



Design and Calculation of Evaluation Index for Urban Road Anti-Blocking Ability

Ronghu ZHOU¹, Qin GE²

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ABSTRACT

Aiming at the lack of an anti-clogging ability index in the road network traffic evaluation index, an anti-clogging ability index was proposed to measure the anti-clogging ability of urban road traffic network: K-anti-clogging coefficient, which is used to measure the shortest path between any pair of starting and ending points on the urban road traffic network. After the current edge of the shortest path is blocked, the shortest path is selected from the current node of the shortest path. If the current edge of the shortest path is blocked again, the selection continues until the shortest path to the ending point is selected. In the case of unrecoverable congestion, the properties of the anti-clogging coefficient vector on any origin-destination pair, a path, and the whole traffic network are analysed, and the algorithm of the anti-clogging coefficient and its complexity are given. Finally, an example analysis is carried out using a local traffic network in a city.

¹ 2009180200@yctei.edu.cn, Economic and Trade Department, Yancheng Polytechnic College

² 20731203@qq.com, Economic and Trade Department, Yancheng Polytechnic College

KEYWORDS

urban road; traffic network; K-anti-clogging coefficient; anti-clogging ability.

1. INTRODUCTION

The urban road traffic network is the link between the city and traffic. Whether or not the layout of the road traffic network is reasonable and appropriate to the characteristics of the city determines to a large extent the efficiency of urban traffic and the development of the traffic structure [1]. Therefore, the question of how to evaluate the performance of urban road traffic networks is becoming increasingly important, which has attracted the attention of scholars and become a hot research topic.

The road traffic network as a whole can be abstracted into a transport point or intersection as a node, the road between the two points is the road section as the edge of the network map. The node represents the origin, destination or intersection; the edge represents the road segment, and the edge length is the actual mileage [2]. For convenience of discussion, the road traffic network diagram is abbreviated as traffic network.

The traffic network is denoted by G(V, E), where $V = \{v_1, v_2, v_3, \dots, v_n\}$ is the node set of G, v_1 is the starting node, v_n is the ending node, E is the set of edges in $G, e_0 \in E, i \neq j$ and e_{ij} are inseparable. The path from v_i to v_j is an alternating sequence $(v_i, e_{ij}, v_j, \dots, v_k, e_{kl}, v_l)$ of points and edges [3].

Due to the lack of research on the anti-clogging ability index of road network traffic evaluation index [4], the anti-clogging ability index, K-anti-clogging coefficient of urban road traffic network is proposed to measure the anti-clogging ability index of urban road traffic network. After the current edge of the shortest path between any pair of origin and destination pairs is blocked [5], the shortest path is selected from the current node of the shortest path. If the current edge of the re-selected shortest path selected again is blocked, selection continues until the shortest path to the endpoint is selected [6]. Assuming that the current node has a total of K congestion, the shortest path obtained after deleting K congestion edges replaces the original shortest path. In the case of irrecoverable congestion, the initial path can be replaced by the shortest path after removing the current blocking edge [7]. The vector properties of the K-anti-blocking coefficient on any origin-destination pair, a path, and the whole traffic network are analysed and the corresponding algorithm of the K-anti-blocking coefficient and its complexity are given. Finally, a local traffic network in a city is used as an example to analyse the anti-blocking ability [8].

By studying the anti-clogging ability of urban traffic networks, the K-anti-clogging coefficient is proposed. This index makes up for the lack of road network traffic evaluation and also improves the urban road network performance evaluation system, which can provide better suggestions for urban planning and road design [4, 8].

2. URBAN ROAD TRAFFIC NETWORK K-ANTI-BLOCKING FACTOR

2.1 Research on the anti-clogging ability of urban roads

In 1982, Corley and Sha [9] first proposed the most critical edge problem of the shortest path, that is, in a 2-edge connected undirected network G, there must be at least one edge e. When e is deleted from G, the shortest path length from s to t will increase to the maximum. The essence of the problem is how to find e efficiently. Since the general algorithm of the most critical edge in the network was proposed in 1989, many scholars have carried out in-depth and continuous research on some special problems and the question of how to improve the effectiveness of the algorithm. With the further application of network technology in transportation, communication, GIS and other fields, the research on network structure has become a hot issue for scholars [10–12]. The most typical representative results such as that of Nardelli et al. [12] proposed several algorithms to solve the most critical edge in the unconstrained basic network and analysed the impact on the shortest path when a routing node fails in the communication network. A computer algorithm based on a heap is given [11]. At the same time, it was pointed out that the path problem of multi-edge, multi-point, edge-point interruption and point-edge with cost needs to be further studied.

In this paper, a relatively static research approach is applied to the case where congestion information is known prior to the departure of the transport vehicle so that alternative routes can be chosen. In practice, transportation vehicles may encounter congestion many times while travelling on any path, making that path invalid, i.e. the different paths will fail [12]. Therefore, different paths cannot solve the problem of sudden blockage of paths in the process of moving. The problem of dissimilar paths is to find a path with spatial differences between the original nodes in a given transportation network, mainly the path decision when the original optimal path is unavailable due to climate and other reasons.

It is proposed that there is a blockage on the current side of the shortest path, and the index of the anti-blocking ability of the traffic network, the anti-blocking coefficient is proposed, i.e., the ratio of the alternative path to the original shortest path between any pair of points on the traffic network when there is a blockage on the current side of the shortest path between the pair of points and traffic congestion cannot be recovered [13]. Then, the anti-blocking coefficients of a series of origin-destination pairs are compared and analysed, and the anti-blocking coefficient vectors of a path and the whole network are obtained to measure the anti-blocking ability of the whole traffic network. The properties, algorithms and complexity of the algorithm, when the leading edge is blocked on the shortest path are given.

