, were written in the middle of the 18th century. Arithmetica Horvatszka (1758), written by Mihajlo Šilobod, and Aritmetika u slavni jezik ilirski (1766) by Mate Zoričić were published in separate parts of Croatia; the former was published in the part of Croatia that was under Austro-Hungarian rule and the latter under the Venetian rule. Although they were not directly intended for school work, the authors wanted to teach the general population the basics of numbers and arithmetic through these books. This paper provides a brief analysis of the calculation methods from that time, as well as the forms of instructions the authors gave to their readers.*

Keywords: *addition algorithm, casting out nines subtraction, algorithm*

INTRODUCTION

From the 10th to the end of the 20th century, Croatia was not an independent state but was a part of various superpowers. The Croatian people have rarely had an influence on the creation of any form of policy, including education policy. Currently, Croatia is considered a young state, created by the disintegration of Yugoslavia, which formerly incorporated Slovenia, Bosnia and Herzegovina, Serbia, Montenegro and Macedonia. Therefore, Croatia's educational structure is a part of the historical heritage shared by the nations of other states of the

former Yugoslavia. Similarities with other countries have historical roots dating back to the 18th century. The northern and central parts of the former Yugoslavia, Slovenia, Croatia and Serbia were largely part of Austro-Hungary, and a smaller part was once under the jurisdiction of the Venetian Republic, only to fall under the Austrian Empire after Napoleon's brief rule. At the time of great turmoil, national consciousness was being born in all three provinces of the Austrian Empire, and this birth was most evident in culture and education.

In the 18th century, part of Croatia along the Adriatic Sea was under the rule of the Venetian Republic and in a short period under French rule during Napoleon, while the continental part of today's Croatia was a part of Austro-Hungary. The majority of the Croatian population lived in rural areas, they were uneducated and oppressed, and their basic income came from agriculture and day labor. According to Karp and Schubring (2014), the period before the introduction of formal compulsory education is called the premodern period in Europe, and it lasts from 1500 to 1800. In the Croatian territory, society started to ideologically shift at the beginning of the 18th century toward the ideas of Enlightenment and the education of the general public. The beginning of modern education in the Croatian territory is considered to be 1777, when the document *Ratio educationis* was adopted for all countries of Austro-Hungary, including Croatia. Until then, primary schools were located in only a few city centers founded by the Catholic Church, primarily with the intention of educating future priests. In smaller communities, education took place under the auspices of the local church. In parish churches, children were taught the basics of reading, writing, and arithmetic, but schooling was not compulsory. Since a large part of the population lived on agriculture and day labor, the help of each member of the household was necessary, so it was not uncommon for children not to go to school to help in the household. If a child was recognized as fit to continue their education, they continued school in city centers only if the family could financially secure a life in the city, which was rare. Primary schools in the city centers were run mainly by the Jesuit order, whose teaching was based on the *Ratio studiorum* curriculum, according to which schools were divided into 'lower' and 'higher' schools. The lower schools lasted for five years and were based on the teaching of Latin, which was used in church rites, grammar, *humanitas* and rhetoric. The high school curriculum included logic, mathematics, physics, ethics, and metaphysics, with continued learning of the Latin language. All classes at both levels were conducted in Latin (Hrobec et al., 2017). Individuals who wanted to strengthen the consciousness of the Croatian people through education did so in various ways, and the most important one was writing books in the Croatian language in a simple vocabulary addressed to the general population, and because of it, the period is also called the Enlightenment. Among others, the first arithmetic textbooks by Mihajlo Šilobod and Mate Zoričić were published in that period. Although they

