ISSN 1846-6168 (Print), ISSN 1848-5588 (Online)
https://doi.org/10.31803/tg-20220921070537

Original scientific paper
Received: 2022-09-21, Accepted: 2022-10-17

Approach of Solving Multi-objective Programming Problem by Means of Probability Theory and Uniform Experimental Design

Maosheng Zheng*, Haipeng Teng, Yi Wang

Abstract: In this paper, an approach to deal with the multi-objective programming problem is regulated by means of probability-based multi-objective optimization, discrete uniform experimental design, and sequential algorithm for optimization. The probability-based method for multi-objective optimization is used to conduct conversion of the multi-objective optimization problem into a single-objective optimization one in the viewpoint of probability theory. The discrete uniform experimental design is used to supply an efficient sampling to simplify the conversion. The sequential algorithm for optimization is employed to carry out further optimization. The corresponding treatments reveal the essence of the multi-objective programming, and consideration of the simultaneous optimization of each objective of multi-objective programming problem rationally. Two examples are conducted to illuminate the rationality of the approach.

Keywords: discretization; favorable probability; multi-objective programming; probability theory; sequential algorithm for optimization

1 INTRODUCTION

Multi-objective programming (MOP) is a branch of mathematical programming, which studies the optimization of more than one objective function [1]. The idea of multi-objective programming sprouts in 1776 in the study of utility theory in economics. In 1896, economist Pareto first proposed the multi-objective programming problem in the study of economic balance, and gave a simple idea, which was later called Pareto optimal solution. In 1947, von Neumann and Morgenstern mentioned the multi-objective programming problem in their work on game theory, which attracted much more attentions to this problem. In 1951, Koopmans proposed the multi-objective optimization problem in the activity analysis of production and distribution, and proposed the concept of Pareto optimal solution for the first time. In the same year, Kuhn and Tucker gave the concept of Pareto optimal solution of vector extremum problem from the perspective of mathematical programming. The sufficient and necessary conditions for the existence of this solution are also studied. Debreu's discussion on evaluation equilibrium in 1954 and Harwicz's research on multi-objective optimization problems in topological vector spaces in 1958 laid the foundation for the establishment of this discipline. In 1968, Johnsen published the first monograph on multi-objective decision-making models. Until the 1970s and 1980s, the basic theory of multi-objective programming was finally established after the efforts of many scholars, which made it a new branch of applied mathematics [2].

There are generally the following methods for solving multi-objective programming: one is the method of transforming multiple objectives into a single objective that is easier to solve, such as main objective method, linear weighting method, ideal point method, etc. The other is called hierarchical sequence method, that is, the target is given a sequence according to its importance, and each time the next target optimal solution is found in the previous target optimal solution set, until the common optimal solution is obtained.

The main target method takes a certain $f_1(x)$ as the main target, and the other p–1 are non-main targets. At this time, it is hoped that the main target will reach the maximum value, and the remaining targets should meet certain conditions; the linear weighting method will assign the same weight to the objective functions $f_1(x)$, $f_2(x)$, ..., $f_p(x)$ respectively. The coefficient ω_j, perform a linear weighted sum to obtain a new evaluation function, $U(x)=\sum_{j=1}^{p}\omega_j f_j(x)$, then the multi-objective problem becomes a single-objective problem, but normalization is required when the dimensions are different; for a linear programming problem with multiple objectives, the decision maker hopes to achieve to, these goals in turn under these constraints by means of minimizing the total deviation from the target values, which is the problem to be solved by goal planning [1].

In practical engineering systems, such as many nonlinear, multi-variable, multi-constraint and multi-objective optimization problems in power systems, the existing mathematical methods have limited ability to optimize these problems, and the obtained solutions are not satisfactory [2].

Above discussions indicate that the normalization and the introductions of subjective factors are indispensable treatment in the above "additive" algorithms to transfer diverse criteria into a "unique criterion", and the final result depends on the normalization process significantly [3]. Different normalization methods could result in complete differences in the consequence. Besides, beneficial performance index and unbeneficial performance index are treated in non-equivalent or inconsistent manners in some algorithms. In addition, the "additive" algorithm in the multi-objective optimization is corresponding to the form of "union" from the viewpoint of set theory. So, above algorithms could be seen as a semi-quantitative approach in some sense.