From the existing research results, it can be concluded that the research on anti-congestion capability is only limited to the situation where the current node on the shortest path between any pairs of points in the traffic network is blocked once and the congestion is not recoverable. If the front edge of the shortest path of the traffic network is blocked and the congestion is not recoverable, it is necessary to select a new shortest path from the current node. If the current edge of the new shortest path is blocked again, and so on, the current node has a total of K times of congestion in turn, what is the anti-blocking ability of the urban road traffic network? How to measure the anti-clogging ability? The original measure of the anti-blocking ability of the urban road network is the index of the anti-blocking coefficient.

In order to make up for the lack of research on anti-blocking ability in road network traffic evaluation, this paper proposes an index to measure the anti-blocking ability of urban road traffic networks, the K-anti-blocking coefficient, based on the actual needs of transportation decision makers, traffic management departments and road design [14]. It is defined from three aspects: (1) the definition of the K-anti-blocking coefficient between any pair of origin and destination on the traffic network, (2) the definition of the K-anti-blocking coefficient vector on a path, (3) the definition of the K-anti-blocking coefficient vector of the whole traffic network. The properties of the K-anti-blocking coefficient are analysed respectively, and the algorithm and complexity of the K-anti-blocking coefficient of the whole traffic network are given. The calculation results of the K-anti-clogging coefficient vector of the urban road traffic network can provide important suggestions for traffic diversion and road construction.

2.2 Definition of K-anti-clogging coefficient of urban road traffic network

In general, the vehicle has a clear destination before departure, and rational decision-makers will choose the shortest path from the origin to the destination [15]. If the vehicle encounters congestion K times during the travel process (such as natural disasters, traffic accidents, etc.) and the congestion cannot be cleared, the vehicle decision maker needs to choose the new travel path again, so the decision maker needs to understand the anti-clogging ability of each pair of origin-destination pairs in the traffic network. In fact, the anti-clogging ability between any pair of origin-destination pairs on the traffic network reflects the anti-clogging ability of the entire traffic network.

The K-anti-blocking coefficient proposed in this paper is used to measure the current edge of the shortest path between any pair of starting and ending points on the urban road traffic network. After the current edge of the shortest path is blocked, the shortest path is selected from the current node of the shortest path. If the current edge of the selected shortest path is blocked again, the selection continues until the shortest path to the endpoint is selected. Assuming that the current node has a total of K blockages, the shortest path obtained after deleting K blockages again replaces the original shortest path [16].

Given a road traffic network G(V, E), $V = \{v_1, v_2, v_3, \dots, v_n\}$ is the node set of $G, E = \{e_i | i=1, 2, \dots, m\}$ is the set of edges in G. For the purpose of discussion, v_i and v_j are a pair of starting and ending points in G, P_{ij} and $S_{(v_i,v_j)}$ are the shortest path and the shortest path length from v_i to v_j , respectively, e_i is the incidence edge of P_{ij} on v_i , e_i^1 is the incidence edge of P_{ij}^1 on v_i , e_i^2 is the incidence edge of P_{ij}^2 on v_i ,..., e_i^k is the incident edge of P_{ij}^k on v_i , h is the number of incident edges of the starting point v_i , then $0 \le k \le h$, $w(e_i)$ are the weights of e_i , as shown in *Figure 1*.

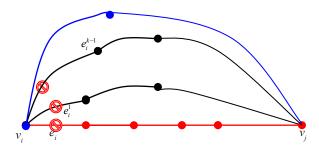


Figure 1 – K-anti-clogging coefficient diagram on the traffic network

 P_{ij}^{1} and $S^{1}(v_{i}, v_{j})$ are respectively the shortest path and the length of the shortest path from v_{i} to v_{j} after deleting e_{i} in G; P_{ij}^{2} and $S^{2}(v_{i}, v_{j})$ are respectively the shortest path and the length of the shortest path from v_{i} to v_{j} after deleting e_{i}^{1} in $G - e_{i}$. Analogy in turn: P_{ij}^{k} and $S^{k}(v_{i}, v_{j})$ are the shortest path and the shortest path length from v_{i} to v_{i} after deleting e_{i}^{k-1} in $G - e_{i} - e_{i}^{k-2}$ respectively [14].

Definition 1: K-anti-blocking coefficient of any origin-destination pair (v_i, v_j) on road traffic network is $\beta^k = \frac{S^k(v_i, v_j)}{V_i}$

$$P_{net} = S(v_i, v_j)$$

The K-anti-blocking coefficient in Definition 1 gives the calculation method of the anti-blocking ability between any origin-destination pairs on the traffic network. If the K-anti-clogging coefficient of any origin-destination pair on the traffic network is larger, the anti-clogging ability between the origin-destination pairs is worse. If the K-anti-blocking coefficient of any origin-destination pair is smaller, the anti-blocking ability between the origin-destination pairs is stronger [17].

Let $P_{1,lp+1} = P(1, l_p + 1)$ be the shortest path from v_i to v_{lp+1} , and let N(i,j) denote the set of nodes on $P(1, l_p + 1)$ except v_i and v_j . Then, $N(i,j) = \{v_1, ..., v_{lp+1}\} \setminus \{v_i, v_j\}$, |N(i,j)| are the number of nodes in N(i,j), v_i and v_j are any points on $P(1, l_p + 1)$.

