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Capital structure optimization: a model of optimal capital structure from the aspect of capital cost and corporate value

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ABSTRACT

The purpose of this study is, firstly, to examine capital structure optimization and secondly, to provide a framework for determining the optimal capital structure from the aspect of capital cost and corporate value. The results of our work provide an innovative model for arriving at a company's optimal capital structure based on the estimation of the effective cost of capital and the determination of the shares of new equity and long-term debt that will both minimize the overall cost of capital and maximize its value. The model can be applied for quantitative estimates of optimal capital structure. This paper contributes to the literature by applying mathematical modeling and mathematical theory of optimization to solve the problem of capital structure optimization, and by providing a framework for determining optimal capital structure. The scientific contribution of this research is development of a model of optimal capital structure from the aspect of capital cost and corporate value, and new equations for calculating the effective costs of long-term financing sources. This model provides explicit advice on optimal long-term debt and equity level and can be applied to produce a firm-specific recommendation about optimal capital structure that a given company should use.

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1. Introduction

The capital structure optimization is extremely important and complex field in corporate financial management, since the performance success, viability and future survivability depend on financing decisions. This paper aims to explore the effective costs of long-term financing sources and to provide a framework for determining the optimal capital structure from the aspect of capital cost and corporate value. Using the principles of mathematical modeling, as well as those of corporate finance, we have developed mathematical models to explain the effective costs of sources of

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long-term financing as well as some aspects of optimal capital structure which have not been explored entirely in previous studies.

In previous models, the trade-off between tax shields and bankruptcy costs associated with debts yields an optimal capital structure. The fundamental problem of earlier models is that those models can not be applied to produce a firm-specific recommendation about optimal capital structure that a given company should use. We develop an innovative model that includes two aspects: the aspects of capital cost and corporate value, and can be applied to produce a firm-specific recommendation about optimal capital structure that a given company should use. Earlier models also considered these aspects, but did not derive closed form solutions. The model of optimal capital structure from the aspect of capital cost and corporate value addresses the choice of optimal capital structure by the corporation and can be applied for quantitative estimates of optimal capital structure.

This study has several contributions to the field. In this study, we develop a new approach which is a model-based approach, a paradigm that emphasizes the application of mathematical modeling principles and optimization theory to solve the problem of capital structure optimization on the one hand, and to provide a framework for determining optimal capital structure from the aspect of capital cost and corporate value on the other. The authors provide a reasonably user-friendly model for arriving at a company's optimal capital structure based on the estimation of the effective cost of capital and the determination of the shares of new equity capital and long-term debt capital that will both minimize the overall cost of capital and maximize its value. In comparison to previous models, this model can be used to produce a firm-specific recommendation of the optimal amount of new long-term debt and new equity that a given company should use. The model gives an answer to how much long-term debt and equity a company should use, and how the use of long-term debt and equity affects the corporate value. Furthermore, the model provides explicit rather than general advice on optimal long-term debt and equity level, and also matches the effective cost of capital quite accurately. We provide formulas that can be used to approximate the effective cost of long-term debt and equity capital, and in turn determine the optimal amount of new long-term debt and equity, for any given corporation. Thus, our findings have potentially important implications for managers and in the literature on capital structure.

The remainder of the paper is organized as follows. Section 2 addresses the theoretical grounds of the research. Section 3 explains the methodology, addresses theoretical grounds for modelling, and develops corresponding models. Section 4 provides a discussion. Finally, section 5 is the conclusion.

2. Literature review

Financial economists have set four various capital structure theories such as trade-off theory (Kraus & Litzenberger, 1973), pecking order theory (Myers & Majluf, 1984), signaling theory (Ross, 1977) and market timing theory (Baker & Wurgler, 2002). The trade-off theory describes the optimal level of debt as a trade-off between the tax benefits of corporate borrowing and the increasing bankruptcy costs associated with additional borrowing. The modern trade-off model of corporate leverage predicts that a corporation's optimal debt level is set by trading off the tax benefits of increasing leverage against the increasingly severe bankruptcy costs and agency costs of heavy debt usage (Graham et al., 2010).

Although a great deal of evidence is consistent with the implications of the trade-off model, one inconsistency is that profitable firms tend to borrow less, rather than more as the model predicts. The pecking order theory predicts that managers will operate their firms in such a way as to minimize the need to secure outside financing, by retaining profits to build up financial slack. Managers will use the safest source of funding as senior debt, when they must secure outside financing. In this theory, there is no well-defined debt-equity ratio, because there are two kinds of equity, internal and external, one at the top of the pecking order and another at the bottom (Brealey et al., 2011). The signaling theory predicts that managers will select their firms' leverage levels to signal that the firm is strong enough to employ high debt and fund its profitable investments (Ross, 1977). The market timing model predicts that firms attempt to time the market by issuing equity when share values are high and issuing debt when share prices are low.

According to the traditional approach to capital structure, a moderate degree of financial leverage can lower the firm's weighted average cost of capital—as cheaper debt is substituted for more expensive equity—thereby increase the total value of the firm. Thus, the traditional position implies that the value of the firm is not independent of its financing mix and that there exists an optimal capital structure (Shapiro & Balbirer, 2000). Traditionalists believe that because debt is cheaper, combining equity with reasonable amounts of debt results in a reduction of the firm's overall cost of capital. The value of the firm is higher when a lower discount rate (the cost of capital) is applied to its cash flow stream (Chamber & Lacey, 2014). Thus, under the traditionalists' argument, the goal of corporate management is to find the level of equity and debt financing that minimizes the firm's cost of capital and maximizes corporate value. To summarize, traditionalists search for the combination of securities that creates a minimum overall cost of capital.

