Economic Research-Ekonomska Istraživanja

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

Technological progress and optimal income taxation

Insook Lee

To cite this article: Insook Lee (2023) Technological progress and optimal income taxation, Economic Research-Ekonomska Istraživanja, 36:2, 2142828, DOI: 10.1080/1331677X.2022.2142828

To link to this article: https://doi.org/10.1080/1331677X.2022.2142828

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2023, VOL. 36, NO. 2, 2142828
https://doi.org/10.1080/1331677X.2022.2142828

Routledge
Taylor & Francis Group

OPEN ACCESS Check for updates

Technological progress and optimal income taxation

Insook Lee

Bay Area International Business School, Beijing Normal University, Guangdong, China

ABSTRACT
Facing technological progress, how should a government reform income taxation? To address this question, optimal capital and labor income taxation is obtained for an economy of heterogeneous individuals. Technological progress raises optimal capital income tax rate and lowers optimal average marginal labor income tax rate if it is capital-biased by increasing relative capital productivity. Technological progress does the opposite if it is labor-biased by decreasing relative capital productivity. Neither capital-biased nor labor-biased technological progress affects optimal slope of labor income tax rate schedule. Technological progress does not affect optimal income taxation if it is unbiased by preserving relative capital productivity.

ARTICLE HISTORY
Received 13 May 2022
Accepted 27 October 2022

KEYWORDS
Capital-biased technological progress; unbiased technological progress; optimal income taxation; relative capital productivity

JEL CODE
H21; O33; H24; E25

1. Introduction

In developed economies, technological advancements have considerably improved productivity. As shown by Figure 1 from the data of OECD economies that represent developed economies, since 1960, total factor productivity has been on upward trend for about sixty years. On the other hand, as demonstrated by Figure 2 from their data, relative capital productivity remained almost same between 1960 and 1978, confirming Kaldor's stylized fact. It was only after 1990 when relative capital productivity started to exhibit upward trend, which is consistent with finding of Karabarbounis and Neiman (2014). Above all, data of both Figures 1 and 2 mean that substantial progresses of production technology took place over the past decades. At the same time, it should be noted that pre-tax income inequality of developed economies also exacerbated especially more severely after 1990, as illustrated by Figure 3. In fact, empirical findings of numerous rigorous studies (e.g., Acemoglu, 2002; Aghion et al., 2019; Caselli, 1999; Foerster & Gyoergy Tóth, 2015; Galor & Moav, 2000) found that technological progress results in widening the income gap between the rich and the poor. This implies that the increases in pre-tax income Gini index (Figure 3) can be attributable to the production technology progresses (Figures 1 and 2).[1] For

CONTACT Insook Lee islee@bnu.edu.cn

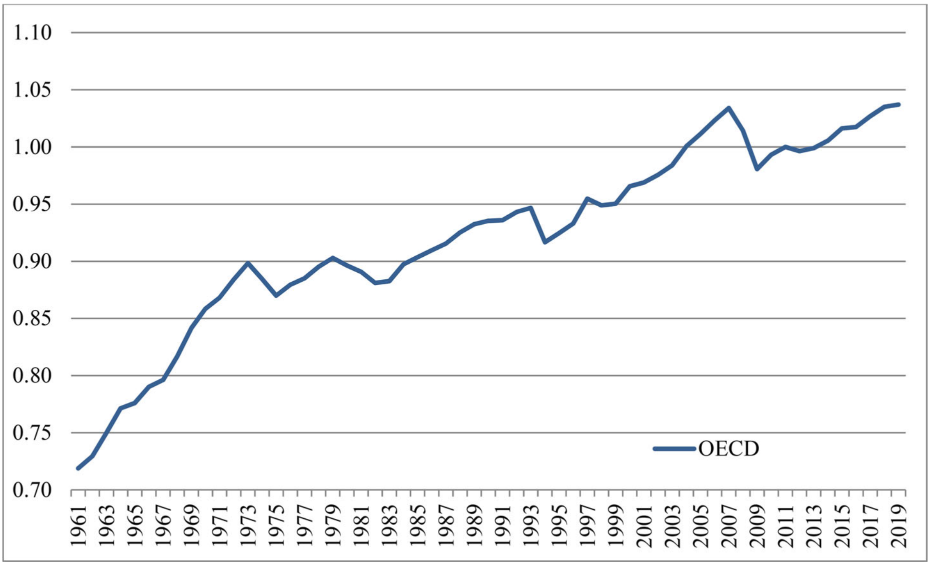

Figure 1. Total factor productivity of OECD economies.
Note: The data of total factor productivity of OECD economies are averaged and secured from the US Federal Reserve Bank which normalizes the data to take the value of one for year 2011.

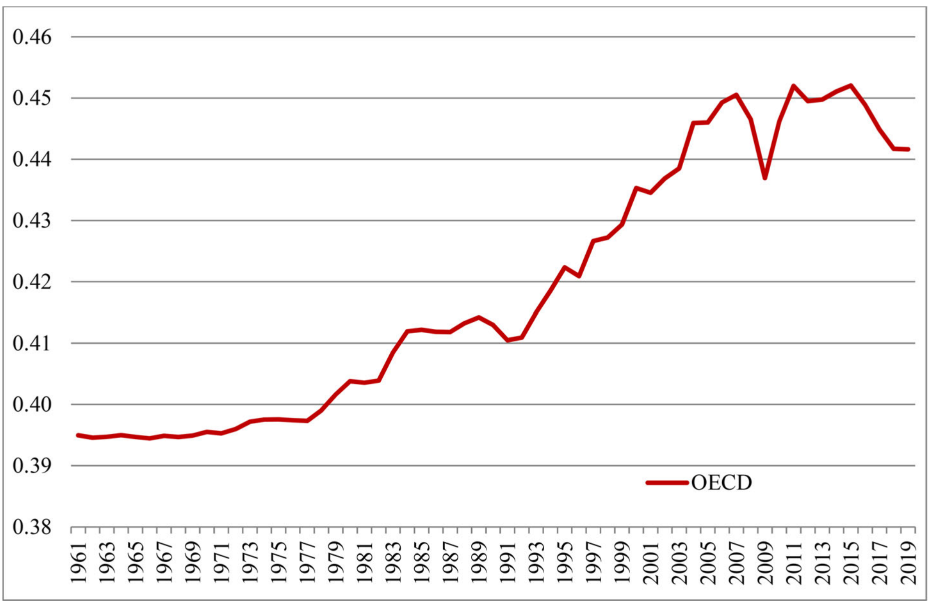

Figure 2. Relative capital productivity of OECD economies.
Note: With standard Cobb-Douglas production function, capital income share is unit-free marginal productivity of capital input (i.e., output elasticity with respect to capital input) and represents capital's relative contribution to output; hence, it indicates relative capital productivity. The income share data of OECD economies are averaged and secured from the US Federal Reserve Bank.

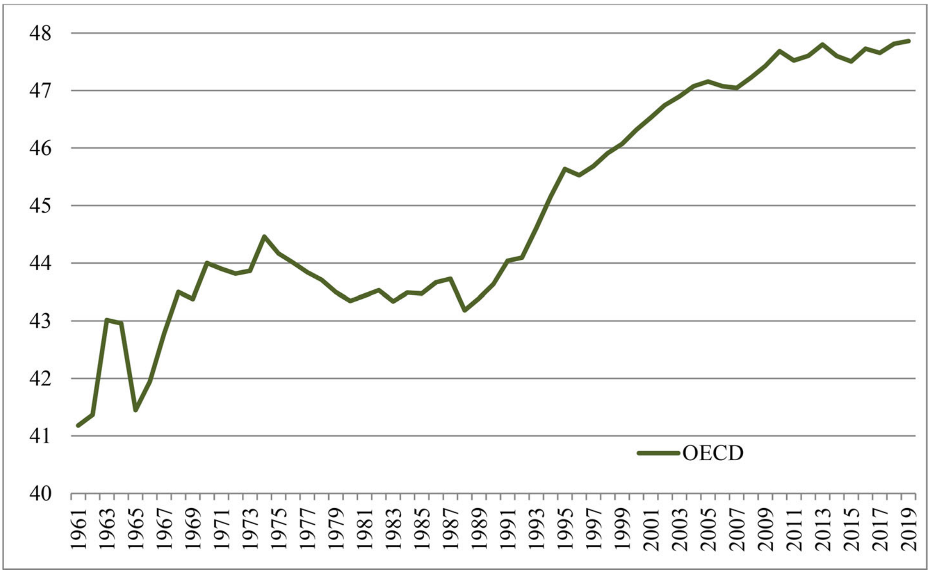

Figure 3. Pre-tax income inequality of OECD economies.
Note: The inequality of pre-tax income is measured by Gini index in % scale. The data of pre-tax income inequality of OECD economies are averaged and secured from Standardized World Income Inequality Database (Solt, 2020).

addressing the re-distributional concerns from technological progress, it is important to understand whether and how a government should adjust rates of income taxes, although it is not yet well studied in the public finance literature. This article analyses effects of technological progress on optimal capital and labor income taxation in a general-equilibrium model with taxpayers of unequal earning abilities.

This article is related to the literature that analyses optimal income tax policy responses to production technology changes. The representative studies of this literature are pioneer studies by Zhu (1992), Chari et al. (1994) and Werning (2007), as they analysed how optimal capital and labor income tax rates respond to technology shocks. However, the technology shocks of these studies are random and adopted for representing business-cycle fluctuations of short-term. Hence, the results of these studies are not necessarily directly applicable for understanding whether or how technological progress, which is long-run changes in production technology, affects optimal capital and labor income taxation. Distinct from and complementary to these studies of the literature, this article makes contribution of analysing how fundamental long-run advances in production technology affect optimal capital and labor income taxation.