The anti-blocking coefficient of any point pair on $P(1, l_p + 1)$ is $\beta_{net}^k = \frac{S^k(v_i, v_j)}{S(v_i, v_j)}$ thereby giving the following definition.

Definition 2: The K-anti-blocking coefficient vector of a path is defined as $\beta_p^k = (\beta_p^{k,1}, ..., \beta_p^{k,p})$, where $\beta_p^{k,h} = \max_{\substack{i \neq j, |N(i,j)=h}} \{\beta_p^{k,h}(i,j) | h = 1, 2, ..., l_p\}.$

The definition of the K-anti-blocking coefficient vector in Definition 2 gives the calculation method of the anti-blocking ability index on a path.

Let l be the number of intermediate nodes on the shortest path with the maximum number of intermediate nodes in G, and then make the following definition.

Definition 3: The K-anti-blocking coefficient vector of the road traffic network is defined as $\beta_{net}^k = \beta_{net}^{k,1}, ..., \beta_{net}^{k,1}$, where $\beta_{net}^{k,\theta} = \max_{i \neq j \mid N(i,j)=m} \{\beta_{net}^{k,\theta}(i,j) \mid \theta = 1,2,3,..., l\}, \theta, \theta$ are the number of nodes except v_i and v_j on the shortest path with v_i and v_j as the starting and ending points [18].

The K-anti-blocking coefficient vector in Definition 3 reflects the anti-blocking ability of the entire traffic network. The K-anti-blocking coefficient of each pair of origin-destination points in the network is classified according to the number of intermediate nodes on the shortest path connecting each pair of origin-destination points. Those with the same number of nodes are classified as one class. The maximum value of the K-anti-blocking coefficient is found in the same class. A vector composed of the maximum values of K-anti-block-ing coefficients of different classes is given, which indicates that if the front edge of the shortest path of the traffic network is blocked and the congestion cannot be recovered, a new shortest path must be selected from the current node. If the current edge of the new shortest path is blocked again, repeat the above steps. The current node has the worst substitution effect of the shortest path and the original shortest path after K times of blocking.

2.3 Properties of K-anti-clogging coefficient of urban road traffic network

A. The properties of the starting and ending point pairs (v_i, v_j) , K-anti-clogging coefficient $\beta_{net}^k(i, j)$

a. Let N(i,j) denote the set of nodes on $P_{i,j}$ except v_i and v_j , and $N(i,j) = \{v_1, ..., v_{lp+1}\} | \{v_i, v_j\}$, then, $P_{i,j} = \{e(v_i, v_1), e(v_1, v_2), ..., e(v_i, v_j)\}$.

Herein, let P_{ij}^k (k = 1, 2, ..., l) denote the shortest path from v_1 to v_j in *G*. At this point, when k = 1, 2, ..., l, and *i*, j do not belong to k.

If $N(i,j) = \emptyset$, then $P_{ij} \{ e(v_i, v_j) \}$, where P_{ij}^1 is the shortest path from v_1 to v_j after removing the edge $e(v_i, v_j)$ in G.

 P_{ij}^2 is the shortest path from v_1 to v_j in G without edges $e(v_i, v_j)$ and $e(v_i, v_1^1)$.

 P_{ij}^k is the shortest path from v_1 to v_j after removing the edge, $e(v_i, v_j)$, $e(v_i, v_1^1)$, ..., $e(v_i, v_1^{k-1})$ in G.

For the study of anti-blocking ability, it is proposed that there is a blockage on the current side of the shortest path, and the anti-blocking coefficient is proposed. The anti-blocking coefficient of a series of origin-destination pairs is compared and analysed, and then the anti-blocking coefficient vector of a path and the entire network is obtained to measure the anti-blocking ability of the entire transportation network, and the properties, algorithms and the complexity of the algorithms are given. The specific lemmas are as follows.

Lemma 2.1. For a given i, j, if $v_r \in N(i, j)$, then $\beta_{net}(i, r) \ge \beta_{net}(i, j)$ holds.

It is proved that the shortest path from v_i to v_j must be on P_{ij} from $P_{ij} = \{e(v_i, v_1), e(v_1, v_2), ..., e(v_l, v_j)\}$, and from the definition of $\beta_{net}^k(i,r) : \beta_{net}^k(i,r) = \frac{S^k(v_1, v_r)}{S(v_1, v_1)}$, because $S^k(v_i, v_r) \ge S(v_i, v_r)$.

From A:
$$\beta_{net}(i,r) = \frac{S^k(P_{ir})}{S(P_{ir})} \ge \frac{S^k(P_{ir}) + S(P_{rj})}{S(P_{ir}) + S(P_{rj})}$$
 because $\frac{S^k(P_{ir}) + S(P_{rj})}{S(P_{ir}) + S(P_{rj})} = \frac{S^k(P_{ir}) + S(P_{rj})}{S(P_{ij})}$.

Based on the definition of P_{ij}^k , it can be obtained that: $S^k(P_{ij}) = S^k(P_{ii+1}) + S(P_{i+1j})$.

For the above expression, there are two cases:

1) if r=i+1 for this case, r is the first node in the point set of N(i,j), therefore there exists: $S^{k}(P_{ij}) = S^{k}(P_{i+1}) + S(P_{i+1j})$ 2) if r=i+1, for this case, r is one of the nodes in the point set of N(i,j) excluding the first node and the last node, therefore there exists: $S^k(P_{i\,i+1}) < S^k(P_{ir})$, further substituting $S^k(P_{ir})$ for $S^k(P_{i\,i+1})$, the following expression can be obtained: $S^k(P_{ij}) < S^k(P_{ir}) + S(P_{rj})$.