The agency cost approach to optimal capital structure assumes that the optimal capital structure minimizes the sum of the agency costs of debt and equity. Total agency costs decrease as some debt is added, but then begin to increase as the debt ratio increases further. Other things being equal, the optimal capital structure would be at the point where total agency costs were minimized (Seitz & Ellison, 1995).

There is extensive literature on the determination of optimal capital structure (Binsbergen et al., 2011; DeMarzo & Fishman, 2007; Ju et al., 2005; Leland & Toft, 1996; Mao, 2003; Mu et al., 2017; Park, 2015; Vilauso & Minkler, 2001). Leland and Toft (1996) developed a model of optimal leverage and risky corporate bond prices for arbitrary debt maturity. The findings suggest that optimal leverage depends upon debt maturity and is markedly lower when the firm is financed by short-term debt. Vilauso and Minkler (2001) created a dynamic model of corporation's financing policy. They suggest that although equity finance reduces transaction costs when assets are highly specific, equity finance also offers bondholders greater protection from excessive risk taking which reduces the agency costs of debt. Thus, the optimal capital structure uses both debt and equity finance to minimize the sum of agency costs. Mao (2003) created a simple model that captures both the risk-shifting and under-investment problems. Ju et al. (2005) examined the optimal capital structure choice using a dynamic capital structure model. In a dynamic model, the firm strategically lowers its initial leverage ratio to avoid bankruptcy. DeMarzo and Fishman (2007) demonstrated that the optimal mechanism can be implemented by a combination of equity, long-term debt and a line of credit. According to Bessler et al. (2011) managers need to evaluate the agency costs of debt against the agency costs of equity in order to arrive at optimal financing decisions.

Binsbergen et al. (2011) investigated optimal capital structure and provide formulas that can be used to approximate the cost of debt and in turn to determine the optimal amount of debt. Craven and Islam (2013) find that the debt-equity ratio does affect the value of the firm, and hence the need for good corporate financial management to maximize the value of the firm, by choosing the optimal debt. Park (2015) finds a firm with high volatility of earnings optimally issues debt of shorter maturity. His results support the dynamic capital structure model in which a firm decides its leverage as a trade-off between bankruptcy costs and tax benefits. Mu et al. (2017) investigated optimal capital structure with moral hazard. The results indicate that the firm issues more debt with higher coupon ex ante and defaults earlier ex post than without manager moral hazard. Palmowski et al. (2020) have studied the extension of the Leland-Toft optimal capital structure model. They have obtained explicitly an optimal bankruptcy strategy and the corresponding equity/debt/firm values. Adeoye et al. (2021) suggest that the optimal capital structure required to minimize the marginal cost of the agency problem is a higher use of debt and lower cost of equity. The study develops a new contract theory model based on the integrated issues of capital structure, corporate governance and agency problems.

Theories on capital structure have failed to resolve numerous disagreements over whether companies can affect their total value and the total cost of capital by changing the capital structure. In previous theoretical research, no equation was found to determine the optimal capital structure (Vidučić, 2001). The fundamental problem of theories of capital structure and earlier models is that those models cannot be applied to produce a firm-specific recommendation about optimal capital structure that a given company should use. Furthermore, they do not explain the effective costs of sources of financing. Despite the extensive increase in the volume of literature on corporate optimal capital structure, relevant questions remain to be unanswered regarding how corporations can evaluate various financing alternatives and establish the optimal capital structure. Consequently, this paper addresses the issue of capital structure optimization and provides an innovative model that can be applied to produce a firm-specific recommendation about optimal capital structure that a given company should use.

3. Research

3.1. Methodology

The methodology applied in this paper includes the application of mathematical modeling and mathematical optimization theory in solving scientific problems. Mathematical models attempt to represent nonmathematical reality by means of equations and other mathematical statements as they translate the essential features of a given situation into mathematical symbols. A mathematical model is thus a formal structure that creates a framework within which a problem can be analyzed (Cook & Russell, 1989). We analyzed various long-term financing sources available to corporations from a cost-effective point of view. The independent variables which impact the effective cost of long-term financing sources are selected and the relations between them are defined. For precise formulation of the relationship between a set of independent variables, mathematical methods have been adapted in order to yield the effective cost of capital.

Mathematical optimization can be described as the science of determining the best solutions to mathematically defined problems, which may be models of physical reality or of manufacturing and management systems (Snyman, 2005). Thus, the optimization approach is used to solve the issue of the optimal capital structure. The linear programming model with the objective of minimizing the overall cost of capital consists of objective function, four constraints and nonnegativity conditions. The objective function is a mathematical expression that measures the effectiveness of a particular solution for the optimal capital structure problem. The constraints are mathematical statements which specify such elements of the problem as the limitations of available long-term sources. The upper and down bounds of third constraint are defined according to the vertical rule of financing, the conservative vertical rule of financing and the results from empirical analyses of capital structure. According to Edwards and Hamson (2007) empirical models are not derived from assumptions concerning the relationships between variables and they are not based on physical laws or financial principles. When we have no principles to guide us and no obvious assumptions suggest themselves, we may turn to the data to find how some of our variables are related. Consequently, we turn to the data of Croatian, Slovenian and Czech joint-stock companies whose financial instruments are listed on the capital market to find how the variables the shares of long-term debt and short-term debt are related.

3.2. Effective cost of sources financing

The cost of capital is used to make decisions which involve raising new capital. This section provides explicit equation models to compute the effective cost of equity and long-term debt financing available to corporations.

