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Partial passive ownership holdings and R&D risk choices in a differentiated duopoly

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ABSTRACT
This study investigates the R&D risk choices in a differentiated duopoly, in which a firm has partial passive ownership holdings (P.P.O.s) in its rival. Firms perform R&D projects with identical expected outcomes but different risk degrees. It mainly finds that: (1) the P.P.O.s make both firms more willing to take R&D risks; (2) compared with the firm which owns a share of its rival, its partially owned rival is more willing to take R&D risks; (3) for both firms, their private incentive for R&D risk is lower than the social incentive. However, the P.P.O.s may make the private optimum closer to the social optimum.

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1. Introduction

Partial passive ownership holdings (P.P.O.s), also known as partial passive shareholding, refer to the phenomenon that a firm holds non-controlling minority shares of other firms (Dong & Chang, 2020; Leonardos et al., 2021). This phenomenon of P.P.O.s began in Japan in the 1950s. Later, there were many cases of firms owning shares in competitors in western developed countries, such as Microsoft's acquisition of a 7% stake in its rival, Apple, in 1997. At present, the P.P.O.s are also becoming more and more extensive in many developing countries (e.g., China). R&D is crucial for manufacturing firms. It is the inexhaustible driving force for the long-term and sustainable development of firms. The P.P.O.s make the interests of firms related, which may affect their R&D incentives. Since a firm that invests resources in developing a new technology (or product) does not determine whether it will succeed, the results of R&D activities are generally uncertain (Silipo & Weiss, 2005). In facing such uncertainty, the optimal choice of risk associated with R&D projects is a variable of interest (Xing, 2019a).

Based on the above background, we intend to answer the following questions in this study: (1) How do the P.P.O.s affect the optimal R&D risk choices in a duopoly?;

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(2) Which firm is more willing to take R&D risks, the firm that owns the rival's shares or its partially owned rival?; (3) Whether the firms can achieve the socially optimal R&D risk level? If not, how do the P.P.O.s affect the extent of inefficiency?

To investigate the above questions, we develop a differentiated duopoly model to explore the optimal R&D risk choices when a firm holds the minority of shares of its rival. The model has two stages. In the first stage, firms determine the type of R&D project from a series of projects with the same expected outcome but different risks. In this stage, they are uncertain about the outcome of R&D, but they know the probability distribution it obeys. In the second stage, firms have already known the R&D outcome and they compete in quantity (i.e., conduct Cournot competition). The main findings are as follows. First, the P.P.O.s have a positive effect on the R&D risk-taking of both firms.[1] Second, the firm that does not own the rival's shares takes higher R&D risks than the one that owns its shares. Third, the private optimum of R&D risk is too low from the perspective of social welfare, while the P.P.O.s may reduce this inefficiency.

The main contributions of this study are as follows. First, to the best of our knowledge, this study is the first to investigate the effect of P.P.O.s on strategic R&D risk choices. Second, the comparison of R&D incentives of firms in an asymmetric situation has been a topic of interest in previous literature. We compare the optimal R&D risks of the firm which owns a share of its rival and its partially owned rival. This study significantly supplements the relevant research. Third, we compare private and social incentives for R&D risk and analyse the possible influence of P.P.O.s on the difference between them.

This paper relates to the literature analysing strategic R&D risk choices in oligopolistic markets. Some scholars focus on the technology invention time and study the R&D risk choices in patent races (Bagwell & Staiger, 1990; Cabral, 1994; Dasgupta & Stiglitz, 1980; Ishibashi & Matsumura, 2006; Klette & de Meza, 1986; O'Donoghue, 1998; and others). In addition, some scholars use the variance of R&D outcome to represent the R&D risk and examine the optimal R&D risk choices (Cabral, 2003; Lee, 2017; Lee & Cho, 2020; Tishler, 2008; Tishler & Milstein, 2006; Xing, 2014, 2017, 2019a, 2019b; Xing et al., 2021; Zhang, 2020; Zhang et al., 2013). However, the issue of P.P.O.s has not received attention in these studies.

This article also relates to the literature that investigates the effects of P.P.O.s on the R&D incentives. Shelegia and Spiegel (2015), López and Vives (2019), Vives (2020), Anton et al. (2021), Bayona and López (2018) and Brito et al. (2020) consider how the P.P.O.s affect cost-reducing R&D (quality-enhancing R&D) and give the conditions under which they have positive effects on this type of R&D investments. In addition, Liu (2019) examines the impact of PPOs on R&D aiming at improving product differentiation in a mixed duopoly and thinks that they enhance the possibility of the private firm to make such R&D investment. However, these studies do not involve the R&D risk and thus do not consider how the PPOs affect the R&D risk choices.

The rest of this study is organised as follows. Section 2 describes the basic model, and section 3 (section 4) analyses the private (social) optimum of R&D risk. Section 5 extends the basic model and the final section concludes.

2. The basic model

Consider an industry with two firms producing differentiated goods. Each firm provides only one type of product and firm i provides product i. Following the studies of Singh and Vives (1984), Qiu (1997) and Lin and Saggi (2002), we give the following inverse demand function:

$$p_i = a-(q_i + rq_j),\ \ i \neq j, i, j = 1, 2 \tag{1}$$

In (1), p_i (q_i) is the price (quantity) of product i, $a>0$ and $0<r<1$. The larger value of parameter r implies the higher (lower) degree of substitutability (differentiation) between two products.

Firms undertake process R&D investment to lower their marginal production cost. After R&D, the marginal production cost of firm i is: $c_i = c-x_i$, where c is the initial marginal production cost and x_i is the R&D effort of firm i.[2] It follows that the firm i's production cost after R&D is: $C_i = c_i q_i = (c-x_i)q_i$. The R&D outcomes are uncertain when firms conduct process innovation (Xing, 2014). We assume that x_i obeys the probability distribution: $F(x_i)$ $(x_i \geq 0)$. We let $\mu_i = E(x_i)$ $(\sigma_i = V(x_i))$ represent the expectation (variance) of x_i. The covariance of x_i and x_j is assumed to be zero (i.e., $Cov(x_i, x_j) = 0,\ i \neq j$). That is, they are independent.[3]

The fixed costs of each firm are only generated by its R&D investment. Therefore, the profit function of firm i can be given by:

$$\pi_i = p_i q_i - C_i - I(\mu_i, \sigma_i),\ \ i = 1,\ 2 \tag{2}$$

In (2), $I(\mu_i, \sigma_i)$ is the R&D cost of firm i.[4] In order to ensure the existence of interior solutions in the R&D stage, $I(\mu_i, \sigma_i)$ is assumed to satisfy: (1) it is strictly convex about σ_i[5]; and (2) it is enough large if σ_i tends to infinity. Following Tishler (2008), Xing (2017), Lee and Cho (2020) and Xing et al. (2021), we use the variance of R&D outcome to represent the risk of R&D project.

Firm 1 has P.P.O. holdings in firm 2, owning a share k $(0<k<0.5)$ of the rival's profits (Fanti, 2015; Leonardos et al., 2021; Papadopoulos et al., 2019). Although firm 1 owns a share of firm 2, it does not have decision-making power over firm 2 (Bárcena-Ruiz & Sagasta, 2021). We assume that firm 1 maximises its total profit. Its objective function is:

$$U_1 = \pi_1 + k\pi_2 \tag{3}$$

In addition, firm 2 only obtains a share $1-k$ of its profit. It follows that the objective function of firm 2 is:

$$U_2 = (1-k)\pi_2 \tag{4}$$

Social welfare comprises the producer surplus $(\pi_1 + \pi_2)$ and consumer surplus (CS), which is given by:

$$SW = \pi_1 + \pi_2 + CS \tag{5}$$

In (5), $CS = (q_1^2 + 2rq_1q_2 + q_2^2)/2$.

We consider a two-stage game. The timing of it is as follows. In the first stage, each firm engages in R&D and determines the R&D risk levels. In this stage, it chooses the type of its R&D project from a series of projects with identical expected outcome but different risks. A firm choosing the type of R&D project is equivalent to choosing the R&D risk level in this study. Both firms are uncertain about the outcome of R&D (i.e., x_i), despite they are aware of the probability distribution it obeys. Therefore, the expected value and variance of R&D outcome (i.e., μ_i and σ_i) are common knowledge for firms 1 and 2 (Xing et al., 2021). Following Tishler (2008), Xing (2017) and Zhang (2020), we assume that firms are risk-neutral. In the second stage, firms compete in quantity (i.e., conduct Cournot competition). In this stage, each firm knows the R&D outcomes of itself and its competitor.

