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GENERAL REDUCTION FORMALISM FOR NLO CALCULATIONS IN PQCD[1]

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We discuss a generally applicable and systematic method to represent an arbitrary tensor one-loop Feynman integral, with N external lines and massless propagators, in terms of a basic set of eight scalar Feynman integrals with 2, 3 and 4 external lines. To demonstrate the practicality of the method, we calculate one of the one-loop Feynman diagrams with 6 external lines, which contribute to the hard-scattering amplitude of the process $\gamma\,\gamma \to \pi^+\,\pi^-$ at high momentum transfer in the context of pQCD.

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1. Motivation

The framework for analyzing exclusive processes (EP) involving large momentum transfer within the context of perturbative QCD (pQCD) is well established. Owing to the fact that the leading order (LO) predictions in pQCD do not have much predictive power, the inclusion of higher-order corrections is essential because they have a stabilizing effect, reducing the dependence of the LO predictions on the renormalization and factorization scales and on the renormalization scheme. However, only a few EP have been analyzed at the next-to-leading order (NLO).

The NLO analysis of EP in pQCD requires the evaluation of one-loop Feynman integrals with massless propagators. These integrals contain IR divergences (both soft and collinear) and need to be regularized. The most suitable regularization method for pQCD calculations is dimensional regularization.

Considerable progress has recently been made in developing efficient approaches for calculating one-loop Feynman integrals with a large number of external lines. As far as the calculation of one-loop N-point massless integrals is concerned, the most

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complete and systematic method has been developed by Binoth et al. [1]. However, it does not apply to all cases of practical interest. Namely, the method [1] usually breaks down during reduction of integrals in which the set of external momenta contains subsets comprised of two or three collinear on-shell momenta. Integrals of this type arise when performing the leading-twist NLO analysis of hadronic EP at large momentum transfer in pQCD.

With no restrictions regarding the external kinematics, in this paper we describe an efficient, systematic and completely general method for reducing an arbitrary one-loop N-point massless integral to a set of basic integrals. A more detailed description is given in Ref. [2].

2. Method

In order to obtain one-loop radiative corrections to physical processes in massless gauge theory, integrals of the following type are required

$$I^N_{\mu_1\cdots\mu_P}(D;\{\nu_i\}) = (\mu^2)^{2-D/2} \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{l_{\mu_1}\cdots l_{\mu_P}}{A_1^{\nu_1}A_2^{\nu_2}\cdots A_N^{\nu_N}}, \qquad A_i = (l+r_i)^2 + \mathrm{i}\,\varepsilon\,. \tag{1}$$

This is a rank P tensor one-loop N-point Feynman integral with massless internal lines in D dimensions, where l is the loop momentum, μ is the usual dimensional regularization scale and $\nu_i \in \mathbf{N}$ are arbitrary powers of the propagators. We denote the corresponding scalar integral by I_0^N. These integrals represent generalizations of the usual integrals in practical calculations, where $\nu_i = 1$. However, the most natural presentation of the reduction method discussed here is in terms of these generalized integrals.

Calculation of Feynman integrals usually proceeds in two steps: decomposition of tensor integrals to scalar integrals and reduction of scalar integrals to a set of basic scalar integrals for which analytic solutions are known.

We perform the tensor decomposition using the method originally proposed in Ref. [3]. On the basis of Feynman parameter representations of the tensor and scalar integrals, the following equation can be derived

$$\begin{aligned} I^N_{\mu_1\cdots\mu_P}(D;\{\nu_i\}) = & \sum_{\substack{k,j_1,\cdots,j_N\geq 0 \\ 2k+\Sigma j_i = P}} \frac{(4\pi\mu^2)^{P-k}}{(-2)^k} \left[\prod_{i=1}^N \frac{\Gamma(\nu_i+j_i)}{\Gamma(\nu_i)}\right] \\ & \times \left\{[g]^k[r_1]^{j_1}\cdots[r_N]^{j_N}\right\}_{\mu_1\cdots\mu_P} I_0^N(D+2(P-k);\{\nu_i+j_i\}), \end{aligned} \tag{2}$$