3.2.1. The effective cost of equity financing

A corporation can raise common equity by issuing new shares of common stock or by reinvesting earnings. The independent variables that have an impact on the effective cost of new common stock are: the net profit (P_r), the rate of retained earnings (k), the dividend for the existing preffered stock (D_{ps}), the agency costs of equity (AC_e), the number of stocks (s), the value of common stock (P_0) and the floating costs (FC).

The effective cost of new common stock can be written as

$$k_e = \frac{\sum costs\ of\ financing\ by\ stocks}{net\ proceeds} \tag{1}$$

Thus, the effective cost of new common stock can be expressed in terms of independent variables as follows:

$$k_e = \frac{[Pr \times (1-k) - D_{ps} + AC_e]}{(s \times P_o - FC)} \tag{2}$$

where Pr is the net profit, k is the rate of retained earnings, D_{ps} is the dividend for the existing preffered stock, AC_e is the agency costs of equity, P_o is the value of common stock, s is the number of common stocks and FC is the flotation costs.

The sales-to-asset ratio is a measure of how effectively the firm's management deploys its assets and equity agency costs are inversely related to the sales-to-asset ratio. (Ang et al., 2000). Thus, the sales-to-asset ratio can be used as a direct measure of equity agency costs which are inversely related to the ratio. This ratio is calculated as follows

$$\text{Sales to Asset Ratio} = \text{Annual Sales/Total Assets} \tag{3}$$

Equity agency costs can be expressed as follows

$$\text{Equity Agency Costs} = \frac{1}{\text{Sales } to \text{ Asset Ratio}} \tag{4}$$

Now, rearranging expression (4) we arrive at the expression

$$\text{Equity Agency Costs} = \frac{1}{\frac{\text{Annual Sales}}{\text{Total Assets}}} = \frac{\text{Total Assets}}{\text{Annual Sales}} \tag{5}$$

Consequently, equity agency costs can be calculated as follows

$$\text{Equity Agency Costs } (AC_e) = \frac{\text{Total Assets } (TA)}{\text{Annual Sales } (AS)} \tag{6}$$

3.2.2. The effective cost of Long-Term debt financing

In our research, we have analyzed simple interest bank loan, simple interest discounted loan, bank loan with compound interest, discounted bank loan with compound interest, and long-term debt obtained by issuing coupon bonds and annuity bonds. The effective interest rate after tax for any long-term bank loan available to corporations can be defined as follows

$$\text{Effective interest rate after tax} = \frac{\text{Nominal interest on face of long-term loan}}{\text{Net proceeds of long-term loan}} \times (100 - \text{profit tax rate}) \tag{7}$$

3.2.2.1. Simple interest bank loan. The independent variables which have an impact on the effective interest rate after tax for a simple interest bank loan with compensating balances or a deposit are: the nominal amount of debt (N_j), the interest rate on debt (i_1), the interest rate on deposit (i_2), the amount of compensating balances (CB), the amount of deposit (Cod), the discount rate (d) and the profit tax rate (t). The generalized model for calculating the effective interest rate after tax can be introduced

$$k_d = \frac{\sum_{j=1}^{n} \frac{(N_j \times i_1/100 - C_{od} \times i_2/100)}{(1+d/100)^j}}{\sum_{j=1}^{n} \frac{(N_j - CB - C_{od})}{(1+d/100)^j}} \times (100 - t) \tag{8}$$

where k_d is the effective interest rate after tax, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, C_{od} is the amount of deposit, d is the discount rate and t is the profit tax rate.

3.2.2.2. Simple interest discounted loan. The independent variables which have an impact on the effective interest rate after tax for a simple interest discounted loan with compensating balances or a deposit are: the nominal amount of debt (N_j), the interest rate on debt (i_1), the interest rate on deposit (i_2), the amount of compensating balances (CB), the amount of deposit (Cod), the discount rate (d) and the profit tax rate (t). The effective interest rate after tax can be expressed as follows

$$k_d = \frac{\sum_{j=0}^{n-1} \frac{(N_j \times i_1/100 - C_{od} \times i_2/100)}{(1+d/100)^j}}{\sum_{j=0}^{n-1} \frac{(N_j - N_j \times i_1/100 - CB - C_{od})}{(1+d/100)^j}} \times (100 - t) \tag{9}$$

where k_d is the effective interest rate after tax, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, Cod is the amount of deposit, d is the discount rate and t is the profit tax rate expressed as percentage.

3.2.2.3. Bank loan with compound interest. The independent variables which have an impact on the effective interest rate after tax for a bank loan with compaund interest and compensating balances or a deposit are: the nominal amount of debt (N_j), the interest rate on debt (i_1), the interest rate on deposit (i_2), the amount of compensating balances (CB), the amount of deposit (Cod), the discount rate (d) and the profit tax rate (t). Therefore, the generalized model for calculating the effective interest rate after tax for a bank loan with compound interest and compensating balances or a deposit can be introduced

$$k_d = \frac{\sum_{j=1}^{n} \left\{ \frac{\left[N_0 \times \frac{r^n(r-1)}{r^n-1} - (N_{j-1} - N_j)\right] - \left[C_{od} \times \left(1 + \frac{i_2}{100}\right)^j - C_{od} \times \left(1 + \frac{i_2}{100}\right)^{j-1}\right]}{(1 + d/100)^j} \right\}}{\sum_{j=0}^{n-1} \frac{(N_j - CB - C_{od})}{(1 + d/100)^j}} \times (100 - t) \tag{10}$$

where $r = 1 + \frac{i_1}{100}$

No is the principal, n is the number of annuities, Cod is the amount of deposit, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, d is the discount rate and t is the profit tax rate.

