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A mathematical approach to network contagion regarding greening banks' policies

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ABSTRACT

Green banking has become dominant in academics' and practitioners' discourse. The purpose of the research is to investigate whether banks' attitudes, in deciding to go green, change under the influence of other banks through their mutual interaction and whether hysteresis plays a part in the process. A mathematical model, described by a differential system with time delay, considering three variables, *i.e.*: green, outsider and undecided banks, is used as a research method. We investigate the local stability of the two equilibrium points. Moreover, we look for the optimal control strategy targeting the undecided banks so that the outsiders' group diminishes. The main contribution is that the paper sheds more light on the qualitative rather than on the quantitative side of the banking business given that banks' behaviours are examined when it comes to implementing green policies. The research has policy implications since bank managers can decide whether to follow the greening trends of other banks and bank regulators can use the instrument for tracking the overall changes in banks' behaviours in this respect.

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1. Introduction

Due to the effects of climate change, the term "green" has become increasingly popular in academics' and practitioners' discourse. In the absence of a largely accepted definition, the concept of "green banking" is compatible with corporate social responsibility, sustainable banking and environmental, social and governance. All these, including reporting and compliance requirements, have an indirect or direct impact on the types and range of activities in which banks engage.

Nevertheless, achieving these goals relies on the determination of bank policymakers and a clear understanding of the implied positive externalities. In this respect,

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the ability and willingness of banks, in collaboration with their clients, to identify their operations' economic, social and environmental impact are determinant factors. As part of the larger concept of sustainable banking, the banks' own *green* business ethics signals to clients their expectations in terms of promoting social inclusion, environmentally friendly procedures, durable profit-making, etc.

There is no one-fits-all definition of green finance, but a rather comprehensive one would be that it is a needed strategy to adjust risks by including the environmental factor (Septina, 2018).

Recent experiences show that green banking has been related to paperless bank transactions (internet banking, mobile banking, etc.), environmentally friendly investments' diversification and support granted for financial inclusiveness. Nevertheless, though expanding, this particular business approach has not been fully embraced. Some banks run full-fledged green operations (or are susceptible to do so), some are not completely aware of the benefits, remaining in a latent state while others are still reluctant to go green knowing that, at least in the short run, profits will be lower.

From another operational standpoint, there is a huge green investment opportunity worldwide, amounting to trillions of dollars as stated by Kludovacz et al. (2018) and International Financial Corporation (2019). Despite the need to convert economies into environmentally friendly ones that claim equally huge costs, this endeavour could trigger future long-term healthy economic growth.

Though evidence shows that banks can stimulate sustainable development and mitigate economic, social and environmental risks, the advancements have been rather slow. Among the many causes that deter the applicability of sustainability requirements is that not all banks fully understand the necessity to change, despite the evidence that *greening* bank investments may lead to higher, sustained performance and increased clients' loyalty.

Before the present paper, the authors have researched the financial risk contagion, verifying the applicability of the Hopf bifurcation with time delay in this particular field (Donath et al., 2020). Another research looks at financial inclusion as a determinant of sustainable banking (Donath & Oprea, 2020). The present paper is a continuation of research, but now we aim to identify the banks' attitudes when it comes to adopting the green investments paradigm.

The objective of the present study is to emphasise the existence of information channelling among banks that manifest different attitudes regarding *greening* their business: either they fully embrace the new paradigm, remain outsiders or are, simply, undecided. When the *greening* information messenger is perceived as trustworthy more banks will embrace the new business strategy.

The purpose of the research is to show that banks' attitudes change under the influence of other banks through their mutual interaction and that hysteresis plays a part in the process. Hence, the research aims to find the appropriate level of time delay at which the optimal control strategy can be identified to diminish the *Outsiders* group of banks.

The main contribution of the paper is that it looks at the attitude of the banks through the lenses of the mathematical epidemiology method, distinguishing three possible reactions of banks when they face the imminent need to adopt green business principles, i.e. *the Green banks* (or on track to go green), *Undecided* (banks that are not fully convinced of the benefits of green investments) and the *Outsiders* that do not venture to embrace the new paradigm (GUO). The GUO triptych allows an in-depth analysis of the manner in which banks accept or reject the new business approach, but, most importantly, it shows the transition from one group to another via the control theory that is fundamental in choosing the correct investment strategy to diminish the number of *Outsiders* at an optimum control level.

To our best knowledge, few works have used this method to explain changes in the financial field, albeit research on green banking uses a variety of methods, most of which rely on surveys and/or best practices which is understandable given the relative novelty of this research topic.

But, acknowledging the complexity of the decision-making process in adopting a green approach to the bank business, the authors of this paper have pursued a different path, more abstract but, at the same time, more inclusive since it encompasses the *greening* process in its entirety. Moreover, based on the proposed simulations, managers can adapt the data to the particular circumstances in which they operate, thus making the correct decisions. The authors also add value to the mathematical models found in the literature by introducing a time delay (τ) which confers more accuracy to the dynamics of banks' changing attitude knowing that implementing the *greening* strategies span over a longer timeframe.

The paper proceeds as follows: Literature review, Prerequisites, Description and dynamical analysis of the mathematical model, Optimal control strategy and Numerical simulation. The remainder of the paper is dedicated to Conclusions and further research.

2. Literature review

The topic of green banking has drawn increased attention over the last decade, albeit the *greening* concept had already emerged in the 1990s by connecting financial performance with the businesses' carbon footprint (Hamilton, 1995). Others (Schmidheiny & Zorraquin, 1996) argued that debt default could have been triggered by the penalties companies faced when not fully realising the environmental impact of investments. The researchers' interest in green banking shows a rise by an average growth rate of 25% in the number of publications discussing the topic (Prerana & Arup, 2021). They group the findings according to *the conceptual, legal, model, stakeholder, financial and green performance of banks aspects.*

As financial intermediaries, traditionally, banks play an active role not only in channelling funds effectively towards the real economy but also in influencing the business philosophy. Two decades ago, Jeucken and Bouma (1999) drew attention to the unique stance of banks that allowed them to promote sustainable businesses. A decade later, Sahoo and Nayak (2007) reiterated the determinant role of banks in supporting sustainable development that was gathering pace, while (Lalon, 2015) supports the concept of green banking as any form of environmentally friendly driven bank operations.

Presently, banks take on a new emerging part that is key to implementing sustainability and green principles, i.e. nudging their clients to embrace a green approach to their businesses. On the other hand, the growing sustainability awareness of the population pressures banks to adjust their lending policies and comply with the new requirements. (Biswas, 2011).

Another strand of literature that mainly regards the Asian countries' experience (Risal & Joshi, 2018) studies the environmental impact of banking concluding that green lending has little impact as compared to green equipment and green policies. Others, (Carlucci et al., 2018) approach the topic from a different perspective, i.e. the mapping of cognitive determinants of sustainable banking, which is important in identifying the main links between them so that the sustainability objectives are attained.

Green banking is regarded as encompassing a large set of values, including the fair distribution of the added value among shareholders and other contributors to the bank's profitability, i.e. employees and clients (Imeson & Sim, 2009). The argument stands since it may help mitigate the inherent bank risks, banks bearing the responsibility to prevent contagion (Scholtens & van Weensven, 2003). In addition, Anagnostopoulos et al. (2018) support the inclusion of sustainability in bank policy and Gelder (2006) summarises the main actions banks can take to become more sustainable.