were not textbooks in the scholastic sense, they were used to teach the reader arithmetic (Dimić, 1958). Under the strong influence of the Enlightenment ideas, numerous books were published, mostly of a literary nature, 16 in Croatian with the purpose of indirectly teaching the population useful and practical knowledge, such as strengthening agricultural skills for better yields and housekeeping. Following this trend, the first two arithmetic textbooks were created in Croatian: *Arithmetitika Hotvatszka* by Mihajlo Šilobod and *Aritmetika u slavni jezik ilirski* by Mate Zoričić. Both addressed the general public, primarily convincing them of the benefits of knowing how to calculate. In the introduction, Zoričić assures the reader that he has seen many young men in Italy and European countries using their calculation skills to become good traders but also to manage money wisely in their own household. It is through examples of the handling money that both authors show all calculations. Money and measures of time, mass, and volume extend through all examples in both textbooks, but since they originate from two different territories under different state administrations, the textbooks use different currencies and different measures. Zoričić worked on the territory of coastal Croatia under Venetian rule, so in his examples he used *cekin*, *groš*, Dalmatian lyre (he calls it *libra*) and *soldin*, while Šilobod uses *talir* (he calls them *ranji*), *groš* and *krajcara* (Kolar-Dimitrijević, 2013, Šilobod, 1758). Similar to Zoričić, Šilobod strived to pass on the knowledge of arithmetic to the reader, thus increasing the quality of life of people in the territory of central Croatia and Slavonia. Šilobod himself was taught in Latin in the Jesuit schools of Austro-Hungary and was aware of the importance of learning in Croatian. Unlike Zoričić, who dedicated his book to young men for successful home economics and those who want to become tradesmen, Šilobod pointed out that his book could be the basis for those who will teach children and hence paved the way for the soon-to-be compulsory secular education, which began in Austro-Hungary with the reform of Maria Theresa in 1774, led by the Jesuit Felbiger (Felbiger, 1777).

THE IMPORTANCE OF THE FIRST ARITHMETIC TEXTBOOKS IN THE CROATIAN LANGUAGE FOR TEACHING MATHEMATICS TODAY

Teaching in primary and secondary schools in the Republic of Croatia today has been regulated by the National Curriculum since 2019. The national curriculum for mathematics, as for all other school subjects, paves the way for modern education. Unlike previous documents that are oriented toward the contents of the subject, the national curriculum is structured according to learning outcomes. Of particular importance to the teaching of mathematics is the dual system of outcomes, focusing both on the subject matter and process competencies such as presentation and communication, connection,

logical thinking, problem solving and technology application. The legislative framework thus directed the teaching of mathematics toward educational reform. However, the documents alone are not enough for a profound and high-quality reform of teaching mathematics. Teachers' attitudes and knowledge are a key factor when implementing changes and making adjustments according to modern needs.

The Croatian school system inherited the structure and division of education from the former Yugoslav era, which Yugoslavia instead inherited from the 18th century. In Croatia, schools are divided into primary and secondary schools. Elementary school lasts eight years and is divided into lower and higher grades, and it is compulsory for every child. Secondary schools are divided into general (the so-called 'gymnasiums') and vocational schools, which can last three to four years. A total of 140 hours a year are devoted to teaching mathematics in primary school, while in secondary school, the number of hours varies depending on the type of school. The teaching of mathematics in the lower grades is taught by a teacher who teaches other subjects as well. Study programs for initial teacher education in the lower grades of primary school do not provide a strong mathematical background. Some research shows that teachers have positive attitudes toward problem-oriented teaching of mathematics, but when faced with the selection of activities for teaching, they always give priority to traditional arithmetic problems (Mišurac Zorica et al., 2013). External evaluations of the knowledge of ten-year-olds in the Republic of Croatia have been implemented since 2008 in the form of national exams and TIMSS (The Trends in International Mathematics and Science Study). They indicate that Croatian students in the lower grades of primary schools are somewhat successful in coping with arithmetic problems, while statistically speaking, the results are significantly below the average of the participating countries in regard to various forms of application of mathematical knowledge. Based on these indicators, it is evident that the teaching of arithmetic is still a Croatian teachers' focus. Arithmetic knowledge, especially knowledge of written algorithms, is considered to be central mathematical knowledge by teachers. They mostly trust the now traditional forms of written algorithms and emphasize their importance, thus placing mathematics itself in second place, which does not depend on the arithmetic process itself. This is not meant to diminish the value of arithmetic itself as part of mathematics, but it is meant to achieve a balance with other educational goals of teaching mathematics. The systematization of the history of teaching mathematics through the centuries provides all participants with insight into the educational process on the transformation of educational goals depending on the needs of modern society. The first arithmetic textbooks remain the individual efforts to improve the society in which they lived through education. In a similar way, it is necessary to understand the importance of following trends in teaching mathematics today

with the aim of preparing the population for life in the modern world, a world of rapid economic and technological growth and development. The analysis of Zoričić's and Šilobod's arithmetic textbooks will give an insight into the forms of performing written algorithms that differ from the ones we have today, which are considered traditional in Croatia.