Consider the above two cases together, i.e. there exists a node *r* for which the following expression exists: $S^{k}(P_{ij}) \leq S^{k}(P_{ir}) + S(P_{rj})$.

Further, dividing $S(P_{ij})$ on both side of the inequality simultaneously, results in the following expression:

$$\frac{S^{k}(P_{ir}) + S(P_{ij})}{S(P_{ij})} \ge \frac{S^{k}(P_{ij})}{S(P_{ij})} \cdot$$

 $\beta_{net}^{k}(i,r) \ge \frac{S^{k}(P_{ij})}{S(P_{ij})} \text{ from (2) because } \beta_{net}^{k}(i,j) \ge \frac{S^{k}(P_{ij})}{S(P_{ij})} \text{ so } \beta_{net}^{k}(i,r) \ge \beta_{net}^{k}(i,r) \text{ (2.2)}.$ Because the meanings of $\beta_{net}(i,j)$ and $\beta_{net}(j,i)$ are different, for a general undirected graph network, if v_{1} and

Because the meanings of $\beta_{net}(i, j)$ and $\beta_{net}(j, l)$ are different, for a general undirected graph network, if v_1 and v_j are non-adjacent vertices, $\beta_{net}(i, j)$ is generally not symmetrical, but when v_i and v_j are adjacent vertices, $\beta_{net}^k(i, j)$ and $\beta_{net}(j, i)$ are symmetrical [9, 15].

Corollary 2-1. If $P_{0,i+1}$ is the shortest path from v_0 to v_{l+1} , let $N(i,j) = \{v_1, v_2, ..., v_l\}$, then $\beta_{net}^k(0,1) \ge \beta_{net}^k(1,2) \ge ... \ge \beta_{net}^k(0,l+1)$ holds.

b. K-anti-blocking coefficient $\beta_{net}^{k}(i,j)$ on the origin-destination pair (v_i, v_j) , then $\beta_{net}^{k}(i,j) \ge \beta_{net}^{k-1}(i,j)$. Proof: $\beta_{net}^{k}(i,j) = \frac{S^{k}v_i, v_j}{S(v_i, v_j)}, \ \beta_{net}^{k-1}(i,j) = \frac{S^{k-1}v_i, v_j}{S(v_i, v_j)}$ According to the definition of K-anti-clogging coefficient of $(v_i, v_j), S^{k}(v_i, v_j) \ge S^{k-1}(v_i, v_j), \ \frac{S^{k-1}(v_i, v_j)}{S^{k}(v_i, v_j)} \le \frac{S^{k}(v_i, v_j)}{S(v_i, v_j)}$ (2.3).

B. Properties of K-anti-clogging coefficient vector β_p^k on a path

Theorem 2-1. If $P_{0,l+1}$ is the shortest path from v_0 to v_{l+1} , then $\beta_p^{k,0} \ge \beta_p^{k,1} \ge ... \ge \beta_p^{k,l}$ holds. Proof: Let $\beta_p^{k,r}$ be the maximum of $\beta_p^k(i-1, i+r)$ and $\beta_p^k(i+r, i-1)$, where i=1,2,...,l-rMake $\beta_p^{k,r} = \beta_p^k(i-1, i+r)$ again.

From Corollary 1, it can be seen that there must be $\beta_p^k(i-1, i+r) \le \beta_p^k(i-1, r-1)$ on a shortest path $P_{i-1,i+r}$ with point pair (v_{i-1}, v_{i+r}) as the starting and ending points [16].

 $\beta_p^{k,r-1} \text{ is the maximum value of } \beta_p^k(i-1, i+r-1) \text{ and } \beta_p^k(i+r-1, i-1), \text{ where } i = 1, 2, ..., l-r+1.$ So $\beta_p^{k,r} = \beta_p^k(i-1, i+r) \le \beta_p^k(i-1, i+r-1)$ is established, that is, $\beta_p^{k,r} \le \beta_p^{k,r-1}$ is established. $\beta_p^{k,r} \le \beta_p^{k,r-1} \le \beta_p^{k,r-2} \le ... \le \beta_p^{k,0}$ is established by analogy.

C. Properties of K-anti-blocking coefficient vector β_{net}^k on road traffic network

Theorem 2-2. The K-anti-blocking coefficient on the traffic network satisfies inequality $\beta_{net}^{k,0} \ge \beta_{net}^{k,l} \ge ... \ge \beta_{net}^{k,l}$, where *l* is the number of intermediate nodes on the shortest path with the largest number of intermediate nodes in *G* [17, 18].

Proof: Let $\beta_{net}^{k,r} = \beta_{net}^{k,r}(i, j)$, then there are *r* intermediate nodes besides (v_i, v_j) on the shortest path between point pairs (v_i, v_j) . Without loss of generality, it can be assumed that j=i+r+1, and the intermediate node between point pairs (v_i, v_j) is $\{(v_i, v_{i+1}, ..., v_{i+r}, v_{i+r+1})\}$.

By Lemma 2-1,
$$\beta_{net}^{k,r} = \beta_{net}^{k,r}(i, i+r+1) \le \beta_{net}^{k,r-1}(i, i+r)$$
 (2.4)
 $\beta_{net}^{k,r}(i, i+r) \le \beta_{net}^{k,r-1}$

So $\beta_{net}^{k,r} = \beta_{net}^{k,r}(i, i+r+1) \le \beta_{net}^{k,r-1}(i, i+r) < \beta_{net}^{k,r-1}$ is established.