Perhaps one of the major issues raised by literature is the significance of the concept of *values* from a corporate perspective (Oladele, 2013). In the context of green banking, one can argue that *value* refers to the positive externalities of green investments on the general welfare. The positive spillover is mainly determined by bank innovation and human resource investment which become relevant when discussing long-term profitability and the market value of the bank (Cabrita & Bontis, 2008). Authors (Ibe-Enwo et al., 2019) discuss green banking from the perspective of bank loyalty stressing the steps to be taken from a managerial perspective. Ellahi et al. (2021) analyse customers' awareness, concluding that it differs according to age, gender and occupation. Afzal et al. (2022) argue the necessity of strong institutions in alleviating environmental damages as well as improving education to further promote sustainable development.

But, literature on green banking does not provide yet a mainstream theoretical framework, authors relying in their research on best practices trying to depict the essential drivers that induce or trigger a change in the bank industry. Over time, sustainable banking has been investigated from various perspectives: religious, ethical, social inclusion, environmental, environmentally friendly internal operations etc. (Weber, 2012).

In this respect, the inclusion of social and environmental aspects in the banks' policies, strategies and products set the trend for the rest of the industries. Though most works insisted on the carbon footprint (D'Amato et al., 2017), one could not ignore the role of banks in funding sustainability-related investments. Nevertheless, even in earlier stages of green banking emergence, Lydenberg (2007) started the debate on the negative impact of short-term profitability and the necessity of a more durable approach while Tonello (2006) stressed the impact of financial volatility and instability due to unsustainable banking. According to Weber (2016), the green business model related to financial institutions should prevent any risks induced by the green transformation of the economy.

Notably, Asian banks have paved the way towards green banking, being aware that it is crucial in their effort to pursue economic growth sustainably. Devkota et al.

(2021) endeavours to provide a more accurate definition of *green banking* stating that it refers to ordinary banks that consider all social and environmental factors needed to preserve natural resources and act based on ethical and sustainable principles, granting low-cost financial support to businesses. Continuing the same line of thought, Bhardwaj's & Malhotra's (2013) investigations show that green/sustainable banking reflects the effort to assure all stakeholders that banking operations are consistent with the preservation of the environment.

Literature is also preoccupied with the green loan - profitability nexus, questioning whether going green supports profitability. Applying various methods, the general conclusion is that green banking leads to long-term, more durable profitability (Hossain & Kalince, 2014). Others argue, though, that there is no direct relationship between the two variables since the process is at an early stage (Taslima & Salina, 2016). Conducting an empirical study, (Bose et al., 2021) conclude that, due to improved cost-effectiveness and regulatory norms, banks that pursue green policies better perform than banks that do not envisage such an approach. Indian authors cited by Ahuja (2015) promote green banking as a possibility to support economic development without a negative impact on the environment. Awino et al. (2012), in an early study, already predicted the positive impact of green banking on Kenyan banks' performance. In his study on Chinese banks, Weber (2016) shows the necessity to include environmental risks when assessing the overall financial risk.

Deciding on the best research method is quite challenging when quantitative and qualitative analyses are needed. Most of the research in the green banking field relies on surveys to find the banks' views of the benefits and obstacles in embedding the new business perspective. Other papers analyse the green banking securities' return (Septina, 2018), or employ an empirical perspective (Rajput et al., 2013) to depict the banks' environmental performance. Not uncommon are studies that look into the banks' customers' perception of green banking (Koiry et al., 2017). Others have analysed bankers' insights into green banking, emphasising solutions to prevent carbon footprint and rewards for pursuing this path (Amir, 2021) and (Arinal et al., 2018).

For the present paper, the authors have chosen a rather heterodox approach to banking, starting from the mathematical models developed for epidemiological studies. These models are quite common in marketing-related research (Lacitignola, 2021), computer virus spread (Chen et al., 2015), in analysing the impact of information, noise and rumours on businesses, etc. (Ghosh et al., 2020) but less used in analysing changes in the financial field.

3. Prerequisites

Starting from the complex nature of the new banking business model, several initiatives were launched aiming to promote *green banking*. The framework for implementing this objective is shown in Figure 1. The main components to be considered are *The sustainability-driven pillars* (i.e. economy, social, environment), the drivers of the new banking model reflecting the client-centred relationship of the banks, and the implementation of a new banking culture based on governance and inclusiveness.

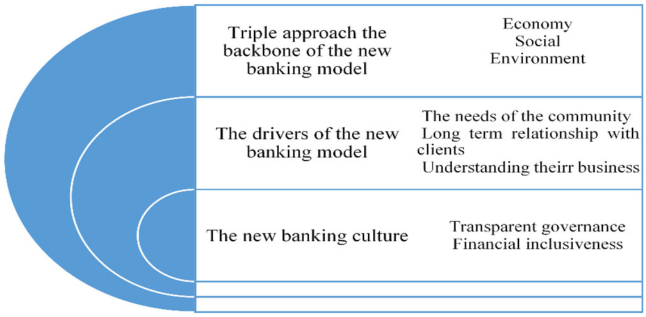

Figure 1. The framework of the new banking model.
Source: authors' representation

A comprehensive statistical view of the green banking stance for 2019 is provided by Whitney et al. (2020). The results show that 67% of established green banks are in high-income countries, 70% being public bank institutions. The results show that most of the surveyed banks (35%) are in the initial stages of adopting green policies, 13% are assessing the market, 23% have initiated institutional design, 13% are recruiting capital, 10% have a start-up and launched green strategies while 6% are tracking the results. Leading by example means that public banks are most committed to adopting green policies. The targets for the present decade are equally ambitious, 30% of loans in 2030 should be green compared to 7% in 2016.

The present stance of green banking raises the question concerning the factors that determine banks either to embrace or reject the greening strategy. As listed by Sustainable Banking Network (Sustainable Banking Network, 2020) a supportive policy environment is the most important and the awareness of low-income countries that deal with environmental and social issues is crucial for sustainable economic development.

Equally important is the voluntary involvement of local financial institutions, the incentives for innovation, competitiveness and the leading example of successful banks. In this respect, bank networks that follow the *trigger–engage-launch–refine* process help coagulate greening initiatives.

As practice around the world shows, there are remarkable *benefits* of sustainable banking (Lalon, 2015). Some of them refer to these banks' ability to grant funding for investments that are climate-friendly and that foster sustained economic development thus complying with the Paris agreement.

From this perspective, green banking coagulates other funding initiatives towards low-carbon activities. Further on, these banks can implement a new financial investment pattern, by changing the mindset of investors. Such attitudes can be further channelled to all types of investments including pension funds. On a national level, these banks may act as leaders in finding local solutions for particular pollution and/or climate problems. By encouraging long-term, durable profits, financial risks can be easily alleviated. In these circumstances, one could detect solutions to mitigate financial vulnerabilities and reverse negative contagions into positive ones.

By recording larger benefits, many banks have started to pursue green investments thus becoming broadcasters among their network and considered drivers of green banking. In this respect, green successful banks have gone through a meticulous design and implementation stage (Sustainable Banking Network, 2020) starting with a rigorous strategy based on their financial capabilities, involving major stakeholders, implementing locally relevant financial schemes, addressing constraints, engaging with their peers in learning and knowledge sharing, integrating sustainability in regulations and monitoring.

Despite the remarkable progress that has been made, some *obstacles* remain relevant: a need for assistance in preparing green investment projects, a lack of nationwide consensus for sustainable development commitment, the inability of the educational system to prepare the newly required competencies, a lack of financial stimuli for investors pursuing green projects.