3. The private optimum

In the second stage, given the R&D outcomes of firms (i.e., x_1 and x_2), each firm chooses the production level (q_i) that maximises its objective function (see (3) and (4)). The first-order conditions (F.O.C.s) for the optimal outputs are given as follows:

$$\frac{\partial U_1}{\partial q_1} = a - c + x_1 - 2q_1 - rq_2 - rkq_2 = 0 \tag{6}$$

$$\frac{\partial U_2}{\partial q_2} = (1-k)(a - c + x_2 - 2q_2 - rq_1) = 0 \tag{7}$$

According to F.O.C.s, we obtain the following best-reply function: $q_1(q_2) = \frac{1}{2}[a - c + x_1 - (1+k)rq_2]$ and $q_2(q_1) = \frac{1}{2}(a - c + x_2 - rq_1)$. Obviously, as the value of k increases, firm 1 is less aggressive in the product market (Papadopoulos et al., 2019). We solve the system of F.O.C.s (see (6) and (7)) and give the following equilibrium outputs:

$$q_1^C = \frac{2(a - c + x_1) - r(1+k)(a - c + x_2)}{4 - (1+k)r^2} \tag{8}$$

$$q_2^C = \frac{2(a - c + x_2) - r(a - c + x_1)}{4 - (1+k)r^2} \tag{9}$$

The second-order conditions (S.O.C.s) are satisfied because of $\partial^2 U_1/\partial q_1^2 = -2<0$ and $\partial^2 U_2/\partial q_2^2 = -2(1-k)<0$. Submitting (8) and (9) into (3) and (4) respectively, we obtain the profit functions on x_1 and x_2 :

$$\pi_1^C = \frac{\left[(2-kr^2)(a - c + x_1) - r(1-k)(a - c + x_2)\right]\left[2(a - c + x_1) - r(1+k)(a - c + x_2)\right]}{\left[4-(1+k)r^2\right]^2} - I(\mu_1, \sigma_1) \tag{10}$$

$$\pi_2^C = \frac{\left[2(a - c + x_2) - r(a - c + x_1)\right]^2}{\left[4-(1+k)r^2\right]^2} - I(\mu_2, \sigma_2) \tag{11}$$

Now we turn to the first stage of the game. Firms choose their project type from a series of R&D projects. The expected outcomes of these projects are the same, but their risks are different (Tishler, 2008; Xing, 2014; Zhang et al., 2013). In this situation, a firm choosing the project type is equivalent to choosing the risk (i.e., variance) of R&D project. Using (10) and (11), we derive the following expected profit of firm i:

$$E(\pi_1^C) = \frac{\begin{pmatrix} [(2-kr^2)(a-c+\mu_1)-r(1-k)(a-c+\mu_2)][2(a-c+ \\ \mu_1)-r(1+k)(a-c+\mu_2)] + 2(2-kr^2)\sigma_1 + r^2(1-k^2)\sigma_2 \end{pmatrix}}{[4-(1+k)r^2]^2} - I(\mu_1, \sigma_1) \quad (12)$$

$$E(\pi_2^C) = \frac{[2(a-c+\mu_2)-r(a-c+\mu_1)]^2 + r^2\sigma_1 + 4\sigma_2}{[4-(1+k)r^2]^2} - I(\mu_2, \sigma_2) \quad (13)$$

According to (12) and (13), we derive the expected gross profit of firm i ($E(\Pi_i^C) = E(\pi_i^C) + I(\mu_i, \sigma_i), \quad i = 1, 2$) and the expected industrial gross profit ($E(\Pi^C) = E(\Pi_1^C) + E(\Pi_2^C)$). By analysing the impact of R&D risks on them, we get the following Lemma.

Lemma 1. *$E(\Pi_1^C)$, $E(\Pi_2^C)$ and $E(\Pi^C)$ increase as σ_1 (or σ_2) increases.*

Proof. See Appendix A.

Lemma 1 states that the R&D risk of a firm has positive effect on its (or its rival's) expected gross profit and the expected industrial gross profit. When investigating how corporate social responsibility (emission tax) affects the R&D (environmental R&D) risk choices, Lee and Cho (2020) and Xing et al. (2021) obtain similar results. However, they do not consider the cross ownership between firms.

The reason for Lemma 1 is as follows. If the marginal production cost changes from symmetry (*ex ante*) to asymmetry (*ex post*), the existence of product substitution will transfer production from the firm with high marginal cost (*ex post*) to the one with low marginal cost (*ex post*) (Kitahara & Matsumura, 2006; Matsumura, 2003). This substitution between products can save the total production costs of the industry. It follows that the higher marginal production cost asymmetry implies the more industrial gross profits. That is, there exists the industry-profit-increasing production substitution effect. A firm that chooses a R&D project with higher risk (given the other firm's choice) is more conducive to the marginal production cost asymmetry (Xing et al., 2021). Thus, each firm's R&D risk has positive effect on the expected industrial gross profit. In addition, if a firm chooses a R&D project with higher risk, it will expect to get more from the total profits of the industry. The competitor of this firm also expects to get more due to it can take advantage of differentiation incentives if the marginal production cost asymmetry is highly volatile (d'Aspremont et al., 1979; Hotelling, 1929). Thus, a firm's R&D risk has positive effect on its (or its rival's) expected gross profits.

In the R&D stage, both firms are uncertain about the R&D outcomes of their project. They choose the R&D risk (σ_i) to maximise the expected value of their objective function. Thus, the F.O.C.s can be given by:

$$\frac{\partial E(U_1^C)}{\partial \sigma_1} = \frac{\partial E(\pi_1^C)}{\partial \sigma_1} + k\frac{\partial E(\pi_2^C)}{\partial \sigma_1} = \frac{4-kr^2}{[4-(1+k)r^2]^2} - \frac{\partial I(\mu_1, \sigma_1)}{\partial \sigma_1} = 0 \tag{14}$$

$$\frac{\partial E(U_2^C)}{\partial \sigma_2} = (1-k)\frac{\partial E(\pi_2^C)}{\partial \sigma_2} = (1-k)\left[\frac{4}{[4-(1+k)r^2]^2} - \frac{\partial I(\mu_2, \sigma_2)}{\partial \sigma_2}\right] = 0 \tag{15}$$

Due to μ_1 and μ_2 are the same constant, only σ_i is variable in $\partial I(\mu_i, \sigma_i)/\partial \sigma_i$ ($i = 1, 2$). For clarity of expression, we set:

$$g(\sigma_i) = \frac{\partial I(\mu_i, \sigma_i)}{\partial \sigma_i}, \; i = 1, 2 \tag{16}$$

Combining with (14) and (15), the equilibrium R&D risks (i.e., σ_1^C and σ_2^C) satisfy the following equations:

$$\frac{4-kr^2}{[4-(1+k)r^2]^2} - g(\sigma_1^C) = 0 \tag{17}$$

$$\frac{4}{[4-(1+k)r^2]^2} - g(\sigma_2^C) = 0 \tag{18}$$

Now we examine the impact of P.P.O.s on equilibrium R&D risks. Using (17) and (18), we can prove the following proposition.

Proposition 1. *(i) σ_1^C increases as k increases, and (ii) σ_2^C increases as k increases.*

Proof. See Appendix B.

