NUMERICAL ANALYSIS OF PLATE THICKNESS EFFECT ON RESIDUAL STRESS DISTRIBUTION AROUND A COLD EXPANDED HOLE

Summary

The process of cold expansion creates compressive stresses around the mounting hole, which can help to reduce stress concentrations during the loading of a structure. A numerical simulation using the finite element method was conducted to analyse the effect of cold expansion on perforated plates of different thicknesses. The simulation considered the thickness of the plate and the tapered mandrel and examined the induced residual stresses. The results showed that the magnitude of the circumferential residual stresses varied depending on the thickness of the plate and that the extent of the zone of compressive stresses was strongly affected by the plate thickness. These findings are particularly relevant to the optimisation of the cold expansion process, as the optimal parameters for a particular plate thickness may not be optimal for some other thickness. Therefore, the results of this study can guide the optimisation of the cold expansion process for different plate thicknesses.

Key words: mounting hole, cold expansion, plate thickness, residual stresses, tapered mandrel

1. Introduction

In the aerospace and automotive industries, the presence of fastener holes for riveting in many sub-assemblies can lead to local stress concentration, resulting in structural weakness due to shock [1] or cyclic loading over time [2]. Various methods are employed to mitigate this issue, with shot peening being an effective technique that creates a compressive residual stress layer on the surface, thereby reducing crack propagation [3]. However, its drawback lies in the fact that the residual stresses induced are limited to the outer layers and penetrate only to a shallow depth, which is insufficient to prevent crack propagation into the bulk material [3]. To overcome this limitation, cold expansion is utilized to generate compressive residual stresses throughout the thickness of the hole. This approach strengthens the affected area and prevents crack propagation into the bulk material, thereby enhancing the longevity and reliability of structures.
In this context, the cold expansion technology of connection holes serves as an alternative mechanical technique widely employed in the assembly of aeronautical structures due to its proven performance in preventing crack initiation and propagation. Numerous experimental and numerical studies indicated that the service life of a structure can increase up to tenfold through the expansion of holes intended for rivets or bolts [4-10]. Researchers consistently highlight the long-term advantages of this technique, as it significantly enhances the service life of structures, resulting in maintenance cost savings [11], while simultaneously improving structural safety and user protection [12].

The cold expansion technique can be implemented in various ways, as evidenced by the existing literature. One approach involves the use of an oversized ball or mandrel, which is forced through the hole to create a localized region of plastic strain [10]. The effectiveness of this process depends on the magnitude and distribution of the residual stresses [13-15]. It is well-established that induced residual stresses are non-uniformly distributed throughout the expanded hole depth. Near the entrance face, minimal residual stresses are produced, while maximal residual stresses are generated around the mid-thickness location [16-18]. Consequently, crack initiation primarily occurs at the entrance face [19-21]. Previous studies have reported the presence of positive stresses near the entrance face of the hole when utilizing a spherical mandrel for cold expansion, further confirming the initiation of fatigue cracks in this region [10, 16]. Additionally, several parameters can influence the desired outcomes of cold expansion, such as the degree of expansion [14], the type and geometric shape of the oversized tool [10, 15], and the coefficient of friction [18].

The effect of reaming quality on fatigue strength has also been investigated, with studies like the one conducted by Liu et al. [22] reporting that reaming after expansion increases the residual compressive stress at the hole edge, consequently significantly enhancing the fatigue life of specimens. In this context, it should be noted that the formation of holes during drilling or milling is vital to ensure surface integrity, thus making a significant contribution to achieving the desired outcome following the cold expansion process. It is also worth mentioning that selecting appropriate cutting parameters, including cutting speed and feed rate, has a substantial impact on the quality of the resultant holes and contributes to the expected result after the cold expansion operation [23].