Let *l* be the number of intermediate nodes on the shortest path with the largest number of intermediate nodes in *G*, and so on, then $\beta_{net}^{k,0} \ge \beta_{net}^{k,1} \ge \dots \ge \beta_{net}^{k,l}$ holds.

3. ALGORITHM FOR THE K-ANTI-BLOCKING COEFFICIENT

3.1 K-anti-clogging coefficient algorithm

For a given road traffic network G(V, E), G is still connected after removing any edge. Let $N(v_i)$ be the set of adjacent nodes of v_i , $N(v_i) = \{v_{i1}, v_{i2}, ..., v_{id(i)}\}$, where d(i) is the degree of v_i , $k \le d(i) - 1$.

Step 1. For any node v_i , apply the Dijkstra labelling method to calculate the shortest path P_{ij} and the shortest path length $S(v_i, v_j)$ from v_i to any node v_j , where j=1,2..., N-1, noting that when applying the Dijkstra labelling method, the following information can be obtained:

- 1) The number |N(i, j)| of intermediate nodes on P_{ij} ;
- 2) The edge $e(v_i, v_{i+1}^k)$ associated with v_i in P_{ij} , where $e(v_i, v_{i+1}^k)$. Step 2.
- 1) After deleting $e(v_i, v_{i+1})$ on P_{ij} in traffic network *G*, the shortest path P_{ij}^1 and the shortest path length $s^1(v_i, v_j)$ from v_i to v_j are calculated by using Dijkstra's labelling method on graph $G e(v_i, v_{i+1})$.
- 2) $e(v_i, v_{i+1}^1 \text{ on } P_{ij}^1 \text{ is deleted in the traffic network } G-e(v_i, v_{i+1}), \text{ and the shortest path } P_{ij}^2 \text{ and the shortest path length } s^2(v_i, v_j) \text{ from } v_i \text{ to } v_j \text{ are calculated in the graph } G-e(v_i, v_{i+1}) e(v_i, v_{i+1}^1) \text{ by using Dijkstra labelling method again. } e(v_i, v_{i+1}^{k-1}) \text{ is deleted in the traffic network } G-e(v_i, v_{i+1}) e(v_i, v_{i+1}^1) \dots e(v_i, v_{i+1}^{k-2}) \text{ by analogy, and Dijkstra's labelling method is used again to calculate the shortest path } P_{ij}^k \text{ and the shortest path length } s^k(v_i, v_i) \text{ from } v_i \text{ to } v_i.$
- 3) Repeating the first step to the second step and taking j=1,2,...,N-1, $i\neq j$, all the shortest path P_{ij}^k and the shortest path length $s^k(v_i,v_j)$ can be obtained.

Step 3. Calculate
$$\beta_{net}^{k} = \frac{S^{k}(v_{i}, v_{j})}{S(v_{i}, v_{j})}, \ \beta_{net}^{k,t}(i) = \max_{|N(i,j)|=t, j\neq i} \{\beta_{net}^{k,t}(i,j) | j = 1, 2, ..., n-1, t = 0, 1, 2, ..., l\}.$$

Step 4. Repeat steps 1 to 3, take $i=1, 2, ..., N$, that is $N=|V|$, and all $\beta_{net}^{k,t}(i,j)$ and $\beta_{net}^{k,t}(i)$ can be obtained.

Step 5. Calculate $\beta_{net}^{k,t} = \max_{1 \le i \le N} \{ \beta_{net}^{k,t}(i) | t = 0, 1, 2, ..., l \}.$

Step 6. Output $\beta_{net}^{k,t}(i,j), \beta_{net}^{k,t}(i), \beta_{net}^{k,t}$, where $i \in \{1, 2, ..., N\}, j \in \{1, 2, ..., N\}, t = 0, 1, 2, ..., l$.

Through the above analysis, it can be seen that *L* is the maximum value $(j \neq i)$ of the number of intermediate nodes on the shortest path from node v_i to any node v_i .

3.2 K-anti-clogging coefficient algorithm complexity analysis

For a traffic network with n vertices and m edges, the complexity of the K-anti-blocking algorithm is as follows:

- 1) The number of calculations in the first step of the algorithm is $O(n^2)$;
- 2) The number of calculations in the second step of the algorithm is $d(i) \cdot O(n^2)$; the number of calculations required to apply the Dijkstra labelling method once is $O(n^2)$. Because it is necessary to apply the Dijkstra labelling method to v_{ii} , r=1,2, ..., d(i), respectively, in the worst case r=n, the number of calculations is $d(i) \cdot O(n^2)$;
- 3) Step 4 of the algorithm can be completed in O(n) time;
- 4) Step 1 to Step 5 of the algorithm completes the calculation of $\beta^k(i,j)$ and $\beta^{k,l}(i)$ starting from node v_i , and the total amount of calculation is $d(i) \cdot O(n^2)$. When v_i takes over any node in *V*, the total amount of calculation

is:
$$\sum_{i=1}^{N} d(i) O(n^2) = O(|E|) n^2 = O(n^4);$$

5) In Step 6, for a given L, only $\beta^{k,\theta}(i)$ needs to be computed, and the total amount of computation is $O(n^4)$.