Mate Zoričić and Mihail Šilobod were educators who were primarily teachers. They listened to their fellow citizens, their needs and problems, and the potential of children whom they taught the basics of literacy. Mate Zoričić, a Franciscan priest, lived and worked in Dalmatia, the southern part of the coastal Croatian territory under Venetian rule, where like many priests, he taught children, including arithmetic (Dadić, 2001). Due to living in the coastal part of Croatian territory, his book was created under the influence of the Italian *scuolla d'abaco*, and he himself stated that the knowledge conveyed in the book he learned from the great teacher Figatelli, referring to Giuseppe Maria Figatelli, who published the book *Trattato Aritmetico* in 1678 in Venice. *Trattato Aritmetico* is an arithmetic textbook used by many teachers at the time and is an integral part of many monastic libraries. It is a template based on which Zoričić wrote, and it is assumed that Šilobod used it as a template as well (Borić, 2016). Although some sources claim that Zoričić simply copied the *Trattato Aritmetico*, the comparison of both books reveals significant differences, as well as the modifications that Zoričić introduced in relation to Figateli's book. Zoričić understood the environment in which he lived and the prior knowledge with which the reader approached the work, which is why he used the everyday situational contexts in examples and applied elements close to his contemporaries. When talking about addition, he uses situations in which the protagonist has to determine the total cost when buying wood, oil, horses and similar things. In contrast, Figatelli adds and subtracts money, and when multiplying and dividing, he shows a procedure without linking it to real life situations. Unlike Figatelli, Zoričić adds and subtracts through dozens of examples, while Figatelli briefly describes addition and subtraction in one procedure for each action and moves very quickly from calculating with natural numbers to fractions, exponentiation, roots, binomial theorem, and the triple rule. It can be seen from Figateli's textbook that it was not written for a beginner in arithmetic but for someone who had mastered the basics of calculus with natural numbers. Mihajlo Šilobod Bolšić, like Mate Zoričić, was a priest, but unlike Zoričić, who was a Franciscan, Šilobod belonged to the Jesuit order. He was born near Zagreb in a noble family and had the opportunity to study, first in Zagreb and then in Vienna. By accepting the priestly service, he acted as a cultural and educational activist incentral Croatia (Veljan, 2020). His book *Arithmatika Horvatszka* is the first mathematical textbook printed in the Croatian language. Although Šilobod's and Zoričić's arithmetic were created independently of each other (Borić, 2016), they were written in the same style through a dialog between the writer and the reader. The same style of dialog was

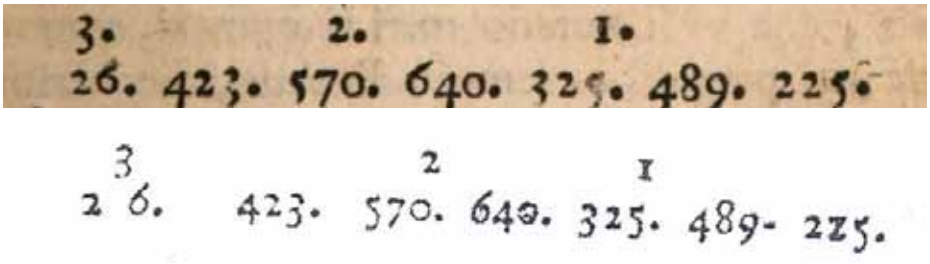
later adopted by Franjo Močnik, whose textbooks are considered an important milestone in the methodics of teaching mathematics (Hladnik, 2015; Hojan, 2014). Like Zoričić, Šilobod relied on Figatelli's book *Trattato Aritmetico* as well, although it is not possible to determine whether this was a result of him taking over the writing style or whether such a style of teaching was common at the time. In Šilobod's *Arithmetika Horvatszka,* it is possible to find some elements that also appear in the *Trattato Aritmetico*. In his book, Figatelli uses the "cast out nines method" to check multiplication calculations, and a similar procedure is used by Šilobod when checking addition. Unlike Zoričić, Šilobod spends less time adding, subtracting, multiplying and dividing natural numbers and devotes most of his time to calculating fractions, the triple rule and problems from everyday life. Comparing Zoričić's and Šilobod's textbooks on arithmetic, we can see that Šilobod, like Figatelli, does not devote much time to calculation with natural numbers, yet it seems that calculation is a form of repetition and introduction to the rest of the content, while the emphasis of Zoričić's arithmetic is on calculation with natural numbers. The reasons behind it can be various, and it is considered that Zoričić intended the book to those who may encounter the basics of numbers and arithmetic for the first time, while Šilobod intended the book to those with a good foundation of mathematical knowledge. Thus, in the introductory part, Zoričić begins with the instruction that each number is composed of many 'ones' and gives an example of the number 3 being composed of 1, 1, 1. His instructions similarly indicate that he is addressing a beginner in dealing with numbers, while Šilobod starts with the use of digits in composing multidigit numbers even without referencing the formation of numbers within the decimal system. Whether it is about the structure of the group of readers whom they address or about the quality of methodical instructions, about the failure to understand the way of teaching - there is not enough data to determine.