Regarding the effect of hole depth, a recent study by Yao et al. [7] proposed a new method based on multiple interference expansion called the multi-spherical rotary cold expansion process. The authors found that gradually expanding the hole with different degrees of expansion yielded positive results. This observation is in agreement with the findings in [17], where the authors proposed an optimized conical mandrel that ensures gradual penetration, resulting in relatively evenly distributed residual stresses throughout the depth of the hole.

The main objective of this paper is to analyse the influence of plate thickness on the residual stresses induced by the previously proposed tapered mandrel. The distribution trends of residual stresses around the expanded holes will be examined and compared. To achieve this, plate models of different thicknesses will be created, and cold expansion will be simulated using the tapered mandrel parameters proposed in [17].

2. Plates and mandrel geometrical model

In this study, our focus lies on the utilization of cold expansion with a conical mandrel fabricated from En24 grade steel. Cold expansion is a widely employed mechanical technique in the aerospace and automotive industries to enhance the service life of structures by
preventing crack initiation and propagation. Figure 1 depicts the geometry and dimensions of the plates under investigation, which vary in thickness. The conical mandrel is designed to penetrate the perforated hole, commencing from one side (entrance) and extending towards the opposite side (exit), as illustrated in Figure 2.

The interference degree is a key parameter in the cold expansion process and influences the induced residual stresses and their distribution [18].

To achieve optimal outcomes, the conical section of the mandrel possesses the smallest base diameter of \( d = 5 \) mm and the largest diameter of \( D = 5.23 \) mm, as illustrated in Figure 3a. This configuration results in an interference degree of \( DE\% = 4.6 \), which is determined by Equation (1) [17, 21]:

\[
DE\% = \frac{D - d}{d} \times 100
\]  

**Fig. 1** Geometrical characteristics of perforated plates with dimensions given in millimetres

It is important to note that the proposed mandrel was previously validated and its tapering parameters were extensively studied in reference [17]. Figure 3 provides the geometrical definition and the shape of the mandrel used in this study.

**Fig. 2** Diagram of the cold expansion principle

**Fig. 3** Mandrel geometry definition:
- a) Geometry and dimensions in mm
- b) Complete model
- c) Quarter model as simulated

The purpose of this study is to investigate the effect of the plate thickness on the residual stresses induced by the conical mandrel. We aim to analyse the distribution trends of residual stresses around the expanded holes of different thicknesses and compare the results.
3. Materials properties

Aluminium alloys have been used widely in the design of aerospace structures due to their superior properties such as lightweight, high strength-to-weight ratio [1], excellent formability and good tribological behaviour [24, 25]. These characteristics make aluminium alloys ideal for various aerospace applications, including aircraft structures, panels, wings and fuselage components. The use of aluminium helps to reduce the overall weight of the aircraft, leading to improved fuel efficiency, increased payload capacity and enhanced performance.

The plates with holes that have been analysed in this study were constructed using the aluminium alloy 7075-T6.

For the cold expansion process, a mandrel made of En 24 grade steel was used. It should be emphasized that these specific materials were already utilized in previous studies too [17, 21 and 26].

To better understand the properties of the plate material, Table 1 has been compiled, outlining the key mechanical features such as Young’s modulus \(E\) in GPa, yield stress \(\sigma_y\) in MPa and Poisson’s coefficient \(\nu\).

<table>
<thead>
<tr>
<th>Mechanical properties</th>
<th>Aluminium alloy 7075-T6</th>
<th>En 24 grade steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (E) (GPa)</td>
<td>72</td>
<td>210</td>
</tr>
<tr>
<td>Yield stress (\sigma_y) (MPa)</td>
<td>500</td>
<td>-</td>
</tr>
<tr>
<td>Poisson’s coefficient (\nu)</td>
<td>0.33</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 4 [21] illustrates the tensile behaviour of Al 7075-T6 in detail, providing a visual representation of its mechanical response under tension.

