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Dynamic pricing and inventory control of online retail of fresh agricultural products with forward purchase behavior

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ABSTRACT

In this paper we formulate and analyze a novel model on a retailer's inventory and pricing decisions for fresh agricultural products with consumers' forward consuming behavior under online channel. We consider a dynamic stochastic setting, where every period consists of two stages, discounting pricing stage and regular sale stage. At the beginning of the period, the retailer decides how much new fresh agricultural products to order and sets discount price for leftover inventories from the previous period which will be disposed otherwise, and determines regular price for fresh products on the second stage, respectively. Since forward purchase consumers may buy the products during discount pricing stage, which may cannibalize future sales at regular price, the retailer needs to make a trade-off decision between regular price and discounting price. We bring forward a dynamic optimization model and use nonlinear programming method of Karush Kuhn Tucker condition to obtain the optimal dynamic strategy, which is comparatively analyzed to dominate the related static strategy. We also show that consumers forward buying behavior will negatively influence the retailer's profit. When the price is set too low in regular or discounting sales, the profit will show an up-down trend if the inventory exceeds a certain threshold. Meanwhile, when fresh goods returns are allowed and resold in the secondary stage, the retailer's profit will increase. We finally conduct numerical studies to further characterize the optimal policy, and to evaluate the loss of efficiency under static policies when compared to the optimal dynamic policy.

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1. Introduction

Since the development of E-commerce, more and more retailers supplement their brick-and mortar presence with an online store by providing cheaper and more efficient distribution channels for their products or services. For example, Amazon launched its business with an e-commerce-based model of online sales and product delivery. The efficiency of

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online channel also attracts a large number of fresh agricultural products sale businesses to enter this field. IResearch consulting company has found that the number of fresh agricultural products (FAPs) transactions in China's e-commerce market exceeded 200 billion yuan in 2018. It is expected that the average annual growth of online retail fresh agricultural products transactions will remain at around 35% in the next three years. In the future, as the scale of mobile users and the penetration rate of mobile online shopping continues to increase, and the concept of online shopping becomes more and more popular, consumers' demand for online fresh purchase will continue to expand.

However, as the perishability nature of fresh agricultural products that make the products deteriorating over time soon, those retailers are currently facing high operating costs and logistics costs even through online channel selling. Such as Stater Bros, a well-known grocery store in Southern California who focuses its business on fresh agricultural products selling, reported its higher commodity costs and competitive pressures in fiscal 2011[1]. Generally, they always set a discount price policy to sell non-fresh agricultural products a period of time for taking back some revenue. It is not a trivial problem for the seller to do that. However, with the development of information technology, some consumers may recognize the promotion sales and adopt strategic purchase policy, accordingly. As Angwin and Mattioli (2012) and Shi (2014) and other scholars mentioned, e-commerce greatly influences the transmission mode of price and changes the behavior pattern of consumers, such as the appearance of forward buying behavior. Some of the consumers who can afford the full price may be attracted by the price discount to buy non-fresh products, which has a negative impact on the revenue from selling at the full price. Meanwhile, pricing is also a double-edged sword in that pricing can not only affect the demand but also have great impact on the sellers' profit, especially a poor pricing policy which can easily damage the seller's benefit. For example, Tesco, the world's third-biggest supermarket chain, failed to revive its sales performance despite spending £500 million on price cuts in Eurozone[2]. Although the discount selling on those units can generate some revenue for the otherwise disposed items, the new and old products will compete together among consumers who select their choice based on its utility from combination of quality and price. In addition, a report from Xinlang Finance on December 12, 2015, claims that about 20% of fresh products could be returned and exchanged within 48 hours without reasons. And the 4th China fresh e-commerce forum in 2017, officially releases the first Chinese community standards, 'fresh e-commerce platform returning and exchange service requirements', which provides consumers with rights to rescission. Since then, high rates of return and exchange of goods in online retailing appears, which increases channel inventory cost and incurs return cost, and then influences the retailer's profit because all the returned products may be resell on a discount price or disposed directly. As Yao and Zhang (2013), Altug and Aydinliyim (2016), Fan and Wang (2018) pointed out, an important difference between online and offline retail is that online retail has logistics costs because of a large number of returns. Therefore, some fundamental decisions are at the store manager's discretion in such an environments, how much to order for the next time and what pricing policies to follow, the regular price and the discount price, respectively.

In this study, we formulate a novel model in which a retailer makes simultaneous decisions on regular and discount price as well as inventory control for fresh agricultural products under online retailing channel, in the presence of consumer's forward buying behavior and demand uncertainty. We consider a dynamic stochastic setting over multi-period planning horizon, in which each period consists of two parts, a discount pricing stage and a regular-sales stage. For the convenience of further study, we label the time periods such that every period starts with the discounting stage. Any unit from the previous period has to either sold or disposed at the end of the discounting stage, while left-over inventory from the regular-sales stage can be carried over to the discounting stage of the next period. In practice, for the sake of feasible retailing operation as the perishabiliy, the return and exchange policy applies only to the new fresh agricultural products except the old ones which still can be given satisfactory price discount compensation. Generally, new fresh products at the regular-sale stage could be returned and resold at the discount stage or disposed directly, while the old ones at the discounting stage are not allowed. And the return cost is assumed to be related to the sale quantity of fresh agricultural products based on previous studies. Demand in each period is random with some consumers forward buying behavior of preferring buy in the discounting stage.