Besides these obvious obstacles that constrain some banks to remain *outsiders* at the prospect of going green, there are other contributing factors, i.e. the insufficiency of correct and full information, the inability to see things in perspective, wariness to comply with new regulations, commitment to short term profits or unwillingness to change.

Another group of banks remain *undecided* about the possibility to adopt a green approach to their business. From our point of view, here, *indecision* mainly refers to the incapacity of the decision-makers to perceive the high costs of remaining in the same traditional business pattern and the lack of understanding of the new banking ecosystem.

It is equally true that no clear definition of *green* is provided, there is limited national expertise to lead sustainable finance, there are constraints on the economy's capacity to adjust, as well as the resentment of new regulations, etc. that hinder the efforts. Regarding the *outsider* and *undecided* banks, an interesting perspective could be envisaged if behavioural factors such as bias (overconfidence, unfounded past experiences, lack of confidence) were introduced in analysing banks' responsiveness to greening their policies. Likewise, hysteresis prevents some banks to acknowledge the benefits of greening policies, hence the high hidden costs of overlooking this new possibility of performing. Under this variety of attitudes, through information and communication channels, banks are, sometimes, induced to change their option, thus joining another group of banks that seem to be more compelling. Hence, the question of information contagion intensity among banks, which can be either positive or negative, is raised.

4. Description and dynamical analysis of the mathematical model

Authors have focused on the theory of differential systems with time delay, contributing to considerable advancements in the field of the biological, economic, neural network, and chemical systems, when time delays frequently occur (Mircea et al., 2012; Rihan, 2021).

For the investigation, we have chosen mathematical modelling given that mathematical models have proven useful as quantitative tools in solving economic and financial problems (Dhar et al. (2010); Sirghi & Neamtu, 2013). Moreover, in recent years, they have widely been exploited in explaining contagion in epidemics, going beyond the quantitative boundaries by including behavioural aspects and thus lending a helping hand to qualitative analysis.

Indeed, communication, interaction and information are among the most important channels that contribute to changing attitudes and behaviours. In bank networks, information contagion impacts the options of banks when deciding whether to adopt a green investment approach. To prove to what extent it is true, epidemiology mathematics was chosen for the present paper since it allows the analysis of corresponding banks' attitudes as *Green, Undecided and Outsiders* (GUO) and the changes that might occur in banks rallying in one group or another under the influence of their peers.

The *G* group are the banks that, being fully informed on green investments, *spread the word* among their network showing the benefits of green banking. The *O* group includes banks that have come across the information, but because of subjective or objective reasons remain outsiders. The *U* group of banks are undecided, probably because full information on the benefits of greening is not available or not sufficiently compelling. The *O* class of banks is targeted so that the number of banks included here diminishes by implementing the correct control strategies to convince them to join the *G* class of banks or at least the *U* class.

For the demonstration, it is considered that a constant number of banks (N) are included in the banking system and parameter a stands for the entry/exit rate on the bank market. Irrespective of the value of parameter a, N should remain unchanged.

The green banks (G) send the information to the outsiders' (O) class at a ρ rate, which can lead to migration to the other groups. The differential equation that characterises the outsiders' group is given by:

$$\dot{O}(t) = aN - \rho O(t)G(t) - aO(t)$$

The outsider (O) banks move to the green (G) group with a probability p and to the undecided (U) class with a probability $(1-p)$. It can be assumed that $p \in [0, 1]$ and has a high value if the message originates from a trusted green bank. Henceforth, $(1-p)$ stands for the undecided class that ignores the messages. The value of $(1-p)$ increases if the message comes from untrustworthy green banks. When the undecided group joins either the green or outsiders', they can still reconsider the decision and join the opposite group. It is also possible that some green banks stop sending the message following the interaction with other green banks (that perhaps were not profitable enough) and choose to join the undecided group at a rate σ. Correspondingly, undecided banks can join the greens' group either individually at a rate λ or by interacting with other green banks at a rate α p, where α is the original backdrop rate and p is called the trust parameter. Because the decision to join another group of banks is not made instantly, we incorporate the time delay in the equations of the green and undecided groups at time t that are impacted by the state variable G at previous time $\tau \leq$ t. Thus, the differential equations that describe the green and undecided banks are:

$$\dot{G}(t) = p\rho O(t)G(t) + \alpha p G(t)U(t) + \lambda U(t-\tau) - \sigma G(t)G(t-\tau) - aG(t)$$

$$\dot{U}(t) = (1-p)\rho O(t)G(t) - \alpha p G(t)U(t) - \lambda U(t-\tau) + \sigma G(t)G(t-\tau) - aU(t)$$

Therefore, the mathematical model, given by a differential system with time delay:

$$\begin{aligned} \dot{O}(t) &= aN - \rho O(t)G(t) - aO(t) \\ \dot{G}(t) &= p\rho O(t)G(t) + \alpha p G(t)U(t) + \lambda U(t-\tau) - \sigma G(t)G(t-\tau) - aG(t) \quad (1) \\ \dot{U}(t) &= (1-p)\rho O(t)G(t) - \alpha p G(t)U(t) - \lambda U(t-\tau) + \sigma G(t)G(t-\tau) - aU(t) \end{aligned}$$

where all parameters are positive real numbers.

To find the equilibrium points of (1) we identify the solutions of the following system:

$$\begin{aligned} aN - \rho OG - aO = 0 \\ p\rho OG + \alpha p GU + \lambda U - \sigma G^2 - aG = 0 \\ (1-p)\rho OG - \alpha p GU - \lambda U + \sigma G^2 - aU = 0. \end{aligned}$$

We suppose that the natural diminishing rate for each class of banks is slower than the *greening* marketing process $a < p\rho N$. Thus, we obtain the green-free banking equilibrium $E_0 = (N, 0, 0)$ and only one positive equilibrium $E_1 = (O_1, G_1, U_1)$, green-standing equilibrium, where:

$$O_1 = \frac{aN}{a + \rho G1}, \;\; U_1 = (\rho N - \rho G_1 - a)G_1,$$

where G_1 is the positive solution of the following equation:

$$\rho(\alpha p + \sigma)x^2 + (\sigma a + a\alpha p + \rho\lambda + \rho a - \rho p N)x + a(a - p\rho N) + \lambda(a - \rho N) = 0.$$

The second-degree equation has only one positive root, because $\rho(\alpha p + \sigma) > 0$ and $a(a - p\rho N) + \lambda(a - \rho N) < 0$.

To study the stability of the equilibrium points we linearize system (1) using the transformation:

$$x_1(t) = O(t) - O^*, \;\; x_2(t) = G(t) - G^*, \;\; x_3(t) = U(t) - U^*$$

and obtain:

$$\begin{aligned} \dot{x}_1(t) &= a_{11}\; x_1(t) + \; a_{12}\; x_2(t) \\ \dot{x}_2(t) &= a_{21}\; x_1(t) + a_{22}\; x_2(t) + b_{22}\; x_2(t-\tau) - a_{23}\; x_3(t) + b_{23}\; x_3(t-\tau) \quad (2) \\ \dot{x}_3(t) &= a_{31}\; x_1(t) + a_{32}\; x_2(t) - b_{22}\; x_2(t-\tau) - a_{23}\; x_3(t) - a - b_{23}\; x_3(t-\tau) \end{aligned}$$

where

$$a_{11} = -(\rho G^* + a), \quad a_{12} = -\rho O^*, a_{21} = p\rho G^*, \;\; a_{22} = p\rho O^* + \alpha p U^* - \sigma G^* - a,$$

$$a_{23} \;\; = \;\; -\alpha\rho G^*, \quad a_{31} \;\; = (1-p)\rho G^*, a_{32} \;\; = (1-p)\rho O^* - \alpha p U^* + \sigma G^*,$$

$$b_{22} = -\sigma G^{*}, \ b_{23} = \lambda.$$

The characteristic equation at the green-free banking $E_0 = (N, 0, 0)$ is given by:

$$(a+z)(a+z)\left(a+z+\lambda e^{-\lambda\tau}\right) = 0. \quad (3)$$

If there is no delay the green-free equilibrium E_0 is asymptotically stable.