![Fig. 4 Tensile curve for Al7075-T6 [21]](image_url)

In this study, an isotropic elasto-plastic model was utilised to simulate the behaviour of the plate, whilst the pin was modelled using an elastic model. To capture the hardening behaviour, a kinematic hardening model was employed, which estimates the hardening response with a linear hardening rate. The yield stress, a critical factor governing the hardening behaviour, was defined as a tabulated function dependent on plastic strain, as presented in Table 2. At specific states, the yield stress is obtained through interpolation from the tabular data, enabling us to accurately represent the material response.
Table 2 Tabulated function of plastic strain

<table>
<thead>
<tr>
<th>Stress (MPa)</th>
<th>Plastic strain (mm/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>660</td>
<td>0.11</td>
</tr>
</tbody>
</table>

4. Finite element model

The expansion of the holed plates was modelled using the finite element method (FEM). This approach enabled us to simulate and analyse the behaviour of the plates during the cold expansion process accurately. To enhance computational efficiency, only a quarter of the structure was modelled. The physical contact between the plate and the mandrel during the expansion process was accurately achieved through the implementation of surface-to-surface contact with a friction coefficient of $\mu = 0.1$. For the analysis, linear brick elements (C3D8R) were selected due to their suitability for plastic deformation calculations and compatibility with contact elements. The volume size of the simulations, given in terms of the number of elements and nodes is summarized in Table 3 for different plate thicknesses.

Table 3 Problem sizes of different simulations

<table>
<thead>
<tr>
<th>Plate thickness</th>
<th>Number of elements</th>
<th>Number of nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm</td>
<td>2452</td>
<td>3335</td>
</tr>
<tr>
<td>2 mm</td>
<td>4904</td>
<td>6003</td>
</tr>
<tr>
<td>3 mm</td>
<td>8652</td>
<td>10153</td>
</tr>
<tr>
<td>4 mm</td>
<td>11536</td>
<td>13277</td>
</tr>
<tr>
<td>5 mm</td>
<td>17020</td>
<td>19257</td>
</tr>
<tr>
<td>6.32 mm</td>
<td>17304</td>
<td>19525</td>
</tr>
</tbody>
</table>

Abaqus 6.14 software was utilized for conducting three-dimensional finite element simulations. Figure 5 presents an overview of various finite element models employed in the cold expansion procedure. The figure provides a visual representation of the meshing configurations performed for different plate thicknesses, illustrating the implemented discretization approach for accurately capturing the structural behaviour during the cold expansion process.

![Finite element models of plates of varying thicknesses](image)

**Fig. 5** Finite element models of plates of varying thicknesses (the mandrel is shown on the 1 mm thick plate as an example).
To capture the material behaviour with greater accuracy, Figure 6 illustrates a refined mesh near the hole region.

![Finite element model view of plate and mandrel near hole edge](image)

**Fig. 6** Finite element model view of plate and mandrel near hole edge

To minimize the computational effort, we utilised the symmetrical geometric and loading conditions along the X-Z and Y-Z planes. Figure 7 depicts the application of symmetry boundary conditions to the planes of symmetry. This approach allowed us to effectively simulate the system while reducing computational complexity.

![Schematic representation of boundary conditions](image)

**Fig. 7** Schematic representation of boundary conditions

5. **Results and discussion**

For each plate thickness simulated, cold expansion of the hole was achieved using a tapered mandrel. Figure 8 illustrates the distribution of circumferential residual stresses in the vicinity of the hole. It can be observed that the circumferential stresses ($S_{22}$) are distributed differently around the hole and in its vicinity, with this effect being more pronounced for greater plate thicknesses. Additionally, iso-values indicate the presence of tensile residual stress away from the hole (the red region). As the plate thickness increases, these tensile stresses become more localized through the thickness, away from the hole edge. In contrast, compressive residual stresses are highly localized in the plate mid-thickness as the thickness increases. For lesser thicknesses, quasi-localized tensile stresses are present throughout the plate thickness, neighbouring the hole edge.
It is important to highlight that the $S_{22}$ stresses, as depicted in the figures, coincide with the circumferential ($S_{00}$) stresses at the specific point of interest. To further illustrate this correspondence, Figure 9 serves as a visual representation, clearly demonstrating how the $S_{22}$ stresses are in agreement with the circumferential ($S_{00}$) stresses at the selected measurement location.