Our model assumes a population of homogeneous (statistically identical and independent) customers making strategic choices from a set of different ages of FAPs. We set two factors to present the proportion of consumers who attempt to make forward purchases and the ratio of return and exchange, respectively. The retailer, through the integration of inventory control and dynamic pricing, influences consumers' choices to his advantage to maximize profit. Obviously, the problem is extremely complicated even if the products are not perishable. Unlike standard inventory models of general products with same attributes, the products' perishability, consumers forward buying portion and the ratio of return and exchange effect plays an important role in driving the dynamic pricing adjustments as it affects not only the overall probability of making a sale but also the sales probability of individual variables within the in-stock offerings/variables. As a result, the original formulation is not jointly concave in the decision variables and is therefore intractable. To deal with the complexity, we work with the portion parameters to transform the problem to be jointly concave and tractable. Therefore, we characterize the optimal policy and develop effective methods to solve it. At last, a numerical study is presented to illustrate the interplay of the price and inventory control of FAP on online channel. Our contribution to the literature as follows. Firstly, we formulate and analyze a novel model on a retailer's dynamic inventory and pricing decisions for fresh agricultural products with consumers' forward consuming behavior under online channel. Characterizing the structure of model, especially with conditions on some parameters, we present an optimal solution to the problem. Second, our numerical results provide insights on the behavior of the optimal dynamic prices, and highlight the complex interplay of inventory scarcity and consumers forward buying behavior. Thirdly, we generalize the regularity conditions of profit functions for lost-sales inventory-pricing models, using the concept of concavity.

The remainder of the paper is structured as follows. Section 2 reviews the literature. We introduce the model formulation and exploit the optimal solution, as well as some numerical experiments in section 3, Finally, section 4 provides concluding remarks.

2. Literature review

The development of online retailing put forward a profound impact on the traditional retail. Although, bringing shopping convenience to consumers, it also changes business model and cost structure of retailers (Chen et al., 2006) as well as consumers purchase behavior, such as forward purchase buying (Shi, 2014). Some studies (Altug & Aydinliyim, 2016; Fan & Wang, 2018; Yao & Zhang, 2013) have pointed out that there is an important difference between online and offline sales, which lies in the logistics cost and return or exchange. In decades of years, studies mainly focus on problems under different market environments, see Table 1 listing the inventory control models of different market demand, which follows different stochastic distribution. With regard to fresh agricultural products retailing, most scholars generally add one or two more variables, such as freshness or time on the market demand function (Chen et al., 2014, 2018; San-José et al., 2015; Tripathi et al., 2015).

In line with offline sales, dynamic pricing strategy can facilitate the sales since the menu cost of online sales is almost zero, especially when demand is random and sensitive (Meng et al.,2010). More scholars also point out that more efficiency and less decay can be achieved through dynamic pricing strategies (Adenso-Díaz et al., 2017; Akçay et al., 2010; Herbon et al., 2014; Lu et al., 2016).See Table 2, that many scholars explore the dynamic pricing of perishable products under offline business environment.

Besides the above offline mode of operation, some studies involve dynamic pricing and inventory control on fresh agricultural products in online retail setting. Cheng (2011) established a Berman recursive equation with dynamic pricing strategy under online sale setting based on the market demand state model, and finally used a Q learning algorithm to solve the dynamic optimization problem. Wang and Dan (2014) assumed that the utility function of consumers' random demand is related to freshness and price under multi-price discount, and constructed an economic order volume model of online sales of fresh products. Cohen et al. (2017) proposed a linear integer programming approximation to solve the dynamic promotional pricing problem and showed that their approach could use sales data from online retailers to increase profits by 3%. Fisher et al. (2018) proposed that online retailers could easily track price changes of competitors and studied competitive dynamic pricing

Table 1. Inventory control models based on different demand functions.

Author(year)	Market demand	Inventory control model
Chen et al. (2014)	The demand depends on the price plus an additional random term	Joint pricing and inventory control strategy
San-José et al. (2015)	Based on fixed costs and variable costs that increase with storage time	Economic order batch inventory control model with partial reverse order
Tripathi et al. (2015)	Requirements and carrying costs are functions of time	Inventory model for quantity discount of perishable products
Chen et al. (2018)	The consumer utility follows the Poisson distribution, and the demand depends on the price and the time	Joint inventory and price optimization problems of deteriorated products with menu costs
Hu et al. (2019)	Demand depends on demand and random variables; Costs fixed costs plus variable costs	Consider joint product pricing and inventory control issues for periodic inspections of individual products

Source: made by authors.

Table 2. Dynamic pricing model under offline setting.