If there is a time delay and $\lambda \leq a$, then (3) becomes:

$$\varphi_1(z) = \varphi_2(z),$$

where $\varphi_1(z) = (a+z)^3$ and $\varphi_2(z) = -(a+z)^2 \lambda e^{-z\lambda}$.

Therefore, using Rouche's theorem (Knoop, 1996), we deduce that E_0 is asymptotically stable for any time delay.

For the local stability of the green-standing equilibrium $E_1 = (O_1, G_1, U_1)$ we are interested in the roots of the characteristic equation:

$$P_3(z) - P_2(z)e^{-z\tau} = 0, \quad (4)$$

where

$$P_3(z) = z^3 + p_{32}z^2 + p_{31}z + p_{30}, \ P_2(z) = p_{22}z^2 + p_{21}z + p_{20}$$

with

$$p_{32} = a - a_{11} + a_{22} + a_{23}, \ p_{22} = -b_{22} - b_{23},$$

$$p_{31} = a_{23}a_{32} + (a_{22} + a_{11})(a_{23} - a) + a_{22}a_{11} - a_{12}a_{21},$$

$$p_{30} = a_{12}a_{23}(a_{31} + a_{21}) - a_{23}a_{11}(a_{22} + a_{32}) + aa_{11}a_{22},$$

$$p_{21} = b_{22}(a + a_{11} + 2a_{23}) + b_{23}(a_{11} + a_{22} + a_{32}),$$

$$p_{20} = -2b_{22}a_{11}a_{23} + b_{23}(a_{11}a_{21} - a_{11}a_{22} + a_{12}a_{31} - a_{11}a_{32}) - aa_{11}a_{22}.$$

If there is no delay equation (4) becomes:

$$z^3 + (p_{32} - p_{22})z^2 + (p_{31} - p_{21})z + p_{30} - p_{20} = 0. \quad (5)$$

Moreover, if the following conditions hold:

$$p_{32} > p_{22}, \quad p_{30} > p_{20} \quad \text{and} \quad (p_{32} - p_{22})(p_{31} - p_{21}) > p_{30} - p_{20}, \quad (6)$$

E_1 is locally asymptotically stable.

We investigate the existence of Hopf bifurcation in Appendix. More precisely, we look for the critical value of parameter τ so that the system's stability changes and a periodic solution occurs (Marsden & McCracken, 2012).

Under conditions (6) and (10) (see Appendix), the green-standing equilibrium is locally asymptotically stable for $\tau \in [0, \tau_0)$. When the time delay crosses the critical value τ_0 a Hopf bifurcation occurs at E_1.

5. Optimal control strategy

In this section, we look for the necessary and sufficient conditions of the optimal control strategy. We introduce the control variable u(t), a continuous function, defined as the control strategy such as effective tools or nudges targeting the undecided banks at time t.

Let t_f, Δ be two given constants that define the admissible control set

$$\mho = \left\{ u(t) \ \ is \ \ measurable, \ \ 0 \le u(t) \le \Delta, \ \ t \in [0, t_f] \right\}.$$

The optimal control problem minimizes the functional

$$J(u) = \int_0^{t_f} \left[O(t) + \frac{B}{2} u(t)^2 \right] dt, \tag{7}$$

subject to

$$\begin{cases} \dot{O}(t) = aN - \rho O(t)G(t) - aO(t) - (1-\epsilon)u(t)U(t), \\ \dot{G}(t) = p\rho O(t)G(t) + \alpha pG(t)U(t) + \lambda U(t-\tau) - \sigma G(t)G(t-\tau) \\ \qquad -aG(t) + \epsilon u(t)U(t), \\ \dot{U}(t) = (1-p)\rho O(t)G(t) - \alpha pG(t)U(t) - \lambda U(t-\tau) + \sigma G(t)G(t-\tau) \\ \qquad -aU(t) - u(t)U(t) \end{cases} \tag{8}$$

where $O(\sigma) > 0$, $G(\sigma) > 0$, $U(\sigma) > 0$, with $\sigma \in [-\tau, 0]$ and B is a positive constant to keep a balance in the size of u(t) and $u(t)^2$ is the impact of the control [Chen et al., 2015].

The corresponding Hamilton function is given by:

$$H(O, G, U, u, \lambda_1, \lambda_2, \lambda_3, t) = O(t) + \frac{B}{2} u(t)^2 + \lambda_1(t) \frac{dO(t)}{dt} + \lambda_2(t) \frac{dG(t)}{dt} + \lambda_3(t) \frac{dU(t)}{dt} \tag{9}$$

where $\lambda_1, \lambda_2, \lambda_3$ are the adjoint functions.

As in [Chen et al., 2015], there is an optimal control $u^*(t)$ such that $J(u^*(t)) = minJ(u(t))$, subject to (8).

Given $u^*(t)$ and the corresponding $O^*(t)$, $G^*(t)$, $U^*(t)$, the variables $\lambda_1(t), \lambda_2(t), \lambda_3(t)$ satisfy:

$$\begin{cases} \dot{\lambda}_1(t) = (a + \rho G^*)\lambda_1(t) - p\rho G^*\lambda_2(t) - (1-p)\rho G^*\lambda_3(t), \\ \dot{\lambda}_2(t) = \rho O^*\lambda_1(t) - [p\rho O^* + \alpha p U^* - \sigma G^* - a]\lambda_2(t) - (1-p)\rho O^*\lambda_3(t) \\ \qquad +(\alpha p U^* - \sigma G^*)\lambda_3(t) - \ [-\sigma G^*\lambda_2(t) + \sigma G^*\lambda_3(t)]\chi_{[0,\, t_f-\tau]}(t) \\ \dot{\lambda}_3(t) = -1 + (1-\epsilon)u\lambda_1(t) - [\alpha p G^* + \epsilon u]\lambda_2(t) + [\alpha p G^* + a + u]\lambda_3(t) \\ \qquad -[\lambda\lambda_2(t) - \lambda\lambda_3(t)]\chi_{[0,\, t_f-\tau]}(t) \end{cases} \tag{10}$$

with the transversality conditions:

$$\lambda_i(t_f) = 0, \quad i = 1, 2, 3$$

and

$$\chi_{[0,\, t_f-\tau]}(t) = \begin{cases} 1, & t \in [0, t_f - \tau] \\ 0, & otherwise \end{cases}$$

Moreover, $u^*(t) = \min\left\{\max\left(0, \frac{O^*(t)}{B}\left((1-\epsilon)\lambda_1(t) - \epsilon\lambda_2(t) + \lambda_3(t)\right)\right), \Delta\right\}$.

6. Numerical simulations

For the numerical simulations we consider the following parameters: $N = 0.5$, $a = 0.02$, $\rho = 0.2$, $p = 0.5$, $\sigma = 1$, $\alpha = 0.8$, $\lambda = 0.2$ and Mathematica and Maple software were used. The green-free banking equilibrium is $E_0 = (0.5, 0, 0)$ and only one positive equilibrium is $E_1 = (0.17067,\ 0.19295,\ 0.13636)$.