Proposition 1 implies that, the P.P.O.s lead both the firm that owns the rival's share and its partially owned rival to be more willing to take R&D risks. Further, when we give the specific form of the R&D cost function, then we can compare the effects of PPOs on σ_1^C and σ_2^C. For example, if $I(\mu_i, \sigma_i) = \frac{1}{2}(\mu_i^2 + \sigma_i^2)$ ($i = 1, 2$), we can prove that $\partial\sigma_1^C/\partial k < \partial\sigma_2^C/\partial k$. In this situation, the positive effect of P.P.O.s on the optimal R&D risk of firm 2 is stronger than that of firm 1. The intuition of Proposition 1 is as follows. First, we consider the part (i). The P.P.O.s can achieve a certain degree of collusion (Azar et al., 2018; Brito et al., 2019; Leonardos et al., 2021), which strengthens the industry-profit-increasing production substitution effect. Combining with Lemma 1, a higher value of k implies higher positive effect of the firm 1's R&D risk on the expected industrial gross profits. In addition, the increase of k leads firm 1 to get more from the industrial gross profits. Thus, firm 1 takes more R&D risks if k increases. Second, we consider the part (ii). With a higher value of k, on the one hand, it leads to higher positive effect of the firm 2's R&D risk on its expected gross profits because of weaker market competition. On the other hand, it leads firm 2 to get less from its expected gross profits. The former effect dominates the latter one. Thus, firm 2 takes more R&D risks if k increases.

Next we turn to analyse which firm (firm 1 or firm 2) is more willing to take R&D risks when it chooses the R&D project. According to (17) and (18), we derive the following result.

Proposition 2. $\sigma_1^C < \sigma_2^C$.

Proof. See Appendix C.

Proposition 2 shows that firm 2 chooses riskier R&D project than firm 1 in equilibrium. This result implies that, compared with the firm that owes the rival's shares, its partially owned rival is more willing to take R&D risks. The intuition is as following. In contrast with firm 2, firm 1 has some shares in its rival and thus can internalise the market competition to a certain extent (Fanti, 2015). It follows that a higher k tends to reduce the firm 1's incentive to produce but improve the firm 2's incentive to produce (Bárcena-Ruiz & Sagasta, 2021). The above effect is considered by each firm when it chooses the type of its R&D project (i.e., determining the R&D risk level). In order to reduce the marginal production cost to a greater extent, firm 2 is more aggressive than its rival. Thus, firm 2 is more willing to take R&D risks than firm 1.

4. The social optimum

This section considers the (second-best) socially optimal R&D risk that maximises the expected social welfare first and then takes a comparison of the private and social incentives for R&D risk. To derive the expected social welfare, we need to give the expected consumer surplus first. Combining (8, 9) and $CS = (q_1^2 + 2rq_1q_2 + q_2^2)/2$ gives the following expected consumer surplus:

$$E(CS^C) = \frac{\begin{pmatrix} [2(a-c+\mu_1)-r(1+k)(a-c+\mu_2)]^2 + 2r[2(a-c+\mu_1)-r(1+k)(a-c+\mu_2)][2(a-c+\mu_2)- \\ r(a-c+\mu_1)] + [2(a-c+\mu_2)-r(a-c+\mu_1)]^2 + (4-3r^2)\sigma_1 + [(1+k)^2r^2-4(1+k)r^2+4]\sigma_2 \end{pmatrix}}{2[4-(1+k)r^2]^2} \tag{19}$$

Lemma 2. *$E(CS^C)$ increases as σ_1 (or σ_2) increases.*

Proof. See Appendix D.

Lemma 2 indicates that each firm's R&D risk has positive effect on the expected consumer surplus. The reason for this lemma is as follows. If the marginal production cost changes from symmetry (*ex ante*) to asymmetry (*ex post*), the firm with low (high) marginal cost (*ex post*) increases (reduces) the output through strategic interaction between two firms (i.e., strategic substitutes) (Kitahara & Matsumura, 2006; Matsumura, 2003). Compared with the situation of cost symmetry (*ex ante*), cost asymmetry (*ex post*) makes the firm with low marginal cost more willing to reduce prices. It follows that the higher marginal cost asymmetry implies the more consumer surplus. That is, there exists consumer-surplus-improving production substitution effect. A firm choosing a project with higher R&D risk (given the other firm's choice) is more conducive to the marginal production cost asymmetry (Xing et al., 2021). Thus, each firm's R&D risk has positive effect on the expected consumer surplus.

Submitting (12, 13) and (19) into (5) yields the following expected social welfare:

$$E(SW^C) = \frac{\left(\begin{array}{c}[(6-2kr^2-2r^2)(a-c+\mu_1)+r(1+k)(a-c+\mu_2)][2(a-c+\mu_1)-r(1+k)(a-c \\ +\mu_2)]+3[2(a-c+\mu_2)-r(a-c+\mu_1)]^2+(12-4kr^2-r^2)\sigma_1+[12-(1+k)^2r^2]\sigma_2\end{array}\right)}{2[4-(1+k)r^2]^2} - I(\mu_1,\sigma_1) - I(\mu_2,\sigma_2) \tag{20}$$

According to (20), we derive the following F.O.C.s:

$$\frac{\partial E(SW^C)}{\partial \sigma_1} = \frac{12-4kr^2-r^2}{2[4-(1+k)r^2]^2} - \frac{\partial I(\mu_1,\sigma_1)}{\partial \sigma_1} = 0 \tag{21}$$

$$\frac{\partial E(SW^C)}{\partial \sigma_2} = \frac{12-(1+k)^2r^2}{2[4-(1+k)r^2]^2} - \frac{\partial I(\mu_2,\sigma_2)}{\partial \sigma_2} = 0 \tag{22}$$

Combining the above F.O.C.s and (16), the socially optimal R&D risk of firm i (i.e., σ_i^{SC}) satisfies the following equations:

$$\frac{12-4kr^2-r^2}{2[4-(1+k)r^2]^2} - g(\sigma_1^{SC}) = 0 \tag{23}$$

$$\frac{12-(1+k)^2r^2}{2[4-(1+k)r^2]^2} - g(\sigma_2^{SC}) = 0 \tag{24}$$

Using (23) and (24), we obtain $\frac{\partial \sigma_1^{SC}}{\partial k} = \frac{r^2(4-2kr^2+r^2)}{g'(\sigma_1^{SC})[4-(1+k)r^2]^3}$ and $\frac{\partial \sigma_2^{SC}}{\partial k} = \frac{4r^2(2-k)}{g'(\sigma_2^{SC})[4-(1+k)r^2]^3}$. It follows that $\frac{\partial \sigma_i^{SC}}{\partial k}>0$ because of $g'(\sigma_i^{SC})>0$, $4-2kr^2+r^2>0$, $2-k>0$ and $4-(1+k)r^2>0$. Thus, σ_i^{SC} increases as k increases. The reason for this result is that the P.P.O.s can achieve collusion to some extent, which enhances the industry-profit-increasing production substitution effect. However, the P.P.O.s may weaken the consumer-surplus-improving production substitution effect. The former effect of k dominates its later effect. With an increase of k, the social gain from R&D risk is higher. Thus, the socially optimal R&D risk level also increases.

Proposition 3. *(i) $\sigma_1^C<\sigma_1^{SC}$; and (ii) $\sigma_2^C<\sigma_2^{SC}$.*

Proof. See Appendix E.

Proposition 3 shows that the equilibrium R&D risk for each firm is too low from the viewpoint of social welfare. In other words, each firm has an insufficient incentive for taking R&D risks. Further, if we give the following form of the R&D cost function $I(\mu_i,\sigma_i) = \frac{1}{2}(\mu_i^2+\sigma_i^2)$ $(i=1,2)$, we can prove that $\partial(\sigma_1^{SC}-\sigma_1^C)/\partial k<0$ and $\partial(\sigma_2^{SC}-\sigma_2^C)/\partial k<0$. In this situation, comparing with the situation without P.P.O.s, the P.P.O.s make the private optimum of each firm closer to the social optimum (i.e., the P.P.O.s lead the level of inefficiency to decrease).

The reason for Proposition 3 is as follows. The private optimum of R&D risk is different from the social optimum because the social planner and firms pursue inconsistent goals when determining the R&D risk level. For firm 1, it only cares for its own expected profit and partially cares for its rival's expected profit. However, it does not consider the positive effect of its R&D risk on the expected consumer surplus (see Lemma 2). In addition, for firm 2, it only cares for its own expected profit in a certain proportion. However, it does not consider the positive effect of its R&D risk on the rival's expected profit and the expected consumer surplus (see Lemmas 1 and 2). The social planner not only considers a given firm's expected profit, but also considers that of its rival and the expected consumer surplus. Thus, from the perspective of social welfare, firm 1 (or firm 2) has an insufficient incentive for taking R&D risks.

5. Extensions

This section considers several extensions of the basic model and checks the robustness of the main findings in Sections 3 and 4.