Authors	year	Dynamic pricing model
Wang Chuanxu	2009	Model of jointly dynamic pricing and inventory of perishable products when demand is sensitive to price and time at different levels of decay.
Zhang and Zhang; Feng et al.; Yi-jiang Liu et al.	2011, 2014, 2015	When perishable goods are not sold or consumed, a Stackelberg game model is constructed, considering product freshness and product supply rate.
Banerjee and Turner; Herbon et al.	2012, 2017	When demand decreases over time and is price sensitive, a model is formulated that takes into account an optimal replenishment plan for fresh agricultural products with dynamic pricing policy.
Lilin et al.	2015	A single cycle decision model considering the decline of agricultural product value and inventory cost.
Hu et al.	2016	Stochastic dynamic inventory and two-stage decrement model of perishable goods
Chen et al.	2018	Under the condition of random demand, a dynamic optimal model of deteriorated products with menu cost considered is constructed

Source: made by authors.

strategies. In addition to the above studies, many scholars conduct consumers behavior into the online sales. For example, based on the newsboy model and considering the strategic behaviors of consumers, Tang et al., (2018) discretized the value decline of fresh agricultural products and bring forward a single-stage and two-stage pricing inventory decision-making model. Li et al. (2015) constructed an inventory system with stochastic perturbation into a continuous time stochastic differential equation model. Using dynamic pricing and production control, a stochastic dynamic programming problem is developed to optimize the total discount profit. Hu et al. (2016) also considered a joint replenishment and discount stochastic dynamic inventory model when facing forward buying customers under the condition of demand uncertainty. Bi et al. (2018) explored an online sales of two interchangeable limited stock perishable products under uncertain demand. Through the above survey of fresh agricultural products' pricing, inventory control, and consumers' purchase behavior under online setting, we can see that most studies concentrate on the offline operation mode with different factors, and constructed a variety of types of inventory control models under different external demand environments. And scholars have gradually integrated strategic consumers' behavior into the model. Although it posed new challenges to the management decision-making, here are only a few studies involved in the strategic behavior of consumers and online retail setting at present. Our work is consistent with above papers of introducing purchase behavior factor to describe consumer ratio so as to optimize the joint pricing and inventory management under online sale setting.

3. Model formulation

Generally, the retailer orders new fresh agricultural products to sell at a determined regular price, and clear the inventory from the regular stage at a discount price due to the perishability, simultaneously deciding a new products order for continuously retailing, when facing consumers forward purchase behavior on online setting, where we call the former sale regular stage and the latter sale discount stage. For

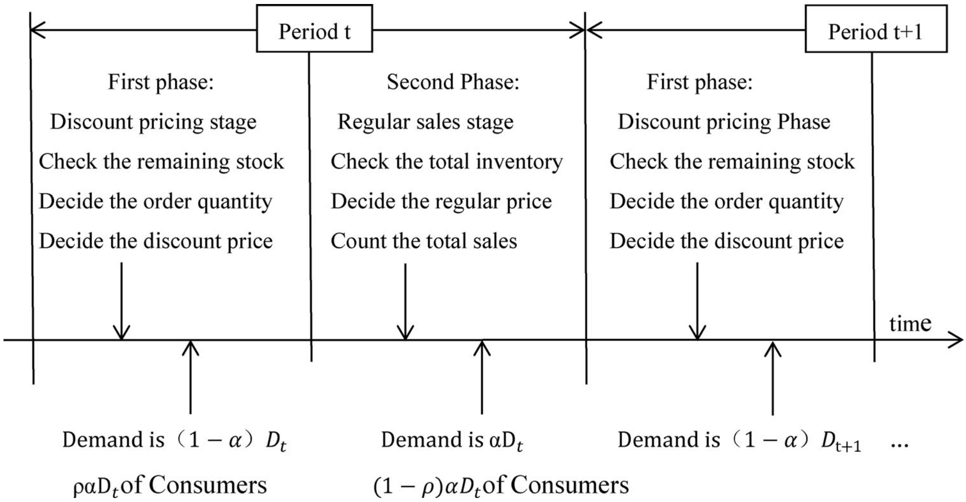

Figure 1. Sequence of sales operation.
Source: made by authors.

Table 3. Variables/parameters in the model.

Symbol	Meaning
Q_t	Order quantity of fresh agricultural products in the regular stage of period t
I_t	Inventory level at period t
r_t	Regular price of fresh agricultural products at period t
p_t	The discount price of fresh agricultural products at period t
D_t	Aggregate Market demand at period t, uniform distribution in (0, M), with pdf $f(.)$ and cmd $F(.)$
α	Portion of the demand in the normal sales period to the total demand
R_t	Loss of return at period t
ρ	Portion of strategic consumers in the whole market
c_t	Unit purchase cost of fresh products at period t
w_t	Unit logistics cost of fresh products at period t
g_t	Unit shortage cost of fresh products at period t
h_t	Unit holding cost of fresh products at period t

Source: made by authors.

convenience of further study, we define a period of sale consisting of two sale stages, a discount stage followed by a regular sales stage, so that the sequence of retailer selling is a dynamic pricing and inventory control issue in T periods, as shown in Figure 1 by setting some variables or parameters in Table 3.