Recently, a probability-based method for multi-objective optimization (PMOO) was proposed to solve the intrinsic problems of subjective factors in previous multi–objective optimizations [3-5]. A brand new idea of favorable probability was proposed to reflect the favorable degree of a performance index in the optimization in the PMOO. The PMOO aims to treat the simultaneous optimization of multiple objectives in the viewpoint of probability theory. In the novel methodology of PMOO, all performance utility indicators of alternatives are preliminarily divided into two types, i.e., beneficial or unbeneficial types according to their functions and preference in the optimization; each performance utility indicator of an alternative contributes to a partial favorable probability quantitatively. Moreover, the product of all partial favorable probabilities produces the total favorable probability of an alternative, which thus transfers the multi-objective optimization problem into a single-objective optimization one rationally.

In this paper, it regulates a rational approach of multi-objective programming by means of probability theory, discrete uniform experimental design, and sequential algorithm for optimization. Furthermore, examples for illumination of this approach are given.

2 NEW APPROACH OF SOLVING MULTI-OBJECTIVE PROGRAMMING PROBLEM

The rational approach of multi-objective programming is conducted by the combination of probability theory, discrete uniform experimental design, and sequential algorithm for optimization integrally.

The probability-based method for multi-objective optimization is used to conduct conversion of the multi-objective optimization problem into a single-objective optimization one in the viewpoint of probability theory.

The discrete uniform experimental design is used to supply an efficient sampling to simplify the conversion, which is especially important for the goal functions in multi-objective programming problem being continuous functions. Sequential algorithm for optimization is employed to carry out further optimization.

2.1 Probability Theory Based Treatment

In the viewpoint of probability theory, the entire event of appearance of "simultaneous optimization of multi-objective" is corresponding to the product of the each individual objective (event). Therefore, the usual term "the higher the better" for the utility index of performance indicator needs to be expressed quantitatively in term of probability theory, which stimulates us to seek a proper expression for the term "the higher the better" in probability theory quantitatively. A brand new idea of "favorable probability" was proposed in [3-5] to interpret the preference degree of the candidate in the selection, i.e., it uses the term "favorable probability" to characterize the preference degree of the utility index of a performance indicator quantitatively in the optimization.

As to the multi-objective programming problem, each goal is indeed an objective of the PMOO. All performance utility indicators of alternatives are preliminarily divided into two types, i.e., beneficial or unbeneficial types according to their functions and preference in the optimization; thus, the subsequent process of PMOO can be employed rationally.

The assessment of the partial favorable probability P_{ij} for the beneficial index is written as [3-5]

$$P_{ij} = \alpha_j \cdot X_{ij};\ i = 1, 2, ..., n;\ j = 1, 2, ..., m;\ \alpha_j = \frac{1}{n \cdot \bar{X}_j}. \quad (1)$$

While the assessment of the partial favorable probability P_{ij} for the unbeneficial index is expressed as

$$P_{ij} = \beta_j \cdot (X_{j\max} + X_{j\min} - X_{ij});\ i = 1, 2, ..., n;$$
$$j = 1, 2, ..., m;\ \beta_j = \frac{1}{n \cdot \left[(X_{j\max} + X_{j\min}) - \bar{X}_j\right]}. \quad (2)$$

In Eqs. (1) and (2), X_{ij} is the value of utility index of performance indicator; n expresses the number of the performance indicator; m expresses the number of the alternative in the evaluation. $\bar{X}_j$ represents the arithmetic average of the value of utility index of performance indicator X_{ij} over index i for specific j; X_{jmax} and X_{jmin} indicate the maximum and minimum values of X_{ij} over index i for specific j, respectively.