3.2.2.4. Discounted bank loan with compound interest. The independent variables which have an impact on the effective interest rate after tax for a discounted bank loan with compaund interest and compensating balances or a deposit are: the nominal amount of debt (N_j), the interest rate on debt (i_1), the interest rate on deposit (i_2), the amount of compensating balances (CB), the amount of deposit (Cod), the discount rate (d) and the profit tax rate (t). Therefore, the generalized model for calculating the effective interest rate after tax for a discounted bank loan with compound interest and with compensating balances or a deposit can be introduced

$$k_d = \frac{\left(\frac{N_0 \times i_1}{100} - \frac{C_{od} \times i_2}{100}\right) + \sum_{j=1}^{n-1} \frac{\left\{\left[N_0 \times \frac{\rho^{n-1}(\rho-1)}{\rho^n - 1} - (N_{j-1} - N_j)\right] - \left[C_{od} \times \left(\frac{100}{100-i_2}\right)^j - C_{od} \times \left(\frac{100}{100-i_2}\right)^{j-1}\right]\right\}}{(1+d/100)^j}}{\sum_{j=0}^{n-1} \frac{\left(N_j - I_j - CB - C_{od}\right)}{(1+d/100)^j}} \times (100 - t) \tag{11}$$

where

$$I_j = N_0 \times \frac{\rho^{n-1}(\rho - 1)}{\rho^n - 1} - (N_{j-1} - N_j)\rho = \frac{100}{(100 - i_1)}$$

No is the principal, n is the number of annuities, C_{od} is the amount of deposit, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, I_j is the interest for period j, d is the discount rate and t is the profit tax rate expressed as percentage.

3.2.2.5. Coupon bonds. The independent variables which have an impact on the effective cost of long-term debt after tax obtained by issuing coupon bonds are: the nominal interest rate, face value of a bond, the bond discount, the bond premium, flotation costs, the discount rate and the profit tax rate. The effective cost of long-term debt after tax, obtained by issuing coupon bonds (k_b), is given by

$$k' = \frac{\sum_{j=1}^{n} \frac{N_0 \times i/100}{(1+d/100)^j}}{\sum_{j=1}^{n} \frac{N_0 \times \left(1 - \frac{d_0}{100} + \frac{p}{100} - \frac{fc_b}{100}\right)}{(1+d/100)^j}} \times (100 - t) \tag{12}$$

Now, rearranging equation (8) we arrive at the expression

$$k_b = \frac{\sum_{j=1}^{n} \frac{i/100}{(1+d/100)^j}}{\sum_{j=1}^{n} \frac{(1 - d_o/100 + p/100 - fc_b/100)}{(1+d/100)^j}} \times (100 - t) \tag{13}$$

where i is the interest rate on debt, d_0 is the bond discount, p is the bond premium, fc_b are flotation costs, d is the discount rate and t is the profit tax rate.

3.2.2.6. Annuity bonds. The independent variables which have an impact on the effective cost of long-term debt after tax obtained by issuing annuity bonds are: the nominal interest rate, face value of a bond, the nominal amount of debt in period j, the bond discount, the bond premium, flotation costs, the discount rate and the tax rate. The effective cost of long-term debt after tax, obtained by issuing annuity bonds (k_b) is given by

$$k_b = \frac{\sum_{j=1}^{n} \frac{\left[N_0 \times \frac{r^n(r-1)}{r^n-1} - (N_{j-1} - N_j)\right]}{(1+d/100)^j}}{\sum_{j=0}^{n-1} \frac{(N_j - N_0 \times d_0/100 + N_0 \times p/100 - N_0 \times fc_b/100)}{(1+d/100)^j}} \times (100 - t) \tag{14}$$

where $r = 1 + \frac{i}{100}$ i is the interest rate on debt, N_0 is the face value of a bond, Nj is the nominal amount of debt in period j, d_0 is the bond discount, p is the bond premium, fc_b are flotation costs, d is the discount rate and t is the profit tax rate.

The real rate of interest as the real cost for these long-term debt financing can be calculated by using these equations to determine which sources of long-term financing to employ, in doing which the financial manager of a company should take into consideration the cost of all available sources expressed as the effective cost of long-term debt or common stock and choose the long-term source in accordance with the effective cost, because this represents the real cost of financing.

3.3. An innovative model of optimal capital structure

The issue of the optimal capital structure is addressed from the aspect of capital cost and corporate value. In building the model, we have used the capital structure strategy which aims to achieve the optimal capital structure from the aspect of capital cost and corporate value. The capital structure strategy that is employed involves the following elements: determining the effective costs of capital structure components and the overall cost of capital, considering the primary imperfections affecting capital structure such as taxes, bankruptcy costs, agency problems and information asymmetry, minimizing the overal cost of capital and making the choice of capital structure components that will result in minimizing the overall cost of capital and maximizing the corporate value.

Based on this capital structure strategy, we have formulated a model of optimal capital structure from the aspect of capital cost and corporate value. The model consists of a homogeneous set of equations: the equation for calculating the amount of capital needed, the equations for calculating the effective cost of financing by equity and long-term debt, the linear programming model with the objective of minimizing the overall cost of capital, and the equation model for calculating the value.