In the first stage of period t, the retailer has some inventory on hand and decides the discount price to sell. The inventory, which comes from the regular stage of previous period and is non-fresh as the perishability, cannot be carried over to the second stage and have zero salvage value if unsold at last. In the meanwhile, the retailer also decides the order quantity of fresh products for the second stage. All unsold in the regular stage are carried to the next period, which incurs holding cost. The aggregated demand in period t, denoted by Dt, is random, stationary, bounded, and independent of each other. As shown in the Figure 1, during the regular sales phase, we introduce $\alpha (0< \alpha <1)$ to represent the portion of consumers who attempts to make purchases in the first stage. Therefore, the market demand would be αDt and $(1-\alpha)$Dt during regular and discounting stage, respectively. This study also adds strategic behaviors of consumers. Among them who try to purchase in the first stage, some can buy the regular price while are attracted by the discounting price in the second stage. In line with Hu et al. (2016) and Shum et al. (2017), let $\rho(0<\rho<1)$ represent the portion of such consumers. If $\rho = 0,$ the two stages of demand are independent of each other. The retailer's pricing and inventory decisions become straightforward. We focus on the case with $\rho > 0,$ that is to say, the issue in the presence of strategic consumers. The inventory at period t only comes from the regular stage since the unsold products at the first stage are all disposed of, denoted as I_t, and obviously, $I_t = [Q_{t-1} - (1-\rho)\alpha\ D_{t-1}]^+$.

Moreover, for convenience of further study, we assume that there is no dynamic demand substitution. That is to say, when the preferred fresh agricultural products are out of stock, customers will not buy the other type of agricultural products. Consumers will choose to buy in the first stage or the second stage, and each consumer will buy at most one kind of fresh agricultural product. We also assume that the loss of returned products is related to the sales quantity of fresh agricultural products when return policy is applied, denoted as $R_t(X_t) = \beta X,$ where β is the ratio of returned products, which is a random variable between [0, 1], and X_t is the amount of returned products.

3.2. Dynamic model of offline retailing

In the offline setting, consumers pickup and buy products on site, so that there is generally no return loss for the retailer. The retailer's goal is to maximize its profit by dynamically adjusting inventory and determining the price in the two sale stages. The profit in phase t is:

$$\begin{aligned}\pi_t(Q_t, r_t, p_t) &= E\hat{\pi}_t(Q_t, r_t, p_t)\\ &= p_t E\left[I_t \cap (1-\alpha)D_t + \rho\alpha D_t\right] + r_t E[Q_t \cap (1-\rho)\alpha D_t] - c_t Q_t\\ &= -\frac{r_t {Q_t}^2}{2M(1-\rho)\alpha} + (r_t - c_t)Q_t + p_t I_t - \frac{p_t {I_t}^2}{2M(1-\alpha+\rho\alpha)}\end{aligned} \tag{1}$$

Obviously, all profit from the start to period T is a total of every period's profit, which is $\sum_{t=1}^{T}\pi_t(Q_t, r_t, p_t)$. We define a discount factor γ and get the recursive function $v_t(I_t)$.

$$v_t(I_t) = \max\left\{\pi_t(Q_t, r_t, p_t) + \gamma E v_{t+1}(I_{t+1})\right\}, \tag{2}$$

where I_t is state variable and (Q_t, r_t, p_t) is the decision variables, generally, $Q_t>0, r_t>p_t$.

Denote function $\phi_t(Q_t, r_t, p_t)$ as

$$\phi_t(Q_t, r_t, p_t) = \pi_t(Q_t, r_t, p_t) + \gamma E v_{t+1}[Q_t - (1-\rho)\alpha D_t]^+ \tag{3}$$

With the state transfer equation $I_{t+1} = [Q_t - (1-\rho)\alpha D_t]^+,$ we can obtain the dynamic programming model as below.

As a result, $Q_t > 0, r_t > \mathrm{p}_t, v_t(I_t) = \max \phi_t(Q_t, \mathrm{r}_t, \mathrm{p}_t)$

$$s.t. v_{T+1} = -kI_{T+1}$$

$$Q_t > 0, \quad r_t > \mathrm{p}_t > 0 \tag{4}$$

The first constraint equation is boundary condition, as we assume that all the unsold product at the last period will be disposed of with unit cost k. In other words, there is no second market for these inventory that can be treated with some salvage value.

Theorem 1 : *With initial inventory at any period, the expected profit function,* $\pi_t(Q_t, r_t, p_t)$*, is quadratic differentiable and jointly concave on* (Q_t, r_t, p_t).

Proof: At the last period, $\mathrm{t} = \mathrm{T}$, the optimal profit function is expressed as:

$$\begin{aligned}\pi_T(Q_T, \mathrm{r}_T, \mathrm{p}_T) &= \mathrm{p}_T E\left[I_T \cap (1-\alpha) D_T + \rho\alpha \mathrm{D}_T\right] + \mathrm{r}_T E[(1-\rho)\alpha \mathrm{D}_T \cap Q_T] - \mathrm{c}_T Q_T \\ &= -\frac{r_T {Q_T}^2}{2\mathrm{M}(1-\rho)\alpha} + (r_T - \mathrm{c}_T) Q_T + p_T I_T - \frac{p_T {I_T}^2}{2\mathrm{M}(1-\alpha+\rho\alpha)}\end{aligned}$$