where $\{[g]^k[r_1]^{j_1}\cdots[r_N]^{j_N}\}_{\mu_1\cdots\mu_P}$ represents a symmetric (with respect to $\mu_1\cdots\mu_P$) combination of tensors, each term of which is composed of k metric tensors and j_i external momenta r_i (for example, $\{gr_1\}_{\mu_1\mu_2\mu_3} = g_{\mu_1\mu_2}r_{1\mu_3}+g_{\mu_1\mu_3}r_{1\mu_2}+g_{\mu_2\mu_3}r_{1\mu_1}$). With the decomposition (2), the problem of calculating the tensor integrals has been reduced to the calculation of the general scalar integrals. In this form of decomposition, it is not assumed that any four, out of N, 4-vectors r_i are linearly independent (non-exceptional kinematics) in the case when $N \geq 5$. Consequently, the number of terms in the decomposition (2) is larger in comparison with other methods (e.g. Ref. [1]), but its advantage is the applicability of Eq .(2) for arbitrary kinematics. Usually, the non-exceptional kinematics is not realized in EP owing to the appearance of collinear external lines in Feynman diagrams for the hard scattering amplitude.

As it is well known, the direct evaluation of the general scalar integral is a nontrivial problem. However, using recursion relations, the problem can be significantly simplified since the calculation of the original scalar integral can be reduced to the calculation of a certain number of simpler basic integrals.

The basis for the scalar reduction is the following identity [4]

$$0 \equiv \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{\partial}{\partial l^\mu} \left(\frac{z_0 l^\mu + \sum_{i=1}^N z_i r_i^\mu}{A_1^{\nu_1} \cdots A_N^{\nu_N}} \right), \tag{3}$$

where z_i $(i = 1 \cdots N)$ are arbitrary constants and $z_0 = \sum_{i=1}^N z_i$. The identity (3) is satisfied owing to the translational invariance of the dimensionally regulated integrals. After the differentiation and some algebraic manipulations [2], the identity (3) takes the form

$$\begin{aligned} \sum_{i,j=1}^N (r_j - r_i)^2 z_i \nu_j I_0^N(D; \{\nu_k + \delta_{kj}\}) &= \sum_{i,j=1}^N z_i \nu_j I_0^N(D; \{\nu_k + \delta_{kj} - \delta_{ki}\}) \\ &\quad - (D - \sum_{j=1}^N \nu_j) z_0 I_0^N(D; \{\nu_k\}) \, , \end{aligned} \tag{4}$$

where it is understood that $I_0^N(D; ..., \nu_l, 0, \nu_{l+1}, ...) \equiv I_0^{N-1}(D; ..., \nu_l, \nu_{l+1}, ...)$.

The relation (4) represents the starting point for the derivation of the recursion relations for scalar integrals. We have obtained the fundamental set of recursion relations by choosing the arbitrary constants z_i $(i = 1 \cdots N)$ so as to satisfy the following system of linear equations:

$$\sum\nolimits_{i=1}^N r_{ij} z_i = C, \quad j = 1, \ldots, N, \qquad r_{ij} = (r_i - r_j)^2, \tag{5}$$

where C is an arbitrary constant. It should be mentioned that, in the literature, the constant C is usually taken to be different from zero. It is precisely this fact that leads to the breakdown of the various scalar reduction methods. Namely, for some kinematics (e.g. collinear on-shell external lines) the system (5) has no solution for $C \neq 0$. However, if the possibility $C = 0$ is allowed, the system (5) will have a solution regardless of kinematics.

If (5) is taken into account, the relation (4), after a few manipulations [2], reduces to the recursion relation

$$C\, I_0^N(D-2; \{\nu_k\}) = \sum_{i=1}^N z_i I_0^N(D-2; \{\nu_k - \delta_{ki}\}) + (4\pi\mu^2)(D-1-\sum_{j=1}^N \nu_j) z_0 I_0^N(D; \{\nu_k\}), \tag{6}$$

where z_i are given by the solution of the system (5). This is a generalized form of the recursion relation which connects the scalar integrals in different number of dimensions [1, 4, 5].