Capital structure refers to the equity and long-term debt that firm uses to finance its assets and future growth while current liabilities are used to finance temporary current assets. The implication of matching principle is that long-term assets and permanent current assets should be financed by long-term sources of financing while temporary short-term assets should be financed by short-term financing. Consequently, the amount of capital needed can be expressed as follows

$$\text{Equity} + \text{long-term debt} = \text{long term assets} + \text{permanent current assets} \tag{15}$$

$$\text{Short-term liabilities} = \text{temporary current assets} \tag{16}$$

The aim of the cost minimization problem is to identify the mix of long-term financing sources that will minimize the overall cost of capital taking into account capital market imperfections. This linear programming model allows us to identify the shares of new equity capital and long-term debt that will minimize the overall cost of capital which is used as a discount rate in the mathematical model for calculating corporate value.

3.3.1. The linear programming model

The optimization approach is used to solve the issue of the optimal capital structure. The formulated linear programming model that will minimize total costs of capital consists of an objective function and a set of constraints. The objective function is a mathematical expression that measures the effectiveness of a particular solution for the optimal capital structure problem. The constraints are mathematical statements which specify such elements of the problem as the limitations of available long-term sources. The following variables and constants are relevant to the linear programming model: the overall cost of capital, the effective cost of existing equity capital, the effective cost of new equity capital, the effective cost of preffered capital, the effective cost of the existing long-term debt capital, the effective cost of new long-term debt capital, the share of new equity capital in total capital, the share of new long-term debt, the share of the existing equity capital, the share of the existing preffered capital and the share of the existing long-term debt in total capital.

Decision variables are the variables w_{e1} and w_{d2}, whose values are determined when the linear programming model is solved. More precisely, w_{e1} represents the share of new equity capital in total capital, while w_{d2} represents the share of new long-term debt capital in total capital. The constants u_1, u_2 and u_3 should be determined and u_1 represents the share of the existing equity capital, u_2 represents the share of the existing preffered capital, and u_3 represents the share of the existing long-term debt capital in total capital. The constants will always be between 0 and 1.

Therefore, the problem of optimal capital structure from the aspect of capital cost can be solved using the following objective function

$$z\ (w_{e1,}w_{d2}) = \min\ \left(k_{e1}x\ u_1 + k_{e2}x\ w_{e1} + k_p x\ u_2 + k_{d1}x\ u_3 + k_{d2}x\ w_{d2}\right) \tag{17}$$

where k_{e1} represents the effective cost of existing equity capital, k_{e2} represents the effective cost of new equity capital, k_p represents the effective cost of preffered capital, k_{d1} represents the effective cost of the existing long-term debt and k_{d2} represents the effective cost of new long-term debt. The variables called w_{e1} and w_{d2} are decision variables, and the variables u_i represent constants.

The objective function defines the overall cost of capital in terms of the share of new equity capital in total capital, the share of new debt capital in total capital and constants. In other words, the total cost of capital represents the objective function and the aim is to minimize the total cost of capital.

We now establish the following constraints:

The first constraint forces the sum of constants, the share of new equity capital and the share of new debt capital in total capital, to equal 1 and can be expressed as

$$\sum_{i=1}^{3} u_i + w_{e1} + w_{d2} = 1 \tag{18}$$

Therefore, the sum of the share of the existing equity capital in total capital (u_1), the share of the existing preffered capital (u_2), the share of the existing long-term debt capital (u_3), the share of new equity capital (we_1) and the share of new long-term debt capital in total capital (wd_2) is equal to 1.

The second constraint refers to the total equity capital. This constraint indicates that the sum of the share of the existing equity capital in total capital (u_1), the share of new equity capital in total capital (w_{e1}) and the share of the existing preffered capital in total capital (u_2) is no less than 0.56 and no more than 0.71, and can be expressed as

$$0.56 \leq u_1 + w_{e1} + u_2 \leq 0.71 \tag{19}$$

The third constraint reflecting the total debt capital indicates that the sum of the share of the existing long-term debt capital in total capital (u_3) and the share of new long-term debt capital in total capital (w_{d2}) is no less than 0.29 and no more than 0.44, and can be expressed as

$$0.29 \leq u_3 + w_{d2} \leq 0.44 \tag{20}$$

The constraints (13) and (14) can be rewritten as follows

$$0.56 - u_1 - u_2 \leq w_{e1} \leq 0.71 - u_1 - u_2 \tag{21}$$

$$0.29 - u_3 \leq w_{d2} \leq 0.44 - u_3 \tag{22}$$

It is important to note that the lower bound and the upper bound on variables w_{e1}, w_{d2} and constants are defined by the vertical financing rules and the determined ratio of long-term to short-term debt in the financial structure, which is 2:3 and represents a reasonably determined ratio. The results of the empirical research of the financial structure of joint-stock companies whose financial instruments are listed on the capital market in the Republic of Croatia, Slovenia and the Czech Republic were used to confirm that the 2:3 ratio of long-term to short-term debt is rationally determined and that it is the ratio used by listed joint-stock companies.

The vertical rule of financing, which determines the debt to equity ratio 2:1, means that the share of total debt in total capital should not be greater than 66.67 percent, while the share of equity capital should not be less than 33.33 percent. The lower limit of total equity capital and the upper limit of total long-term debt are defined using this vertical rule of financing and the determined ratio of long-term to short-term debt in the financial structure, which is 2:3. Consequently, we may define the equity to long-term debt ratio as 0.56: 0.44 . This assumption means that the share of total equity capital should be greater than 56 percent, which represents the lower bound of total equity capital, while the share of total long-term debt should be less than 44 percent, which represents the upper bound of total long-term debt capital.