We have $\lambda \leq a$ and E_0 is asymptotically stable for any $\tau \geq 0$. The trajectory (t, O(t)) is displayed in Figure 2:

If there is no delay $\tau = 0$, conditions (6) are satisfied and E_1 is locally asymptotically stable. The orbits (t, O(t)), (t, G(t)), (t, U(t)) are shown in Figure 3:

Conditions (6) and (A4) are satisfied. Equation (A2) has only one positive root $\omega_0 = 0.412094$ and the critical value of the time delay is $\tau_0 = 3.94678$. A Hopf bifurcation takes place, a limit cycle occurs. The orbits (t, O(t)), (t, G(t)), (t, U(t)) can be observed in Figure 4:

To find the optimal strategy solution for (8) subject to (7) we consider the following values of the parameters: $t_f = 300$, $\Delta = 0.8$, $\varepsilon = 0.8$, $B = 20$, $\tau = mh$, with the step size $h = 0.1$ and different values for m: 20, 39, 80.

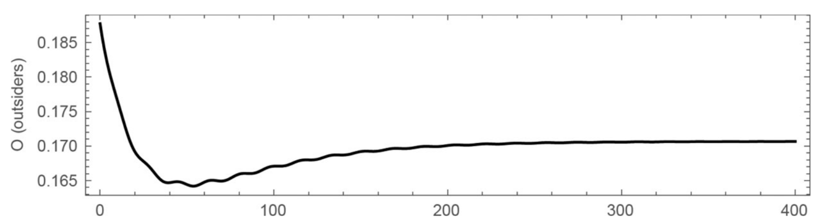

Figure 2. The evolution of O(t) in the case of a discrete time delay $\tau = 6$ choosing an initial condition in a neighbourhood of the green-standing equilibrium point (0.5, 0, 0), that is locally asymptotically stable.
Source: Authors' own representation.

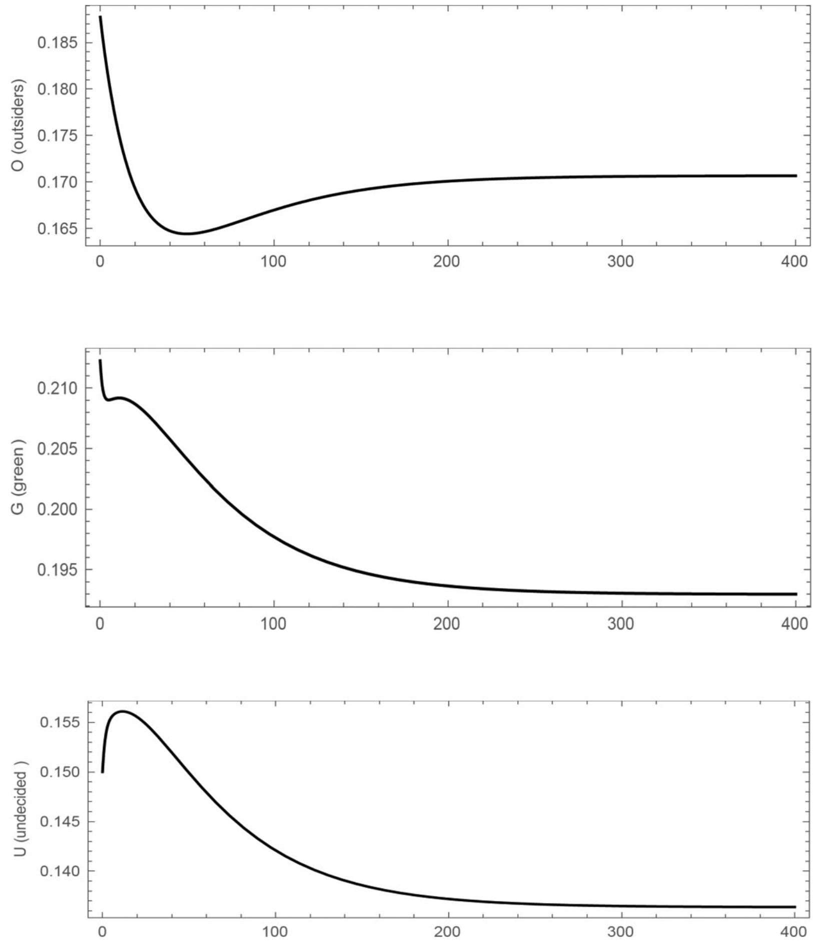

Figure 3. The evolution of O(t), G(t), U(t), if there is no time delay and the initial condition is in a neighbourhood of the green-standing equilibrium point $(0.17067, 0.19295, 0.13636)$ that is locally asymptotically stable.
Source: Authors' own representation.

For the time delay $\tau = 2$ we display, in Figures 5–7, the control variable and the evolution of outsiders, green class and undecided class with and without control.

Figure 7 shows that for $\tau = 2$ the G group of banks slightly decreases remaining in the same range, the U group remains fairly stable, but the O group diminishes, proving that the optimum strategy control value that was found (Figure 6) is the appropriate one to reach the pursued target prompting the narrowing the O group of banks that are reluctant to adopt the green investment strategy. We can notice that the control is effective.

For the time delay $\tau = 3.9$ we display, in Figures 8–10, the control variable and the evolution of outsiders, green class and undecided class with and without control.

As Figures 9 and 10 show, for $\tau = 3.9$ and the optimum control (Figure 10), the number of G banks remains in the same range as in the previous case, but the U and O group of banks are more volatile, meaning that the impact of the adopted strategies is not accurate enough to induce stability in deciding whether to pursue or not a green path. Nevertheless, there is a potential to lower the O group.

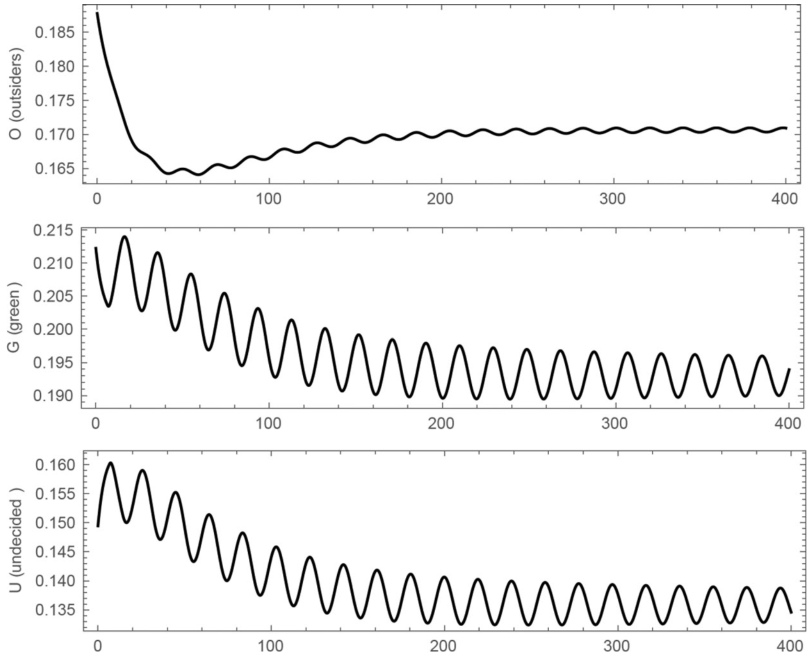

Figure 4. The evolution of O(t), G(t), U(t) in the case of $\tau = 6.9$, choosing an initial condition in a neighbourhood of $(0.17067,\ 0.19295,\ 0.13636)$. A stable limit cycle takes place.
Source: Authors' own representation.