By directly choosing the constants z_i in (4), so that $z_i = \delta_{ik}$, for $k = 1, \cdots, N$, we arrive at a system of N equations which is always valid

$$\sum_{j=1}^{N}(r_k - r_j)^2 \nu_j I_0^N(D; \{\nu_i + \delta_{ij}\}) = \sum_{j=1}^{N} \nu_j I_0^N(D; \{\nu_i + \delta_{ij} - \delta_{ik}\}) - (D - \sum_{j=1}^{N} \nu_j) I_0^N(D; \{\nu_i\}). \tag{7}$$

The solution of the system (7) with respect to $I_0^N(D; \{\nu_i + \delta_{ij}\})$, $j = 1, \cdots, N$, if it exists, represents a set of recursion relations.

By appropriate use of Eqs. (6) and (7), any one-loop Feynman integral can be reduced to a basic set of eight scalar integrals. The use of these equations in practical calculations depends on whether the kinematic determinants

$$\det(R_N) = \det\left(r_{ij}\right)_{N\times N}, \quad \det(S_N) = \det\begin{pmatrix} 0 & 1 \\ 1 & r_{ij} \end{pmatrix}_{(N+1)\times(N+1)} \tag{8}$$

are equal zero or not. We distinguish the following four different types of recursions:

- Case I: $\det(S_N) \neq 0$, $\det(R_N) \neq 0$; $(C \neq 0$ and $z_0 \neq 0)$,
- Case II: $\det(S_N) \neq 0$, $\det(R_N) = 0$; $(C = 0$ and $z_0 \neq 0)$,
- Case III: $\det(S_N) = 0$, $\det(R_N) \neq 0$; $(C \neq 0$ and $z_0 = 0)$,
- Case IV: $\det(S_N) = 0$, $\det(R_N) = 0$; $(C \neq 0$ and $z_0 = 0)$ or $(C = 0$ and $z_0 = 0)$,

where we have indicated the necessary choice for the constants C and z_0 in a way that the system (5) has a solution, and the most useful recursion relations emerge from (6). In practical calculations, the relation (7) is used only in Case I.

Making use of Eqs. (6) and (7), any scalar integral $I_0^N(D; \{\nu_i\})$ can be represented as a linear combination:

$$I_0^N(D; \{\nu_k\}) = \sum_i c_i(D, r_{ij})\, I_0^{N-1}(D^{(i)}; \{\nu_k^{(i)}\}) + \lambda\, I_0^N(D'; \{1\}), \tag{9}$$

where the dimension D' is usually chosen to be $4 + 2\varepsilon$ or $6 + 2\varepsilon$. The infinitesimal parameter ε regulates the divergences. The parameter λ equals 0 for Cases II, III and IV [2]. It follows that in all cases, with the exception of Case I, the integrals with N external lines can be represented in terms of the integrals with smaller number of external lines. Consequently, then, there exists a fundamental set of integrals of the form $I_0^N(4 + 2\varepsilon; \{1\})$, in terms of which all integrals can be represented as a linear combination.

Therefore, any dimensionally regulated one-loop N-point Feynman integral can be represented in terms of six box integrals ($N = 4$), one triangle integral ($N = 3$)

and a general (arbitrary D, ν_1 and ν_2) two-point integral ($N = 2$) [2]. Five of the six basic box integrals are IR divergent in $D = 4$, while the basic triangle integral is finite. However, all basic box integrals are finite in $D = 6$. Thus, an alternative fundamental set of integrals is comprised of five box integrals in $D = 6$, one box and triangle integral in $D = 4$ and a general two-point integral. This set of integrals is particularly interesting because the integral I_0^2 is the only divergent one. In the final result, all divergences, IR as well as UV, are contained in the general two-point integrals and associated coefficients. The expressions for all relevant basic integrals can be found in the literature [2, 5 − 7].