Moreover, the total / overall favorable probability of an alternative is written as

$$P_i = P_{i1} \cdot P_{i2} \cdot \ldots \cdot P_{im} = \prod_{j=1}^{m} P_{ij}\ . \quad (3)$$

The total / overall favorable probability of an alternative, i.e. Eq. (3), thus transfers the multi-objective optimization problem into a single-objective optimization one in viewpoint of probability theory for the simultaneous optimization of multiple objectives rationally.

2.2 Discrete Uniform Experimental Design and Sequential Algorithm for Optimization

Since the goal functions in multi-objective programming problem are usually continuous ones, discretization can be used to conduct the simplified treatment for the simplicity.

As was stated in [6], the methodologies of good lattice point (GLP) and uniform experimental design (UED) make the discretization possible and practical. The methodologies of GLP and UED are based on number theory, which could supply effective assessment for a definite integral with finite sampling points [6, 7]. The finite sampling points are uniformly distributed within the integral domain with low-discrepancy [8, 9]. The characteristic of the uniformly distributed point set makes the convergence much faster than Monte Carlo sampling [8, 9], which thus has been promising a very good algorithm in approximate calculations with a surname – "quasi – Monte Carlo Method". Fang specially developed uniform design and uniform design table for the proper using of UED [10]. Sequential uniform design or sequential algorithm for optimization (SNTO) can be used to conduct further optimization for the multi-objective programming problem due to its similarity to problem of multi-objective optimization [6, 8].

Finally, the multi-objective programming problem is conducted by means of the probability - based multi-objective optimization and discrete uniform experimental design straightforward.

3 APPLICATIONS

In this section, two examples are given to illuminate the applications of the regulated approach in solving multi-objective programming problem by means of probability theory and discrete uniform experimental design.

3.1 Production with Maximum Profits and Least Pollutions

A factory produces two kinds of products α and β during the planning period. Each product consumes three different resources, A, B, and C [1]. The unit consumption of resources for each product, the limit of various resources, the unit price, unit profit and unit pollution caused by each product are shown in Tab. 1 [1]. Assume that all products can be sold. Now, the problem is how to arrange production which can maximize profit and output value, and cause the least pollution.

Table 1 Unit consumption of resources, unit profit and pollution for each product

Content	Product		Limit of resource (ton)
	α	β	
Unit consumption of resource A (ton)	9	4	240
Unit consumption of resource B (ton)	4	5	200
Unit consumption of resource C (ton)	3	10	300
Unit price (¥RMB /ton)	400	600	
Unit profit (¥RMB /ton)	70	120	
Unit pollution (CO_2, kg /ton)	3	2	

Solution:

Assume the output of products α and β are x_1 and x_2, respectively, the mathematical model of the problem is as following with s. t. (restraint) conditions,

$\text{Max}\, f_1(x) = 70x_1 + 120x_2$,

$\text{Max}\, f_2(x) = 400x_1 + 600x_2$,

$\text{Min}\, f_3(x) = 3x_1 + 2x_2$,

s. t.

$9x_1 + 4x_2 \le 240$,

$4x_1 + 5x_2 \le 200$,

$3x_1 + 10x_2 \le 300$,

$x_1, x_2 \ge 0$.

Since this problem is with two input variables, says, x_1 and x_2, according to literatures [6] and [10], at least 17 uniformly distributed sampling points are needed to conduct the discretization with uniform experimental design within the working domain. Here we try to employ the uniform table $U^*24(24^9)$ to perform the discretization, the consequences are shown in Tab. 2.

From Tab. 2, it can be seen that 5 sampling points are excluded due to the restraint of the s. t. conditions, and 19 sampling points are within the working domain of the s. t. conditions, which meets the requirement of at least 17 uniformly distributed sampling points within the domain of the s. t. conditions. In this problem, both the goal functions $f_1(x)$ and $f_2(x)$ are beneficial indexes, while the goal functions $f_3(x)$ is an unbeneficial index.