Obviously, the efficient $\frac{r_T {Q_T}^2}{2\mathrm{M}(1-\rho)\alpha}$ is more than zero, which leads the function to be concave, ceteris paribus, and the optimal ordering quantity is $Q_T^* = \frac{\mathrm{M}\alpha r_T - c_T(1-\rho)}{r_T}$. The Hessian matrix over (Q_T, r_T, p_T). is as below.

$$\begin{pmatrix} -\dfrac{r_T}{M(1-\rho)\alpha} & \dfrac{M(1-\rho)\alpha - \mathrm{Q}_T}{M(1-\rho)\alpha} & 0 \\ \dfrac{M(1-\rho)\alpha - \mathrm{Q}_T}{M(1-\rho)\alpha} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

When $Q_T > \mathrm{M}(1-\rho)\alpha$, the matrix is negative semi-definite. In this case, the profit function is jointly concave on $(Q_T, \mathrm{r}_T, \mathrm{p}_T)$. That is to say, there could be an optimal solution that makes the retailer earn the highest profit. Furthermore, given pricing variables, Q_T is proportional to r_T. This suggests that the retailer could raise or cut down the regular price and ordering quantity simultaneously. Similarly, when $t = 1, 2, 3 \ldots T-1$, the expected profit function is also jointly concave on (Q_t, r_t, p_t).

Theorem 2: *With initial inventory at any period,* $v_t(I_t)$*is concave and decreases on* I_t *and* $\phi_t(Q_t, r_t, p_t)$ *is continuously quadratic differentiable and jointly concave function on* (Q_t, r_t, p_t)

Proof: We prove it by induction. First, we prove that $\phi_T(Q_T, \mathrm{r}_T, \mathrm{p}_T)$ is continuously quadratic differentiable. When $\mathrm{t} = \mathrm{T}$, $\mathrm{E}(\mathrm{I}_{\mathrm{T}+1}) = \mathrm{E}[\mathrm{Q}_\mathrm{T} - (1-\rho)\alpha \mathrm{D}_\mathrm{T}]^+ = \int_0^{\frac{\mathrm{Q}_\mathrm{T}}{(1-\rho)\alpha}} [\mathrm{Q}_\mathrm{T} -$

$(1-\rho)\alpha D_T]f(x)dx = \frac{Q_T{}^2}{2M(1-\rho)\alpha}>0$, which is quadratic differentiable and convex on Q_T. Substituting the boundary condition, $v_{T+1} = -kI_{T+1}$, into $\phi_T(Q_T, r_T, p_T)$, we can get

$$\begin{aligned}\phi_T(Q_T, r_T, p_T) &= \pi_T(Q_T, r_T, p_T) + \gamma E v_{T+1}[Q_T - (1-\rho)\alpha D_T]^+ \\ &= \pi_T(Q_T, r_T, p_T) - \gamma\ k * E[Q_T - (1-\rho)\alpha D_T]^+ \\ &= \pi_T(Q_T, r_T, p_T) - \frac{Q_T{}^2}{2M(1-\rho)\alpha}\end{aligned}$$

According to theorem 1, we know $\phi_T(Q_T, r_T, p_T)$ is continuously quadratic differentiable and jointly concave on (Q_T, r_T, p_T). Assume now that $\phi_k(Q_k, r_k, p_k)$ is jointly concave for some t = 1,2, … ,T-1 and that $\pi_t(I_t)$ is also concave and non-increasing. Then, $\phi_{k-1}(Q_{k-1}, r_{k-1}, p_{k-1}) = \pi_{k-1}(Q_{k-1}, r_{k-1}, p_{k-1}) + \gamma\ E v_{k-1}[Q_{k-1} - (1-\rho)\alpha D_{k-1}]^+$, is jointly concave: joint concavity of the first term is verified according thereom1 as well as for the case $t = T$ above.

According to the structural properties of the model, the following theorems can be obtained.

Theorem 3: *With initial inventory I_t at any period t, there exists an optimal value $(Q_t{}^*, r_t{}^*, p_t{}^*)$ that maximizes the retailer's profit, i.e. $(Q_t{}^*, r_t{}^*, p_t{}^*) \in \text{argmax}\phi_t$ $(Q_t{}^*, r_t{}^*, p_t{}^*)$.*

Proof: On the basis of the model analyzed above, we can construct an optimization problem of any period T with constraints, as shown below:

$$\min -\phi_t(Q_t, r_t, p_t)$$

$$\text{s. t.} \begin{cases} p_t - r_t < 0 \\ -Q_t < 0 \\ -r_t < 0 \\ -p_t < 0 \end{cases} \tag{5}$$

Through KKT conditions and the previous analysis, we can get the best pricing strategy and order quantity decision of multi-variables nonlinear dynamic programming.

$$L\big(Q_t(r_t, p_t), (\lambda_{t1}, \lambda_{t2})\big) = -\ \phi_t(Q_t, r_t, p_t\big) + \lambda_{t1}(p_t - r_t\big) - \lambda_{t2}Q_t$$

$$\text{s. t. } \lambda_{t1}、\lambda_{t2} \geq 0 \tag{6}$$

Assuming that $(Q_t{}^*, r_t{}^*, p_t{}^*)$ is the optimal strategy, KKT conditions can be constructed as follows:

$$-\nabla\phi_t(Q_t, r_t, p_t) = 0$$

$$\begin{array}{ll} p_t - r_t < 0 & \lambda_{t1}(r_t - p_t) = 0 \\ -Q_t < 0 & \lambda_{t2} Q_t = 0 \end{array}$$

The first equation represents the first order condition of the Lagrange function, and the last two equations represent the condition of complementary relaxation. It is worth noting that the optimal strategy of order quantity depends on parameters I, c, α, ρ, which means that enterprises need to consider these parameters to make the optimal strategy.