This article is also related to Guerreiro et al. (2022) since both studies address optimal tax policy response to a change in production technology. Guerreiro et al. (2022) analysed whether linear robot tax on the firms should be introduced for responding to technology change of automation. Guerreiro et al. (2022) defined automation as an exogenous decrease in the price of robots and assumed that robots are only capital input available for production; thus, they took a partial-equilibrium approach. Guerreiro et al. (2022) did not elaborate on whether or how automation affects optimal capital and labor income taxation. Differentiating from Guerreiro et al. (2022), this article examines whether and how technological progress affects optimal capital and labor income taxation in a general-equilibrium model where input prices are endogenously determined. Because taxes on capital and labor incomes have been the most consequential fiscal policy instruments available, this article makes a meaningful contribution.

In addition, this article is also relatable to the Ramsey optimal income taxation literature that rationalizes taxing capital income (e.g., Aiyagari, 1995; Chamley, 2001; Conesa, Kitao & Krueger, 2009; Mattauch et al., 2018; Park, 2014) against the zero-capital-tax result of Chamley (1986) and Judd (1985). Although the main purpose of this article is not for discussing whether a strictly positive rate of capital income tax is desirable or not, in the middle of paving the way to identify effects of technological progress on optimal income taxation, this article obtains optimal capital and labor income tax rates before introducing a change in production technology and finds a new rationale for taxing capital income. As such, this article also contributes to the literature on whether to tax capital income or not.

This article is organized as follows. Section II delineates the model economy and types of technological progress. Section III characterizes allocation of stationary competitive general equilibrium. Utilizing this, Section IV obtains optimal capital and labor income taxation, based on which Section V examines the effect of each type of technological progress on optimal capital and labor income taxation. Section VI concludes the article.

2. Economic environment

Consider an economy of individuals who are heterogeneous only in terms of earning ability[2] and are indexed by i. Individuals are born with equal amount of capital endowment and unequal levels of earning ability. The population size stays as one. This economy is in its steady state; so, time subscripts are not necessary. For any given i, the lifetime utility of individual i is

$$u = (1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}\left[\log(c_i) - \frac{l_i^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \chi\log(G)\right] \tag{1}$$

where $\beta \in (0,1)$ is time preference; $c_i>0$ and $l_i \in [0,1]$ are private goods consumption and labor supply, respectively, of individual i; $\eta>0$ is Frisch elasticity of labor supply; G is public goods provided by the government of this economy; $\chi>0$ is preference for public goods. For tractability, the logarithm consumption utility of (1) is chosen which is consistent with empirical findings on labor supply (e.g., Kimball & Shapiro, 2008) as income effect of wage rate is cancelled out with its substitution effect. In maximizing the utility of (1), individual i meets the following inter-temporal budget constraint:

$$w\theta_i l_i - T_L(w\theta_i l_i) + (k_i + rk_i - T_K(rk_i)) \geq c_i + k'_i \tag{2}$$

where w and r are given wage and interest rates that are determined by competitive factor markets, respectively; $T_L(w\theta_i l_i)$ and $T_K(rk_i)$ are labor and capital income taxes, respectively, paid by individual i; θ_i denotes the earning ability of individual i; and, k_i

and k'_i are capital investment (i.e., savings) made by individual i in the previous period and in the current period, respectively. Furthermore, standard transversality conditions are met so that we can find a unique solution of the maximization problem of each individual over his lifetime. In this line, individuals of this economy cannot play a Ponzi game.

In particular, for any given i, earning ability of individual i is

$$\theta_i = \exp(\pi_i + \varepsilon_i) \tag{3}$$

where π_i and ε_i are given at birth and stays unchanged for lifetime. π_i is distributed according to $Exponetial\left(\frac{1}{\sigma_\pi}\right)$, and ε_i is distributed according to $Normal(-\frac{\sigma_\varepsilon}{2}, \sigma_\varepsilon)$. The support of earning-ability distribution is denoted by $\Theta \subset \mathbb{R}^+$. Consequently, $\exp(\pi_i)$ follows a Pareto distribution; and, $\exp(\varepsilon_i)$ follows a Lognormal distribution. The specification of earning-ability distribution of (3) closely resembles actual income distributions (e.g., Armour et al., 2016) by generating a thick upper tail in the earnings distribution. Thus, (3) was also adopted by other previous studies too (e.g., Heathcote et al., 2017). Notice that both $\sigma_\varepsilon > 0$ and $\sigma_\pi > 0$ represent the degree of inequality of individuals' earning ability.

With their earning ability known, in each period, each individual chooses their own labor supply and private goods consumption from maximizing their own utility for the remaining lifetime. For any given $\theta_i \in \Theta$, the maximization problem that individual i solves for each period is stated, in a recursive way, as

$$v(k_i;\theta_i) = v_i(k_i) = \max \ \left\{(1-\beta)\left[\log(c_i) - \frac{l_i^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \chi\log(G)\right] + \beta v_i(k'_i) \ \text{s.t} \ (2)\right\}. \tag{4}$$

In this economy, a representative firm produces output that can be used for private goods and public goods consumption. The production function is standard Cobb-Douglas function because it has been most widely adopted and proven to be well fitted to long-run data (León-Ledesma & Satchi, 2019).[3] Thus, with $1 > \alpha > 0$,

$$Y = F(K, L) = z_T(z_K K)^\alpha (z_L L)^{1-\alpha} \tag{5}$$

where Y is total output; K is aggregate capital; and, L is aggregate labor in efficiency unit. In each period, the representative firm maximizes its profit solving $\max z_T(z_K K)^\alpha (z_L L)^{1-\alpha} - rK - wL$. Some scholars (e.g., Romer, 1990) assumed that firms can control technology innovations by treating technology as one of freely choosable inputs for production. However, in reality, technology breakthroughs are not easily controllable to be a choice variable, unlike labor or capital input decisions. Therefore, we describe a change in production technology with an exogenous change in one of the parameters of production function. That is, we define technological progress as an increase in z_T, z_K, z_L or α of production function (5).

An increase in z_T is technological progress that improves total factor productivity; an increase in z_K is capital-augmenting technological progress; and, an increase in z_L is labor-augmenting technological progress. Obviously, these three forms of technological progress are different from each other. For example, as $1>\alpha>0$, an increase in z_K by 1.5 yields strictly different amounts of increases in marginal factor productivity and total output than an increase in z_T or z_L by 1.5 does. While they are definitively different from each other, increases in z_T, z_K and z_L have the following common feature: They preserve relative factor productivity with raising unit-free marginal productivities of both capital and labor together at the *same* rate. As relative capital productivity is $\frac{F_K(K,L)\frac{K}{Y}}{F_L(K,L)\frac{L}{Y}} = \frac{\alpha}{1-\alpha}$ and its reciprocal $(\frac{1-\alpha}{\alpha})$ is relative labor productivity, an increase in the value of any one of z_T, z_K and z_L does not favour the relative capital productivity or the relative labor productivity. Therefore, based on this common feature, the three forms of technological progress (total factor productivity improvement, capital-augmenting technological progress and labor-augmenting technological progress) are classified into one type of technological progress: 'unbiased technological progress'. Moreover, the relative factor productivity does not change whenever the relative capital productivity does not. Thus, technological progress is unbiased when it preserves relative capital productivity.

By contrast to an increase in z_T, z_K and z_L, an increase or a decrease in the technology parameter α does not preserve the relative factor productivity but favours the relative productivity of capital or labor. Thus, a change in the value of α represents another type of technological progress: 'biased technological progress'. An increase in α is capital-biased technological progress, as it raises the relative capital productivity (lowers the relative labor productivity). A decrease in α is labor-biased technological progress, as it raises the relative labor productivity (lowers the relative capital productivity). As shall be shown later, it turns out that the effect of labor-biased technological progress (a decrease in α) is simply opposite to the effect of capital-biased technological progress (an increase in α). Hence, it is sufficient for analysing the effect of biased technological progress to examine the effect of capital-biased technological progress based on which it is trivial to identify the effect of labor-biased technological progress.

Obviously, like the other three technology parameters of z_T, z_K and z_L, the value of α does not depend on firm's input decisions or factor markets. Nonetheless, after factor market clearing, unlike any of z_T, z_K and z_L, the value of the technology parameter α can also indicate the capital income share (i.e., the ratio of the total capital income to the total output) that is directly observable with aggregate income data. In this light, unlike most of previous studies that assumed no change α in the light of Kaldor's stylized fact from the data of 1960s, we allow that the value of α can change in the long run for reflecting the recently observed increases in the capital income share[4] of numerous countries (e.g., Karabarbounis & Neiman, 2014). Their empirical findings of the increases make capital-biased technological progress more relevant than labor-biased technological progress. Above all, the four technology parameters of z_T, z_K, z_L and α, whose changes represent unbiased or biased technological progress, are not time-variant and will be later allowed to increase for identifying the effect of each type of technological progress.

To finance public goods provision, the government of this economy levies taxes on labor and capital incomes, meeting the following fiscal budget constraint in each period.

$$\int_{\Theta}\{T_L(w\theta l_i(\theta)) + T_K(rk_i(\theta))\}dF_\theta = G = gY \tag{6}$$

where $g \in (0,1)$ is the portion of the total output used for public goods that the government decides to provide. The government chooses capital income tax rate of $\tau_K \in [0,1]$;

$$T_K(rk_i) = \tau_K rk_i. \tag{7}$$

At the same time, the government selects labor income tax rate schedule among the schedules that take a form of the following function:

$$T_L(y_i) = y_i - \rho_L(y_i)^{1-\mu_L} \tag{8}$$

where $y_i = w\theta_i l_i$ (pre-tax labor income). If $1-\mu_L \leq 0$ or $\rho_L \leq 0$, more labor supply entails less or no disposable income causing individuals not to work. Thus, in order to induce individuals to supply labor and earn a positive amount of taxable income,

$$1-\mu_L>0 \text{ and } \rho_L>0 \tag{9}$$

Notice that marginal labor income tax rate increases with pre-tax labor income if $\mu_L > 0$ and decreases with pre-tax labor income if $\mu_L<0$. If $\mu_L = 0$, then labor income tax is linear, as capital income tax[5] of (7). Thus, μ_L is the slope of labor income tax rate schedule, indicating the labor income tax progressivity. While individuals are endowed with equal amount of capital, they have unequal abilities to earn labor incomes, which is consistent with equal rate of tax on capital incomes (linear capital income tax) and unequal rates of tax on labor incomes (nonlinear labor income tax). The government can address the earning-ability inequality by properly choosing μ_L. Furthermore, corresponding to the average marginal capital income tax rate τ_K, the average marginal labor income tax rate τ_L is defined as

$$\tau_L = \int_{\Theta}\{T_L{}'(y_{i,t}(\theta))(\frac{y_{i,t}(\theta)}{Y_t})\}dF_\theta. \tag{10}$$

With any given μ_L that determines the slope of labor income tax rate schedule, ρ_L (or equivalently τ_L) determines the level of labor income tax rates.