3. Example

As an ilustration of the tensor decomposition and scalar reduction methods, we now evaluate an one-loop 6-point Feynman diagram shown in Fig. 1. This is one (out of 456) diagram contributing to the NLO hard-scattering amplitude for the exclusive process $\gamma(k_1, \varepsilon_1)\,\gamma(k_2, \varepsilon_2) \to \pi^+(P_+)\,\pi^-(P_-)$, (with both photons on-shell) at large momentum transfer.

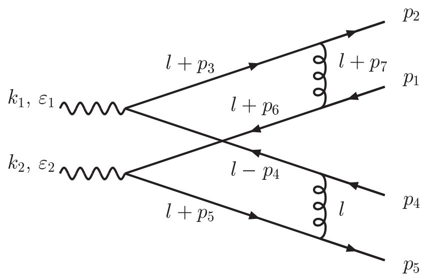

Fig. 1. One of the diagrams contributing to the hard scattering amplitude of the process $\gamma\,\gamma \to \pi^+\,\pi^-$ at NLO.

In the $\gamma\,\gamma$ centre-of-mass frame, the 4-momenta of the incoming and outgoing particles are

$$k_{1,\,2} = \sqrt{s}/2\,(1, \mp\sin\theta_{\mathrm{c.m.}}, 0, \pm\cos\theta_{\mathrm{c.m.}}), \quad P_\pm = \sqrt{s}/2\,(1, 0, 0, \pm 1), \tag{10}$$

while the polarization states of the photons are

$$\varepsilon_1^\pm = \varepsilon_2^\mp = \mp 1/\sqrt{2}\,(0, \cos\theta_{\mathrm{c.m.}}, \pm\mathrm{i}\sin\theta_{\mathrm{c.m.}}), \tag{11}$$

where $\sqrt{s}$ is the total centre-of-mass energy of the $\gamma\,\gamma$ system (or the invariant mass of the $\pi^+\,\pi^-$ pair).

For example, taking $\theta_{\mathrm{c.m.}} = \pi/2$ and assuming that the photons have opposite helicities, the amplitude corresponding to the Feynman diagram of Fig. 1. is proportional to the integral

$$\mathcal{I} = \frac{(\mu^2)^{-\varepsilon}}{2} \int \frac{\mathrm{d}^{4+2\varepsilon} l}{(4\pi)^{4+2\varepsilon}} \frac{\mathrm{Tr}\,[\gamma_\mu\gamma_5 \not{P}_+ \gamma^\mu (\not{l} + \not{p}_3) \not{\varepsilon}_1 (\not{l} - \not{p}_4) \gamma_\nu \gamma_5 \not{P}_- \gamma^\nu (\not{l} + \not{p}_5) \not{\varepsilon}_2 (\not{l} + \not{p}_6)]}{l^2 (l + p_3)^2 (l - p_4)^2 (l + p_5)^2 (l + p_6)^2 (l + p_7)^2}, \tag{12}$$

with the momenta p_i $(i = 1, \ldots, 7)$

$$
\begin{aligned}
&p_1 = \overline{x}\, P_+, \quad p_2 = x\, P_+, \quad p_3 = k_1 - \overline{y}\, P_-, \quad p_4 = \overline{y}\, P_-, \\
&p_5 = y\, P_-, \quad p_6 = y\, P_- - k_2, \quad p_7 = \overline{x}\, P_+ + y\, P_- - k_2.
\end{aligned} \tag{13}
$$

The quantities x and $\overline{x} \equiv 1-x$ (y and $\overline{y} \equiv 1-y$) are the fractions of the momentum P_+ (P_-) shared between the quark and the antiquark in the π^+ (π^-).

With the aim of regularizing the IR divergences, the dimension of the integral is taken to be $D = 4 + 2\varepsilon$.