INFLUENCE OF MARIO GIUSEPPE FIGATTELI ON ŠILOBOD'S AND ZORIČIĆ'S ARITHMETIC

As mentioned earlier, Zoričić describes numbers to the reader as a set of 'ones', after which he tries to explain the system of decimal values by introducing digits and weighting factors of the decimal system, although he does not explain the grouping of ones into tens, tens into hundreds, etc. Along with the Croatian names for numbers and arithmetic operations, he also cites Italian terms, with the aim of the reader effectively participating in trade with Italian merchants, who, according to Zoričić, used their knowledge to exploit the local population for their ignorance of numbers and calculating. In the introductory part of reading multidigit numbers, it is possible to see an example of the number 26 423 570 640 325 489 225, which is also in the *Trattato Aritmetico* (figure 1).

Figure 1

Figatelli's (upper figure) and Zoričić's (down figure) example



This makes it possible to conclude that Zoričić used it when writing his arithmetic, but this is the only such example. After explaining the formation, notation, and reading of numbers, he introduces addition as the union of many numbers and links addition to gain and cost. Following this brief explanation of the example of calculating the total cost for wood, meat, wine and horse, he describes the written procedure of addition. After a few more examples of cost, he tries to separate the calculation from the situational context, while stating that we ought to write down the cost for anything we want, while in the next example he doesn't even use the term cost. From the educational aspect, he starts from concrete examples and moves toward generalization but never separates concreate activity such as sharing from mathematical operation in this case division.

The structure of Zoričić's arithmetic does not differ much from Šilobod's. In the first part, he explains reading and writing numbers, as well as the decimal number system and arithmetic operations. In the second part he introduces the reader to fractions, but he does it more vaguely than Šilobod, and the third part is dedicated to trade arithmetic, while the fourth part briefly brings the celestial circle emphasizing the importance of knowing the position of stars for good agricultural yields.

In the first part, he introduces the reader to numbers, their writing and reading. Unlike Šilobod, Zoričić begins the first part with the fact that a number is a multitude of 'ones', meaning that 3 becomes composed of 1, 1 and 1. He places the arithmetic operations of addition, subtraction, multiplication and division in the situational context of everyday life related to trading and use of money and solves the stated problems via written algorithms. Zoričić, unlike Šilobod, emphasizes arithmetic operations with natural numbers, which he repeats in approximately twenty examples, and he begins multiplication with a multiplication table that he calls "one time one", while Šilobod refers to it only as a "table". We also find the difference between these two types of arithmetic in the written subtraction algorithm. As stated, Šilobod uses the algorithm of changing the place value, while Zoričić uses the algorithm of "constancy of

difference" for subtraction, while for multiplication and division, they both use the same algorithms described earlier.

He introduces fractions to the extent to which they serve merchants, says that in trade business, they serve only to denote portions of the money, and links them with the remainder in division. He does not go further into the elaboration of fractions because he believes that the trader does not need it. Zoričić's arithmetic is vaguer in the part about the triple rule and only shows it on a few basic examples.

ARITHMETICA HORVATSZKA BY MIHAIL ŠILOBOD

The structure of Šilobod's arithmetic does not differ much from similar books on arithmetic of that time (Bjarnadottir, 2014). It consists of four parts: the introduction to natural numbers alongside arithmetic operations, the second part talks about fractions, the third part is about business arithmetic, and the fourth part includes mixed content.