5.1. Allowing firms to hold each other's shares

In the basic model, only one firm owns some shares of its rival (i.e., unilateral cross ownership). However, two firms may hold each other's shares (i.e., bilateral cross ownership) in reality. We now consider the situation that each firm is allowed to have a minority of shares in its rival. In this section, we assume that firm i acquires equities on a share k_j $(0<k_j<0.5)$ of firm j's profits $(i \neq j,\ i,j = 1,2)$. Thus, the objective function of firm i is given by: $U_i = (1-k_j)\pi_i + k_i\pi_j\ \ (i \neq j,\ i,j = 1,2)$ (Bayona & López, 2018). We assume that $k_1 \neq k_2$. That is, we extend into the asymmetric P.P.O.s where each firm owns different shares of its rival. The analysis in the main context is one of examples where a firm has partial P.P.O.s in its rival while the rival does not have. Other assumptions are the same as those in the basic model. Similar to the derivation of the private optimum (i.e., $\sigma_i^{*^{\mathrm{I}}}$) and social optimum (i.e., $\sigma_i^{s^{\mathrm{I}}}$) and then prove the following results.

Proposition 4. *(i)* $\sigma_i^{*^{\mathrm{I}}}$ *increases with* k_i $(i = 1,2)$*; (ii)* $\sigma_i^{*^{\mathrm{I}}}<\sigma_j^{*^{\mathrm{I}}}$ *if* $k_i>k_j$ $(i \neq j,\ i,j = 1,2)$*; and (iii)* $\sigma_i^{*^{\mathrm{I}}}<\sigma_i^{s^{\mathrm{I}}}$ $(i = 1,2)$*.*

Proof. See Appendix F.

This implies that when two competing firms own different shares of its rival, the results of Propositions 1–3 in the benchmark case still hold.

5.2. Allowing firms to compete in price in market stage

In the second stage of the basic model, two firms compete in quantity (i.e., conduct Cournot competition), whose choices are strategic substitutes. However, the firms' choices are strategic complements in price competition (i.e., conduct Bertrand competition). Are the main results of the basic model (Propositions 1∼3) robust under price competition? To answer this question, we need to consider the situation that firms compete in price in the market stage. The only difference between this section and the basic model is the form of market competition. We assume that firms decide on price in the second stage in this section. According to (1), we give the following demand function: $q_i = \frac{1}{1-r^2}[(1-r)a - p_i + rp_j]$ $(i \neq j, i, j = 1, 2)$. Similar as in sections 3 and 4, we derive the private optimum (i.e., σ_i^B) and social optimum (i.e., σ_i^{SB}) and then prove the following proposition.

Proposition 5. *(i)* σ_i^B *(*$i = 1, 2$*) increases with* k*; (ii)* $\sigma_1^B < \sigma_2^B$*; and (iii)* $\sigma_i^B < \sigma_i^{SB}$ *(*$i = 1, 2$*).*

Proof. See Appendix G.

This implies that the main results of Propositions 1–3 in the benchmark case are robust to the mode of competition (Cournot or Bertrand).

Proposition 6. $\sigma_i^C < \sigma_i^B$ *(*$i = 1, 2$*).*

Proof. See Appendix H.

In the literature on the comparisons of R&D investments between Cournot and Bertrand, it is already well-known that Cournot firms invest more in R&D than Bertrand firms (Chen & Lee, 2022; Hinloopen & Vandekerckhove, 2009; Qiu,1997).[6] However, this phenomenon can be reversed in this study where the R&D outcome (i.e., x_i) is a parameter that has a probability distribution with the same expected mean while the R&D risk is a choice variable (see Proposition 6). That is, given the same expectation of R&D outcome between Cournot and Bertrand, the strategic effect of R&D on the profit disappears, and only the level of R&D risk matters in determining the firm's choices.

The reason for Proposition 6 is as follows. The higher marginal production cost asymmetry implies more industrial gross profits (see the explanation of lemma 1). Due to the fact that the outputs are strategic substitutes (the prices are strategic complements) under Cournot (Bertrand), the cost asymmetry between firms increases industrial gross profits more significantly under Bertrand (Xing et al., 2021). It follows that Bertrand firm's R&D risk has a higher positive effect on the expected industrial gross profit. Similar reason as in Lemma 1, for a given r and k, if a firm chooses a R&D project with higher risk, it will expect to get more from the total profits of the industry. Thus, Bertrand firm's R&D risk has a higher positive effect on its expected gross profits. This leads Bertrand firm to choose higher R&D risk behaviour, compared to Cournot firm.

Lee and Cho (2020) and Xing et al. (2021) analyse the R&D risk choices under different competition modes by considering corporate social responsibility and emission tax respectively. They think that the R&D risk level in the Bertrand competition case is higher than in the Cournot competition case. Proposition 6 states that their result still holds even in the situation of P.P.O.s.

5.3. Allowing R&D spillovers

There might exist research spillovers between the firm's R&D investments. This section tests the robustness of the main propositions when considering R&D spillovers.

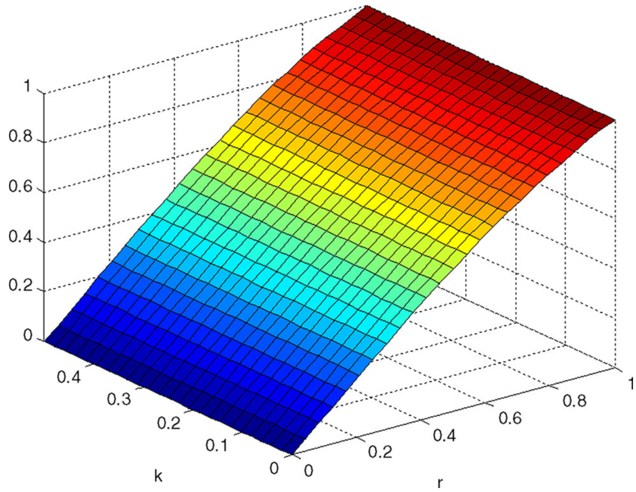

Figure 1. Value range of $\hat{\beta}$ when $0<r<1$ and $0<k<0.5$.
Source: Author's own calculation.

We consider the following marginal production cost of firm i is: $c_i = c - x_i - \beta x_j$ $(i \neq j,\ i,j = 1,2)$, where c is the initial marginal production cost, x_i (x_j) is the R&D effort of firm i (firm j), and β $(0 \leq \beta \leq 1)$ is the R&D spillover parameter (D'Aspremont & Jacquemin, 1988). In the basic model, $\beta = 0$. Other assumptions are the same as those in the basic model. Similar to the derivation of the basic model, we obtain the private optimum (i.e., $\sigma_i^{*^{\text{II}}}$) and social optimum (i.e., $\sigma_i^{s^{\text{II}}}$). We set $\overset{\frown}{\beta} = \frac{r}{2}$, $\breve{\beta} = \frac{r[4+(1-k)r^2]}{8-(1+k)(2-r^2)r^2}$ and $\hat{\beta} = \frac{r[4-(1+k)r^2+\sqrt{32-4(2+3k)r^2+(1+k)^2r^4}]}{2(4-kr^2)}$. $0<\overset{\frown}{\beta}<1$ and $0<\breve{\beta}<1$ because $0<\frac{r}{2}<1$, $0<r[4+(1-k)r^2]<5$ and $8-(1+k)(2-r^2)r^2>5$. In addition, $\hat{\beta}$ also lies in $(0,1)$ (see Figure 1). Then, we can prove the following results.

Proposition 7. *(i) $\sigma_1^{*^{\text{II}}}$ increases (decreases) with k if $\beta \in [0, \overset{\frown}{\beta}) \cup (\breve{\beta}, 1]$ $((\overset{\frown}{\beta}, \breve{\beta}))$, and $\sigma_2^{*^{\text{II}}}$ increases with k for all $\beta \in [0,1]$; (ii) $\sigma_1^{*^{\text{II}}} < (>) \sigma_2^{*^{\text{II}}}$ if $\beta \in [0, \hat{\beta})$ $((\hat{\beta}, 1])$; and (iii) $\sigma_i^{*^{\text{II}}} < \sigma_i^{s^{\text{II}}}$ $(i = 1,2)$ for all $\beta \in [0, 1]$.*

Proof. See Appendix I.