In fact, the labor income tax function of (8) has been adopted by various studies such as Feldstein (1969), Benabou (2002), Corneo (2002), Heathcote et al. (2014, 2017), Guerreiro et al. (2022), and the like. Moreover, Heathcote et al. (2017) showed that the tax function (8) is remarkably well fitted to the US data. Likewise, the linear capital income tax of (7) has been most widely adopted in Ramsey taxation literature and macroeconomics literature. Being consistent with these literatures, the linearity of (7) is necessary for tractability of this analysis.

Without specifying the functions of utility, production and taxes, it is not feasible to identify how technological progress affects optimal income taxation, although specifying the functions does not attain the highest level of generality. Nonetheless, the analytically tractable functions of (1), (5), (7) and (8) still can yield policy-relevant implications because these functions are corroborated by various empirical data. The logarithm consumption utility of (1) is also shown to be consistent with empirical findings on labor supply (e.g., Kimball & Shapiro, 2008).

Above all, the government chooses capital and labor income tax rates from maximizing the social welfare SW.

$$SW = \int_{\Theta} u(\theta) dF_{\theta}. \tag{11}$$

According to (6), (7), (8) and (10), once the government chooses τ_K, τ_L and μ_L, the values of g and ρ_L are automatically determined. Hence, in the social-welfare maximization, the effective number of the government choice variables is actually three, instead of five. Notice that the level of the social welfare is determined by private consumption and labor supply which are actually chosen by individuals, not by the government. For the government to implement a welfare-maximizing allocation of consumption and labor, the firm and individuals need to voluntarily choose the welfare-maximizing allocation with taking the government's policies given. Thus, the welfare-maximizing allocation should be supported as a competitive general equilibrium of this economy.

With the government's tax policies given, a competitive general equilibrium of this economy is defined as a set of allocation decision rules $\{c_i(\theta_i), l_i(\theta_i), k_i(\theta_i)\}_{\theta_i \in \Theta}$, prices of labor and capital (w and r) and g, which satisfies the following three conditions for each period:

i. With the government's policies and prices given, all individuals maximize their own lifetime utility meeting their own budget constraints.
ii. The representative firm maximizes its profit with factor markets being cleared as

$$K = \int_{\Theta} k_i(\theta) dF_{\theta}, \tag{12}$$

$$L = \int_{\Theta} \theta l_i(\theta) dF_{\theta}. \tag{13}$$

iii. The government's budget constraint of (6) is satisfied.

Once the budget constraints of all individuals and the government are met with factor markets being cleared, according to Walras' law, the aggregate resource constraint of this economy is automatically met, clearing goods market as well. As this economy is in its steady state, for any given $\theta_i \in \Theta$, the amount of k_i stays the same over time. Thus, for any given $\theta_i \in \Theta$, $k_i = k'_i > 0$ at a stationary competitive general equilibrium of this economy.

Under the conventional approach of Ramsey optimal taxation, for a welfare-maximizing allocation of labor supply and consumption to be chosen voluntarily by individuals and the firm as a competitive general equilibrium, the government needs to maximize the social welfare with externally meeting the conditions (i), (ii) and (iii). Thus, optimal income tax rates, derived from this conventional primal approach, inevitably contain Lagrange multiplier(s) on the conditions for implementing the welfare-maximizing allocation via market mechanism. As a result, without imposing *arbitrary* assumptions on how the four technology parameters (z_T, z_K, z_L and α) affect the Lagrange multiplier(s), the conventional approach cannot identify the effect of technological progress on optimal income taxation. For properly identifying the effect of technological progress without such arbitrary assumptions, we need to remove the Lagrange multiplier(s) from derivation of optimal income tax rates. To this purpose, we modify the conventional approach by translating competitive-equilibrium allocation of individuals' labor supply and consumption in terms of the government's policy variables. By plugging the translated competitive-equilibrium allocation of labor supply and consumption into the social welfare function of (11), the implementability conditions (i), (ii) and (iii) are imbedded into the restated social welfare function. Then, the government maximizes the restated social welfare function with no external constraint (and thus with no Lagrange multiplier), while it still can ensure that the consequent welfare-maximizing allocation is supported as a competitive general equilibrium.

3. Competitive market equilibrium allocation in terms of policy variable

To find a competitive-equilibrium allocation that meets the implementability conditions (i), (ii) and (iii), we begin with analysing the optimal decision rules of private consumption, labor supply and savings of each individual. To this purpose, for any given $\theta_i \in \Theta$, let us denote the ratio of post-tax labor income to the sum of post-tax capital income and change in capital investment (savings) of individual i by

$$s_i(\theta_i) = \frac{\rho_L (w\theta_i l_i)^{1-\mu_L}}{rk_i(1-\tau_K) + k_i - k'_i} \tag{14}$$

The method of using this auxiliary variable is also adopted by other studies like Benabou (2002) as it streamlines the analysis. As the budget constraint of (2) binds at a competitive general equilibrium, $c_i(\theta_i) = \left(1 + \frac{1}{s_i}\right)\rho_L (w\theta_i l_i)^{1-\mu_L}$. By plugging this into individual i's value function of (4) and then taking a derivative with respect to l_i and s_i, we can find his optimal labor supply and his optimal ratio of post-tax labor income to the sum of post-tax capital income and change in capital investment, which together completely define optimal private consumption of individual i. Thus, first, for any given $\theta_i \in \Theta$, the optimal labor supply of individual i is defined by

$$(1-\beta)\frac{(1-\mu_L)}{l_i} - (1-\beta) l_i^{\frac{1}{\eta}} + \beta \frac{dv_i(k'_i)}{dk'_i}\frac{dk'_i}{dl_i} = 0. \tag{15}$$

Second, the optimal ratio of post-tax labor income to the sum of post-tax capital income and change in capital investment of individual i is defined by

$$(1-\beta)\frac{1}{\left(1+\frac{1}{s_i}\right)}(-1)\left(\frac{1}{s_i}\right)^2+\beta\frac{dv_i(k'_i)}{dk'_i}\frac{dk'_i}{ds_i}=0. \tag{16}$$

By the same token, with (14) and the budget constraint (2) binding at a competitive general equilibrium, $c_i(\theta_i)=\left(1+\frac{1}{s_i}\right)\rho_L(w\theta_i l_i)^{1-\mu_L}=(1+s_i)(rk_i(1-\tau_K)+k_i-k'_i)$ for any given $\theta_i\in\Theta$. By plugging this into the value function of (4) and taking a derivative with respect to k_i and k'_i, we get the following Euler equation that defines the optimal inter-temporal allocation of individual i.

$$\frac{1}{rk_i(1-\tau_K)+k_i-k'_i}=\beta[1+r(1-\tau_K)]\frac{1}{rk'_i(1-\tau_K)+k'_i-k''_i} \tag{17}$$

where k''_i is capital investment made by individual i in the next period; and, at the steady state, $k_i=k'_i=k''_i>0$. The above three optimality conditions allow us to identify each individual's optimal allocation at a competitive general equilibrium of this economy in its steady state.

Lemma 1. *At a stationary competitive general equilibrium, with the government's policies and prices given, for any given $\theta_i\in\Theta$, the optimal labor supply, private consumption, and the optimal ratio of post-tax labor income to the sum of post-tax capital income and change in capital investment of individual i are as follows:*

$$l_i(\theta_i)=\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}, \tag{18}$$

$$c_i(\theta_i)=(\theta_i)^{1-\mu_L}\frac{\rho_L(w)^{1-\mu_L}\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}(1-\mu_L)}}{1-\beta}, \tag{19}$$

$$s_i(\theta_i)=\frac{1-\beta}{\beta}. \tag{20}$$

For proof, see Appendix A1.

Notice that Lemma 1 characterizes the condition (i) for competitive-equilibrium allocation. As shown by (18), steeper slope of labor income tax rate schedule reduces individuals' labor supply by decreasing marginal post-tax return on their labor supply.[6] With the logarithmic utility of consumption, income and substitution effects on labor supply of differences in the effective wage rate ($w\theta_i$) cancel out each other. As a result, as shown in (18) and (20), the optimal labor supply and the optimal ratio of post-tax labor income to the sum of post-tax capital income and change in capital investment of individuals do not depend on earning ability. Moreover, (18) implies that as μ_L approaches one, pre-tax labor incomes of high- and low- ability individuals are equalized. As such, an increase in the slope of labor income tax rate schedule reduces the extent to which the earning-ability inequality develops into the labor income inequality. On the other hand, as appears in (19), individuals of higher earning ability enjoy more private goods consumption. In this line, (14), (18), (19) and (20) imply that individuals of higher ability have more capital to consume more private goods, although all individuals are born with equal amount of capital.

Aggregating individuals' optimal allocation (Lemma 1) yields total supplies of labor and capital, from which we obtain marginal products of labor and capital. Then, to satisfy the implementability condition (ii), market-clearing wage and interest rates are equated with the marginal products of labor and capital, respectively. To meet the implementability condition (iii), the government's budget constraint of (6) is restated in terms of the obtained optimal allocation of individuals, market-clearing wage and interest rates. From this, the solution of ρ_L that meets the implementability condition (iii) is obtained. Furthermore, because of (20) and $k_i = k'_i$ for any given $\theta_i \in \Theta$ at the steady state of this economy, ρ_L is uniquely determined once μ_L and τ_K are decided with the government's budget constraint binding. This enables us to define the steady-state aggregate competitive-equilibrium allocation that meets the implementability conditions (i) (ii) and (iii) as below.