Furthermore, the equivalent stresses, which offer a comprehensive representation of the effective stress distribution, exhibit axial symmetry, as depicted in Figure 10.
more compressive for the plate thickness of 1 mm. The absolute value of the resulting residual stress is 383.53 MPa. This value becomes less compressive for the plate thicknesses of 2 mm and 3 mm. For the thicknesses greater than 3 mm, the circumferential residual stresses increase with an increase in the plate thickness, with the maximum compressive stress of 363.8 MPa determined for the plate thickness of 6.32 mm. Furthermore, higher compressive values (517.7 MPa) were noticed in the vicinity of the mid-thickness position.

![Fig. 11](image1.png) Comparison of circumferential stresses at various locations throughout the plate thickness along the distance from the hole edge

In order to accurately emphasize the impact of plate thickness, we conducted calculations to determine the variations in residual stresses at different locations based on the hole depth. Figure 12 provides a clear illustration of how the range of residual stresses throughout the hole depth is significantly influenced by the plate thickness. Moreover, it is apparent that the disparity between the entrance face and the mid-position expands as the plate thickness increases. Conversely, the difference between the entrance face and the exit face follows an ascending pattern as the plate thickness escalates from 1 mm to 3 mm, but subsequently decreases with further increases in thickness.

![Fig. 12](image2.png) Extent of difference between residual stresses obtained at different locations for different plate thicknesses
Figure 13 offers a comparative examination of the distributions of compressive stresses across the hole edge, considering different plate thicknesses. The findings emphasise the notable influence of varying plate thicknesses on the stress distribution along the hole.

![Graph showing residual circumferential stresses versus normalized hole depth with reference to the plate thickness.](image)

**Fig. 13** Generated circumferential residual stresses versus normalized hole depth with reference to the plate thickness.

Another influential parameter examined in this study is the extent of the zone of compressive residual stresses $Z_{CRS}$ surrounding the hole. $Z_{CRS}$, as illustrated in Figure 14, defines the region where compressive stresses are present. By analysing the parameter, we gain valuable insights into the size and magnitude of the compressive stress zone surrounding the hole, enabling a better understanding of the material response to the applied hole expansion.

![Diagram of residual compression zone](image)

**Fig. 14** Definition of residual compression zone.

Figure 15 depicts the curves representing the variation of the zone of compressive residual stresses ($Z_{CRS}$) at different locations across the thickness of the plates, namely the entrance face, mid-thickness and exit face, as a function of plate thickness. Upon analysing these curves, two distinct trends emerge.

Firstly, on the entrance face and mid-thickness, $Z_{CRS}$ increases as the plate thickness increases until reaching a plateau beyond 4 mm. Conversely, on the exit face, $Z_{CRS}$ decreases as the plate thickness decreases until it reaches 3mm, beyond which it exhibits a slight increase and then stabilizes beyond 4 mm. Furthermore, it is noteworthy that the exit face exhibits higher $Z_{CRS}$ values (up to 4.6 mm) compared to the entrance face and mid-thickness (2.5 mm). These values correspond to the least thickness observed on the exit face and the greatest thicknesses
observed on the entrance face and mid-thickness. These observations are in good agreement with the findings presented in Figure 12. Importantly, regardless of the plate thickness, the entrance face consistently exhibits the lowest $Z_{CRS}$ values.

![Graph showing variation of extent of the compression residual stress zone with change in plate thickness](image)

**Fig. 15** Variation of extent of the compression residual stress zone with change in plate thickness

In order to consolidate these findings and concurrently assess the suitability of the selected mandrel geometry in this study, it is prudent to examine the impact of the mandrel geometry. Consequently, we performed similar simulations using a wider tapered mandrel (conical length $L=3$). The results depicted in Figure 16 exhibit comparable stress distributions throughout the hole depth for the wider tapered mandrel. However, the compressive residual stresses obtained with this mandrel are lower when compared to those obtained using the proposed mandrel ($L=6$ mm).