Table 2 Consequences of discretization with $U^*24(24^9)$

No.	Input variable		Value of goal function			Notes
	x_1	x_2	f_1	f_2	f_3	
1	0.5556	13.125	1613.889	8097.222	27.9167	
2	1.6667	26.875	3341.667	16791.67	58.75	
3	2.7778	9.375	1319.444	6736.111	27.0833	
4	3.8889	23.125	3047.222	15430.56	57.9167	
5	5	5.625	1025	5375	26.25	
6	6.1111	19.375	2752.778	14069.44	57.0833	
7	7.2222	1.875	730.5556	4013.889	25.4167	
8	8.3333	15.625	2458.333	12708.33	56.25	
9	9.4444	29.375				Excl.
10	10.5556	11.875	2163.889	11347.22	55.4167	
11	11.6667	25.625	3891.667	20041.67	86.25	
12	12.7778	8.125	1869.444	9986.111	54.5833	
13	13.8889	21.875	3597.222	18680.56	85.4167	
14	15	4.375	1575	8625	53.75	
15	16.1111	18.125	3302.778	17319.44	84.5833	
16	17.2222	0.625	1280.556	7263.889	52.9167	
17	18.3333	14.375	3008.333	15958.33	83.75	
18	19.4444	28.125				Excl.
19	20.5556	10.625	2713.889	14597.22	82.9167	
20	21.6667	24.375				Excl.
21	22.7778	6.875	2419.444	13236.11	82.0833	
22	23.8889	20.625				Excl.
23	25	3.125	2125	11875	81.25	
24	26.1111	16.875				Excl.

Table 3 Results of the assessments with PMOO with $U^*24(24^9)$

No.	Favorable probability			
	Partial favorable probability			Total
	P_{f1}	P_{f2}	P_{f3}	$P_t \times 10^5$
1	0.0365	0.0349	0.0858	10.9197
2	0.0755	0.0723	0.0539	29.452
3	0.0298	0.0290	0.0867	7.5014
4	0.0689	0.0665	0.0548	25.0748
5	0.0232	0.0232	0.0875	4.6962
6	0.0622	0.0606	0.0556	20.9791
7	0.0165	0.0173	0.0884	2.5242
8	0.0556	0.0547	0.0565	17.1850
9	0	0	0	0
10	0.0489	0.0489	0.0574	13.7128
11	0.0880	0.0863	0.0254	19.3228
12	0.0423	0.0430	0.0582	10.5826
13	0.0813	0.0805	0.0263	17.2122
14	0.0356	0.0372	0.0591	7.81463
15	0.0747	0.0746	0.0272	15.1322
16	0.0289	0.0313	0.0510	5.4291
17	0.0680	0.0687	0.0280	13.1031
18	0	0	0	0
19	0.06135	0.0629	0.0289	11.145
20	0	0	0	0
21	0.0547	0.0570	0.0298	9.2784
22	0	0	0	0
23	0.0480	0.0512	0.0306	7.5231
24	0	0	0	0

Tab. 3 shows the results of the assessments with PMOO, P_{f1}, P_{f2} and P_{f3} represent the partial favorable probabilities of functions f_1, f_2 and f_3 at the corresponding discretized sampling points, respectively; P_t expresses the total / overall favorable probability of each alternative. From Tab. 3, it can be seen that the sampling point No. 2 exhibits the maximum value of total favorable probability. Therefore, further optimization by using sequential uniform design is conducted around the sampling pint No. 2 of the Tab. 2.

Tab. 4 shows the results of the assessments by using sequential uniform design for further optimization, in which $c^{(t)}$ = (Max $P_t(i–1)$ – Max $P_t(i)$)/Max $P_t(i–1)$ expresses the relative error of the maximum total favorable probability at i^{th} sequential step. If we assume a pre-assigned value $\delta = 2\%$ for $c^{(t)}$, then the final optimal consequences for this multi-objective optimization problem are $f_{1Opt.}$ = 3591.927, $f_{2Opt.}$ = 17962.24 and $f_{3Opt.}$ = 59.9609 at the 5th step with "coordinates" x_1^* = 0.0521 and x_2^* = 29.9023. Obviously, x_1^* and x_2^* approach to 0 and 30 at ultimate limit, respectively, which corresponds to optimum values of $f_{1Opt.}$ = 3600, $f_{2Opt.}$ = 18000 and $f_{3Opt.}$ = 60, individually.