Lemma 2. *At a stationary competitive general equilibrium, the total labor, capital and output are as follows:*

$$L = \frac{1}{(1-\sigma_\pi)}\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}, \tag{21}$$

$$K = z_T^{\frac{1}{(1-\alpha)}} z_K^{\frac{\alpha}{(1-\alpha)}} z_L \left(\frac{\beta}{1-\beta}\right)^{\frac{1}{(1-\alpha)}} \alpha^{\frac{1}{(1-\alpha)}} (1-\tau_K)^{\frac{1}{(1-\alpha)}} \frac{1}{(1-\sigma_\pi)}\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}, \tag{22}$$

$$Y = z_T^{\frac{1}{(1-\alpha)}} z_K^{\frac{\alpha}{(1-\alpha)}} z_L \left(\frac{\beta}{1-\beta}\right)^{\frac{\alpha}{(1-\alpha)}} \alpha^{\frac{\alpha}{(1-\alpha)}} (1-\tau_K)^{\frac{\alpha}{(1-\alpha)}} \frac{1}{(1-\sigma_\pi)}\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}. \tag{23}$$

At a stationary competitive general equilibrium, the portion of the total output used for public goods provision is

$$g = 1-\frac{\alpha}{\beta}(1-\tau_K). \tag{24}$$

For proof, see Appendix A2.

Because individuals supply capital input from their post-tax labor incomes, the slope of labor income tax rate schedule affects both inputs of aggregate capital and aggregate labor supply, as shown by (21) and (22). In fact, with the fiscal balance of (6) and (20) of Lemma 1, the competitive-equilibrium provision of public goods also can be equivalently stated in terms of μ_L or ρ_L, which is much more complicated and less tractable than (24) but shows that an increase in μ_L raises the level of g as an increase in τ_K does. Thus, (23) and (24) suggest that an increase in the capital income tax rate or the slope of labor income tax rate schedule reduces aggregate output by lowering individuals' saving or labor-supply incentives, while the increase leads to more public goods provision.

Incorporating the implementability conditions (i), (ii) and (iii), which are delineated by Lemmas 1 and 2 in terms of policy variables, into (1) and (11), the social welfare function at a stationary competitive general equilibrium is stated in terms of the government's decision variables as below.

$$\begin{aligned}SW = (1+\chi)\{&\frac{1}{(1-\alpha)}\log(z_T) + \frac{\alpha}{(1-\alpha)}\log(z_K) + \log(z_L)\} + (1-\mu_L)\left(\sigma_\pi - \frac{\sigma_\varepsilon}{2}\right)\\ &+ (1+\chi)\frac{\eta}{\eta+1}\log\{(1-\mu_L)(1-\beta)\} - \frac{\eta}{\eta+1}\{(1-\mu_L)(1-\beta)\}\\ &+ \left(\frac{\alpha\chi+1}{1-\alpha}\right)\log(1-\tau_K) + \log\left(\frac{1-\sigma_\pi(1-\mu_L)}{1-\sigma_\pi}\right) + \frac{\sigma_\varepsilon\mu_L(1-\mu_L)}{2}\\ &+ \chi\log\left(1-\frac{\alpha}{\beta}(1-\tau_K)\right) + \Gamma_{SW}\end{aligned} \tag{25}$$

where $\Gamma_{SW} = \frac{\alpha(1+\chi)}{(1-\alpha)}\log\left(\frac{\beta}{1-\beta}\right) - \log(\beta) + \left(\frac{\alpha\chi+1}{1-\alpha}\right)\log(\alpha) - \chi\log(1-\sigma_\pi)$ is the non-kernel part that does not contain the government's decision variables. The first three terms of (25) illustrate that unbiased technological progress (an increase in z_T, z_K or z_L) always raises the social welfare, which is independent of the government policies. In contrast, whether and how much biased technological progress (an increase or a decrease in α) raises the social welfare is not independent of the government policies.

Notably, the fourth, fifth and seventh terms of (25) show that an increase in the capital income tax rate or in the slope of labor income tax rate schedule reduces individuals' incentives of supplying capital or labor, to reduce the total output. The sixth term of (25) represents the ensuing reduction in labor-supply disutility. The efficiency loss on the total output from capital and labor income taxation is traded off for the redistributive welfare gains that are represented by the last three terms of the kernel part of (25). The eighth and ninth terms of (25) capture the welfare gain that an increase in the slope of labor income tax rate schedule can bring by controlling the extent to which earning-ability inequality evolves into consumption inequality. The tenth term of (25) shows that an increase in the capital income tax rate also can improve the social welfare by decreasing consumption inequality (i.e., inequality in consumption of private goods and public goods in total) with financing public goods provision. Notably, it is the concavity of utility function that makes a reduction in consumption inequality be welfare improving; hence, as shown in the tenth term of (25), the redistributive welfare gain from capital income taxation does not depend on σ_ε or σ_π.

Because all the implementability conditions (i), (ii) and (iii) are embedded into the social welfare function of (25), when the government obtains capital and labor income tax rates that maximize (25) with no constraint, the obtained income tax rates induce individuals and the firm to voluntarily implement the welfare-maximizing allocation through competitive markets. The optimal capital and labor income tax rates derived from this unconstrained maximization of the government do not contain a Lagrange multiplier, which paves the way for identifying the effects of technological progress on optimal capital and labor income taxation.

4. Optimal capital and labor income taxation

This section characterizes the optimal capital and labor income taxation. In particular, from maximizing the social welfare function of (25), we obtain the following formulae for the optimal capital and labor income tax rates.

Proposition 1. *The optimal capital and labor income taxation (τ_K^*, τ_L^*, μ_L^* and ρ_L^*) is defined as follows.*

$$\tau_K^* = \begin{cases} 1 - \dfrac{\beta}{\alpha}\left(\dfrac{\alpha\chi + 1}{\chi + 1}\right) & \text{if } \alpha(\chi+1) > \beta(\alpha\chi+1) \\ 0 & \text{if } \alpha(\chi+1) \le \beta(\alpha\chi+1). \end{cases} \tag{26}$$

$$\tau_L^* = (1-\alpha) - (1-\mu_L^*)\left(\frac{\alpha\chi+1}{\chi+1}\right). \tag{27}$$

$$\frac{\sigma_\pi^2(1-\mu_L^*)}{1-\sigma_\pi(1-\mu_L^*)} + \sigma_\varepsilon(1-\mu_L^*) = \frac{\eta}{\eta+1}\frac{(\chi+1)}{(1-\mu_L^*)} - \frac{\eta}{\eta+1}(1-\beta). \tag{28}$$

$$\rho_L^* = z_T^{\frac{\mu_L^*}{(1-\alpha)}} (z_K)^{\frac{\alpha\mu_L^*}{(1-\alpha)}} (z_L)^{\mu_L^*} \left(\frac{\beta}{1-\beta}\right)^{\frac{\alpha\mu_L^* - 1 + \alpha}{(1-\alpha)}} \alpha^{\frac{1-\alpha\left(1-\mu_L^*\right)}{1-\alpha}} \left(\frac{1}{1-\alpha}\right)^{\left(1-\mu_L^*\right)} \left(1-\tau_K^*\right)^{\frac{1-\alpha\left(1-\mu_L^*\right)}{1-\alpha}}$$
$$\times \left\{\left(1-\mu_L^*\right)(1-\beta)\right\}^{\frac{\eta}{\eta+1}\mu_L^*} \exp\left[\frac{\sigma_\varepsilon(1-\mu_L)(\mu_L)}{2}\right] \frac{\{1-\sigma_\pi(1-\mu_L)\}}{(1-\sigma_\pi)}. \tag{29}$$

For proof, see Appendix A3.

Fundamentally, Proposition 1 shows that the government sets capital and labor income tax rates for improving social welfare with addressing pre-tax income inequality, even if income taxation distorts individuals' incentive to work and save (invest). Taxing capital income attains welfare gains not only by directly reducing post-tax capital income inequality, but also by indirectly reducing consumption inequality with financing public goods that are provided equally to all individuals. At a competitive equilibrium of this economy, α indicates the share of aggregate pre-tax capital income in aggregate pre-tax income; hence, α determines the contribution of pre-tax capital income inequality to pre-tax income inequality. At the same time, capital income taxation lowers post-tax return on individuals' capital investment, which dampens their incentive to save and entail a decrease in the total output. The optimal capital income tax rate is set to take a balance between the welfare gain and the efficiency loss on total output. As a result, it is straightforward from (26) that the level of optimal capital income tax rate increases when more welfare weights are put on the public goods consumption (an increase in χ) or when pre-tax capital income inequality plays a larger role in pre-tax income inequality (an increase in α). Furthermore, because the concavity of utility and social welfare function by itself causes reducing inequality to yield welfare gains and because all individuals are endowed with equal amount of capital in the first place, the optimal capital income tax rate does not depend on σ_ε or σ_π.

Because no provision of public goods yields the lowest level of social welfare, zero tax rate on both capital and labor incomes is not optimal for the benevolent government, although income taxation is distortionary to incur efficiency loss. In addition, the difference in the ability to earn labor incomes creates the pre-tax income inequality among individuals. Hence, to minimize the consequent efficiency loss for any given amount of welfare gain, first, the government collects labor income tax even when it does not collect capital income tax. Only when redistributive welfare gain from taxing capital income is large enough, it collects capital income tax.