Therefore, the complexity of K-anti-blocking coefficient vector algorithm on road traffic network is (n^4) . In the actual situation, the road traffic network is similar to the plane network. In the plane network, the calculation amount of the shortest path between any two points is $O(n^4)$ by Dijkstra labelling method, and there is $\sum_{i=1}^{N} d(i) = O(|E|) = O(n)$. The calculation amount from Step 1 to Step 5 in the algorithm is

 $n(n-1) \cdot O(n^2) = O(n^4)$. Therefore, on a planar network, the algorithm can calculate the K-anti-blocking coefficient vector β_{net}^k in $O(n^4)$ time[18].

4. A CASE STUDY OF LOCAL TRAFFIC NETWORK

Using the anti-clogging ability index of the urban road traffic network anti-clogging coefficient algorithm, the anti-clogging ability of the local road network in a new urban area of a city is calculated and analysed. The case analysis of the local traffic network in a new urban area of a city is mainly from two perspectives. In Section 4.1 we calculated the anti-blocking ability of two local road networks in a new urban area and compared the corresponding conclusions. In Section 4.2 we calculated the change in anti-blocking ability before and after the addition of new road sections in a new urban area of a city and drew conclusions from the comparison.

4.1 Comparison of anti-blocking ability

In the following section, the properties and algorithm of the K-anti-blocking coefficient are given in the first two subsections. Taking K=2 as an example, the anti-blocking ability of two local road networks in a new urban area is calculated, as listed in *Figure 2*.

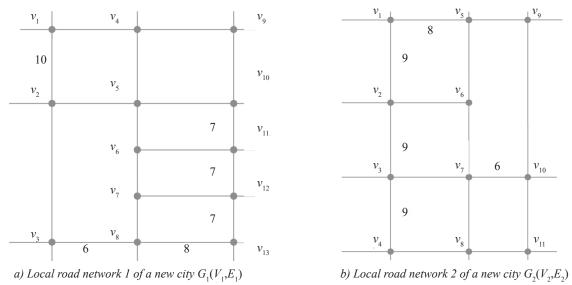


Figure 2 – Abstract map of two local traffic networks in a city

The local road network 1 of the new urban area mainly includes Wanshou Road, Hansen Road, Hekangle Road, etc. and the local road of the new urban area. The network 2 mainly includes the East Second Ring Road, Changying East Road, Xinhe Road and Changle Middle Road. The calculation results of the 2-anti-blocking coefficient of local road network 1 and local road network 2 in the new urban area are analysed and compared.

Suppose that all nodes of $m \le 2$ are not considered in the two local road networks of a new urban area of a city, and *m* is the number of edges associated with a node. In order to facilitate the calculation and analysis, the local road network 1 of a new city and the local road network 2 of a new city are abstracted into traffic network diagrams $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$, as shown in *Figure 2*.

In the traffic network graph $G_1(V_1, E_1)$, $V_1 = (v_1, v_2, ..., v_{13})$ is the set of nodes of $G_1(V_1, E_1)$, E_1 is the set of edges of $G_1(V_1, E_1)$. In the traffic network graph $G_2(V_2, E_2)$, $V_2 = (v_1, v_2, ..., v_{11})$ is the set of nodes of $G_2(V_2, E_2)$, E_2 is the set of edges of $G_2(V_2, E_2)$. The distance measured on the map is used as the weight between two points.

The anti-blocking ability of local road network 1 in a new urban area of a city is calculated by the calculation method of the K-anti-blocking coefficient. When K=2, the results of $\beta_{net}^2 (v_{i}, v_j)$ are shown in *Table 1*, and $\beta_{net}^2 = (\beta_{net}^{2,0}, \beta_{net}^{2,1}, \beta_{net}^{2,2}, \beta_{net}^{2,3}) = (8,29/8,7/3,47/27)$ can be obtained.

$\beta_{net}^2 = \left(v_i, v_j\right)$	v ₂	v_4	<i>v</i> ₅	v ₆	v_7	$v_8^{}$	v_{10}	$v_{_{11}}$	v ₁₂
v ₂		29/8	8	41/13	2	47/27	4	7/3	12/7
v_4	2		13/5	57/17	25/12	48/31	5/3	37/25	11/8
v ₅	8	13/5		47/7	20/7	37/7	25/4	43/15	18/11
V ₆	41/13	57/17	47/7		37/7	20/7	29/15	11/4	29/5
v ₇	17/10	25/12	20/7	47/7		47/7	18/11	29/15	11/4
v_8	43/17	47/31	37/21	20/7	47/7		41/29	21/11	11/3
v_{10}	4	16/9	7/2	7/3	21/11	49/29		43/7	25/7
<i>v</i> ₁₁	7/3	39/25	29/15	11/4	29/15	18/11	37/7	_	37/7
<i>v</i> ₁₂	3/2	27/16	18/11	29/15	11/4	29/15	22/7	37/7	

Table 1 – 2-anti-clogging coefficient $\beta_{net}^2 = (v_i, v_j)$ between arbitrary point pairs of local road network 1 in the new urban area

From the above calculation results, it can be seen that, in the traffic network $G_1(V_1, E_1)$ of the abstract graph of local road network 1 in a new urban area of a city, the maximum 2-anti-blocking coefficient of the point to (v_2, v_5) is 8, that is, the shortest path obtained after removing the upper (v_2, v_5) and (v_1, v_2) of the traffic network is 8 times of the original shortest path. It shows that the anti-blocking ability between the point pairs is the worst. If the shortest path between the point pairs contains 1 or more nodes, the 2-anti-blocking coefficient of the point pairs is less than 8, which indicates that the anti-blocking ability between the point pairs is stronger than that of (v_2, v_5) .