Combing with propositions in the basic model, we find that: (1) the part (ii) of Proposition 1 and both parts (i) and (ii) of Proposition 3 still hold for all $\beta \in [0, 1]$; (2) the part (i) of proposition 1 still holds for most β (for example, given $r = 0.2$ $(r = 0.9)$, $\beta \in [0, 0.1000) \cup (0.1020, 1]$ $(\beta \in [0, 0.4500) \cup (0.6153, 1])$); and (3) if r is large, proposition 2 still holds for most β (for example, given $r = 0.9$, $\beta \in [0, 0.9344)$).

It is worth noting that: (i) $\sigma_1^{*^{\text{II}}}$ may decrease with k if β is moderate $(\beta \in (\dot{\beta}, \ddot{\beta}))$. The reason is as follows. Similar reason as Lemma 1, the R&D risk of firm 1 has a positive effect on its (or its rival's) expected gross profit (i.e., $\frac{\partial E(\Pi_1)}{\partial \sigma_1} > 0$ and $\frac{\partial E(\Pi_2)}{\partial \sigma_1} > 0$) when considering the R&D spillovers. Further, we find that when β is moderate, the P.P.O.s have a negative (or weak positive) effect on $\frac{\partial E(\Pi_1)}{\partial \sigma_1}$, and have a negative effect on $\frac{\partial E(\Pi_2)}{\partial \sigma_1}$. It follows that the P.P.O.s have a negative effect on $\frac{\partial [E(\Pi_1)+E(\Pi_2)]}{\partial \sigma_1}$. With an increase of k, firm 1 pays more attention to industry profits when choosing R&D projects, and thus it may choose lower risk level of R&D project if β is moderate;

and (ii) $\sigma_1^{*^{\mathrm{II}}}$ may be higher than $\sigma_2^{*^{\mathrm{II}}}$ if r is small and β is large. The reason is as follows. When r is small, the degree of product differentiation is large and the competition in the market stage is weak. Firm 1 partially considers the expected profit of firm 2 when selecting R&D projects. In the situation of weak market competition, once its R&D is successful, it will also bring great benefits to firm 2 if β is large, which in turn will increase firm 1's total profit. The above effect does not exist for firm 2. Thus, firm 1 may take higher risks than firm 2 if r is small and β is large.

The above results remind us that, if the research spillovers are significant, we should be cautious when analysing the impact of P.P.O.s on the R&D risk choices of the firm who owns a share of its rival, or comparing the R&D risk levels of the firm who owns a share of its rival and its partially owned rival.

5.4. Allowing products to be complementary

In the basic model, we assume that the products of different firms have a certain degree of substitutability. However, there also exist the P.P.O.s in some industrial chains. In this situation, the products of upstream and downstream firms are complementary. This section allows products to have a certain degree of complementarity, which is also measured by parameter r. However, unlike the basic model r is assumed to satisfy $-1 \leq r<0$ in this section. Other assumptions are the same as those in the basic model. We can prove that the privately optimal R&D risk levels of firms 1 and 2 satisfy (17) and (18) respectively, and their socially optimal R&D risk satisfy (23) and (24) respectively. Obviously, we can find similar results as in the basic model.[7] That is, the results of Propositions 1∼3 are robust in the situation that the products provided by firms have a certain complementarity.

5.5. Allowing R&D risks correlation

In the basic model, we assume that the covariance of x_i and x_j is zero (i.e., $Cov(x_i, x_j) = 0,\ \ i \neq j,\ \ i,j = 1,2$). This assumption implies that x_i and x_j are independent. However, x_i and x_j may be dependent under certain conditions. In this situation, there may be $Cov(x_i, x_j) \neq 0\ \ (i \neq j,\ \ i,j = 1,2)$. Even in the R&D competitive environment, the P.P.O.s may lead to a certain positive correlation between the R&D risks of the firms.[8] In this section, we assume that $Cov(x_i, x_j) = \theta>0\ \ (i \neq j,\ i,j = 1,2)$, where θ is a constant. Note that θ is not very large to ensure that the expected profits of each firm are positive. Other assumptions are the same as those in the basic model. We can prove that the privately (socially) optimal R&D risk levels of firms 1 and 2 satisfy (17) and (18, 23) and (24)), respectively. Obviously, Propositions 1∼3 in the benchmark case still hold in the situation that each firm's R&D risks have correlations according to the firm's R&D investment levels.[9]

6. Conclusions

This study examines the optimal risk choices from a series R&D projects with different risks in a differentiated duopoly market when one firm has a minority of its rival's shares but the other one does not. It indicates that P.P.O. holdings increase the optimal R&D risk chosen by both firms, while the firm that does not own the rival's shares is more willing to choose the higher R&D risk than the one that owns its shares. In addition, for each firm its privately optimal risk is always lower than the socially optimal risk, and the P.P.O.s play an important role in determining the extent of this inefficiency. Finally, it extends to allow bilateral cross-ownership, Bertrand competition, R&D spillovers, product complementarity and R&D risks correlation. The practical implications are as follows. Firm managers should consider the P.P.O.s when they choose the R&D projects in a competitive environment: the larger level the P.P.O.s, the more volatile should be the outcomes of their selected R&D projects. In addition, policymakers should be aware that the P.P.O.s are conducive to promoting firms to choose high-risk R&D projects and may reduce the inefficiency of private R&D incentives.

In future studies, we can consider the government policy such as R&D risk sharing program or R&D subsidies, and consider the heterogeneous firms such as mixed duopoly or firms with different level of C.S.R.

Notes

1. For example: (1) in May 2009, Daimler, a German carmaker, said that the company had acquired nearly 10% of Tesla Motors Inc, an electromagnetic drive car manufacturer in California. After that, both companies aimed at the research and development of new battery-driven cars with higher risk; and (2) after Microsoft held the stock of apple in 1997, both of them carried out the research and development of more advanced operating systems (Windows 98 and Mac OS X) with higher risk.
2. Note that we do not consider the R&D spillover effects. This simplifying assumption aims to show the only impact of PPOs on the R&D risk choices. However, there might exist research spillovers between the firm's R&D investments, which is also an important key factor in the literature of R&D (Banal-Estañol et al., 2022; Zhuang & Zhao, 2022). We will test the robustness of the main propositions when considering R&D spillovers in section 5.3.
3. We think that the assumption of $Cov(x_i, x_j) = 0 \ (i \neq j)$ is reasonable in the R&D competition environment. The reason is as follows. To prevent the disclosure of R&D secrets, firms generally formulate strict R&D confidentiality regulations, which prevent competitors from obtaining relevant information (Shen et al., 2010). When there is R&D competition among firms, they will avoid the disclosure of R&D information. However, x_i and x_j may be dependent under certain conditions. We will test the robustness of the main propositions when $Cov(x_i, x_j) \neq 0 \ (i \neq j)$ in section 5.5.
4. Zhu (2011) considers the risk costs of technological innovation.
5. We further assume that the function $I(\mu_i, \sigma_i)$ is twice continuously differentiable about σ_i and meets $\frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2} > 0$.
6. This is because the strategic effect of R&D on the profit in relation to its rival's output is positive under Cournot competition (due to the fact that the outputs are strategic substitutes, if Cournot firm invests more and produces more, its rival firm invests less and produces less) while that effect of R&D on the profit in relation to its rival's output is negative under Bertrand competition (due to the fact that the prices are strategic complements, if Bertrand firm invests more and produces more or sets lower price, it rival firm invest more and produces more or sets lower price) (Chen & Lee, 2022).
7. The proof of the main results in this section is the same as that of Propositions $1 \sim 3$. To avoid repetition, we omitted their proof.

8. Note that the conceptual difference exists between R&D spillovers (Section 5.3) and R&D risk correlation (Section 5.5). Following D'Aspremont and Jacquemin (1988), we think that the R&D spillovers imply that some benefits of each firm's final R&D results (e.g. successful inventions) flow without payment to other firms. This refers to the spillovers of the final R&D results of a firm, rather than the leakage (or disclosure) of relevant information in the R&D process. Thus, the R&D spillovers do not imply that x_1 and x_2 are dependent in this study. In addition, 'each firm's R&D risks have correlations according to the firm's R&D efforts' means that x_1 and x_2 are not independent of each other. However, if each firm obtains the R&D information of its competitor in the R&D process, the final R&D results may be related and this may lead to $Cov(x_1, x_2) \neq 0$.
9. The proof of the main results in this section is the same as that of Propositions $1 \sim 3$. To avoid repetition, we omitted their proof.