The conservative vertical rule of financing, which determines the ratio of total debt and equity capital 1:1, means that the share of total debt in total capital should not be greater than 50.00 percent, while the share of total equity capital should not be less than 50.00 percent. The upper limit of total equity capital and the lower limit of total long-term debt are defined using the conservative vertical rule of financing and the determined ratio of long-term to short-term debt in the financial structure, which is 2:3. Consequently, we may define the equity to long-term debt ratio as 0.71: 0.29 . This assumption means that the share of total equity capital should be less than 71 percent, which represents the upper bound of total equity capital, while the share of total long-term debt should be greater than 29 percent, which represents the lower bound of total long-term debt capital.

The fourth constraint reflecting the share of new equity capital in total capital indicates that it should be greater than or equal to 5 percent of the share of new long-term debt in total capital. Thus, it can be expressed as

$$w_{e1} \geq 0.05 \ w_{d2} \tag{23}$$

It is important to realise that the optimal capital structure of a corporation should involve long-term debt and equity financing to ensure that the sum of agency costs is minimized.

We have also the following nonnegativity conditions

$$w_{e1} > 0 \tag{24}$$

$$w_{d2} \geq 0. \tag{25}$$

Thus, the issue of an optimal capital structure from the aspect of capital cost can be modelled as the following linear programming model

$$z\ (w_{e1,} w_{d2}) = \min\ (k_{e1}x\ u_1 + k_{e2}x\ w_{e1} + k_p x\ u_2 + k_{d1}x\ u_3 + k_{d2}x\ w_{d2}) \tag{26}$$

subject to the constraints

$$\sum_{i=1}^{3} u_i + w_{e1} + w_{d2} = 1 \tag{27}$$

$$0.56 - u_1 - u_2 \leq w_{e1} \leq 0.71 - u_1 - u_2 \tag{28}$$

$$0.29 - u_3 \leq w_{d2} \leq 0.44 - u_3 \tag{29}$$

$$w_{e1} \geq 0,05\ w_{d2} \tag{30}$$

nonnegativity constraints

$$w_{e1} > 0 \tag{31}$$

$$w_{d2} \geq 0 \tag{32}$$

where k_{e1} is the effective cost of existing equity capital, k_{e2} is the effective cost of new equity capital, k_p is the effective cost of preffered capital, k_{d1} is the effective cost of the existing long-term debt capital, k_{d2} is the effective cost of new long-term debt capital, w_{e1} is the share of new equity capital in total capital, w_{d2} is the share of new long-term debt, u_i are the constants – the shares of the existing equity capital and preffered capital, and the share of the existing long-term debt in total capital.

The solution to the problem is the value of the decision variables which are the share of new equity capital in total capital (w_{e1}) and the share of new long-term debt capital (w_{d2}) for which the overall cost of capital is minimized. The optimal capital structure from the aspect of capital cost is a capital structure for which the overall cost of capital is minimized.

The overall cost of capital (OCC_{real}) that is minimized can be calculated as follows

$$OCC_{real} = \ k_{e1}x\ u_1 + \ k_{e2}x\ w_{e1} + \ k_p x\ u_2 + k_{d1}x\ u_3 + \ k_{d2}x\ w_{d2} \tag{33}$$

where k_{e1} is the effective cost of existing equity capital, k_{e2} is the effective cost of new equity capital, k_p is the effective cost of preffered capital, k_{d1} is the effective cost of the existing long-term debt, k_{d2} is the effective cost of new long-term debt, w_{e1} is the share of new equity capital, w_{d2} is the share of new long-term debt in total capital, u_i are the constants.

3.3.2. Calculation of corporate value

The value of corporation is expressed as a function of the expected cash flows, discounted at the overall cost of capital that is minimized. We can define the value function as

$$V = -I + \frac{FCF_1}{(1+OCC_{real})^1} + \frac{FCF_2}{(1+OCC_{real})^2} + \frac{FCF_3}{(1+OCC_{real})^3} + \frac{FCF_4}{(1+OCC_{real})^4} + \frac{FCF_5}{(1+OCC_{real})^5} + \frac{Terminal\ value_{\ after\ five\ years}}{(1+OCC_{real})^5} \quad (34)$$

where I is the investment, FCF_n represents cash flow in year n, TV is the terminal value and OCC_{real} is the overall cost of capital.

Equation (34) implies that the value of corporation which is equal to the present value of expected net cash flows, discounted at the overall cost of capital which is minimized, presents its maximized value.

Finally, we can rewrite the model of optimal capital structure from the aspect of capital cost and corporate value in an explicit form

1. The amount of capital needed can be calculated as follows

$$\text{Equity} + \text{long-term debt} = \text{long term assets} + \text{permanent current assets} \quad (35)$$

$$\text{Short-term liabilities} = \text{temporary current assets} \quad (36)$$

2. The effective cost of capital can be calculated as follows
3. The effective cost of new common stock

$$k_e = \frac{[Pr \times (1-k) - D_{ps} + AC_e]}{(s \times P_o - FC)} \quad (37)$$

where

$$AC_e = \frac{Total\ Assets\ (TA)}{Annual\ Sales\ (AS)}$$

Pr is the net profit, k is the rate of retained earnings, D_{ps} is the dividend for the existing preffered stocks, AC_e is the agency costs of equity, P_o is the value of common stock, s is the number of common stocks and FC is the flotation costs.