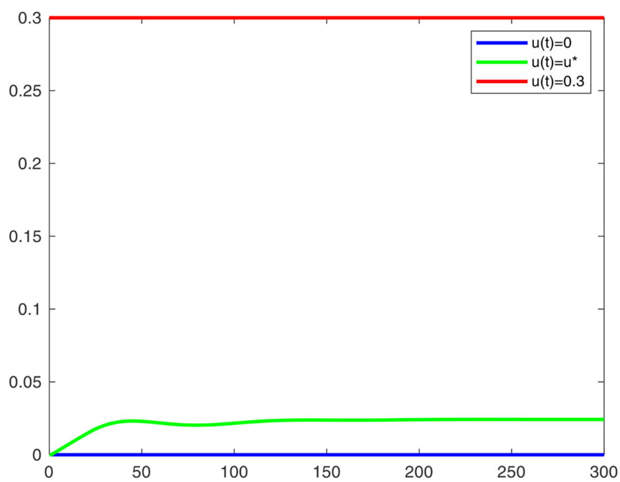

Figure 5. The control variable u(t) for the discrete time delay $\tau = 2$.
Source: Authors' own representation.

For the time delay $\tau = 8$ we display, in Figures 11–13, the control variable and the evolution of outsiders, green and undecided groups with and without control.

If $\tau = 8$, as Figures 12 and 13 show the trends of all three groups of banks become volatile for the optimum control value and the target to lower the number of the O group is not reached. Hence, a time delay of 8 is perceived as too long and induces even more inconsistencies in the approach of green banking meaning that the control

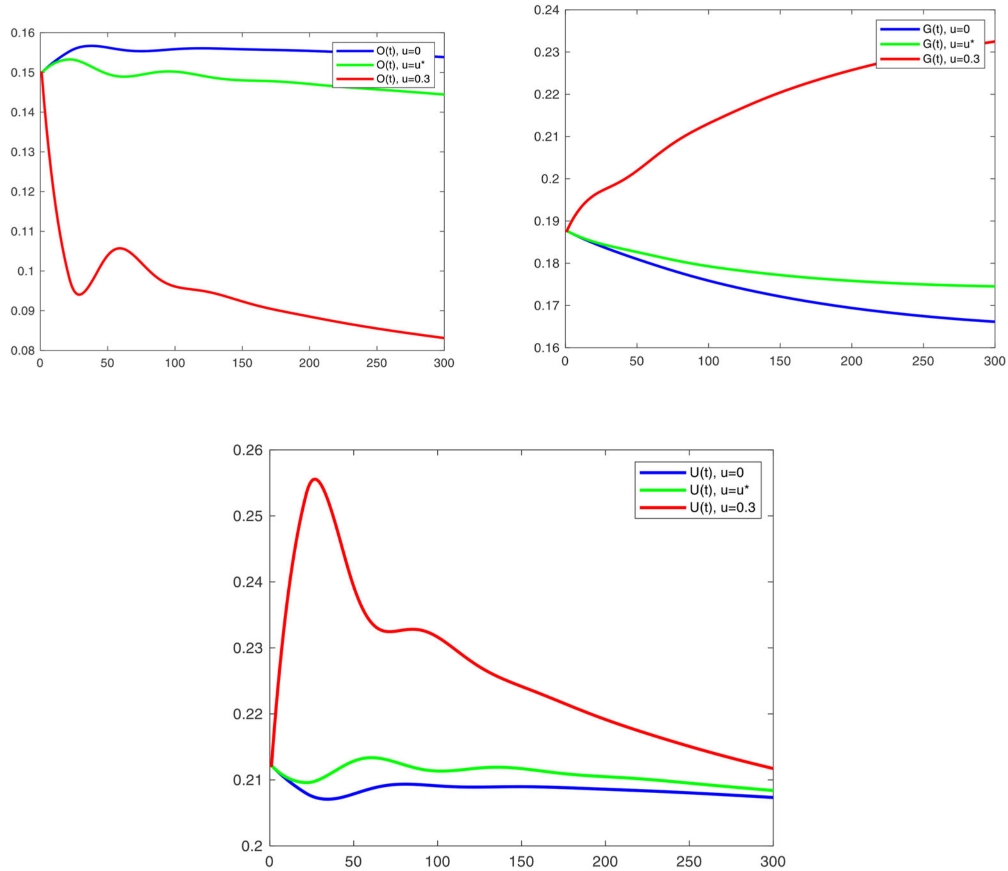

Figure 6. The orbits (t, O(t)), (t, G(t)), (t, U(t)) corresponding to the control strategies: $u = 0$, $u = u^*$, $u = 0.3$ and the discrete time delay $\tau = 2$.
Source: Authors' own representation.

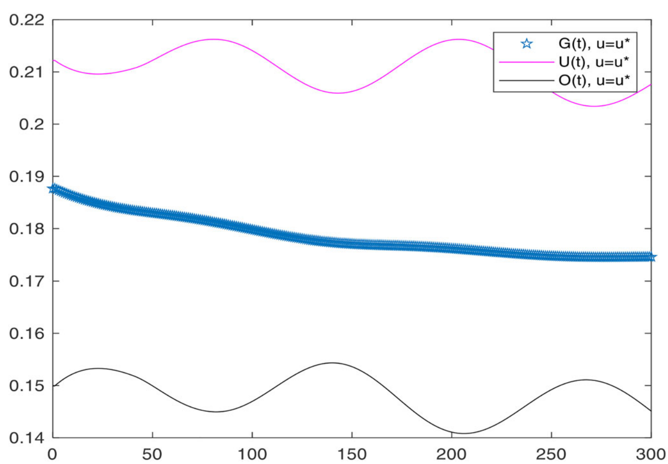

Figure 7. The evolutions of O(t), corresponding to the optimal control and the discrete time delay $\tau = 2$.
Source: Authors' own representation.

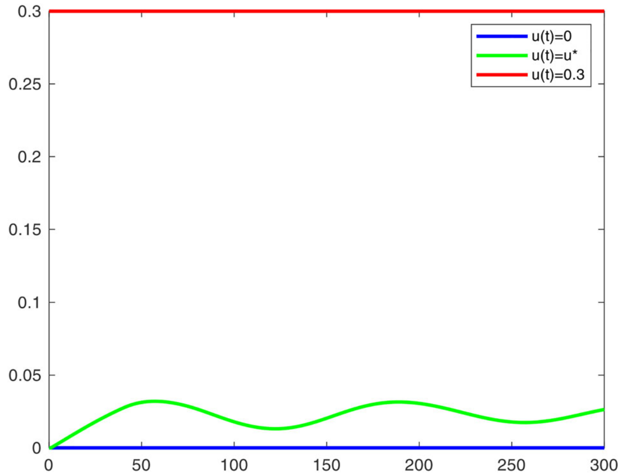

Figure 8. The control variable u(t) for the discrete time delay $\tau = 3.9$.
Source: Authors' own representation.