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Appendix A

Proof of Lemma 1. Because $\frac{\partial E(\Pi_1^C)}{\partial \sigma_1} = \frac{2(2-kr^2)}{[4-(1+k)r^2]^2} > 0$, $\frac{\partial E(\Pi_1^C)}{\partial \sigma_2} = \frac{r^2(1-k^2)}{[4-(1+k)r^2]^2} > 0$, $\frac{\partial E(\Pi_2^C)}{\partial \sigma_1} = \frac{r^2}{[4-(1+k)r^2]^2} > 0$, $\frac{\partial E(\Pi_2^C)}{\partial \sigma_2} = \frac{4}{[4-(1+k)r^2]^2} > 0$, $\frac{\partial E(\Pi^C)}{\partial \sigma_1} = \frac{2(2-kr^2)+r^2}{[4-(1+k)r^2]^2} > 0$ and $\frac{\partial E(\Pi^C)}{\partial \sigma_2} = \frac{r^2(1-k^2)+4}{[4-(1+k)r^2]^2} > 0$.

Appendix B

Proof of Proposition 1. (i) According to (17), $g(\sigma_1^C) = \frac{4-kr^2}{[4-(1+k)r^2]^2}$. Therefore, $g'(\sigma_1^C)\frac{\partial \sigma_1^C}{\partial k} = \frac{(4+r^2-kr^2)r^2}{[4-(1+k)r^2]^3} > 0$. Because $g'(\sigma_1^C) = \frac{\partial^2 I(\mu_1, \sigma_1)}{\partial \sigma_1^2}\Big|_{\sigma_1=\sigma_1^C} > 0$, $\frac{\partial \sigma_1^C}{\partial k} > 0$; and (ii) According to (18), $g(\sigma_2^C) = \frac{4}{[4-(1+k)r^2]^2}$. Therefore, $g'(\sigma_2^C)\frac{\partial \sigma_2^C}{\partial k} = \frac{8r^2}{[4-(1+k)r^2]^3} > 0$. Because $g'(\sigma_2^C) = \frac{\partial^2 I(\mu_2, \sigma_2)}{\partial \sigma_2^2}\Big|_{\sigma_2=\sigma_2^C} > 0$, $\frac{\partial \sigma_2^C}{\partial k} > 0$.

Appendix C

Proof of Proposition 2. According to (17) and (18), $g(\sigma_1^C) - g(\sigma_2^C) = -\frac{kr^2}{[4-(1+k)r^2]^2}$. There exists $\widehat{\sigma}$ ($\widehat{\sigma} > 0$) satisfying $(\sigma_1^C - \sigma_2^C)g'(\widehat{\sigma}) = -\frac{kr^2}{[4-(1+k)r^2]^2} < 0$. Because $g'(\widehat{\sigma}) = \frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2}\Big|_{\sigma_i=\widehat{\sigma}} > 0$, $\sigma_1^C - \sigma_2^C < 0$.

Appendix D

Proof of Lemma 2. Because $\frac{\partial E(CS^C)}{\partial \sigma_1} = \frac{4-3r^2}{2[4-(1+k)r^2]^2} > 0$ and $\frac{\partial E(CS^C)}{\partial \sigma_2} = \frac{(1+k)^2 r^2 - 4(1+k)r^2 + 4}{2[4-(1+k)r^2]^2} > 0$.

Appendix E

Proof of Proposition 3. (i) According to (17) and (23), $g(\sigma_1^{SC}) - g(\sigma_1^C) = \frac{4-2kr^2-r^2}{2[4-(1+k)r^2]^2}$. There exists $\tilde{\sigma}$ ($\tilde{\sigma}>0$) satisfying $(\sigma_1^{SC} - \sigma_1^C)g'(\tilde{\sigma}) = \frac{4-2kr^2-r^2}{2[4-(1+k)r^2]^2} > 0$. Because $g'(\tilde{\sigma}) = \frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2}|_{\sigma_i=\tilde{\sigma}} > 0$, $\sigma_1^{SC} - \sigma_1^C > 0$; and (ii) According to (18) and (24), $g(\sigma_2^{SC}) - g(\sigma_2^C) = \frac{4-(1+k)^2 r^2}{2[4-(1+k)r^2]^2}$. There exists $\hat{\sigma}$ ($\hat{\sigma}>0$) satisfying $(\sigma_2^{SC} - \sigma_2^C)g'(\hat{\sigma}) = \frac{4-(1+k)^2 r^2}{2[4-(1+k)r^2]^2} > 0$. Because $g'(\hat{\sigma}) = \frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2}|_{\sigma_i=\hat{\sigma}} > 0$, $\sigma_2^{SC} - \sigma_2^C > 0$.

Appendix F

Proof of Proposition 4. Similar to the derivation of the basic model, we obtain that $\sigma_i^{*^{I}}$ and $\sigma_i^{s^{I}}$ satisfy:

$$\frac{(1-k_2)\left[4(1-k_1)^2(1-k_2) - k_1(1-k_1+k_2)^2 r^2\right]}{\{4(1-k_1)(1-k_2) - \left[1-(k_1-k_2)^2\right]r^2\}^2} - g(\sigma_1^{*^{I}}) = 0 \tag{25}$$

$$\frac{(1-k_1)\left[4(1-k_2)^2(1-k_1) - k_2(1-k_2+k_1)^2 r^2\right]}{\{4(1-k_1)(1-k_2) - \left[1-(k_1-k_2)^2\right]r^2\}^2} - g(\sigma_2^{*^{I}}) = 0 \tag{26}$$

$$\frac{(1-k_2)\{12(1-k_1)^2(1-k_2) - \left[4k_1^3 - (5k_2+7)k_1^2 + 2(1+k_2)^2 k_1 + (1+k_2)^2(1-k_2)\right]r^2\}}{2\{4(1-k_1)(1-k_2) - \left[1-(k_1-k_2)^2\right]r^2\}^2} - g(\sigma_i^{s^{I}}) = 0 \tag{27}$$

$$\frac{(1-k_1)\{12(1-k_2)^2(1-k_1) - \left[4k_2^3 - (5k_1+7)k_2^2 + 2(1+k_1)^2 k_2 + (1+k_1)^2(1-k_1)\right]r^2\}}{2\{4(1-k_1)(1-k_2) - \left[1-(k_1-k_2)^2\right]r^2\}^2} - g(\sigma_i^{s^{I}}) = 0 \tag{28}$$

Thus, we can prove the following results: (i) According to (25) and (26), $g(\sigma_1^{*^{I}}) = \frac{(1-k_2)[4(1-k_1)^2(1-k_2) - k_1(1-k_1+k_2)^2 r^2]}{\{4(1-k_1)(1-k_2) - [1-(k_1-k_2)^2]r^2\}^2}$ and $g(\sigma_2^{*^{I}}) = \frac{(1-k_1)[4(1-k_2)^2(1-k_1) - k_2(1-k_2+k_1)^2 r^2]}{\{4(1-k_1)(1-k_2) - [1-(k_1-k_2)^2]r^2\}^2}$. Therefore, $g'(\sigma_1^{*^{I}})\frac{\partial \sigma_1^{*^{I}}}{\partial k_1} = \frac{(1-k_2)r^2(1-k_1-k_2)\{(1-k_1+k_2)^3 r^2 + 4(1-k_2)[(1-k_1)k_2 + (1-2k_1) + 2k_2 + k_1^2]\}}{\{4(1-k_1)(1-k_2) - [1-(k_1-k_2)^2]r^2\}^3} > 0$ and $g'(\sigma_2^{*^{I}})\frac{\partial \sigma_2^{*^{I}}}{\partial k_2} = \frac{(1-k_1)r^2(1-k_2-k_1)\{(1-k_2+k_1)^3 r^2 + 4(1-k_1)[(1-k_2)k_1 + (1-2k_2) + 2k_1 + k_2^2]\}}{\{4(1-k_1)(1-k_2) - [1-(k_1-k_2)^2]r^2\}^3} > 0$. Because $g'(\sigma_i^{*^{I}}) = \frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2}\Big|_{\sigma_i=\sigma_i^{*^{I}}} > 0$ $(i = 1, 2)$, $\frac{\partial \sigma_i^{*^{I}}}{\partial k_i} > 0$; (ii) According to (25) and (26), $g(\sigma_1^{*^{I}}) - g(\sigma_2^{*^{I}}) = \frac{r^2(k_2-k_1)(1-k_1-k_2)^2}{\{4(1-k_1)(1-k_2) - [1-(k_1-k_2)^2]r^2\}^2}$. If $k_2 > (<) k_1$, there exists $\bar{\sigma}$ ($\bar{\sigma}>0$) satisfying $(\sigma_1^{*^{I}} - \sigma_2^{*^{I}})g'(\bar{\sigma}) = \frac{r^2(k_2-k_1)(1-k_1-k_2)^2}{\{4(1-k_1)(1-k_2) - [1-(k_1-k_2)^2]r^2\}^2} > (<) 0$. Because $g'(\bar{\sigma}) = \frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2}|_{\sigma_i=\bar{\sigma}} > 0$, $\sigma_1^{*^{I}} - \sigma_2^{*^{I}} > (<) 0$ if