It shows that the ratio of the alternative path to the shortest path is the largest when there are two congestions on the shortest path between (v_2, v_5) point pairs in the whole traffic network. (v_2, v_5) cannot be blocked, and the performance of the traffic network is the worst once the congestion occurs.

The anti-clogging ability of traffic network graph $G_2(V_2, E_2)$ is calculated by the algorithm of the K-anti-clogging coefficient. The calculation result of $\beta_{net}^2(v_{i,}v_j)$ is shown in *Table 2*, and $\beta_{net}^2 = (\beta_{net}^{2,0}, \beta_{net}^{2,1}, \beta_{net}^{2,2}) = (7,57/15,25/7)$ can be obtained.

$\beta_{net} = (i, j)$	v ₂	v ₃	<i>v</i> ₅	v ₆	v ₇	$v_8^{}$	v_{10}
v ₂		43/9	35/17	17/9	35/17	22/13	41/23
v ₃	43/9	_	22/13	35/17	13/4	35/17	16/7
v ₅	47/17	19/13		39/9	34/18	43/27	5/3
v ₆	13/4	41/17	25/9	_	39/9	47/18	33/15
v ₇	35/17	13/4	17/9	39/9		25/9	40/6
v ₈	19/13	29/17	43/27	17/9	25/9		31/9
<i>v</i> ₁₀	41/23	25/7	21/12	33/15	7	57/15	

Table 2 – 2-anti-clogging coefficient $\beta_{net}^2 = (v_i, v_j)$ between 2 arbitrary point pairs of local road network in the new urban area

From the above calculation results, it can be seen that in the given local road network 2, the 2-anti-blocking coefficient of the point pair (v_{10}, v_7) is larger than 7, that is, the shortest path obtained after removing (v_{10}, v_7) and (v_{10}, v_{11}) is 7 times the original shortest path, indicating that the anti-blocking ability between (v_{10}, v_7) point pairs is relatively poor. When the shortest path between other pairs of nodes in the network contains more than 1 node, the 2-anti-blocking coefficient is less than (v_{10}, v_7) 's 2-anti-blocking coefficient 7, indicating that the anti-blocking ability between other pairs of nodes is better than (v_{10}, v_7) 's.

By comparing the 2-anti-blocking coefficient vectors of the new urban road network 1 and the new urban road network 2, it can be found that:

- 1) When the number of nodes contained between pairs is 0, the 2-anti-blocking coefficient of local road network 1 is greater than that of local road network 2, indicating that the anti-blocking ability of local road network 2 is stronger.
- 2) When the number of nodes between the pairs is 1, the 2-anti-blocking coefficient of local road network 1 is less than that of local road network 2, which indicates that the anti-blocking ability of local road network 1 is stronger.
- 3) When the number of nodes between the pairs is 2, the 2-anti-blocking coefficient of local road network 1 is smaller than that of local road network 2, which indicates that the anti-blocking ability of local road network 1 is stronger.

From the comparison results of the K-anti-blocking coefficient of the two different local road networks, it can be seen that the comparison of the K-anti-blocking coefficient of the traffic network with the same number of nodes is relatively comparable.

4.2 Comparison of anti-clogging ability before and after adding road sections

With the development of the economy, the original urban roads can no longer meet the needs of society for roads. Every year, large cities will continue to add new road sections or extend the original roads to connect with other roads in the city, forming a well-developed urban road traffic network, which provides great convenience for people's passage and cargo transportation. What is the anti-blocking ability of the city after adding new road sections through the urban road traffic network? Is it strengthened or weakened?

This paper takes the local traffic network of a new city as an example to study the anti-blocking ability of urban road traffic networks due to the increase of new road sections. Assuming K=2, the 2-anti-clogging coefficient of the region in 2020 and 2021 is calculated and compared.

By comparing the 2020 and 2021 maps of the local road network in the new urban area, it can be found that new road sections are added to the area in 2021. In order to facilitate calculation and simplification, the above map of the local road network in the new urban area is abstracted into an urban road traffic network map, as shown in *Figure 3*. In the abstract map of the local road network traffic network in the new urban area, the bold edges are the newly added road sections in 2021, and the actual measured distance on the map is used as the weight of the edges in the traffic network map.

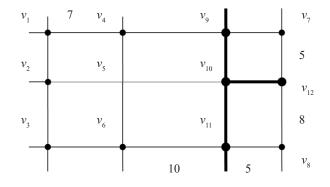


Figure 3 – Abstract map of traffic network of local road network in different years

When no new roads are added to the local road network of a new city, the 2-anti-clogging coefficient $\beta_{net}^2 = (i, j)$ is calculated, as shown in *Table 3*.

Table 3 – The 2-anti-clogging coefficient $\beta_{net}^2 = (i, j)$ of the local road network in the new urban area of a city in 2020

$\beta_{net}(ij)$	v ₂	v_4	v ₅	v ₆	
v ₂		7/3	23/7	5/3	
v_4	7/2		7	43/13	
v ₅	23/7	27/5		19/4	
v ₆	3	43/13	19/4		

It can be seen from *Table 3* that when there is no new road section, the 2-anti-blocking coefficient of the local road network is $\beta_{net}^2(i, j) = (\beta_{net}^{2,0}, \beta_{net}^{2,1}) = (7,7/2)$, and the 2-anti-blocking coefficient of the local road network (v_4, v_5) is 7, which indicates that the shortest path after removing the edges (v_4, v_5) and (v_1, v_4) on the traffic network is 7 times that of the original shortest path. When the shortest path between the point pairs contains more than 1 node, the 2-anti-blocking coefficient of the point pair is less than 7, which indicates that the anti-blocking ability of other point pairs is stronger than that of (v_4, v_5) .