Table 4 Results of the assessments by using sequential uniform design with U*24(24⁹)

Step	Domain	Optimum "coordinates"		Value of goal			Max. total favorable probability P_t×10⁵	$c^{(t)}$
		x_1^*	x_2^*	$f_{1Opt.}$	$f_{2Opt.}$	$f_{3Opt.}$		
0	[0, 26.6667] × [0, 30]	1.6667	26.8750	3341.6670	16791.6700	58.7500	29.4520	
1	[0, 13.3333] × [15, 30]	0.8333	28.4375	3470.8330	17395.8300	59.3750	14.4243	0.5102
2	[0, 6.6667] × [22.5, 30]	0.4167	29.2188	3535.4170	17697.9200	59.6875	12.4265	0.1385
3	[0, 3.3333] × [26.25, 30]	0.2083	29.6094	3567.7080	17848.9600	59.8438	11.5726	0.0687
4	[0, 1.6667] × [28.125, 30]	0.1042	29.8047	3585.8540	17924.4800	59.9219	11.1760	0.0343
5	[0, 0.8333] × [29.0625, 30]	0.0521	29.9023	3591.9270	17962.2400	59.9609	10.9848	0.0171

3.2 Production with Maximum Profits and One Output

A factory produces two products: *A* and *B*. The profit of producing each piece of *A* is 4 ¥RMB, and the profit of producing each piece of *B* is 3 ¥RMB. The processing time of each piece of *A* is twice as long as that of each piece of *B*. If the whole time is used to process *B*, 500 pieces of *B* can be produced for per day. The factory's daily supply of raw materials is only enough to produce a total of 400 pieces of *A* and *B*. Besides, the product *A* is a tight-fitting product that sells very well.

Now, the problem is how to arrange the daily outputs of *A* and *B* so that the factory can obtain the maximum profit under the existing conditions.

Solution:

Let's first set x_1 = daily output of product *A*, x_2 = daily output of product *B* [11]. Then, it gets following mathematical model,

Max $f_1(x) = 4x_1 + 3x_2$,

Max $f_2(x) = x_1$,

s. t.

$x_1 + x_2 \le 400$,

$2x_1 + x_2 \le 500$,

$x_1, x_2 \ge 0$.

Since this problem is with two input variables, says, x_1 and x_2, again at least 17 uniformly distributed sampling points could be used to conduct the discretization with uniform experimental design within the working domain [6, 10]. Here we try to use the uniform table U*31(31¹⁰) to perform the discretization, the consequences are shown in Tab. 5. From Tab. 5, it can be seen that 14 sampling points are excluded due to the restraint of the s. t. conditions, and 17 sampling points luckily are within the domain of the s. t. conditions, which satisfies the requirement of at least 17 uniformly distributed sampling points within the domain of the s. t. conditions. In this problem, both the goal functions $f_1(x)$ and $f_2(x)$ are beneficial indexes.

Table 5 Consequences of discretization with U*31(31¹⁰)

No.	Input variable		Value of goal function		Notes
	x_1	x_2	f_1	f_2	
1	4.0323	109.6774	345.1613	4.0323	
2	12.0968	225.8065	725.8065	12.0968	
3	20.1613	341.9355	1106.452	20.1613	
4	28.2258	45.1613	248.3871	28.2258	
5	36.2903	161.2903	629.0323	36.2903	
6	44.3548	277.4194	1009.677	44.3548	
7	52.4194	393.5484			Excl.
8	60.4839	96.7742	532.2581	60.4839	
9	68.5484	212.9032	912.9032	68.5484	
10	76.6129	329.0323			Excl.
11	84.6774	32.2581	435.4839	84.6774	
12	92.7419	148.3871	816.129	92.7419	
13	100.8065	264.5161	1196.774	100.8065	
14	108.871	380.6452			Excl.
15	116.9355	83.8710	719.3548	116.9355	
16	125	200	1100	125	
17	133.0645	316.129			Excl.
18	141.129	19.3548	622.5806	141.129	
19	149.1935	135.4839	1003.226	149.1935	
20	157.2581	251.6129			Excl.
21	165.3226	367.7419			Excl.
22	173.3871	70.9677	906.4516	173.3871	
23	181.4516	187.0968			Excl.
24	189.5161	303.2258			Excl.
25	**197.581**	**6.4516**	**809.677**	**197.581**	
26	205.6452	122.5806			Excl.
27	213.7097	238.7097			Excl.
28	221.7742	354.8387			Excl.
29	229.8387	58.06452			Excl.
30	237.9032	174.1935			Excl.
31	245.9677	290.3226			Excl.