$k_2>(<)k_1$; and (iii) According to (25) and (27), $g(\sigma_1^{s^{\mathrm{I}}})-g(\sigma_1^{*^{\mathrm{I}}}) = \frac{(1-k_2)\{4(1-k_1)^2(1-k_2)-(1-k_1+k_2)[(1-k_2)k_1+(1-2k_1^2-k_2^2)]r^2\}}{2\{4(1-k_1)(1-k_2)-[1-(k_1-k_2)^2]r^2\}^2}$. Because $4(1-k_1)^2(1-k_2)-(1-k_1+k_2)[(1-k_2)k_1+(1-2k_1^2-k_2^2)]r^2>(3-2k_1+k_2)(1-k_1-k_2)^2>0$, $g(\sigma_1^{s^{\mathrm{I}}})-g(\sigma_1^{*^{\mathrm{I}}})>0$. There exists $\underline{\sigma}$ ($\underline{\sigma}>0$) satisfying $(\sigma_1^{s^{\mathrm{I}}}-\sigma_1^{*^{\mathrm{I}}})g'(\underline{\sigma})>0$. Because $g'(\underline{\sigma}) = \frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}|_{\sigma_i=\underline{\sigma}}>0$, $\sigma_1^{s^{\mathrm{I}}}-\sigma_1^{*^{\mathrm{I}}}>0$. In addition, according to (26) and (28), $g(\sigma_2^{s^{\mathrm{I}}})-g(\sigma_2^{*^{\mathrm{I}}}) = \frac{(1-k_1)\{4(1-k_2)^2(1-k_1)-(1-k_2+k_1)[(1-k_1)k_2+(1-2k_2^2-k_1^2)]r^2\}}{2\{4(1-k_1)(1-k_2)-[1-(k_1-k_2)^2]r^2\}^2}$. Because $4(1-k_2)^2(1-k_1)-(1-k_2+k_1)[(1-k_1)k_2+(1-2k_2^2-k_1^2)]r^2>(3-2k_2+k_1)(1-k_2-k_1)^2>0$, $g(\sigma_2^{s^{\mathrm{I}}})-g(\sigma_2^{*^{\mathrm{I}}})>0$. There exists σ ($\sigma>0$) satisfying $(\sigma_2^{s^{\mathrm{I}}}-\sigma_2^{*^{\mathrm{I}}})g'(\sigma)>0$. Because $g'(\sigma) = \frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}|_{\sigma_i=\sigma}>0$, $\sigma_2^{s^{\mathrm{I}}}-\sigma_2^{*^{\mathrm{I}}}>0$.

Appendix G

Proof of Proposition 5. Similar as in sections 3 and 4, we can prove that σ_i^B and σ_i^{SB} satisfy:

$$\frac{(2-r^2)^2-(1-r^2)r^2k}{(1-r^2)[4-(1+k)r^2]^2}-g(\sigma_1^B)=0 \tag{29}$$

$$\frac{(2-r^2)^2}{(1-r^2)[4-(1+k)r^2]^2}-g(\sigma_2^B)=0 \tag{30}$$

$$\frac{12-9r^2+2r^4+2r^4k-4r^2k}{2(1-r^2)[4-(1+k)r^2]^2}-g(\sigma_1^{SB})=0 \tag{31}$$

$$\frac{12-9r^2+2r^4+r^4k^2-r^2k^2-2r^2k}{2(1-r^2)[4-(1+k)r^2]^2}-g(\sigma_2^{SB})=0 \tag{32}$$

Thus, we can prove the following results: (i) According to (29) and (30), $g(\sigma_1^B) = \frac{(2-r^2)^2-(1-r^2)r^2k}{(1-r^2)[4-(1+k)r^2]^2}$ and $g(\sigma_2^B) = \frac{(2-r^2)^2}{(1-r^2)[4-(1+k)r^2]^2}$. Therefore, $g'(\sigma_1^B)\frac{\partial\sigma_1^B}{\partial k} = \frac{r^2\{(1-k)r^2(1-r^2)+2[2(1-r^2)+r^4]\}}{(1-r^2)[4-(1+k)r^2]^3}>0$ and $g'(\sigma_2^B)\frac{\partial\sigma_2^B}{\partial k} = \frac{2r^2(2-r^2)^2}{(1-r^2)[4-(1+k)r^2]^3}>0$. Because $g'(\sigma_i^B) = \frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}\Big|_{\sigma_i=\sigma_i^B}>0$, $\frac{\partial\sigma_i^B}{\partial k}>0$ $(i=1,2)$; (ii) According to (29) and (30), $g(\sigma_1^B)-g(\sigma_2^B) = -\frac{kr^2}{[4-(1+k)r^2]^2}$. There exists $\breve{\sigma}$ ($\breve{\sigma}>0$) satisfying $(\sigma_1^B-\sigma_2^B)g'(\breve{\sigma}) = -\frac{kr^2}{[4-(1+k)r^2]^2}<0$. Because $g'(\breve{\sigma}) = \frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}\Big|_{\sigma_i=\breve{\sigma}}>0$, $\sigma_1^B-\sigma_2^B<0$; and (iii) According to (29) and (31), $g(\sigma_1^{SB})-g(\sigma_1^B) = \frac{1+(1-r^2)+2(1-r^2k)}{2(1-r^2)[4-(1+k)r^2]^2}$. There exists $\overrightarrow{\sigma}$ ($\overrightarrow{\sigma}>0$) satisfying $(\sigma_1^{SB}-\sigma_1^B)g'(\overrightarrow{\sigma}) = \frac{1+(1-r^2)+2(1-r^2k)}{2(1-r^2)[4-(1+k)r^2]^2}>0$. Because $g'(\overrightarrow{\sigma}) = \frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}\Big|_{\sigma_i=\overrightarrow{\sigma}}>0$, $\sigma_1^{SB}-\sigma_1^B>0$. In addition, according to (30) and (32), $g(\sigma_2^{SB})-g(\sigma_2^B) = \frac{4-r^2(1+k)^2+r^4k^2}{2(1-r^2)[4-(1+k)r^2]^2}$. There exists $\overleftarrow{\sigma}$ ($\overleftarrow{\sigma}>0$) satisfying $(\sigma_2^{SB}-\sigma_2^B)g'(\overleftarrow{\sigma}) = \frac{4-r^2(1+k)^2+r^4k^2}{2(1-r^2)[4-(1+k)r^2]^2}>0$. Because $g'(\overleftarrow{\sigma}) = \frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}\Big|_{\sigma_i=\overleftarrow{\sigma}}>0$, $\sigma_2^{SB}-\sigma_2^B>0$.