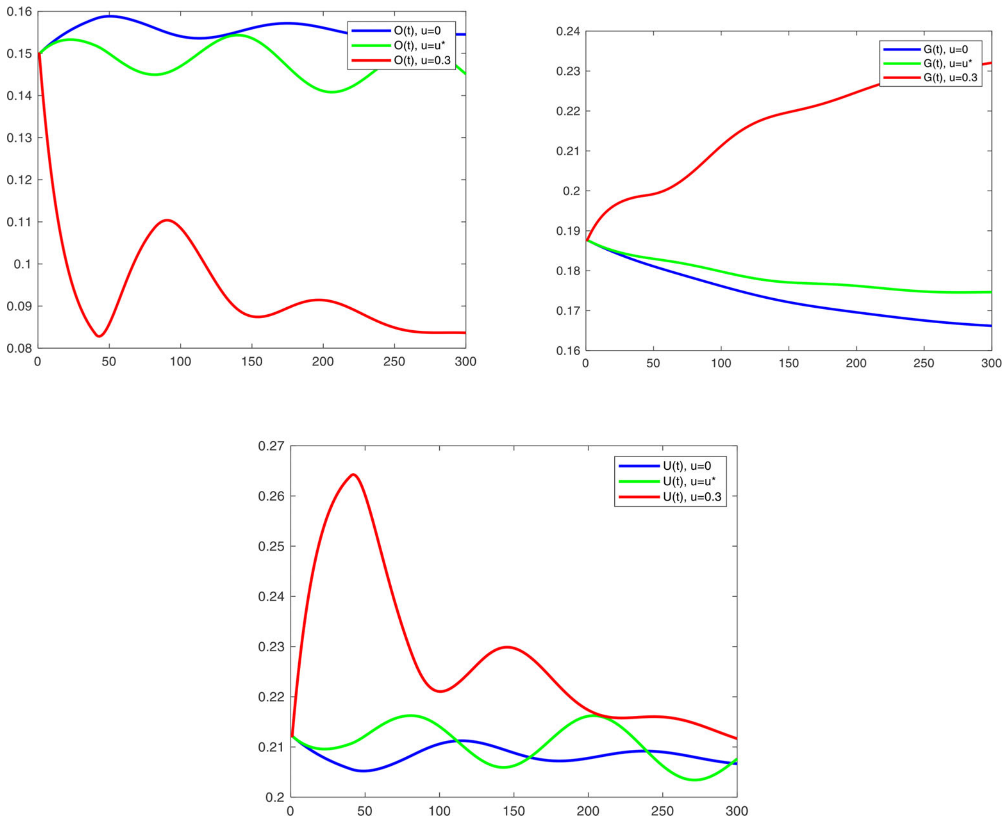

Figure 9. The orbits (t, O(t)), (t, G(t)), (t, U(t)) corresponding to the control strategies: $u = 0$, $u = u^*$, $u = 0.3$ and the discrete time delay $\tau = 3.9$.
Source: Authors' own representation.

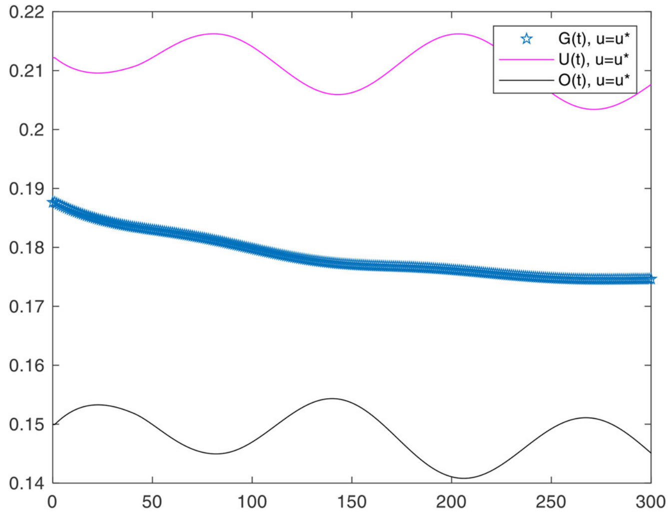

Figure 10. The evolution of (t, O(t)), (t, G(t)), (t, U(t)) corresponding to the optimal control and the discrete time delay $\tau = 3.9$.
Source: Authors' own representation.

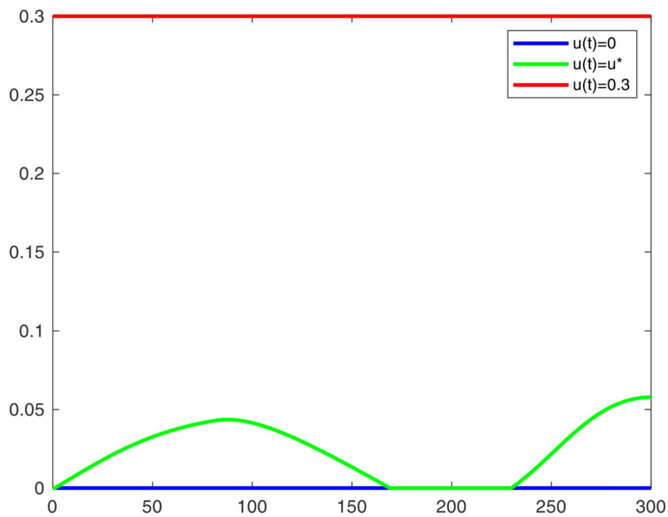

Figure 11. The control variable u(t) for the discrete time delay $\tau = 8$.
Source: Authors' own representation.

strategies, though at an optimum level, are considered either ineffective or too costly to implement.

7. Conclusion and further research

Green banking has become a reality that cannot be ignored. Banks have become very powerful drivers of the new *greening* paradigm, providing their clients with the necessary funding to convert their own activity towards sustainability. At the same time, it is important to stress the fact that not only climate-related challenges must be

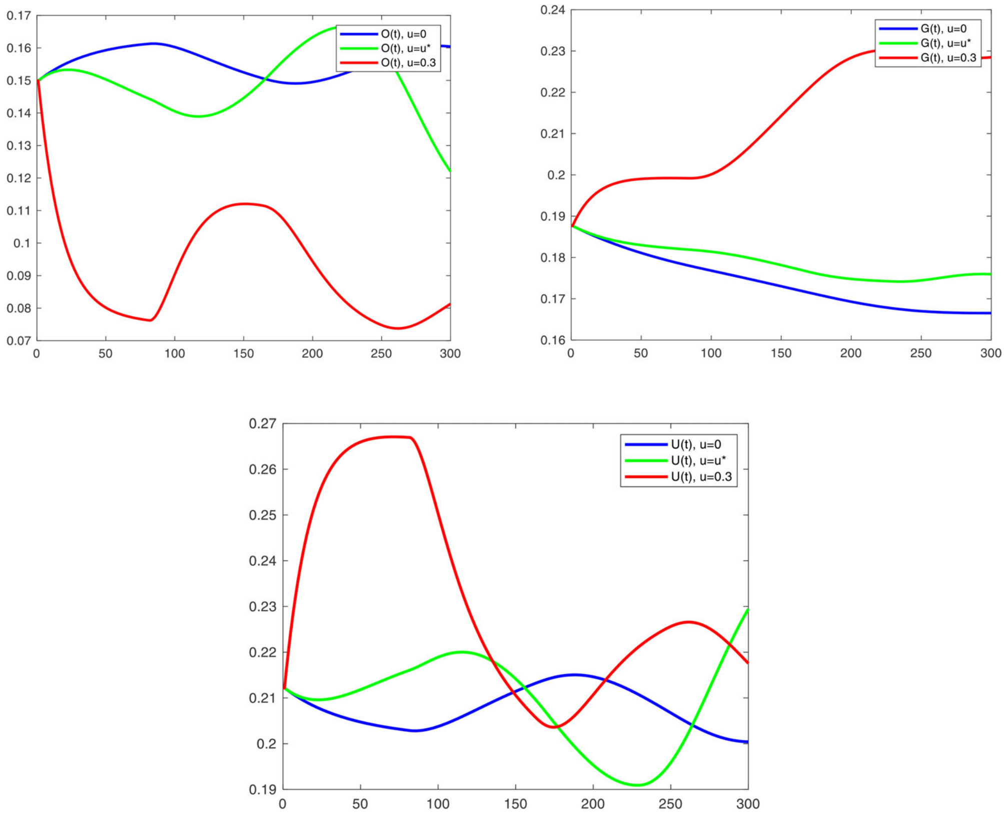

Figure 12. The orbits (t, O(t)), (t, G(t)), (t, U(t)) corresponding to the control strategies: $u = 0$, $u = u^{*}$, $u = 0.3$ and the discrete time delay $\tau = 8$.
Source: Authors' own representation.