Introduction to numbers starts from the description of the formation of numbers in the decimal system, and it is mostly based on reading and writing numbers. The author introduces decimal place values, without referring to the fact that all numbers are composed of 'ones', which the reader probably needed for an in-depth understanding of numbers. When introducing arithmetic operations, there is no elementary addition of numbers up to 20; instead, before the first addition he argues that one must know how to add single-digit numbers, and in two examples he states that the reader must know that 3 and 5 make 8, that 10 and 6 make 16, which implies that he is addressing the reader who has mastered counting, writing numbers, and number facts for addition. He introduces all four arithmetic operations with natural numbers through concrete situational contexts understandable to people of the time, such as buying, selling, borrowing, and measuring land and time. He focuses on instructions for performing a written arithmetic algorithm with a detailed explanation of the procedure. He does not use particular terms for arithmetic operations but instead assigns the names of physical actions to them: *zbrajanje – dodavanje, oduzimanje – odvajanje, množenje – povećavanje, dijeljenje – razdjeljavanje* (engl. Addition – adding, subtraction – separation, multiplication – increase, division – dividing). Such practice remained in Croatian textbooks until the 1970s (Muhvić, 1972), and in teaching practice, it remains even today. Problems concerning terminology relate primarily to the narrowing of the structure of the concept of arithmetic operation but also to misconceptions that often occur among students, such as equating the arithmetic operation of division with the physical action of sharing, which is not necessarily fair sharing.

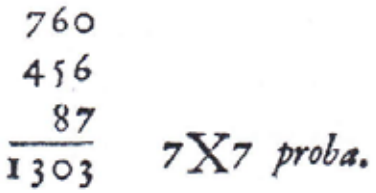
If we look at the situational contexts that Šilobod uses for addition and subtraction, in the additive structure, we find a join – unknown final state and

part-part-whole for items of addition and separate – unknown final state and separate – unknown change for subtraction.

Šilobod, however, focuses on the procedures of written calculation, in which he instructs the reader of the manner of writing and speech that accompanies the calculation, but he does not go into the reasons that justify these procedures. The written addition algorithm begins with an instruction on how to inscribe the addends, as well as a note to make sure that the numbers are aligned on the right side. Šilobod uses the *casting out nines* method to check the accuracy of the calculation, a similar version of which can be found in Figatelli. The verification consists of comparing the remainder after dividing the sum of the digits of all the addends by 9, as well as the remainder after dividing the sum of the digits of the addition results by 9. In the example (figure 2), the sum of the digits of the addends is 43, while the sum of the digits of the total sum is 7; 43mod9 = 7 and 7mod9 = 7, whereby Šilobod proves the accuracy of the addition.

Figure 2

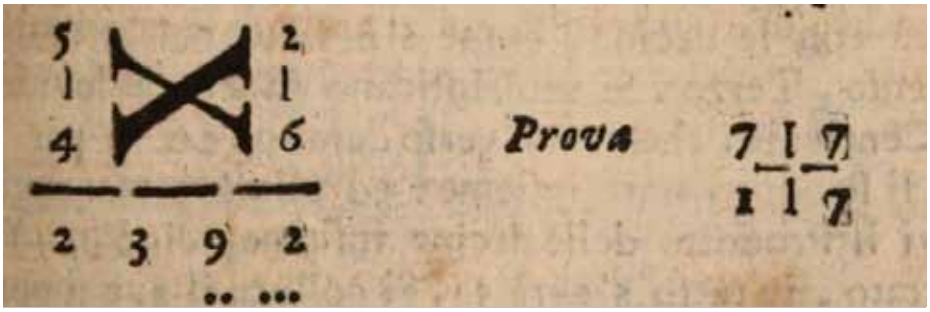
Casting out nines method in Šilobod's arithmetic



Figatelli uses a similar version of the *casting out nines* method to verify multiplication (figure 3), where the top number in the cross is the remainder of dividing the first factor by 9, the bottom number in the cross is the remainder of dividing the second factor by 9, the left number in the cross is the remainder of dividing products by 9, and the right number in the cross is the product of the upper and lower numbers in the cross. If the left and right numbers in the cross are equal, Figtelli concludes that the multiplication is correct.

Figure 3

Casting out nines method in Figatelli's arithmetic



Casting out nines method is over 1000 years old (Bruckheimer et al., 1995) and was common in the literature during the Renaissance, although there was evidence even then that such a method was not certified to be accurate in general, because there are counterexamples that prove it to be incorrect, which is what Zoričić states in his arithmetic.

Šilobod also verifies the subtraction calculation with the usual sum of the difference and the subtrahend. He introduces subtraction in the same way as addition, with 2 examples of subtraction of numbers up to 10, after which he immediately proceeds to the explanation of the written procedure, where he uses the algorithm of "constancy of difference".