Markedly, since (26) of Proposition 1 shows that optimal capital income tax rate can be strictly positive, this article makes a contribution for the optimal income taxation literature that rationalizes taxing capital income against the zero-capital-tax result of Chamley (1986) and Judd (1985). In fact, the existing studies in this literature showed that optimal linear capital income tax rate can be strictly positive by introducing borrowing constraints (Aiyagari, 1995), limited enforcement of private insurance (Park, 2014), uninsurable idiosyncratic risk on earnings (Chamley, 2001; Conesa, Kitao & Krueger, 2009), and the like. This article differs from these studies with regard to the key rationale. Because capital income taxation attains welfare gain by reducing consumption inequality with more public goods provision[7] as well as from reducing capital income inequality, optimal capital income tax rate can be strictly positive. This rationale for taxing capital income – welfare gain from indirect redistribution through public goods provision – is newly presented by this article.[8]

At the same time, taxing labor income also attains welfare gains not only by directly reducing post-tax labor income inequality, but also by indirectly reducing consumption inequality with financing public goods that are provided equally to all individuals. This redistributive welfare gain is obtained at the cost of the efficiency loss of dampening labor-supply incentive, which shapes the optimal labor income taxation. Moreover, $1-\alpha$ determines the contribution of pre-tax labor income inequality to pre-tax income inequality at a competitive equilibrium of this economy. Hence, (27) of Proposition 1 shows that optimal average marginal labor income tax rate depends on preference for public goods (χ) and $1-\alpha$, as optimal average marginal capital income tax rate of (26) depends on χ and α.

The average marginal labor income tax rate represents the *entire* schedule of labor income tax rates, which depends on μ_L and ρ_L. As noted above, for any given value of μ_L, with the government budget being balanced, deciding ρ_L is equivalent to deciding τ_L. As shown in Section III, labor-income gap between individuals of different abilities is controlled by the slope of labor income tax rate schedule μ_L. The left-hand side of (28) of Proposition 1 refers to the social marginal benefit that an increase in the slope of labor income tax rate schedule brings by controlling the extent to which earning-ability inequality develops into labor income inequality. On the other hand, the right-hand side of (28) shows the social marginal cost from an increase in the slope of labor income tax rate schedule. By taking a balance between the social marginal benefit and cost, optimal slope of labor income tax rate schedule is determined.

Essentially, Proposition 1 shows that optimal capital and labor income taxation tackles pre-tax income inequality, for which efficiency loss of distorting incentives for capital investment and labor supply is traded off. Notably, Proposition 1 also demonstrates that optimal capital and labor income tax rates depend on the four technology parameters (z_T, z_K, z_L and α). Above all, the closed-form formulae of optimal capital and labor income tax rates of Proposition 1 enable us to analyse how technological progress affects optimal capital and labor income taxation.

5. Effect of technological progress on optimal income taxation

So far, we obtain optimal capital and labor income tax rates with production technology *fixed*, this section now introduces an increase in each of the four technology parameters (z_T, z_K, z_L and α) to analyse how technological progress affects optimal tax rates on capital and labor incomes. To begin, one of the technology parameters z_T, z_K and z_L is increased respectively to entail unbiased technological progress, without changing all the other parameters.

Proposition 2. *Unbiased technological progress does not affect optimal capital income tax rate or optimal average marginal labor income tax rate. Likewise, unbiased technological progress does not affect optimal slope of labor income tax rate schedule, either.*

For proof, see Appendix A4.

Unbiased technological progress (an increase in z_T, z_K or z_L) does not affect the extent to which earning-ability inequality develops into labor income inequality. Hence, unbiased technological progress (capital-augmenting technological progress, labor-augmenting technological progress and total factor productivity improvement) does not affect optimal slope of labor income tax rate schedule, as implied from (28) of Proposition 1.

On the other hand, unbiased technological progress (an increase in z_T, z_K or z_L) increases competitive-equilibrium prices of labor and capital at the *same* rate. Thus, unbiased technological progress does not alter relative incentives for individuals to supply labor and capital. When technological progress is unbiased, it entails proportional scale-up of pre-tax capital and labor incomes. Although this scale-up enlarges absolute difference in high-ability and low-ability individuals' pre-tax incomes, it does not change relative difference in their pre-tax capital and labor incomes. Unbiased technological progress does not change the relative contribution of pre-tax labor income inequality to pre-tax income inequality or that of pre-tax capital income inequality to pre-tax income inequality at a competitive equilibrium of this economy. Hence, redistributive welfare gain from raising the level of labor income tax rate or capital income tax rate remains unaffected by unbiased technological progress. At the same time, because unbiased technological progress preserves relative factor productivity, it does not change relative contribution of capital or labor for production. Therefore, unbiased technological regress entails no change in optimal capital income tax rate, optimal average marginal labor income tax rate or optimal slope of labor income tax rate schedule.

As demonstrated by the social welfare function of (25), all the three technology parameters of z_T, z_K and z_L are additively separable from the government's tax policy variables; hence, optimal capital and labor income tax rates that maximize social welfare are independent of an increase in z_T, z_K or z_L. Thus, the maximum level of social welfare after an increase in z_T, z_K or z_L is attained by the *same* values of τ_K^*, τ_L^*, μ_L^* and ρ_L^* before the increase, while it is higher than before the increase. Thus, unbiased technological progress (an increase in z_T, z_K or z_L) does not affect the optimal capital and labor income taxation, while it improves social welfare. In light of Proposition 2, the result of Zhu (1992), Chari et al. (1994) and Werning (2007) that optimal capital and labor income tax rates do not change in responding to random technology shocks can be extended for long-run technology changes *only if* technology changes preserve the relative factor productivity.

Having investigated the effect of unbiased technological progress on optimal income taxation, we now move on analysing the effect of biased technological progress (an increase or a decrease in α). As the effect of capital-biased technological progress is the opposite to the effect of labor-biased technological progress, for an efficient analysis, we elaborate on the former only. While an increase in z_T, z_K or z_L always increases total output, an increase in α does not always do so. If a technological change results in decreasing total output, it not a progress. To rule this out, we focus on the case where an increase in α increases total output by meeting the following condition:

$$\frac{dF(K,L)}{d\alpha} = z_T(z_K K)^{\alpha}(z_L L)^{(1-\alpha)} \ln\left(\frac{z_K K}{z_L L}\right) > 0 \tag{30}$$

at a stationary competitive equilibrium before an increase in α. In fact, the condition of (30) is not restrictive at all, because the theoretical findings of this article hold regardless of whether (30) is met or not. In addition, while capital-biased technological progress (an increase in α) always raises interest rate, it does not always raise wage rate. Nonetheless, regardless of whether an increase in α lowers or raises wage rate, so long as an increase in α increases total output, it is capital-biased technological progress. Moreover, the effect of capital-biased technological progress on optimal income taxation is independent of whether capital-biased technological progress raises wage rate or not.

By introducing an increase in the technology parameter α without changing all the other parameters, we find how capital-biased technological progress affects optimal income taxation as follows.

Proposition 3. *The effect of capital-biased technological progress on optimal capital income tax rate is positive, while the effect of capital-biased technological progress on optimal average marginal labor income tax rate is strictly negative. However, capital-biased technological progress does not affect optimal slope of labor income tax rate schedule.*

For proof, see Appendix A5.

Biased technological progress (an increase or a decrease in α) does not alter how earning-ability inequality evolves into labor income inequality. Therefore, neither capital-biased technological progress nor labor-biased technological progress affects optimal slope of labor income tax rate schedule, which can be shown from (28) of Proposition 1.

In contrast to unbiased technological progress, biased technological progress does not increase competitive-equilibrium prices of labor and capital at the same rate. Thus, when technological progress is biased, it entails disproportional increases of pre-tax capital and labor incomes. In particular, capital-biased technological progress (an increase in α) disproportionately increases competitive-equilibrium interest rate, which in turn increases relative incentives for individuals to supply capital. By increasing the relative productivity of capital, capital-biased technological progress (an increase in α) raises relative pre-tax return to capital investments. As a result, capital-biased technological progress increases the relative contribution of pre-tax capital income inequality to pre-tax income inequality while decreasing the relative contribution of pre-tax labor income inequality to pre-tax income inequality at a competitive equilibrium of this economy. Capital-biased technological progress enlarges absolute difference as well as relative difference in high-ability and low-ability individuals' pre-tax incomes. Thus, redistributive welfare gain which raising capital income tax rate can bring is positively affected by capital-biased technological progress.

By increasing the relative contribution of pre-tax capital income inequality to pre-tax income inequality, capital-biased technological progress causes raising the level of capital income tax rate to bring greater redistributive welfare gains, whereas it makes labor income taxation yield smaller redistributive welfare gains. Because capital-biased technological progress raises pre-tax rate of return on capital, even when capital income tax rate is increased after capital-biased technological progress, post-tax rate of return on capital can be higher to make individuals invest more than before capital-biased technological progress. At the same time, because capital-biased technological progress raises relative capital productivity and lowers relative labor productivity, capital-biased technological progress makes relatively *less* capital necessary to produce one unit of output. Therefore, capital-biased technological progress raises optimal capital income tax rate but lowers optimal average marginal labor income tax rate. By the same logic, the effect of labor-biased technological progress on optimal capital income tax rate is negative, while its effect on optimal average marginal labor income tax rate is positive. As noted above, like unbiased technological progress, both capital-biased and labor-biased technological progresses do not affect optimal slope of labor income tax rate schedule.

In contrast to z_T, z_K and z_L, the technology parameter of α is not additively separable from the government's tax policy variables in the social welfare function of (25). Thus, the maximum level of social welfare after capital-biased technological progress is no longer attainable by the same values of τ_K^*, τ_L^*, μ_L^* and ρ_L^* before the progress. Basically, unlike unbiased technological progress, biased technological progress alters redistributive welfare gains from capital and labor income taxation while changing the relative contribution of capital and labor inputs for production as well. At the same time, like unbiased technological progress, biased technological progress does not alter the evolution from earning-ability inequality to labor income inequality. Thus, facing capital-biased or labor-biased technological progress, the benevolent government adjusts levels of capital income tax rate and average marginal labor income tax rate to re-maximize social welfare for responding to the progress, without altering the slope of labor income tax rate schedule.