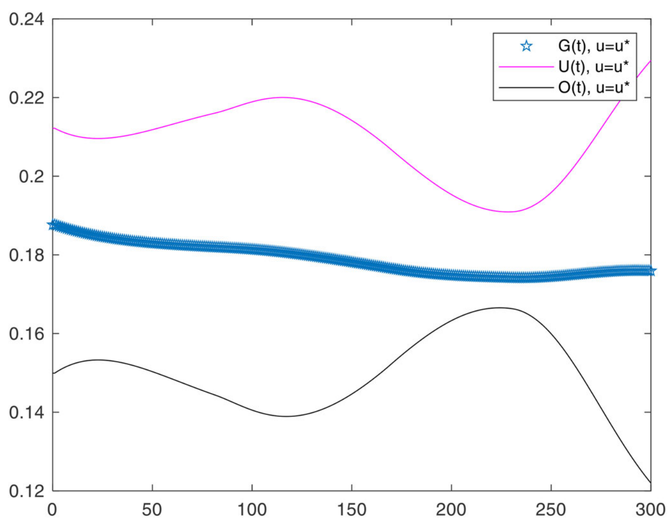

Figure 13. The evolution of (t, O(t)), (t, G(t)), (t, U(t)) corresponding to $u = u^{*}$ and the discrete time delay $\tau = 8$.
Source: Authors' own representation.

addressed by banks, through their investments, but also the other sustainability components, e.g. financial inclusion and mitigating frequent macroeconomic imbalances. By meeting these challenges, banks can also better resolve the traditional risks that have negatively impacted banking systems, culminating in crises and failures.

Nevertheless, as surveys shows, not all banks have embedded the green approach in their businesses, either because they are not fully informed or the transition process is considered too costly.

The purpose of the present research is to show that banks' attitudes to going green change under the influence of the other banks in the network.

We consider a mathematical model, described by a differential system with time delay, including three variables named: green, outsider and undecided banks. Two equilibrium points are determined: green-free and green-standing and the local stability of them is investigated. The numerical simulation shows that banks' attitudes can change under the influence of the network.

Hence the impact of our findings is at least two folded:

On one hand, we provide a managerial tool that can be used in the decision of banks to go green and find the appropriate moment to do so. Our research complements the findings of the cited literature, which demonstrate the benefits of green banking, by offering a more inclusive instrument in the decision-making process.

On the other hand, it provides a useful tool for central bankers and regulators. For them, it is the Undecided group of banks that are targeted, because, depending on the ability of bank authorities to implement effective strategies and nudges, they will join either the Green group or the Outsiders one. It is equally true, that the Undecided and/or Outsiders' group of banks often refer to a series of challenges in adopting greening strategies, i.e. technical capacity constraints, limited or insufficiently accurate knowledge, shareholders' favouring profits rather than long-term sustainable business and engagement with their clients, wariness of transition bureaucracy, etc. Therefore, the monitoring of day-to-day bank activities can be conducted in a pre-established framework that strongly relies on transparency and disclosure principles. At the same time, raising banks' awareness through an effective training system and technical endowment can contribute to their understanding of the climate risks assessment. In addition, setting up simple guidelines, knowledge-sharing platforms, providing taxonomies, exchanging views with successful *green* banks, etc. can contribute to the success of this endeavour. Last but not least, empowering bank networks and wide capacity building is probably the most powerful in persuading more banks to join the broadcasters' group not only showing the benefits but also laying out the process for a smooth transition to *green* banking.

Specifically, we analyse the reaction of the banks to the greening prospect at different levels of the time delay. It is proven that the appropriate delay can be found at which the optimum control strategy value is most effective, i.e. the O group shrinks. The policymakers and regulators may identify the best path and the delay at which the O group of banks narrows. The approach is an abstract one, but more inclusive, suggesting that based on a simulation, the best solution can be found, and adapted to the particularities of each economy and banking system. Since the literature on green banking is still exploratory, we consider that the suggested mathematical approach provides a useful tool. In the numerical simulation, we find that the optimum strategy leads to the diminishes of the outsiders' group, although the green group slightly decreases.

To complement the present findings, since greening the bank systems is a long process, further empirical research is needed. We intend to extend the research by analysing various groups of countries, finding the determinants that drive banks to green their business and identifying the economic and behavioural incentives for adopting green policies.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix. The existence of Hopf bifurcation

We assume that conditions (6) are satisfied. We are looking for $z = \pm i\omega(\omega > 0)$ such that

$$e^{i\omega\tau} = \frac{P_2(i\omega)}{P_3(i\omega)}. \tag{A1}$$

Taking the absolute value of (A1) we have:

$$|P_3(i\omega)| = |P_2(i\omega)|$$

that leads to:

$$\omega^6 + r_2\omega^4 + r_1\omega^2 + r_0 = 0 \tag{A2}$$

$$r_2 = p_{32}^2 - p_{22}^2,\ \ r_1 = (p_{31}^2 - 2p_{30}p_{32} - 2p_{31} + 2p_{20}p_{22} - p_{21})^2,\ \ r_0 = p_{30}^2 - p_{20}^2.$$

Considering $\omega^2 = x$, equation (A2) becomes:

$$x^3 + r_2x^2 + r_1x + r_0 = 0. \tag{A3}$$

We assume $Q(x) = x^3 + r_2x^2 + r_1x + r_0$ satisfies one of the following conditions:

i. $r_0 < 0$ or
ii. $r_0 \geq 0,\ \ \Delta = r_2^2 - 3r_1 > 0,\ \ x^* = \frac{-r_2+\sqrt{\Delta}}{3}$ with $Q(x^*) = 0$. (A4)

In this case equation (A3) has at least one positive root. When there are three positive roots of (A3) denoted by $x_{10},\ x_{20}, x_{30}$, the corresponding positive roots of (A2) are $\omega_{k0} = \sqrt{x_{k0}},\ \ k = 1, 2, 3$. For each ω_{k0} we obtain

$$\tau_k^j = \frac{1}{\omega_{k0}}, j = 0, 1, 2, \ldots \tag{A5}$$

We define $\tau_0 = \min\left\{\tau_k^j\right\},\ \ j = 0, 1, 2, \ldots, k = 1, 2, 3$. Let $z(\tau)$ be the root of (A1) with $z(\tau_0) = i\omega_0$. It satisfies $e^{\tau z(\tau)} = \frac{P_2(i\omega)}{P_3(i\omega)}$ and then

$$\frac{dz(\tau)}{d\tau} = \frac{z(\tau)}{\frac{P_3(z(\tau))}{P_2(z(\tau))}\left(\frac{P_2(z(\tau))}{P_3(z(\tau))}\right)' - \tau}$$

with $\frac{dRe(z(\tau))}{d\tau}|_{\tau=\tau_0} \neq 0$.