The integral $\mathcal{I}$ is very complicated and is rather difficult to be evaluated with the help of previously known methods. It is composed of one-loop 6-point tensor integrals of rank 0, 1, 2, 3 and 4. Performing the tensor decomposition using (2) and evaluating the trace, we obtain the integral $\mathcal{I}$ in the form

$$
\begin{aligned}
\mathcal{I} \quad = \quad & -2(1+\varepsilon)^2 \Big[24\, s^3\, x\, \overline{x}\, y\, \overline{y} \left(4\,\pi\,\mu^2\right)^4 I_0^6(12 + 2\,\varepsilon, \{1,1,1,1,1,5\}) \\
& + 6\, s^3\, \overline{y}\, y \left(4\,\pi\,\mu^2\right)^4 I_0^6(12 + 2\,\varepsilon, \{1,4,1,1,2,1\}) \\
& + 2\, s^3\, \overline{y}\, (y - \overline{y}) \left(4\,\pi\,\mu^2\right)^4 I_0^6(12 + 2\,\varepsilon, \{1,3,2,1,2,1\}) \\
& + 8\, s^3\, y\, \overline{y} \left(4\,\pi\,\mu^2\right)^4 I_0^6(12 + 2\,\varepsilon, \{1,3,1,1,3,1\}) \\
& + 2\, s^3\, x\, y\, \overline{y} \left(4\,\pi\,\mu^2\right)^4 I_0^6(12 + 2\,\varepsilon, \{1,2,2,2,1,2\}) \\
& + s^3\, \overline{y}\, (y - \overline{y}) \left(4\,\pi\,\mu^2\right)^3 I_0^6(10 + 2\,\varepsilon, \{1,2,2,1,2,1\}) \\
& + s^2\, y\, \overline{y}\, (1+\varepsilon) \left(4\,\pi\,\mu^2\right) I_0^6(6 + 2\,\varepsilon, \{1,1,1,1,1,1\}) \\
& + s\, (1+\varepsilon)\, (2+\varepsilon) \left(4\,\pi\,\mu^2\right)^2 I_0^6(8 + 2\,\varepsilon, \{1,1,1,1,1,1\}) \\
& + \ldots 75 \text{ similar terms} \Big].
\end{aligned} \tag{14}
$$

Next, performing the scalar reduction using the method described above, we arrive at the following expression for the integral written in terms of the basic integrals

$$
\begin{aligned}
\mathcal{I} = 8\,(1+\varepsilon)^2 \Big\{ & (4\,\pi\,\mu^2) \Big[\frac{\varepsilon}{\overline{x}}\, I_4^{1m}(6 + 2\,\varepsilon; -s/2, -s\,\overline{y}/2; -s\,y/2) \\
& + \frac{1+\varepsilon}{\overline{x}}\, I_4^{1m}(6 + 2\,\varepsilon; -s\,y/2, -s/2; -s\,\overline{y}/2) \\
& + \left(1 + \varepsilon\left(1 - \frac{x}{\overline{x}}\right)\right) I_4^{1m}(6 + 2\,\varepsilon; -s\,x/2, -s\,\overline{y}/2; -s\,(\overline{x}\,\overline{y} + x\,y)/2) \\
& + \left(-\frac{x}{\overline{x}} + \varepsilon\left(1 - \frac{x}{\overline{x}}\right)\right) I_4^{2me}(6 + 2\,\varepsilon; -s\,\overline{x}/2, -s\,\overline{y}/2; -s/2, -s\,(\overline{x}\,\overline{y} + x\,y)/2) \Big] \\
& + \frac{1}{s} \left[\frac{1}{(\overline{x} - x)\,y} \left(\frac{2\,\overline{x}}{\varepsilon\,(\overline{x} - x)} + 2 - \frac{\overline{x}}{x} \right) I_2(4 + 2\,\varepsilon; -s\,\overline{x}/2) \right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{1}{(x-\overline{x})\,\overline{y}} \left(\frac{2\,x}{\varepsilon\,(x-\overline{x})} + 2 - \frac{x}{\overline{x}} \right) I_2(4+2\,\varepsilon; -s\,x/2) \\
&+ \frac{1}{(\overline{y}-y)\,x} \left(\frac{2\,\overline{y}}{\varepsilon\,(\overline{y}-y)} + 2 - \frac{\overline{y}}{y} \right) I_2(4+2\,\varepsilon; -s\,\overline{y}/2) \\
&+ \frac{1}{(y-\overline{y})\,\overline{x}} \left(\frac{2\,y}{\varepsilon\,(y-\overline{y})} + 2 - \frac{y}{\overline{y}} \right) I_2(4+2\,\varepsilon; -s\,y/2) \\
&+ \left(\frac{(1-x\,\overline{y}-3\,y\,\overline{x})(1-y\,\overline{x}-3\,x\,\overline{y})}{x\,\overline{x}\,y\,\overline{y}\,(x-\overline{x})(y-\overline{y})} + \frac{2(\overline{x}\,\overline{y}+x\,y)(8\,x\,\overline{x}\,y\,\overline{y}-x\,\overline{x}-y\,\overline{y})}{\varepsilon\,x\,\overline{x}\,y\,\overline{y}\,(x-\overline{x})^2(y-\overline{y})^2} \right) \\
&\times I_2(4+2\,\varepsilon; -s\,(\overline{x}\,\overline{y}+x\,y)/2) + \frac{(\overline{x}\,\overline{y}+x\,y)}{x\,\overline{x}\,y\,\overline{y}} \, I_2(4+2\,\varepsilon; -s/2) \Big] \Big\} . \qquad (15)
\end{aligned}
$$