Šilobod labels the multiplication process as 'increase' and introduces it through the example of two times three, without explaining why it is six, so it is possible to assume that the name increase itself indicates that the number three needs to be increased two times. From the situational contexts of multiplicative structures in the examples of multiplication and division, he uses equal groups - unknown whole and equal groups - unknown group size (partitive division). In the written multiplication procedure, the factors are written one below the other; he starts by multiplying the digit from 'one place' of the second factor with the first factor and writes the partial products with a shift to the left.

Unlike addition and subtraction, where he does not refer to number facts, multiplying is accompanied by a multiplication table; although not at the beginning, toward the end of the multiplication unit, the reader is instructed to learn how to multiply single-digit numbers according to the multiplication table so that they can eventually multiply in a fast and efficient way.

In the unit about fractions, he begins his discussion with:

- Reading fractions, using the fraction line and the names numerator and denominator

- Presenting the ‘one’ in the form of a fraction
- Fractions with ‘one’ as the denominator
- Writing natural numbers in the form of a fraction
- Equality of two fractions

He does not use the equality sign again but places an X sign that indicates that the fractions are verified for equality. Furthermore, he introduces mixed numbers, determines the part (expressed by a fraction) of a fraction, expands and shortens fractions, reduces fractions to a common multiple, adds, subtracts, multiplies, and divides fractions. When reducing to a common denominator, all denominators are rather prime numbers, which makes the common denominator always a product of the denominators. When expanding and reducing fractions, Šilobod indicates the equality of the initial and the reduced fraction, but calls them “larger” and “smaller” fractions, which creates unnecessary misconceptions among students, as it does with the names of arithmetic operations. With reduction, the greatest common divisor is determined by the Euclidean algorithm and the method of gradual reduction of the fraction to the irreducible fraction. He performs the multiplication of fractions by multiplying the numerator by numerator and denominator by denominator, and by the same analogy, he performs the division of fractions by dividing the numerator by numerator and denominator by denominator through examples in which they are divisible. In examples where numerators and denominators are not divisible, he multiplies the denominator of the dividend by the numerator of the divisor and the denominator of the divisor by the numerator of the dividend and then divides these multiplications by writing the result in the form of a mixed number or a fraction.

After becoming acquainted with natural numbers and fractions, Šilobod moves on to the section on business arithmetic in which he applies arithmetic theory. With business arithmetic, he starts with the proportional and inversely proportional quantities, citing their Latin terms: *regula trium directa* and regula trium inversa, and continues with *regula trium composita directa*, *regula societatis*, *regula mistionis alligationis, regula falsi* and *regula caci*. The examples he uses in this section on business arithmetic and that he separates into numerous rules also appear in today’s teaching practice; however, they are not separated according to the rules of solving but are solved by applying mathematical knowledge.

Business arithmetic is followed by a section on arithmetic and geometric sequences and mixed problems. Mixed problems apply knowledge of numbers, logic, combinatorics, and measurements. Here are the following problems as an example:

- How to guess who imagined which number
- How to guess who stole what

- How to find the wicked among the good
- How to determine the width of the lake
- How one can one think it's Saturday and the other Sunday
- How can you write 10 without a zero
- How to get the same number on the tips of a triangle
- How to make a magic square - *Quadratum magicum*

ARITMETIKA U SLAVNI JEZIK ILIRSKI BY MATE ZORIČIĆ

Zoričić begins teaching by introducing the reader to the concept of numbers; he writes and reads numbers in both Croatian and Italian. Unlike Šilobod, he devotes more time to writing numbers and tries to explain decimal values, but he does not mention their relationship. He continues by introducing the four arithmetic operations, which is the focus of his arithmetic. He presents the arithmetic operation of addition under the name *skupljanje* (engl. collection), and, much like Šilobod, links it with physical actions. Following the name of the arithmetic operation of *skupljanje*, he introduces addition through situations from the category join - unknown final value determining the total cost. Since his main goal is to prepare the reader for everyday problems, he uses it to justify the application of the term cost, even though he also uses the term cost in subtraction problems. He says that the addition is a union of numbers and does not go into the explanation of adding up to 20, but immediately starts with word problems in which, in order to determine the total cost for wood, meat, wine and horses, he adds four three-digit numbers. In the next three examples, he calculates the cost of specific products, further generalizing and claiming that addition can be performed in the same way, regardless of what is consumed. He links subtraction, as well as addition, with the physical action of separation, thus calling it odvajanje (engl. separation). In word problems that are all within the separate category - unknown final value, he associates the initial value with earnings or benefits while associating change with cost. He uses written algorithms to subtract and add and calls them *računanje s perom* (engl. *plume arithmetic*). He subtracts in writing according to the principle of constancy of difference. He verifies subtraction with addition, and unlike Šilobod, who verifies addition using the cast-out nines method, Zoričić mentions this method while verifying multiplication, as well as Figatelli. Unlike Figatelli, he criticizes this method, claims it to be unreliable and finally proves it with one example.