Appendix H

Proof of Proposition 6. According to (17) and (29), $g(\sigma_1^C)-g(\sigma_1^B) = -\frac{r^4}{(1-r^2)[4-(1+k)r^2]^2}$. There exists $\vec{\sigma}$ ($\vec{\sigma}>0$) satisfying $(\sigma_1^C-\sigma_1^B)g'(\vec{\sigma}) = -\frac{r^4}{(1-r^2)[4-(1+k)r^2]^2}<0$. Because $g'(\vec{\sigma}) =$

$\frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}\Big|_{\sigma_i=\vec{\sigma}}>0$, $\sigma_1^C-\sigma_1^B<0$. In addition, according to (18) and (30), $g(\sigma_2^C)-g(\sigma_2^B)=-\frac{r^4}{(1-r^2)[4-(1+k)r^2]^2}$. There exists $\overleftarrow{\sigma}$ ($\overleftarrow{\sigma}>0$) satisfying $(\sigma_2^C-\sigma_2^B)g'(\overleftarrow{\sigma})=-\frac{r^4}{(1-r^2)[4-(1+k)r^2]^2}<0$. Because $g'(\overleftarrow{\sigma})=\frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}\Big|_{\sigma_i=\overleftarrow{\sigma}}>0$, $\sigma_2^C-\sigma_2^B<0$.

Appendix I

Proof of Proposition 7. Similar to the derivation of the basic model, we obtain that $\sigma_i^{*^{\mathrm{II}}}$ and $\sigma_i^{s^{\mathrm{II}}}$ satisfy:

$$\frac{\left[(1-k^2)r^2+4k\right]\beta^2-(1+k)r(4-kr^2)\beta+4-kr^2}{[4-(1+k)r^2]^2}-g(\sigma_1^{*^{\mathrm{II}}})=0 \tag{33}$$

$$\frac{(2-r\beta)^2}{[4-(1+k)r^2]^2}-g(\sigma_2^{*^{\mathrm{II}}})=0 \tag{34}$$

$$\frac{\left[12-(1+k)^2r^2\right]\beta^2+\left[2(1+k)^2r^3-4(4+k)r\right]\beta+12-(1+4k)r^2}{2[4-(1+k)r^2]^2}-g(\sigma_1^{s^{\mathrm{II}}})=0 \tag{35}$$

$$\frac{\left[12-(1+4k)r^2\right]\beta^2+\left[2(1+k)^2r^3-4(4+k)r\right]\beta+12-(1+k)^2r^2}{2[4-(1+k)r^2]^2}-g(\sigma_2^{s^{\mathrm{II}}})=0 \tag{36}$$

Thus, we can prove the following results: (i) According to (33) and (34), $g(\sigma_1^{*^{\mathrm{II}}})=\frac{[(1-k^2)r^2+4k]\beta^2-(1+k)r(4-kr^2)\beta+4-kr^2}{[4-(1+k)r^2]^2}$ and $g(\sigma_2^{*^{\mathrm{II}}})=\frac{(2-r\beta)^2}{[4-(1+k)r^2]^2}$. Therefore, $g'(\sigma_1^{*^{\mathrm{II}}})\frac{\partial\sigma_1^{*^{\mathrm{II}}}}{\partial k}=\frac{(2b-r)\{[8-(1+k)r^2(2-r^2)]b-r[4+(1-k)r^2]\}}{[4-(1+k)r^2]^3}>(<)0$ if $\beta\in[0,\overset{\frown}{\beta})\cup(\breve{\beta},1]$ $((\overset{\frown}{\beta},\breve{\beta}))$ ($\overset{\frown}{\beta}=\frac{r}{2}$ and $\breve{\beta}=\frac{r[4+(1-k)r^2]}{8-(1+k)(2-r^2)r^2}$), and $g'(\sigma_2^{*^{\mathrm{II}}})\frac{\partial\sigma_2^{*^{\mathrm{II}}}}{\partial k}=\frac{2r^2(2-r\beta)^2}{[4-(1+k)r^2]^3}>0$ for all $\beta\in[0,1]$. Because $g'(\sigma_i^{*^{\mathrm{II}}})=\frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}\Big|_{\sigma_i=\sigma_i^{*^{\mathrm{II}}}}>0$ $(i=1,2)$, $\frac{\partial\sigma_1^{*^{\mathrm{II}}}}{\partial k}>(<)0$ if $\beta\in[0,\overset{\frown}{\beta})\cup(\breve{\beta},1]$ $((\overset{\frown}{\beta},\breve{\beta}))$, and $\frac{\partial\sigma_2^{*^{\mathrm{II}}}}{\partial k}>0$ for all $\beta\in[0,1]$; (ii) According to (33) and (34), $g(\sigma_2^{*^{\mathrm{II}}})-g(\sigma_1^{*^{\mathrm{II}}})=\frac{k[-(4-kr^2)\beta^2+(4-kr^2-r^2)r\beta+r^2]}{[4-(1+k)r^2]^2}$. If $\beta\in[0,\hat{\beta})$ $((\hat{\beta},1])$ ($\hat{\beta}=\frac{r[4-(1+k)r^2+\sqrt{32-4(2+3k)r^2+(1+k)^2r^4}]}{2(4-kr^2)}$), there exists $\dot{\sigma}$ ($\dot{\sigma}>0$) satisfying $(\sigma_2^{*^{\mathrm{II}}}-\sigma_1^{*^{\mathrm{II}}})g'(\dot{\sigma})=\frac{k[-(4-kr^2)\beta^2+(4-kr^2-r^2)r\beta+r^2]}{[4-(1+k)r^2]^2}>(<)0$. Because $g'(\dot{\sigma})=\frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}|_{\sigma_i=\dot{\sigma}}>0$, $\sigma_2^{*^{\mathrm{II}}}-\sigma_1^{*^{\mathrm{II}}}>(<)0$ if $\beta\in[0,\hat{\beta})$ $((\hat{\beta},1])$; and (iii) According to (33) and (35), $g(\sigma_1^{s^{\mathrm{II}}})-g(\sigma_1^{*^{\mathrm{II}}})=\frac{(12+k^2r^2-2kr^2-8k-3r^2)\beta^2+(4kr+2r^3+2kr^3-8r)\beta-r^2-2kr^2+4}{2[4-(1+k)r^2]^2}$. Because $(12+k^2r^2-2kr^2-8k-3r^2)\beta^2+(4kr+2r^3+2kr^3-8r)\beta-r^2-2kr^2+4\geq\frac{(3-2k-r^2)(4-r^2-kr^2)^2}{12+k^2r^2-2kr^2-8k-3r^2}>0$, $g(\sigma_1^{s^{\mathrm{II}}})-g(\sigma_1^{*^{\mathrm{II}}})>0$. There exists $\ddot{\sigma}$ ($\ddot{\sigma}>0$) satisfying $(\sigma_1^{s^{\mathrm{II}}}-\sigma_1^{*^{\mathrm{II}}})g'(\ddot{\sigma})>0$. Because $g'(\ddot{\sigma})=\frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}|_{\sigma_i=\ddot{\sigma}}>0$, $\sigma_1^{s^{\mathrm{II}}}-\sigma_1^{*^{\mathrm{II}}}>0$. In addition, according to (34) and (36), $g(\sigma_2^{s^{\mathrm{II}}})-g(\sigma_2^{*^{\mathrm{II}}})=\frac{(12-4kr^2-3r^2)\beta^2+(-4kr+2r^3k^2+2r^3+4r^3k-8r)\beta-r^2-k^2r^2+4-2kr^2}{2[4-(1+k)r^2]^2}$. Because $(12-4kr^2-3r^2)\beta^2+(-4kr+2r^3k^2+2r^3+4r^3k-8r)\beta-r^2-k^2r^2+4-2kr^2\geq\frac{(3-r^2k^2-2r^2k-r^2)(4-r^2-kr^2)^2}{12-4kr^2-3r^2}>0$, $g(\sigma_2^{s^{\mathrm{II}}})-g(\sigma_2^{*^{\mathrm{II}}})>0$. There exists $\dddot{\sigma}$ ($\dddot{\sigma}>0$) satisfying $(\sigma_2^{s^{\mathrm{II}}}-\sigma_2^{*^{\mathrm{II}}})g'(\dddot{\sigma})>0$. Because $g'(\dddot{\sigma})=\frac{\partial^2 I(\mu_i,\sigma_i)}{\partial\sigma_i^2}|_{\sigma_i=\dddot{\sigma}}>0$, $\sigma_2^{s^{\mathrm{II}}}-\sigma_2^{*^{\mathrm{II}}}>0$.