Here, I_2 is the two-point scalar integral in $D = 4 + 2\,\varepsilon$ with $\nu_i = 1$, while I_4^{1m} and I_4^{2me} are box scalar integrals in $D = 6 + 2\,\varepsilon$. Analytic expressions for these integrals are given in Ref. [2]. Expanding Eq. (15) up to order $\mathcal{O}(\varepsilon^0)$, we finally get

$$
\begin{aligned}
\mathcal{I} = \frac{\mathrm{i}}{(4\,\pi)^2} \frac{8}{s} \Big\{ &-\frac{1}{x\,\overline{y}} \mathrm{Li}_2(\overline{x}-x) - \frac{1}{y\,\overline{x}} \mathrm{Li}_2(x-\overline{x}) - \frac{1}{y\,\overline{x}} \mathrm{Li}_2(\overline{y}-y) - \frac{1}{x\,\overline{y}} \mathrm{Li}_2(y-\overline{y}) \\
&+ \frac{x\,\overline{y}+y\,\overline{x}}{x\,\overline{x}\,y\,\overline{y}} \mathrm{Li}_2\left(-(x-\overline{x})(y-\overline{y})\right) + \frac{\pi^2}{6} \frac{x\,\overline{y}+y\,\overline{x}}{x\,\overline{x}\,y\,\overline{y}} - \frac{\overline{x}\,\overline{y}+x\,y}{x\,\overline{x}\,y\,\overline{y}} \ln\left(\frac{s}{2\,\mu^2} \right) \\
&+ \frac{(\overline{x}-2\,x)}{x\,y\,(\overline{x}-x)} \ln\left(\frac{s\,\overline{x}}{2\,\mu^2} \right) + \frac{(x-2\,\overline{x})}{\overline{x}\,\overline{y}\,(x-\overline{x})} \ln\left(\frac{s\,x}{2\,\mu^2} \right) \\
&+ \frac{(\overline{y}-2\,y)}{x\,y\,(\overline{y}-y)} \ln\left(\frac{s\,\overline{y}}{2\,\mu^2} \right) + \frac{(y-2\,\overline{y})}{\overline{x}\,\overline{y}\,(y-\overline{y})} \ln\left(\frac{s\,y}{2\,\mu^2} \right) \\
&- \frac{x}{\overline{y}\,(x-\overline{x})^2} \ln^2\left(\frac{s\,x}{2\,\mu^2} \right) - \frac{\overline{x}}{y\,(x-\overline{x})^2} \ln^2\left(\frac{s\,\overline{x}}{2\,\mu^2} \right) \\
&- \frac{y}{\overline{x}\,(y-\overline{y})^2} \ln^2\left(\frac{s\,y}{2\,\mu^2} \right) - \frac{\overline{y}}{x\,(y-\overline{y})^2} \ln^2\left(\frac{s\,\overline{y}}{2\,\mu^2} \right) \\
&- \frac{(1-x\,\overline{y}-3\,y\,\overline{x})(1-y\,\overline{x}-3\,x\,\overline{y})}{x\,\overline{x}\,y\,\overline{y}\,(x-\overline{x})(y-\overline{y})} \ln\left(\frac{s\,(\overline{x}\,\overline{y}+x\,y)}{2\,\mu^2} \right) \\
&- \frac{(\overline{x}\,\overline{y}+x\,y)(8\,x\,\overline{x}\,y\,\overline{y}-x\,\overline{x}-y\,\overline{y})}{x\,\overline{x}\,y\,\overline{y}\,(x-\overline{x})^2(y-\overline{y})^2} \ln^2\left(\frac{s\,(\overline{x}\,\overline{y}+x\,y)}{2\,\mu^2} \right) \\