More importantly, this article discovers that relative capital productivity ($\frac{\alpha}{1-\alpha}$) plays an important role in redistributive welfare gains steering optimal capital and labor income tax rates. Previous studies on optimal income taxation have shown that only earning-ability inequality determines redistributive welfare gains to derive optimal income tax rates (e.g., Golosov, Troshkin, & Tsyvinski, 2016; Werning, 2007). These previous studies assumed that pre-tax income inequality is completely determined by earning-ability inequality. In contrast, this article relaxes this assumption by allowing technological progress to affect pre-tax income inequality. This article finds that while pre-tax income inequality from earning-ability difference always affects optimal income taxation, pre-tax income inequality from technological progress does so *only* if relative capital productivity increases by technological progress. As such, relative capital productivity is crucial for whether technology-driven inequality of pre-tax incomes alters optimal capital and labor income taxation.

6. Conclusion and policy implication

In sum, this article analyses effects of technological progress on optimal capital and labor income taxation. To this end, optimal capital and labor income taxation is obtained in a steady-state economy where individuals are heterogeneous only in terms of earning ability. Based on the obtained optimal income taxation, the effects of technological progress on optimal capital and labor income taxation are identified. First, this article shows that technological progress raises optimal capital income tax rate but lowers optimal average marginal labor income tax rate if it is capital-biased by increasing relative capital productivity. Technological progress lowers optimal capital income tax rate but raises optimal average marginal labor income tax rate if it is labor-biased by decreasing relative capital productivity. Nonetheless, capital-biased or labor-biased technological progress does not affect optimal slope of labor income tax rate schedule. Second, this article also shows that technological progress does not affect optimal capital and labor income tax rates or optimal slope of labor income tax rate schedule if it is unbiased by preserving relative capital productivity. These findings of this article highlight the importance of relative capital productivity that determines whether technological progress affects optimal capital and labor income tax rates or not.

In addition, we can apply the findings of this article for tax reform debate from Fourth Industrial Revolutions. This article suggests that only when relative capital productivity increases as appears in Figure 2, a government should raise capital income tax rate and lower average marginal labor income tax without changing the slope of labor income tax rate schedule.

Although the above functions of utility, tax schedule, earning-ability distributions and production are chosen to be consistent with empirical findings so that we can draw policy implications, the specificity of those chosen functions is not of full-degree generality and can be limitation of this study. In this light, to overcome this limitation, future studies can give up tractability for identifying the effect of technological progress on optimal income taxation or use different forms of functions with simulation exercise.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes

1. Notably, increases refer to upward trend. This study focuses on the overall trend, instead of yearly variation, by elaborating on a steady-state economy. Because incomes and GDP are fluctuating around the trend over the business cycles, the data of Figure 1, 2, 3 show yearly ups and downs. However, this volatility is out of the scope of this article.
2. Although some studies (e.g. Guerreiro et al., 2022; Mattauch et al., 2018) assumed the status inequality that individuals are born as workers vs. non-working capitalist (or born as routine workers vs. non-routine workers), we do not impose such dichotomous assumption of the ascribed-status inequality.
3. The price elasticity of substitution between labor and capital of Cobb-Douglas production function is one. Reconciling the dispute over whether the price elasticity of substitution between labor and capital is one or not (e.g., Alvarez-Cuadrado et al. 2018; DeCanio, 2016; Karabarbounis & Neiman, 2014), León-Ledesma and Satchi (2019) showed that the price elasticity of substitution becomes one in the long run although the price elasticity is not always one in the short run and found that Cobb-Douglas production function is well fitted to the long-run data of US.
4. To offer a theoretical explanation on the observed increases in the capital income share, Karabarbounis and Neiman (2014) modeled technological innovation as a decrease in the price of investment goods (machinery, factory and equipment), similar to Guerreiro et al. (2022). Different from these two studies, this article took a general-equilibrium approach by describing technological innovation as a change in the parameter of production function. On the other hand, similar to this article, Acemoglu (2002) also utilized a change in the parameter of production function for describing technological innovation. In particular, Acemoglu (2002) modeled technological progress as a change in the elasticity of substitution between capital and labor. By the definition, elasticity of substitution between capital and labor depends on how responsive firms' own decision of the two inputs is to a change in the input prices. However, this article seeks to measure how technology changes from advancements of science affect productivity of the two inputs, instead of how responsive firms or factor markets are to the technology change. Furthermore, in a general-equilibrium model, an increase in the elasticity of substitution between capital and labor does not necessarily entail an increase in the capital income share.
5. Being differentiated from the most studies of Ramsey taxation literature, which assumed labor income tax to be linear, this article relaxes this assumption so that labor income tax can be nonlinear. Allowing capital income tax to be nonlinear too costs tractability of the model of this study as many of other studies that use a Ramsey model.
6. As β increases, individuals put more weights on the future so that they save more by decreasing the value of (20). In turn, more savings lead to more capital wealth of an individual, which exerts wealth effect on the individual's optimal labor supply, as appears in (18).
7. In fact, the existing studies (e.g., Aiyagari, 1995; Chamley, 2001; Conesa, Kitao & Krueger, 2009; Mattauch et al., 2018; Park, 2014) did not allow public goods provision to affect utility of individuals by treating government expenditures (financed by tax revenue) consumed by nobody in their model economies. This approach could be more tractable than otherwise, while it is less realistic. While some studies assumed government expenditures to be consumed by nobody, other studies like Heathcote et al. (2014, 2017) assumed otherwise. This article adopts the latter approach, different from the existing studies of the literature that showed the desirability of taxing capital income.
8. Mattauch et al. (2018) also showed that optimal capital income tax rate can be strictly positive because capital income taxation reduces wealth inequality through public capital investment. However, their model is quite different from the model of this article. Mattauch et al. (2018) assumed that the government can levy only capital income tax (and thus no labor income tax); individuals are born as either infinitely-lived capitalists or two-period-lived workers; labor supply of workers is exogenously given.

References

Acemoglu, D. (2002). Technical change, inequality, and the labor market. *Journal of Economic Literature*, *40*(1), 7–72. https://doi.org/10.1257/jel.40.1.7

Aghion, P., Akcigit, U., Bergeaud, A., Blundell, R., & Hemous, D. (2019). Innovation and top income inequality. *The Review of Economic Studies*, *86*(1), 1–45. https://doi.org/10.1093/restud/rdy027

Aiyagari, R. (1995). Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting. *Journal of Political Economy*, *103*(6), 1158–1175. https://doi.org/10.1086/601445

Alvarez-Cuadrado, F., N. Van Long, N, & Poschke, M. (2018). Capital-labor substitution, structural change and the labor income share. *Journal of Economic Dynamics and Control*, *87*, 206–231.

Armour, P., Burkhauser, R., & Larrimore, J. (2016). Using the pareto distribution to improve estimates of topcoded earnings. *Economic Inquiry*, *54*(2), 1263–1273. https://doi.org/10.1111/ecin.12299

Benabou, R. (2002). Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, *70*(2), 481–517. https://doi.org/10.1111/1468-0262.00293

Caselli, F. (1999). Technological revolutions. *American Economic Review*, *89*(1), 78–102. https://doi.org/10.1257/aer.89.1.78

Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica*, *54*(3), 607–622. https://doi.org/10.2307/1911310

Chamley, C. (2001). Capital income taxation, wealth distribution and borrowing constraints. *Journal of Public Economics*, *79*(1), 55–69. https://doi.org/10.1016/S0047-2727(00)00095-5

Chari, V., Christiano, L., & Kehoe, P. (1994). Optimal fiscal policy in a business cycle model. *Journal of Political Economy*, *102*(4), 617–652. https://doi.org/10.1086/261949

Conesa, J. C., Kitao, S., & Krueger, D. (2009). Taxing capital? Not a bad idea after all!. *American Economic Review*, *99*(1), 25–48. https://doi.org/10.1257/aer.99.1.25

Corneo, G. (2002). The efficient side of progressive income taxation. *European Economic Review*, *46*(7), 1359–1368. https://doi.org/10.1016/S0014-2921(01)00162-3

DeCanio, S. (2016). Robots and humans–complements or substitutes? *Journal of Macroeconomics*, *49*, 280–291. https://doi.org/10.1016/j.jmacro.2016.08.003

Feldstein, M. (1969). The effects of taxation on risk taking. *Journal of Political Economy*, *77*(5), 755–764. https://doi.org/10.1086/259560

Foerster, M., & Gyoergy Tóth, I. (2015). Cross-country evidence of the multiple causes of inequality changes in the OECD area. In *Handbook of income distribution* (Vol. 2, pp. 1729–1843). Elsevier.

Galor, O., & Moav, O. (2000). Ability-biased technological transition, wage inequality, and economic growth. *The Quarterly Journal of Economics*, *115*(2), 469–497. https://doi.org/10.1162/003355300554827

Golosov, M., Troshkin, M., & Tsyvinski, A. (2016). Redistribution and social insurance. *American Economic Review*, *106*(2), 359–386. https://doi.org/10.1257/aer.20111550

Guerreiro, J., Rebelo, S., & Teles, P. (2022). Should robots be taxed? *The Review of Economic Studies*, *89*(1), 279–311. https://doi.org/10.1093/restud/rdab019

Heathcote, J., Storesletten, K., & Violante, G. (2017). Optimal tax progressivity: An analytical framework. *The Quarterly Journal of Economics*, *132*(4), 1693–1754. https://doi.org/10.1093/qje/qjx018

Heathcote, J., Storesletten, K., & Violante, G. L. (2014). Consumption and labor supply with partial insurance: An analytical framework. *American Economic Review*, *104*(7), 2075–2126. https://doi.org/10.1257/aer.104.7.2075

Judd, K. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics*, *28*(1), 59–83. https://doi.org/10.1016/0047-2727(85)90020-9

Karabarbounis, L., & Neiman, B. (2014). The global decline of the labor share. *The Quarterly Journal of Economics*, *129*(1), 61–103. https://doi.org/10.1093/qje/qjt032