Tab. 6 shows the results of the assessments with PMOO, P_{f1} and P_{f2} indicate the partial favorable probabilities of functions f_1 and f_2 at the corresponding discretized sampling points, individually; P_t represents the total / overall favorable probability of each alternative. From Tab. 6, it can be seen that the sampling point *No. 25* exhibits the maximum value of total favorable probability. Therefore, further optimization by using sequential uniform design is conducted around the sampling point No. 25 of the Tab. 5.

Table 6 Results of the assessments with PMOO with U*31(31[10])

No.	Favorable probability		
	Partial favorable probability		Total
	P_{f1}	P_{f2}	$P_t \times 10^3$
1	0.0263	0.0028	0.0729
2	0.0553	0.0083	0.4598
3	0.0843	0.0139	1.1681
4	0.0189	0.01934	0.3671
5	0.0479	0.0249	1.1954
6	0.0770	0.0305	2.3451
7	0	0	0
8	0.0406	0.0416	1.6858
9	0.0696	0.0471	3.2768
10	0	0	0
11	0.0332	0.0582	1.9310
12	0.0622	0.0637	3.9634
13	0.0912	0.0693	6.3173
14	0	0	0
15	0.0548	0.0803	4.4048
16	0.0838	0.0859	7.2000
17	0	0	0
18	0.0475	0.0970	4.6009
19	0.0765	0.1025	7.8376

Table 6 Results of the assessments with PMOO with U*31(31[10]) (continuation)

No.	Favorable probability		
	Partial favorable probability		Total
	P_{f1}	P_{f2}	$P_t \times 10^3$
20	0	0	0
21	0	0	0
22	0.0691	0.1191	8.2299
23	0	0	0
24	0	0	0
25	**0.0617**	**0.1357**	**8.377**
26	0	0	0
27	0	0	0
28	0	0	0
29	0	0	0
30	0	0	0
31	0	0	0

Tab. 7 shows the results of the assessments by using sequential uniform design for further optimization. Again, set a pre-assigned value $\delta = 2\%$ for $c^{(t)}$, then the final optimal consequences for this multi-objective optimization problem are $f_{1\text{Opt.}} = 1000.56$ and $f_{2\text{Opt.}} = 249.597$ at the 6th step with "coordinates" $x_1^* = 249.597$ and $x_2^* = 0.7258$. Analogically, the tendencies of x_1^* and x_2^* are 250 and 0 at ultimate limit, respectively, which leads to optimum values of $f_{1\text{Opt.}}$ = 1000 and $f_{2\text{Opt.}}$ = 250, separately.

Table 7 Results of the assessments by using sequential uniform design with U*31(31[10])

Step	Domain	Optimum coordinates		Value of goal		Max. total favorable probability $P_t \times 10^3$	$c^{(t)}$
		x_1^*	x_2^*	$f_{1\text{Opt.}}$	$f_{2\text{Opt.}}$		
0	[0, 250] × [0, 400]	197.5810	6.4516	809.6770	197.5810	8.3770	
1	[100, 250] × [0, 200]	218.5484	3.2258	883.8710	218.5484	4.1994	0.4987
2	[170, 250] × [0, 100]	233.2260	1.6129	937.7420	233.2260	3.3800	0.1951
3	[210, 250] × [0, 50]	241.6130	0.8065	968.8710	241.6130	3.0381	0.1011
4	[230, 250] × [0, 20]	248.3870	2.9032	1002.2600	248.3870	1.7052	0.4387
5	[240, 250] × [0, 10]	249.1935	1.4516	1001.1290	249.1935	1.6512	0.0317
6	[245, 250] × [0, 5]	249.5970	0.7258	1000.5600	249.5970	1.6253	0.0157