![Graph showing circumferential residual stresses through normalized plate thicknesses](image)

**Fig. 16** Circumferential residual stresses through normalized plate thicknesses with reference to the tapered mandrel

Additionally, Figure 17 shows how plate thickness influences the distribution of residual stresses along the hole depth using a wider tapered mandrel. This illustration reveals that the distribution of residual stresses remains consistent, with higher stresses observed at the mid-plane position and minimal stresses at the entrance face. Moreover, it confirms that employing a wider tapered mandrel leads to lower residual stresses.
In general, the findings of this study highlight the influence of plate thickness on the distribution of residual stresses induced by the cold expansion technique. In particular, the circumferential residual stresses \( S_{22} \) exhibit different patterns around the hole for varying plate thicknesses, with more pronounced variations observed in thicker plates. The iso-values analysis further reveals the presence of tensile residual stresses away from the hole, which become more localized through the plate thickness as the plate thickness increases. Conversely, compressive residual stresses are highly localized in the mid-thickness region as the plate thickness increases, while lower thicknesses result in quasi-localized tensile stresses neighbouring the hole edge.

The extent of \( Z_{CRS} \) indicates the following trends: an increasing \( Z_{CRS} \) with an increase in the plate thickness on the entrance face and mid-thickness, reaching a plateau beyond 4 mm; a decreasing \( Z_{CRS} \) with a decrease in the plate thickness on the exit face until 3 mm, followed by a slight increase and then \( Z_{CRS} \) stabilizes beyond 4 mm. Furthermore, it is noteworthy that the exit face consistently exhibits higher \( Z_{CRS} \) values compared to the entrance face and mid-thickness.

Furthermore, the utilization of a wider tapered mandrel leads to reduced levels of compressive residual stresses. This observation emphasises the significance of the mandrel geometry selection in governing both the intensity and the pattern of residual stresses generated by the cold expansion method.

By conducting further investigations that include fatigue tests and simulations and consider various mandrel geometries and plate thicknesses during the cold expansion process, valuable insights can be acquired, that contribute to the technique optimisation enhancing durability of cold-expanded components.

6. Conclusions

A FEM simulation using Abaqus 6.14 was performed to investigate the impact of plate thickness on the distribution of induced residual stresses during cold expansion. The results obtained led to the following main conclusions:

- Cold expansion using a tapered mandrel was successfully achieved for all simulated thicknesses.
- Circumferential residual stresses \( S_{22} \) were distributed differently around the hole for different plate thicknesses and were more pronounced for greater thicknesses. Tensile residual stresses were observed away from the hole, and these stresses were more localized through the plate thickness as the plate thickness increased.
The compressive residual stresses at the entrance face became more compressive for a plate thickness of 1 mm and then became less compressive for plate thicknesses of 2 mm and 3 mm. In the case of plate thicknesses greater than 3 mm, the circumferential residual stresses increased as the plate thickness increased, with the maximum compressive stress determined for the plate thickness of 6.32 mm. Higher compressive values were observed in the vicinity of the mid-thickness position.

The extent of the compression residual stress zone (ZCRS) increased with an increase in the plate thickness on the entrance face and mid-thickness, but became almost constant beyond the plate thickness of 4 mm. On the exit face, ZCRS decreased with a decrease in the plate thickness until 3 mm, after which it slightly increased and became constant beyond the plate thickness of 4 mm. The exit face had higher ZCRS values compared to the entrance face and mid-thickness. The entrance face consistently had the lowest ZCRS values regardless of the plate thickness.

To optimize the parameters of the cold hole expansion process, it is important to take the depth of the hole to be expanded into consideration.

REFERENCES