Kimball, M., & Shapiro, M. (2008). Labor supply: Are the income and substitution effects both large or both small?, *NBER Working Paper* No. 14208.
León-Ledesma, M and., & Satchi, M. (2019). Appropriate technology and balanced growth. *The Review of Economic Studies*, *86*(2), 807–835. https://doi.org/10.1093/restud/rdy002
Mattauch, L., Klenert, D., Stiglitz, J., & Edenhofer, O. (2018). Overcoming wealth inequality by capital taxes that finance public investment. *NBER Working Paper* No. 25126.
Park, Y. (2014). Optimal taxation in a limited commitment economy. *The Review of Economic Studies*, *81*(2), 884–918. https://doi.org/10.1093/restud/rdt038
Romer, P. (1990). Endogenous technological change. *Journal of Political Economy*, *98*(5, Part 2), S71–S102. https://doi.org/10.1086/261725
Solt, F. (2020). Measuring income inequality across countries and over time: The standardized world income inequality database. *Social Science Quarterly. SWIID Version*, *9*(3), 1183–1199
Werning, I. (2007). Optimal Fiscal policy with redistribution. *The Quarterly Journal of Economics*, *122*(3), 925–967. https://doi.org/10.1162/qjec.122.3.925
Zhu, X. (1992). Optimal Fiscal policy in a stochastic growth model. *Journal of Economic Theory*, *58*(2), 250–289. https://doi.org/10.1016/0022-0531(92)90055-M

Appendix

A1. Proof of Lemma 1

First of all, the value function of (4) at a stationary competitive general equilibrium is identified using the Guess and Verify method. To this end, for any given $\theta_i \in \Theta$, guess that the value function is $v_i(k) = A\log(k) + B$ with A and B unknown. Because the budget constraint (2) binds at a stationary competitive general equilibrium, $c_i = (1+s_i)(rk_i(1-\tau_K) + k_i - k'_i)$ for any given $\theta_i \in \Theta$, the FOC with respect to capital-investment decision is

$$(1-\beta)\frac{-1}{(rk_i(1-\tau_K)+k_i-k'_i)} + \beta A\frac{1}{k'_i} = 0, \tag{A1}$$

which implies that

$$k'_i = \frac{\beta A(r(1-\tau_K)+1)}{(1-\beta+\beta A)}k_i. \tag{A2}$$

Utilizing (A2), to verify the coefficient A and B, the value function v_i is restated as

$$A\log(k_i) + B = (1-\beta)\log(1+s_i) + (1-\beta)\log\left(\frac{(1-\beta)(r(1-\tau_K)+1)k_i}{(1-\beta+\beta A)}\right) - (1-\beta)\frac{l_i^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + (1-\beta)\chi \log(G) + \beta A\log\left(\frac{\beta A(r(1-\tau_K)+1)k_i}{(1-\beta+\beta A)}\right) + \beta B. \tag{A3}$$

Comparing the coefficient of $\log(k_i)$ in the left- and right-hand sides of (A3) yields that $A = 1$, for any given $\theta_i \in \Theta$. Likewise, for any given $\theta_i \in \Theta$, we obtain that

$$B = \log(1+s_i) + \log(1-\beta) - \frac{l_i^{1+\frac{1}{\eta}}}{\left(1+\frac{1}{\eta}\right)} + \chi\log(G) + \frac{1}{1-\beta}\left[\beta\log(\beta) + \log(r(1-\tau_K)+1)\right]. \tag{A4}$$

Because $k_i = k'_i > 0$ takes a finite value at a steady-state competitive general equilibrium, the value function with these finite coefficients of A and B also takes a finite value at a steady-state competitive general equilibrium. Now, with the identified value function v_i, for any given $\theta_i \in \Theta$, the optimal labor-supply condition of (15) is

$$(1-\beta)\frac{(1-\mu_L)}{l_i} - (1-\beta)l_i^{\frac{1}{\eta}} - \beta l_i^{\frac{1}{\eta}} = 0 \tag{A5}$$

which means that the optimal labor supply of individual i is equal to (18) for any given $\theta_i \in \Theta$. At the same time, the optimality condition of (16) is

$$(1-\beta)\frac{1}{\left(1+\frac{1}{s_i}\right)}(-1)\left(\frac{1}{s_i}\right)^2 + \beta\frac{1}{(1+s_i)} = 0 \tag{A6}$$

which means that the optimal ratio of post-tax labor income to the sum of post-tax capital income and change in capital investment of individual i is equal to (20) for any given $\theta_i \in \Theta$. Moreover, as $c_i = \left(1+\frac{1}{s_i}\right)\rho_L(w\theta_i l_i)^{1-\mu_L}$ at a stationary competitive general equilibrium, (A5) and (A6) imply that the optimal private consumption of individual i is equal to (19) for any given $\theta_i \in \Theta$. ■

A2. Proof of Lemma 2

First, due to (18) of Lemma 1, the aggregate labor supply of (13) at a stationary competitive general equilibrium is

$$L = \{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}\int_{\Theta}\theta dF_\theta = \frac{1}{(1-\sigma_\pi)}\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}, \tag{A7}$$

which shows that the total labor is (21). Second, with (5), (7), (8) and (18), the government's budget constraint of (6) at a stationary competitive general equilibrium is stated as

$$\int_{\Theta}\{w\theta\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}} - \rho_L\left(w\theta\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}\right)^{1-\mu_L} + \tau_K r k_i(\theta)\}dF_\theta = gY. \tag{A8}$$

Because $(1-\alpha)Y = wL$ and $\alpha Y = rK$, with the conditions (12) and (13) being met at a stationary competitive general equilibrium, (A8) is restated as

$$(1-\alpha)Y + \tau_K\alpha Y - \int_{\Theta}\rho_L\left(w\theta\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}\right)^{1-\mu_L}dF_\theta = gY. \tag{A9}$$

Furthermore, at a stationary competitive general equilibrium, $k_i = k'_i$ for any given $\theta_i \in \Theta$; hence, $s_i = \frac{\rho_L(w\theta_i l_i)^{1-\mu_L}}{rk_i(1-\tau_K)}$. Thus, due to (18) and (20) of Lemma 1, (A9) is simplified into

$$(1-\alpha)Y + \tau_K\alpha Y - \left(\frac{1-\beta}{\beta}\right)\alpha Y + \left(\frac{1-\beta}{\beta}\right)\tau_K\alpha Y = gY. \tag{A10}$$

As $Y>0$, we can divide both sides of (A10) by Y without affecting the equality, to obtain that $(1-\alpha) + \tau_K\alpha - \left(\frac{1-\beta}{\beta}\right)\alpha + \left(\frac{1-\beta}{\beta}\right)\tau_K\alpha = g$, which is equal to (24). Third, at a stationary competitive general equilibrium, $k_i(\theta_i) = \frac{\rho_L(w\theta_i l_i)^{1-\mu_L}}{s_i r(1-\tau_K)}$ for any given $\theta_i \in \Theta$, which is aggregated over the entire population to entail

$$K = \frac{\rho_L (w)^{1-\mu_L} \{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}(1-\mu_L)}}{r(1-\tau_K)} \left(\frac{\beta}{1-\beta}\right) \exp\left[\frac{\sigma_\varepsilon (1-\mu_L)(-\mu_L)}{2}\right] \frac{1}{1-\sigma_\pi(1-\mu_L)} \quad \text{(A11)}$$

due to (12) as well as (20). The inter-temporal optimality condition of (17) at a stationary competitive general equilibrium, where $k_i = k'_i = k''_i$ for any given $\theta_i \in \Theta$, implies that

$$r = \left(\frac{1-\beta}{\beta}\right) \frac{1}{(1-\tau_K)}. \quad \text{(A12)}$$

Moreover, at a stationary competitive general equilibrium, $w = z_T z_L (1-\alpha)(\frac{z_K K}{z_L L})^\alpha$. Plugging this market-clearing wage rate, (A12) and (A7) into (A11), and then solving for K entails

$$K = \rho_L^{\frac{1}{1-\alpha(1-\mu_L)}} z_T^{\frac{(1-\mu_L)}{1-\alpha(1-\mu_L)}} (z_K)^{\frac{\alpha(1-\mu_L)}{1-\alpha(1-\mu_L)}} (z_L)^{\frac{(1-\mu_L)(1-\alpha)}{1-\alpha(1-\mu_L)}} (\frac{\beta}{1-\beta})^{\frac{2}{1-\alpha(1-\mu_L)}} (1-\alpha)^{\frac{1-\mu_L}{1-\alpha(1-\mu_L)}} (\frac{1}{1-\sigma_\pi})^{\frac{-\alpha(1-\mu_L)}{1-\alpha(1-\mu_L)}}$$

$$\{(1-\mu_L)(1-\beta)\}^{\frac{\eta(1-\mu_L)(1-\alpha)}{(\eta+1)\{1-\alpha(1-\mu_L)\}}} (\frac{1}{1-\sigma_\pi(1-\mu_L)})^{\frac{1}{1-\alpha(1-\mu_L)}} \exp\left[\frac{\sigma_\varepsilon (1-\mu_L)(-\mu_L)}{2}\right]^{\frac{1}{1-\alpha(1-\mu_L)}}. \quad \text{(A13)}$$

Now, to solve for ρ_L, as $\tau_K r k_i = r k_i - \frac{1}{s_i} \rho_L (w\theta_i l_i)^{1-\mu_L}$ at a stationary competitive general equilibrium, with (20), (A8) is simplified to

$$\left(\frac{1}{1-\beta}\right) \int_\Theta \rho_L \left(w\theta\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}}\right)^{1-\mu_L} dF_\theta = (1-g)Y. \quad \text{(A14)}$$

Using (5) and (24), (A14) becomes

$$\rho_L \left(\frac{1}{1-\beta}\right) \{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}(1-\mu_L)} (z_T z_L (1-\alpha)(\frac{z_K K}{z_L L})^\alpha)^{1-\mu_L} \exp\left[\frac{\sigma_\varepsilon (1-\mu_L)(-\mu_L)}{2}\right] \frac{1}{1-\sigma_\pi(1-\mu_L)}$$

$$= \frac{\alpha}{\beta}(1-\tau_K) z_T (z_K K)^\alpha (z_L L)^{1-\alpha}. \quad \text{(A15)}$$