- The effective interest rate after tax for a simple interest bank loan with compensating balances or a deposit

$$k_d = \frac{\sum_{j=1}^{n} \frac{(N_j \times i_1/100 - C_{od} \times i_2/100)}{(1+d/100)^j}}{\sum_{j=1}^{n} \frac{(N_j - CB - C_{od})}{(1+d/100)^j}} \times (100 - t) \quad (38)$$

where k_d is the effective interest rate after tax, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, C_{od} is the amount of deposit, d is the discount rate and t is the profit tax rate expressed as percentage.
- The effective interest rate after tax for a simple interest discounted loan with compensating balances or a deposit

$$k_d = \frac{\sum_{j=0}^{n-1} \frac{(N_j \times i_1/100 - C_{od} \times i_2/100)}{(1+d/100)^j}}{\sum_{j=0}^{n-1} \frac{(N_j - N_j \times i_1/100 - CB - C_{od})}{(1+d/100)^j}} \times (100 - t) \quad (39)$$

where k_d is the effective interest rate after tax, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, Cod is the amount of deposit, d is the discount rate and t is the profit tax rate.
- The effective interest rate after tax for a bank loan with compound interest and compensating balances or a deposit

$$k_d = \frac{\sum_{j=1}^{n} \left\{ \frac{\left[N_0 \times \frac{r^n(r-1)}{r^n-1} - (N_{j-1} - N_j)\right] - \left[C_{od} \times \left(1+\frac{i_2}{100}\right)^j - C_{od} \times \left(1+\frac{i_2}{100}\right)^{j-1}\right]}{(1+d/100)^j} \right\}}{\sum_{j=0}^{n-1} \frac{(N_j - CB - C_{od})}{(1+d/100)^j}} \times (100 - t) \quad (40)$$

where r is defined by $r = 1 + \frac{i_1}{100}$
No is the principal, n is the number of annuities, Cod the amount of deposit, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, d is the discount rate and t is the profit tax rate.
- The effective interest rate after tax for a discounted bank loan with compound interest and compensating balances or a deposit

$$k_d = \frac{\left(\frac{N_0 \times i_1}{100} - \frac{C_{od} \times i_2}{100}\right) + \sum_{j=1}^{n-1} \frac{\left\{\left[N_0 \times \frac{\rho^{n-1}(\rho-1)}{\rho^n - 1} - (N_{j-1} - N_j)\right] - \left[C_{od} \times \left(\frac{100}{100-i_2}\right)^j - C_{od} \times \left(\frac{100}{100-i_2}\right)^{j-1}\right]\right\}}{(1+d/100)^j}}{\sum_{j=0}^{n-1} \frac{\left(N_j - I_j - CB - C_{od}\right)}{(1+d/100)^j}} \times (100 - t) \tag{41}$$

where I_j and ρ are defined by $I_j = \ N_0 \times \frac{\rho^{n-1}(\rho-1)}{\rho^n - 1} - (N_{j-1} - N_j) \rho = \frac{100}{(100-i_1)}$
No is the principal, n is the number of annuities, Cod the amount of deposit, i_1 is the interest rate on debt expressed as percentage, i_2 is the interest rate on deposit, Nj is the nominal amount of debt in period j, CB is the amount of compensating balances, I_j is the interest for period j, d is the discount rate and t is the profit tax rate.

- The effective cost of long-term debt after tax (k_b), obtained by coupon bonds

$$k_b = \frac{\sum_{j=1}^{n} \frac{i/100}{(1+d/100)^j}}{\sum_{j=1}^{n} \frac{(1 - d_o/100 + p/100 - fc_b/100)}{(1+d/100)^j}} \times (100 - t) \tag{42}$$

where i is the nominal interest rate, d_0 is the bond discount, p is the bond premium, fc_b are flotation costs, d is the discount rate and t is the profit tax rate.

- The effective cost of long-term debt after tax (k_b), obtained by annuity bonds

$$k_b = \frac{\sum_{j=1}^{n} \frac{\left[N_0 \times \frac{r^n(r-1)}{r^n - 1} - (N_{j-1} - N_j)\right]}{(1+d/100)^j}}{\sum_{j=0}^{n-1} \frac{\left(N_j - N_0 \times d_0/100 + N_0 \times p/100 - N_0 \times fc_b/100\right)}{(1+d/100)^j}} \times (100 - t) \tag{43}$$

where r is defined by $r = 1 + \frac{i}{100}$
i is the interest rate on debt, N_0 is the face value of a bond, Nj is the nominal amount of debt in period j, d_0 is the bond discount, p is the bond premium, fc_b are flotation costs, d is the discount rate and t is the profit tax rate.

2. The optimal capital structure from the aspect of capital cost as well as the shares of new equity and new long-term debt, for which the overall cost of capital is minimized, are found by the linear programming model

$$\mathrm{z}\ \left(\mathrm{w_{e1,}}\, \mathrm{w_{d2}}\right) = \min\ \left(\mathrm{k_{e1}x\ u_1 + k_{e2}x\ w_{e1} + k_p x\ u_2 + k_{d1}x\ u_3 + k_{d2}x\ w_{d2}}\right) \tag{44}$$

subject to

$$\sum_{i=1}^{3} u_i + \ w_{e1} + w_{d2} = 1 \tag{45}$$

$$0.56 - u_1 - u_2 \le w_{e1} \le 0.71 - u_1 - u_2 \tag{46}$$

$$0.29 - u_3 \le w_{d2} \le 0.44 - u_3 \tag{47}$$

$$w_{e1} \ge 0.05 \ \ w_{d2} \tag{48}$$

$$\mathrm{w}_{e1} > 0 \tag{49}$$

$$\mathrm{w}_{d2} \ge 0 \tag{50}$$

where k_{e1} is the effective cost of existing equity, k_{e2} is the effective cost of new equity, k_p is the effective cost of preffered capital, k_{d1} is the effective cost of the existing long-term debt capital, k_{d2} is the effective cost of new long-term debt capital, w_{e1} is the share of new equity capital in total capital, w_{d2} is the share of new long-term debt, u_i are the constants – the shares of the existing equity capital, preffered capital and long-term debt in total capital.
The overall cost of capital ($OCC_{\ real}$), that is minimized, is found by

$$\mathrm{OCC_{real}} = \mathrm{k_{e1}x\ \ u_1 + k_{e2}x\ \ w_{e1} + k_p x\ \ u_2 + k_{d1}x\ \ u_3 + k_{d2}x\ \ w_{d2}} \tag{51}$$