4 DISCUSSION

In the past, the multi-objective programming problem was solved usually by using "linear weighting method" [1, 2], i.e., "additive algorithm" in the previous approaches to transfer the multiple objectives into a single objective one, which is the intrinsic problem in principle in the viewpoint of probability theory with "union set" in essence [3]. Or some approaches even took certain objectives as restraint conditions to solve the multi-objective programming problem [1, 2], which obviously deviates from the original intention of multi-objective programming problem in the spirit of the "simultaneous optimization of multiple objectives" essentially.

While, the probability-based method for multi – objective optimization attempts to treat the simultaneous optimization of multiple objectives in the viewpoint of probability theory, which is the proper methodology for multi-objective optimization [3-5]. Therefore, the consequences of the previous approaches are incomparable to the results of the probability-based method for multi-objective optimization due to their intrinsic problem.

5 CONCLUSION

By using probability-based multi-objective optimization for the simultaneous optimization of multiple objectives, discrete uniform experimental design for performing simplification, and the sequential algorithm for conducting further optimization, the multi-objective programming problem can be conducted rationally. The approach properly takes the simultaneous optimization of each objective of multi-objective programming problem into account, which reflects the essence of the multi-objective programming naturally, and creates a new way.

Conflict Statement

There is no conflict of interest.

6 REFERENCES

[1] Chang, H., Wu, Q., & Liu, X. (2019). *Operations Research Tutorial*. Tianjin Science and Technology Press, Tianjin, China.
[2] Liu, S. (2014). *Theoretical Method and Application of Multi-objective Programming*. Shanghai Jiaotong University Press,

Shanghai, China.
[3] Zheng, M., Wang, Y., & Teng, H. (2021). A new "Intersection" method for multi-objective optimization in material selection. *Tehnicki Glasnik, 15*(4), 562-568. https://doi.org/10.31803/tg-202109 01142449
[4] Zheng, M., Wang, Y., & Teng, H. (2022). A novel method based on probability theory for simultaneous optimization of multi-object orthogonal test design in material engineering. *Kovove Materialy, 60*(1), 45-53.
[5] Zheng, M., Wang, Y., & Teng, H. (2022). A novel approach based on probability theory for material selection. *Materialwissenschaft und Werkstofftechnik, 53*(6), 666-674. https://doi.org/10.1002/ mawe.202100226
[6] Zheng, M., Teng, H., Yu, J., Cui, Y., & Wang, Y. (2022). *Probability-based multi-objective optimization for material selection*, Springer, Singapore.
[7] Hua, L.-K. & Wang, Y. (1981). *Applications of Number Theory to Numerical Analysis*. Science Press, Springer, Berlin; New York. I
[8] Fang, K-T. & Wang, Y. (1994). *Number-theoretic Methods in Statistics*, Chapman & Hall, London, UK. https://doi.org/10.1007/978-1-4899-3095-8
[9] Fang, K.-T., Liu, M.-Q., Qin, H., & Zhou, Y-D. (2018). *Theory and Application of Uniform Experimental Designs*, Science Press & Singapore: Springer Nature, Beijing. https://doi.org/10.1007/978-981-13-2041-5
[10] Fang, K.T. (1994). *Uniform Design and Uniform Design Table*. Science Press, Beijing, China.
[11] Ying, J. (1988). *Multi-objective planning*. People's Education Press, Beijing, China.

Authors' contacts:

Maosheng Zheng
(Corresponding author)
School of Chemical Engineering, Northwest University,
Xi'an, 710069, China
E-mail: mszhengok@aliyun.com

Haipeng Teng
School of Chemical Engineering, Northwest University,
Xi'an, 710069, China

Yi Wang
School of Chemical Engineering, Northwest University,
Xi'an, 710069, China