We conduct numerical analysis to illustrate the solution. Let $T = 10$, $\rho = 0.5$, $\alpha = 0.5, \gamma = 0.9$ and initial inventory be 0. By using the optimization toolbox of Matlab software, a series of order quantity and the optimal price can be obtained, as shown in Table 4

According to the final profit margin of the two decision-making strategies, the two-stage dynamic pricing is obviously better than the ordinary single pricing. And the offline retailer's profit can increase by 6.9%, which further proves the correctness of the above theoretical analysis.

3.3. Dynamic model of online retailing

Contrary to offline setting, consumers can not observe and feel the quality of products on site, which some orders would return at last. Therefore, the retailer's profit should consider the logistics cost and return cost except for the shortage cost and holding cost, which will make the study more comprehensive. For convenience of study, the return cost is set down as being related to the sales quantity of fresh agricultural products based on the previous research. Meanwhile, in what follows, we consider the situation where the return of goods is allowed to be resold in discount stage.

As noted above, we formulate the retailer's profit function as below.

$$\begin{aligned} \grave{\pi}_t(Q_t, r_t, p_t) &= E\hat{\pi}_t(Q_t, r_t, p_t) = p_t E\{I_t \cap (1-\alpha)D_t + \rho\alpha D_t\} \\ &\quad + r_t(1-\beta)E[Q_t \cap (1-\rho)\alpha D_t] - c_t Q_t - w_t E(D_t \cap I_t + Q_t) \\ &\quad - h_t E[Q_t - (1-\rho)\alpha D_t]^+ - g_t E(D_t - Q_t - I_t)^+ \\ &= \frac{h_t - (1-\beta)r_t - (w_t - g_t)(1-\rho)\alpha}{2M(1-\rho)\alpha} {Q_t}^2 \\ &\quad + \left[(1-\beta)r_t - c_t + w_t - g_t - \frac{(w_t - g_t)I_t}{M}\right] Q_t \\ &\quad + \left(p_t + w_t - g_t - \frac{w_t - g_t}{2M}\right) I_t - \frac{p_t {I_t}^2}{2M(1-\alpha+\rho\alpha)} - \frac{M}{2} \end{aligned} \tag{7}$$

Similar to previous section, we define $\grave{v}_t(I_t)$ as a function of the total expected profit with discount factor γ from t to T, that is, when $Q_t > 0, r_t > p_t$, the dynamic programming equation is expressed as:

$$\grave{v}_t(I_t) = \max\left\{\grave{\pi}_t(Q_t, r_t, p_t) + \gamma\ E\grave{v}_{t+1}(I_{t+1})\right\} \tag{8}$$

Table 4. Optimal solution of dynamic pricing and regular pricing in offline retail.

	Two-stage dynamic pricing				Common single pricing			
Periods	Order quantity	Regular price	Discount price	Profit a	Order quantity	Regular price	Profit b	The profit margin (a - b)
1	12.50	6.00	6.00		13.00	6.50		
2	13.46	6.55	5.83		13.00	6.50		
3	14.29	7.00	6.26		13.00	6.50		
4	15.00	7.49	6.49		13.00	6.50		
5	13.10	6.28	5.89		13.00	6.50		
6	13.80	6.70	6.00		13.00	6.50		
7	14.58	7.19	5.99		13.00	6.50		
8	15.38	7.80	6.46		13.00	6.50		
9	14.15	6.86	6.12		13.00	6.50		
10	15.07	7.56	6.84	303.21	13.00	6.50	283.62	19.59

Source: made by authors.

Table 5. Optimal solutions of dynamic pricing and single pricing in online retail.

	Two-stage dynamic pricing				Common single pricing			
Periods	Order quantity	Regular price	Discount price	Profit a	Order quantity	Regular price	Profit b	The profit margin (a - b)
1	18.84	10.00	10.00		16.00	12.63		
2	19.82	11.68	10.20		16.00	12.63		
3	20.41	13.50	9.88		16.00	12.63		
4	20.81	14.82	9.70		16.00	12.63		
5	21.03	15.66	12.07		16.00	12.63		
6	20.14	12.82	10.52		16.00	12.63		
7	18.51	9.68	7.31		16.00	12.63		
8	19.00	11.13	8.42		16.00	12.63		
9	20.43	13.47	9.88		16.00	12.63		
10	21.23	16.37	11.19	695.52	16.00	12.63	604.14	91.38

Source: made by authors.