3. The corporate value (V) can be calculated as

$$V = -I + \frac{FCF_1}{(1 + OCC_{real})^1} + \frac{FCF_2}{(1 + OCC_{real})^2} + \frac{FCF_3}{(1 + OCC_{real})^3} + \frac{FCF_4}{(1 + OCC_{real})^4} + \frac{FCF_5}{(1 + OCC_{real})^5} + \frac{Terminal\ \ value\ _{after\ \ five\ \ years}}{(1 + OCC_{real})^5} \tag{52}$$

where I is the investment, FCF_n represents cash flow in year n, TV is the terminal value and OCC_{real} is the overall cost of capital.

4. Discussion

This paper addresses the issue of capital structure optimization and provides a framework for determining the optimal capital structure from the aspect of capital cost and corporate value. The authors provide an innovative model for arriving at a company's optimal capital structure based on the estimation of the effective cost of capital, the determination of the shares of new equity and long-term debt capital that will both minimize the overall cost of capital and maximize its value, and the calculation of corporate value. It can be used to produce an estimate of a company's optimal amount of new equity and new long-term debt. In many cases it is difficult to make a specific recommendation on how much long-term debt and equity a given company should use. However, this model can be used to produce a firm-specific recommendation of the optimal amount of new long-term debt and new equity that a given company should use.

Hundreds of research papers investigate corporate capital structure in an attempt to answer the following questions: how much debt and equity should a company use, how does the use of debt and equity affect corporate value, and how can corporations evaluate various financing alternatives as well as establish the optimal capital structure from the aspect of capital cost and corporate value. Despite all this research, a consensus view of optimal capital structure has yet to emerge. This model gives an answer to how much long-term debt and equity a company should use, and how the use of long-term debt and equity affects corporate value. The model provides explicit advice on optimal long-term debt and equity level and can be applied to produce a firm-specific recommendation about optimal capital structure that a given company should use.

The managers can use the model of optimal capital structure to produce an estimate of a company's optimal shares of new long-term debt and new equity. Thus, the model can be implemented widely and used by practitioners as it focuses on the effective cost of capital and the optimal choice of long-term debt and equity. The model can be used to evaluate various financing alternatives and establish the optimal capital structure from the aspect of capital cost and corporate value. Finally, this model presents a sound financial model that can help managers understand the impact of their capital structure decisions on corporate value before they actually make them. The optimal capital structure is reached as the choice of capital structure components that implies the overall cost of capital that is minimized as well as the corporate value that is maximized. The corporate value that is maximized is based on the present value of expected cash flows, discounted at the overall cost of capital that is minimized.

The linear programming model allows us to identify the shares of new equity capital and long-term debt that will minimize the overall cost of capital. The lower bound and the upper bound on variables, the share of new equity capital w_{e1}, the share of new long-term debt capital w_{d2}, and constants are defined by the vertical rules of financing and the determined ratio of long-term to short-term debt in the financial structure, which is 2:3. This ratio is prudently determined because the use of short-term debt for the acquisition of long-term assets is risky, while the use of long-term debt to purchase short-term assets represents a strategy with high financing costs. The results of the previously conducted empirical research of the financial structure and the share of individual components in the financial structure of joint-stock companies whose financial instruments are listed on the capital market in Croatia, Slovenia and the Czech Republic were used to confirm that the 2:3 ratio of long-term to short-term debt is rationally determined and that it is the ratio used by listed joint-stock companies. To confirm this ratio, future research should extend its analysis to a representative sample of listed companies in the European Union.

5. Conclusion

A new approach to the long-term financing management, used in this research, is a model-based approach, a paradigm that emphasizes the application of mathematical modeling principles and the optimization theory. The major findings are the new equation models for calculating the effective cost for long-term financing sources and a new model of optimal capital structure from the aspect of capital cost and corporate value.

This study has several contributions to the field. In this study, we develop a new approach which is a model-based approach, a paradigm that emphasizes the application of mathematical modeling principles and optimization theory to solve the problem of capital structure optimization on the one hand, and to provide a model for determining optimal capital structure from the aspect of capital cost and corporate value on the other. The authors provide a novel model for arriving at a company's optimal capital structure based on the estimation of the effective cost of capital and the determination of the shares of new equity capital and long-term debt capital that will both minimize the overall cost of capital and maximize its value. This model can be used to produce a firm-specific recommendation of the optimal amount of new long-term debt and new equity that a given company should use. This model gives an answer to how much long-term debt and equity a company should use, and how the use of long-term debt and equity affects the corporate value. Furthermore, the model provides valuable advice on optimal long-term debt and equity level. Finally, we provide formulas that can be easily applied to calculate the effective cost of long-term debt and equity capital, and in turn determine the optimal amount of new long-term debt and equity, for any given corporation.

A new approach to the long-term financing management based on mathematical modeling and the optimization theory motivates future research in the area of long-term financing, and it would be interesting to extend the model of optimal capital structure from the aspect of capital cost and corporate value in the future.

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