Multiplication is the only arithmetic operation that he does not link with any physical operation but simply calls it multiplication, according to the Italian term *moltiplicatione*. After the example '22 times 2', he notes that, for multiplication, it is important to learn to use abacus at least up to 10 times. The abacus he mentions is a multiplication table, but not in the same form as

Šilobod's table, but as a list of products of numbers up to 10. He begins the list with the squares of the numbers, then starts with the multiplication of number 1, then number 2 and so on, until the number 10. Although he never mentions the commutative property, in the list of products, he does not repeat those he had already stated. It is interesting that in the subsequent multiplication operations, he emphasizes that the first number in the arithmetic inscription should be the greater of the two numbers he multiplies. When listing factors in the written operation, he writes them one below the other, while below them he writes partial products, with a shift to the left; he does not justify this mathematically, but gives the same example where partial products are not moved to the left, which he finds to be sufficient proof of a correct procedure. He continues to give examples of multiplication by 10, 100, etc., and multiples of 10, 100, etc., while claiming that no time should be wasted on these calculations.

Upon division, he reintroduces the name *razdjeljivanje* (engl. dividing), linking the arithmetic operation to the physical operation of sharing, and describes it: *how one number can be divided into so many equal parts*. In word problems, he uses only partitive division situations, in which, for example, 15 coins should be divided among 3 people so that each gets an equal amount. In the very first example, he links division with multiplication because to determine how 15 is divided by 3, it is necessary to see many times 3 s fit into 15. After that, he introduces the concept of the remainder after the division operation, and he also introduces a written form of the division operation. In the written operation, he initially uses a longer method, which involves partial division, multiplication, and subtraction. In later examples, he states that two more forms of written division can be used that are shorter but also more difficult because some actions have to be done 'in the head'. He verifies the accuracy of division by multiplication. Although in the next chapter on fractions he links fractions with division, in the chapter on division he clearly states that the dividend must not be greater than the divisor, as it is not possible to divide the numbers in that case.

Zoričić devotes little time to fractions. He writes and names the numerator and denominator of a fraction and the proper and improper fractions. After the introduction, he writes fractions equal to 'one', followed by fractions that can be equated with an integer. Without connecting to real contexts, he then follows by comparing fractions and determining a fraction of a fraction. This concludes the chapter on fractions because, in his opinion, an ordinary trader and landlord does not need anything more than fractions.

As briefly as with fractions, he talks about the triple rule, ratios and conversions of money, measures of volume and mass. In the end, he gives a mathematical circle, which in the manner of astrology states which year will be fertile according to the planetary position and which will be nonfertile.

CONCLUSION

The value of these two arithmetic textbooks is indisputable; they were the first to be written in the Croatian language with the aim of providing the general population with opportunities to develop arithmetic skills and acquire knowledge to improve the quality of their own lives. These textbooks also provide insight into the etymology of terminology in mathematics, which is important in the formation of high-quality mathematical concepts and their scope. The terms themselves can cause the development of misconceptions among students, for example, linking the arithmetic operation of division with the physical operation of sharing (which is not necessarily fair sharing). Thus, Šilobod calls multiplication – *povećavanje* (engl. increase), which directly indicates that he believes that multiplication increases, which is a common misconception among students and the reason why many have difficulty acquiring knowledge of rational numbers (Carpenter et al., 1996). From the overview of terminology given in Table 1, we can see that both authors link arithmetic operations with physical operations but also that they use different versions of the term, which indicates that both textbooks were created independent of each other but also that they were written according to a similar or the same template.