Utilizing (A7) and (A13), we solve for ρ_L to obtain

$$\rho_L = z_T^{\frac{\mu_L}{(1-\alpha)}} z_K^{\frac{\alpha\mu_L}{(1-\alpha)}} z_T^{\mu_L} (\frac{\beta}{1-\beta})^{\frac{\alpha\mu_L - 1 + \alpha}{(1-\alpha)}} \alpha^{\frac{1-\alpha(1-\mu_L)}{1-\alpha}} (\frac{1}{1-\alpha})^{(1-\mu_L)} (1-\tau_K)^{\frac{1-\alpha(1-\mu_L)}{1-\alpha}} \{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}\mu_L}$$

$$\exp\left[\frac{\sigma_\varepsilon (1-\mu_L)(\mu_L)}{2}\right] \frac{\{1-\sigma_\pi(1-\mu_L)\}}{(1-\sigma_\pi)}. \quad \text{(A16)}$$

Plugging (A16) into (A13) yields the total capital of (22). Fourth, applying (21) and (22) to the production function of (5) shows that the total output at a stationary competitive general equilibrium is equal to (23). ■

A3. Proof of Proposition 1

First, since the social welfare function of (25) is concave in τ_K for $\forall \tau_K \in [0,1]$, the FOC with respect to τ_K is sufficient to define optimal capital income tax rate τ_K^* that maximizes social welfare. Thus,

$$\frac{(\alpha\chi+1)}{(1-\alpha)}\frac{-1}{(1-\tau_K^*)}+\frac{\chi}{\frac{\beta}{\alpha}-(1-\tau_K^*)}=0. \tag{A17}$$

Solving (A17) for τ_K^*, we get

$$\tau_K^*=1-\frac{\beta}{\alpha}\left(\frac{\alpha\chi+1}{\chi+1}\right). \tag{A18}$$

Because the social welfare function of (25) is concave in τ_K, when $\alpha(\chi+1)\leq\beta(\alpha\chi+1)$, the highest feasible level of the social welfare is attained by $\tau_K^*=0$. On the other hand, when $\alpha(\chi+1)>\beta(\alpha\chi+1)$, the right-hand side of (A18) itself is optimal capital income tax rate (τ_K^*). When the right-hand side of (A18) is greater than zero, it never exceeds one because $\alpha\in(0,1)$, $\chi>0$ and $\beta\in(0,1)$. Putting these two cases of $\alpha(\chi+1)>\beta(\alpha\chi+1)$ and $\alpha(\chi+1)\leq\beta(\alpha\chi+1)$ together entails the optimal capital income tax rate of (26).

Second, since the social welfare function of (25) is also concave in μ_L for all the feasible values in the range of (9), the optimal slope of labor income tax rate schedule μ_L^* is defined fully by the FOC with respect to μ_L. Thus,

$$\frac{\eta}{\eta+1}\frac{-(1+\chi)}{(1-\mu_L^*)}+\frac{\eta}{\eta+1}(1-\beta)+\frac{\sigma_\pi}{1-\sigma_\pi(1-\mu_L^*)}+\frac{\sigma_\varepsilon(1-2\mu_L^*)}{2}+\frac{\sigma_\varepsilon}{2}-\sigma_\pi=0 \tag{A19}$$

which is equivalently stated as (28).

Third, (A16) of Lemma 2 shows that at a stationary competitive general equilibrium, where the government's budget is balanced, once the government chooses the value of μ_L and τ_K, then the value of ρ_L is automatically determined. Hence, the value of ρ_L^* is automatically defined by τ_K^* and μ_L^* from (26) and (28), which proves (29).

Fourth, recall that average marginal labor income tax rate τ_L is defined according to (10). Based on Lemmas 1 and 2, plugging (23) of Y and $y_i(\theta)=w\theta_i l_i(\theta_i)$ where $w=z_T z_L(1-\alpha)\left(\frac{z_K K}{z_L L}\right)^\alpha$ as well as (A16) into (10) yields average marginal labor income tax rate τ_L at a stationary competitive general equilibrium as follows.

$$\tau_L=(1-\alpha)-\frac{\alpha}{\beta}(1-\mu_L)(1-\tau_K) \tag{A20}$$

which shows that average marginal labor income tax rate τ_L is automatically determined once the government chooses μ_L and τ_K at a stationary competitive general equilibrium. As (A20) means $(1-\tau_K)=\frac{\beta}{\alpha}\frac{1}{(1-\mu_L)}(1-\alpha-\tau_L)$, plugging this into the social welfare function of (25) and then obtaining the FOC for τ_L^* yields

$$\frac{(\alpha\chi+1)}{(1-\alpha)}\frac{-1}{(1-\alpha-\tau_L^*)}+\frac{\chi}{(1-\mu_L^*)-(1-\alpha-\tau_L^*)}=0. \tag{A21}$$

Solving (A21) for τ_L^* entails the optimal average marginal labor income tax rate of (27). ■

A4. Proof of Proposition 2

As described in Section II, with the standard production function of (5), unbiased technological progress is represented by an increase in one of the three technology parameters z_T, z_K and z_L. That is, unbiased technological progress is realized as one of the following three cases: (i) total factor productivity improvement (an increase in z_T), (ii) capital-augmenting technological progress (an increase in z_K) and (iii) labor-augmenting technological progress (an increase in z_L).

First, consider an increase in z_T with all the other parameters being fixed, which represents technological progress of improving total factor productivity. The effect of improving total factor productivity on optimal capital income tax rate is identified with $\frac{d\tau_K^*}{dz_T}$. According to (26) of Proposition 1, $\frac{d\tau_K^*}{dz_T}=0$ showing that technological progress of increasing total factor productivity has no effect on optimal capital income tax rate. By the same token, the effect of improving total factor productivity on optimal average marginal labor income tax rate is identified with $\frac{d\tau_L^*}{dz_T}$ which is equal to zero, according to (27) of Proposition 1. Thus, unbiased technological progress of increasing total factor productivity does not affect optimal average marginal labor income tax rate. Likewise, an increase in z_T has no effect on optimal slope of labor income tax rate schedule, since $\frac{d\mu_L^*}{dz_T}=0$ due to (28) of Proposition 1.

Second, now let us introduce an increase in z_K with all the other parameters being fixed to represent capital-augmenting technological progress. The effect of capital-augmenting technological progress on optimal capital income tax rate is identified with $\frac{d\tau_K^*}{dz_K}$ which is equal to zero, according to (26) of Proposition 1. Similarly, its effect on optimal average marginal labor income tax rate is $\frac{d\tau_L^*}{dz_K}=0$, due to (27) of Proposition 1. Thus, capital-augmenting technological progress does not affect optimal capital income tax rate or optimal average marginal labor income tax rate. Moreover, capital-augmenting technological progress does not affect optimal slope of labor income tax schedule either, since $\frac{d\mu_L^*}{dz_K}=0$ due to (28).

Third, consider an increase in z_L with all the other parameters fixed, which represents labor-augmenting technological progress. Labor-augmenting technological progress also makes no difference in optimal capital income tax rate and optimal average marginal labor income tax rate, since $\frac{d\tau_K^*}{dz_L}=0$ and $\frac{d\tau_L^*}{dz_L}=0$ according to (26) and (27) of Proposition 1. Likewise, labor-augmenting technological progress has no effect on optimal slope of labor income tax rate schedule, as $\frac{d\mu_L^*}{dz_L}=0$ according to (28) of Proposition 1.

Taking the above three cases together shows that unbiased technological progress does not affect τ_K^*, τ_L^* and μ_L^*. Thus, unbiased technological progress makes no difference in optimal capital and labor income taxation. ■

A5. Proof of Proposition 3

As described in the Section II, capital-biased technological progress is represented by an increase in the technology parameter of α with all the other parameters being fixed. First, the effect of capital-biased technological progress on optimal capital income tax rate is identified with $\frac{d\tau_K^*}{d\alpha}$. If $\alpha(\chi+1)>\beta(\alpha\chi+1)$ before capital-biased technological progress, according to (26) of Proposition 1, capital-biased technological progress strictly raises optimal capital income tax rate, because

$$\frac{d\tau_K^*}{d\alpha}=\frac{\beta}{\alpha^2(\chi+1)}>0. \tag{A22}$$

Moreover, as α increases, the condition for τ_K^* to be strictly positive is more likely to be satisfied, because

$$\frac{d\{\alpha(\chi+1)-\beta(\alpha\chi+1)\}}{d\alpha}=(1-\beta)\chi+1>0. \tag{A23}$$

If $\alpha(\chi+1)\le\beta(\alpha\chi+1)$ to have zero capital income tax rate before capital-biased technological progress, (A22) and (A23) imply that capital-biased technological progress raises the level of optimal capital income tax rate sufficiently high to be strictly positive or insufficiently to remain as zero. Putting these two comprehensive cases together, the effect of capital-biased technological progress on optimal capital income tax rate is positive.

Second, the effect of capital-biased technological progress on optimal slope of labor income rate tax schedule is

$$\frac{d\mu_L^*}{d\alpha}=0 \tag{A24}$$

according to (28) of Proposition 1. Thus, capital-biased technological progress does not affect optimal slope of labor income tax rate schedule.

Third, the effect of capital-biased technological progress (an increase in α) on optimal average marginal labor income tax rate is identified with $\frac{d\tau_L^*}{d\alpha}$. Based on (27) of Proposition 1,

$$\frac{d\tau_L^*}{d\alpha} = (-1) - (1 - \mu_L^*) \frac{\chi}{(\chi + 1)} < 0 \tag{A25}$$

due to (A24), (9), and $\chi > 0$. Thus, capital-biased technological progress lowers optimal average marginal labor income tax rate, while it does not affect optimal slope of labor income tax rate schedule. Notice that each step of this proof holds regardless of whether the condition of (30) is met or not. In addition, because labor-biased technological progress is represented by a decrease in α with all the other parameters being fixed, it is straightforward from this proof that labor-biased technological progress lowers optimal capital income tax rate and raises optimal average marginal labor income tax rate, while it does not affect optimal slope of labor income tax rate schedule. ■