Through the state transfer equation $I_{t+1} = [Q_t - (1-\rho)\alpha D_t]^+$, define $\grave{\phi}_t(Q_t, r_t, p_t)$ as

$$\grave{\phi}_t(Q_t, r_t, p_t) = \grave{\pi}_t(Q_t, r_t, p_t) + \gamma\ E\grave{v}_{t+1}[Q_t - (1-\rho)\alpha D_t]^+ \tag{9}$$

Then, when $Q_t > 0,\ r_t > p_t, \grave{v}_t(I_t) = \ \max\grave{\phi}_t(Q_t, r_t, p_t)$

$$\text{s. t. } \grave{v}_{T+1} = -kI_{T+1}, k > 0, \tag{10}$$

where γ is the discount factor, $\gamma \in (0,1)$, $t \in [1,2,3\ldots T]$.We still put forward the above boundary condition that all inventory unsold at last will be disposed of. That is to say, there is no second market in which the inventory can be treated with salvage value. Instead, businesses pay unit cost k to clear inventory, even of unexpired inventory products.

What is different from offline retail is the change of profit function, we first analyze the property of the model.

Theorem 4: *With initial inventory at any period, the expected profit function is continuously quadratic differentiable and jointly concave on*(Q_t, r_t, p_t).

Proof: When $t = T$, the profit function at the last period is expressed as:

$$\begin{aligned}\grave{\pi}_T(Q_T, r_T, p_T) &= p_T E\{I_T \cap (1-\alpha)D_T + \rho\alpha D_T\} + r_T(1-\beta)E[Q_T \cap (1-\rho)\alpha D_T] \\ &\quad - c_T Q_T - w_T E(D_T \cap I_T + Q_T) - h_T E[Q_T - (1-\rho)\alpha D_T]^+ \\ &\quad - g_T E(D_T - Q_T - I_T)^+ \\ &= \frac{h_T - (1-\beta)r_T - (w_T - g_T)(1-\rho)\alpha}{2M(1-\rho)\alpha} {Q_T}^2 \\ &\quad + \left[(1-\beta)r_T + w_T - c_T - g_T - \frac{(w_T - g_T)I_t}{M}\right] Q_T \\ &\quad + \left(p_T + w_T - g_T - \frac{w_T - g_T}{2M}\right) I_T - \frac{p_T {I_T}^2}{2M(1-\alpha+\rho\alpha)} - \frac{M}{2}\end{aligned}$$

The Hessian matrix on (Q_T, r_T, p_T) is

$$\begin{pmatrix} \dfrac{h_T - (1-\beta)r_T - (w_T - g_T)(1-\rho)\alpha}{M(1-\rho)\alpha} & -\dfrac{(1-\beta)Q_T}{M(1-\rho)\alpha} & 0 \\ -\dfrac{(1-\beta)Q_T}{M(1-\rho)\alpha} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The Hessian matrix is obviously negative semi-definite if $r_T > \frac{h_T - (w_T - g_T)(1-\rho)\alpha}{(1-\beta)}$. In this case, the profit function is concave, that is, there is a maximum value to maximize the profits of fresh agricultural products retailers. Meanwhile, ceteris paribus, the optimal ordering quantity, $Q_T^* = \frac{M\alpha(1-\rho)[c_T + g_T - (1-\beta)r_T - w_T]}{h_T - (1-\beta)r_T - (w_T - g_T)(1-\rho)\alpha}$. As shown in the solution, the regular negatively correlated with the order quantity. Similarly, when $t = 1, 2, 3 \ldots T-1$, the result holds by the profit equation and related Hessian matrix analysis.

Theorem 5: *With initial inventory at any period, the function* $\grave{\phi}_t(Q_t, r_t, p_t)$ *is continuously quadratic differentiable and jointly concave function on* (Q_t, r_t, p_t), *and* $\grave{v}_t(I_t)I_t$ *is concave and decreases on*I_t.

Proof: At period T, $\grave{v}_{T+1} = -kI_{T+1}$, then

$$\grave{\phi}_T(Q_T, r_T, p_T) = \grave{\pi}_T(Q_T, r_T, p_T) + \gamma\ E\grave{v}_{T+1}[Q_T - (1-\rho)\alpha D_T]^+ = \grave{\pi}_T(Q_T, r_T, p_T) -$$

And $E[Q_T - (1-\rho)\alpha D_T]^+ = \int_0^{\frac{Q_T}{(1-\rho)\alpha}} [Q_T - (1-\rho)\alpha D_T] f(x)dx = \frac{{Q_T}^2}{2M(1-\rho)\alpha} > 0$, $\grave{v}_T(I_T)$ is quadratic differentiable and concave on Q_T. With property of $\grave{\pi}_T(Q_T, r_T, p_T)$ according to theorem 1, $\grave{\phi}_T(Q_T, r_T, p_T)$ is continuously quadratic differentiable and jointly concave on (Q_T, r_T, p_T). That is there is an optimal decision that maximizes the profit, and it is a decreasing function. $\grave{v}_T(I_T)$ is also decreasing on I_T. And then suppose that when $t = k$, $\grave{\phi}_k(Q_k, r_k, p_k)$ are concave and, $\grave{v}_k(I_k)$ are decreasing on I_t. When $t = k-1$, $\phi_{k-1}(Q_{k-1}, r_{k-1}, p_{k-1}) = \pi_{k-1}(Q_{k-1}, r_{k-1}, p_{k-1}) + \gamma\ Ev_{k-1}[Q_{k-1} - (1-\rho)\alpha D_T]^+$, which property also holds based on theorem 1.