Table 1

Comparison of the nomenclature of mathematical operation by Šilobod and Zoričić

	Aritmetika horvatzka	***Aritmetika u slavni jezik illiricki***
Addition	Pridavanje	Skupljanje
Subtraction	Odnimanje	Izdvajanje
Multiplication	Povećavanje	Množenje
Multiplicand	Povečitelj	Broj od množenja
Multiplier	Povećan	Umnožitelj
Product	/	Prod
Division	Razdjeljivanje	Razdjeljenje
Dividend	Djeljeni (onaj koji je dijeljen)	Broj od razdjeljenja
Divisor	Razdjelitelj	Razdjelitelj
Quotient	Razdjeljen	Prod
Numerator	Brojtel	Brojanoč
Denominator	Imenitelj	Imenovaoc
Operation	/	Djelovanje

Whether it is Giuseppe Maria Figatelli's *Trattato Aritmetico* or some other textbook of the time is difficult to say, but it is indisputable that the models

of arithmetic are similar. The same model of writing an arithmetic textbook of that time is visible in *The New Serbian Arithmetic or a Simple Way toward Reckoning*, published in 1767 in Venice in the Serbian language and in Cyrillic alphabet (Nikolić, 2009). In addition to the terms mentioned earlier, which, in educational terms, can create obstacles for students in mastering the integrity of mathematical concepts, Šilobod also uses the terms 'larger' and 'smaller fraction' for reduced or extended fraction, which is also a common misconception that students have regardless of their success in mathematics.

The style in which both Zoričić and Šilobod write is a conversational style. They write all operations as if they were to orally express it while solving a problem. For example, when writing algorithms, they record in detail the procedures for each of the problems from which it is possible to see the use of the constancy of difference method in the written subtraction algorithm. They do not use mathematical signs for arithmetic operations, nor do they use the sign of equality, but they do use straight horizontal and vertical lines, which have the meaning of equality. Šilobod also uses the increased X sign, which he uses for verifications and as a sign for cross-multiplication in fractions.

By considering the legacy that these two arithmetic textbooks leave to today's elementary mathematics teaching, which is dominated by arithmetic, we can see the roots of some procedures, forms of work, methods and principles of teaching mathematics in general. One of the forms of work that marks both textbooks is the dogmatic - instructional approach in which, without any evidence and explanation, the presented calculation procedures are expected to be accepted and rehearsed a priori by the reader. Such an approach is not surprising for the time in question, but even today, it may be encountered in teaching practice. Šilobod's and Zoričić's arithmetic are undoubtedly the roots of the initial teaching of mathematics in Croatia, and to be able to follow and encourage the evolution of mathematics education in accordance with the needs of modern society, it is necessary to clearly determine where its beginnings lie. These beginnings should be observed from the standpoint of both times (their era and modern times) and the concomitant circumstances. In light of those circumstances, we can see the justification of the dogmatic instructional approach, which currently cannot be justified only by the fact that it is a traditional approach.

Today's mathematics textbooks for lower grades of elementary schools largely follow the traditional model that begins with an introductory task from everyday life, followed by a demonstration of the calculation procedure, practicing that procedure, and in the final part, the procedure that is applied in word tasks. Šilobod and Zoričić approach arithmetic calculation through introductory word tasks through which they demonstrate the calculation procedure. After the introductory example, they demonstrate in the same way several more examples, which are specific due to the selection of numbers

they use. In their books, it is not possible to find tasks that are directed to the reader for independent work, and only Šilobod has the application of calculus through problem tasks in a separate chapter. In contrast to today's textbooks, which, although they have a traditional approach, the authors try to explain to the readers the mathematical and logical justification of a particular procedure, Zoričić and Šilobod do not engage in explanations. For example, in the written subtraction procedure 103 - 86, based on the constancy of the difference, both instruct readers to say "13 less 6 is 7, 10 less 9 is 1 and 1 less 1 is 0" without explaining why we use 13 instead of 3, 10 instead of 0, etc. Through the teaching of mathematics today, judging by textbooks, teachers use the place values and constancy of the difference, which is supported by the statement that "if the same number is added to the minuend and subtrahend, the difference will not change". As a result of the above, we can determine that arithmetics teaching, regardless of the traditional model in the approach structure, has progressed in supporting the understanding of the computation process.

Regardless of their attitudes toward teaching mathematics, teachers in the process of making decisions about teaching methods more often resort to traditional models, which reflects on their students. They rarely resort to research and problem-oriented teaching because they are unsure of its outcome. Transferring such models from teacher to student-future teacher helped keep the parts of Zoričić's and Šilobod's arithmetic alive even today. It is also interesting that some parts that would be an excellent contribution to problem-based and research-oriented teaching in terms of modern mathematics education have disappeared from contemporary teaching practice, such as Šilobod's problem questions at the end of the book, the examples of magic squares, or the operation of multiplying numbers from 6 to 9 using fingers as described by Zoričić in the section on multiplication.

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