&-\frac{2}{\hat{\varepsilon}} \Big[\frac{x}{\overline{y}\,(x-\overline{x})^2} \ln\left(\frac{s\,x}{2\,\mu^2} \right) + \frac{\overline{x}}{y\,(x-\overline{x})^2} \ln\left(\frac{s\,\overline{x}}{2\,\mu^2} \right) \\
&+ \frac{y}{\overline{x}\,(y-\overline{y})^2} \ln\left(\frac{s\,y}{2\,\mu^2} \right) + \frac{\overline{y}}{x\,(y-\overline{y})^2} \ln\left(\frac{s\,\overline{y}}{2\,\mu^2} \right)
\end{aligned}
$$

$$+\frac{(\overline{x}\,\overline{y}+x\,y)(8\,x\,\overline{x}\,y\,\overline{y}-x\,\overline{x}-y\,\overline{y})}{x\,\overline{x}\,y\,\overline{y}\,(x-\overline{x})^2(y-\overline{y})^2}\ \ln\left(\frac{s\,(\overline{x}\,\overline{y}+x\,y)}{2\,\mu^2}\right)\Bigg]\Bigg]\Bigg\}\,, \tag{16}$$

where $1/\hat{\varepsilon} = 1/\varepsilon + \gamma - \ln(4\,\pi)$.

4. *Conclusion*

Through the tensor decomposition and the scalar reduction procedure presented, any massless one-loop Feynman integral with generic 4-dimensional momenta can be expressed as a linear combination of a fundamental set of scalar integrals: six box integrals in $D = 6$, a triangle integral in $D = 4$ and a general two-point integral. All the divergences present in the original integral are contained in the general two-point integral and associated coefficients.

As an illustration, an one-loop 6-point Feynman diagram is evaluated in detail. This diagram gives NLO contributions to the hard-scattering amplitude for the process $\gamma\,\gamma \to \pi^+\,\pi^-$ at large momentum transfer in pQCD.

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OPĆA METODA SVOĐENJA ZA NLO RAČUNE U PQCD-U

Raspravljamo sistematičnu metodu koja se može općenito primijeniti za svođenje proizvoljnih tenzorskih Feynmanovih integrala s jednom petljom, na osnovni skup od osam skalarnih Feynmanovih integrala s 2, 3 i 4 vanjske linije. Da bismo objasnili pogodnost ove metode, računamo jedan od Feynmanovih dijagrama s jednom petljom i 6 vanjskih linija koji doprinose amplitudi tvrdog raspršenja procesa $\gamma\,\gamma \to \pi^+\,\pi^-$ za velike prijenose impulsa i u okviru pQCD-a.