Theorem 6 : *With initial inventory* I_t *at period t, there exists an optimal solution*, i.e. $(Q_t{}^*, r_t{}^*, p_t{}^*) \in \text{argmax}\grave{\phi}_t(Q_t{}^*, r_t{}^*, p_t{}^*)$, $Q_t > 0, r_t > p_t$.

Proof: On the basis of dynamic programming model, an optimization problem of arbitrary period t under constraint conditions is constructed, as shown below:

$$\min -\grave{\phi}_t(Q_t, r_t, p_t)$$

$$\text{s. t.} \begin{cases} p_t - r_t < 0 \\ -Q_t < 0 \\ -r_t < 0 \\ -p_t < 0 \end{cases} \tag{11}$$

Through KKT conditions and the previous analysis, we can get the best pricing strategy and order quantity decision of multivariables nonlinear dynamic programming.

$$\grave{L}\left(Q_t(r_t, p_t), (\lambda_{t1}, \lambda_{t2})\right) = -\ \grave{\phi}_t(Q_t, r_t, p_t) + \lambda_{t1}(p_t - r_t) - \lambda_{t2}Q_t$$

$$\text{s. t. } \lambda_{t1} \text{、} \lambda_{t2} \geq 0 \tag{12}$$

KKT conditions can be constructed as follows:

$$-\nabla\grave{\phi}_t(Q_t, r_t, p_t) = 0$$

$$\begin{array}{cc} p_t - r_t < 0 & \lambda_{t1}(r_t - p_t) = 0 \\ -Q_t < 0 & \lambda_{t2}Q_t = 0 \end{array}$$

The first equation represents the first order condition of the Lagrange function, and the last two equations represent the condition of complementary relaxation. It is not hard to find the optimal solution, $(Q_t{}^*, r_t{}^*, p_t{}^*)$. It is worth noting that the optimal strategy of online retail quantity much more complicated than offline retail since more parameters, such as $I, c, \alpha, \rho, w, g,$ and $h,$ are considered. Similar to previous section, we conduct some numerical experiments analysis to illustrate the performance. Besides the parameters noted earlier, we add $w_t = 1, h_t = 0.5, g_t = 0.5$. With the help of the optimization toolbox using Matlab software, a series of order quantities and optimal prices can be obtained, as shown in Table 4.

Not surprisingly, the data shows that the profit under dynamic pricing and inventory control could increase by about 15.13%. This is because the discounted price in the two-stage pricing changes the demand realization, conforms to different consumers' preferences on freshness and price, and promotes sales. This further unfolds that the retailer should adopt more dynamic strategies on online retail of fresh agricultural products when there are strategic consumers. From revenue management perspective, dynamic pricing and inventory control can enhance the competitiveness of fresh e-commerce.

4. Concluding remark

This paper investigates a problem of fresh agricultural products on online retail setting with strategic consumer behavior. We formulate a stochastic dynamic optimization model on a retailer's inventory and pricing decisions by analyzing forward purchase behavior analysis and online setting. Then exploiting the characteristics of the model and using KKT condition of nonlinear programming method we obtain the dynamic optimal policy or solution, which is comparatively analyzed to dominate the related static strategy. Some numerical experiments are conducted to illustrate the performance on the solution and confirm that there exists effective pricing and inventory strategy to maximize the profits of retailers on online selling fresh agricultural products. The results show that the profit with dynamic strategy increases, to a certain extent, compared that with single pricing of static strategy. This is because that discount pricing attracts more strategic consumers to buy, especially the dynamic pricing policy can adjust the inventory level accordingly. Meanwhile, real-time adjustment of orders can also reduce holding costs, out-of-stock costs and other operating costs, so as to improve the profit. We also show that consumers forward buying behavior will negatively influence the retailer's profit. When the price is set too low in regular or discounting sales, the profit will show an up-down trend if the inventory exceeds a certain threshold. Meanwhile, when fresh goods returns are allowed and resold in the secondary stage, the retailer's profit will increase.

Furthermore, this paper is a preliminary attempt to study the dynamic pricing and inventory strategy of online retail of fresh agricultural products with forward purchase behavior. By now, online sale of fresh agricultural products is developing fast. The strategic behaviors of consumers are becoming more complicated and has much impact on the market demand. In this paper we only represent it with one parameter, the portion. Meanwhile, there is no specific analysis of the relationship between the degree of freshness loss and consumers' choice, nor a more comprehensive analysis on market demand. In the future research, studies can investigate the problem by expanding the consumer behavior and/or the stochastic distribution of market demand, or some other situations, so as to provide decision-making reference for the e-commerce industry of fresh agricultural products.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes

1. http://www.pe.com/articles/million-634066-store-year.html
2. http://www.ft.com/intl/cms/s/0/dee4d20c-2172-11e1-a1d8-00144feabdc0.html#axzz3Iua2R9jr

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