After adding new sections to the local road network in the new urban area, the 2-anti-blocking coefficient $\beta_{net}^2 = (\beta_{net}^{2,0}, \beta_{net}^{2,1}, \beta_{net}^{2,2}) = (26/5, 8/3, 19/4)$ of the local road network in the new urban area of a city in 2021 is calculated. The results are listed in *Table 4*.

$\beta_{net}(i,j)$	<i>v</i> ₂	v_4	<i>v</i> ₅	v ₆	v_9	v_{10}	<i>v</i> ₁₁	<i>v</i> ₁₂
v ₂	_	5/3	23/7	5/3	19/11	33/17	7/5	19/4
v_4	8/3		5	33/13	17/5	29/15	37/23	17/10
v ₅	23/5	5		7/2	29/15	13/5	16/9	31/15
v ₆	7/3	33/13	7/2		37/23	16/9	4	37/23
<i>v</i> ₉	16/11	3	5/3	33/23		5	33/13	3
V ₁₀	33/17	31/15	13/5	14/9	5	_	7/2	23/5
<i>v</i> ₁₁	7/5	33/23	14/9	31/10	23/10	7/2		33/13
<i>v</i> ₁₂	19/11	9/5	31/15	33/23	13/5	26/5	23/13	

Table 4 – 2-anti-clogging coefficient $\beta_{net}(i,j)$ of local road network in the new urban area of a city in 2021

From the calculation results, it can be seen that after the new road section is added to the local road network in 2021, the 2-anti-clogging coefficient of the point pair (v_{10}, v_9) is 26/5, that is, the shortest path of (v_{10}, v_9) after removing the edge (v_{10}, v_{12}) and (v_{10}, v_9) is 26/5 times that of the original shortest path. When the number of nodes included in the shortest path between other point pairs on the traffic network exceeds 1, the 2-anti-clogging coefficient is less than 26/5, indicating that the anti-clogging ability of these point pairs is stronger than that of (v_{10}, v_9) .

By comparing the 2-anti-clogging coefficient before and after the addition of new sections to the local road network, the following conclusions are drawn:

- When the number of nodes between the traffic network nodes is 0, the 2-anti-blocking coefficient of the traffic network before the new road section is not added to the local road network is 7, which is greater than the 2-anti-blocking coefficient of the traffic network after the new road section is added. In addition, 26/5 indicates that the local road network has improved its anti-blocking ability after the addition of new road sections.
- 2) When the number of nodes between the traffic network node pairs is 1, the 2-anti-blocking coefficient of the traffic network before the new road section is not added to the local road network is 7/2, which is greater than the 2-anti-blocking coefficient of the traffic network after the new road section is added. 8/3, indicating that the anti-blocking ability of the local road network is improved after the new road section is added.

By comparing the K-anti-clogging coefficient vector of the local traffic network in the new urban area before and after the increase of the road section, it can be seen that the anti-clogging ability of the region after the increase of the new road section is stronger than the anti-clogging ability before the increase of the road section. Therefore, each big city should constantly build new roads to reduce the possibility for urban road traffic network congestion, reduce the economic and time loss caused by congestion, and accelerate the development of the social economy.

In practice, the urban road traffic network is generally a grid network or a star network. Each vector in the anti-blocking coefficient vector of this network is relatively small, so the anti-blocking ability of the traffic network is still strong. In the actual urban road network, there are many nodes, and the data must be processed by a computer to obtain the K-anti-blocking coefficient vector.

5. CONCLUSION

In this paper, the anti-clogging ability index K-anti-clogging coefficient of urban traffic network is proposed to measure the anti-clogging ability of urban traffic network from the needs of transportation decision makers to choose path schemes, traffic personnel to formulate traffic flow allocation schemes and road planning and design departments to formulate road planning. Firstly, the definition of the K-anti-clogging coefficient between any origin and destination pairs on the traffic network, the anti-clogging coefficient vector on a path, and the K-anti-clogging coefficient vector on the whole traffic network are given. After that, the properties of the K-anti-clogging coefficient are analysed, and the algorithm and algorithm complexity of the K-anti-clogging coefficient vector of two local traffic networks to compare the anti-clogging ability of the two local traffic networks, (2) calculating and analysing the K-anti-clogging coefficient vector before and after adding new sections in a city. Compare the anti-blocking ability before and after adding new road sections, it is found that the anti-blocking ability after adding a new road section is stronger than that before adding a new road section.

By applying the research results of this paper, decision makers can consider the factor of traffic network congestion when selecting the path, that is, decision makers can select the path with a small anti-clogging coefficient through the analysis results of the anti-clogging coefficient of the traffic network. For the analysis of the anti-blocking ability of a single traffic network, if each vector in the K-anti-blocking coefficient vector of the traffic network is relatively small, and the size of each vector does not change much, it shows that the anti-blocking ability of the traffic network is strong. On the contrary, if each vector in the K-anti-blocking coefficient vector ficient vector of the traffic network is relatively large, and the size of each vector changes greatly, it shows that the anti-blocking ability of the traffic network is poor. If the anti-blocking ability of the two traffic networks is compared, the K-anti-blocking coefficient vectors of the two traffic networks are calculated respectively.

By calculating the K-anti-blocking coefficient vector of the urban road traffic network, the anti-blocking ability of the urban road traffic network is analysed. Due to the large number of nodes and edges in the urban traffic network, the calculation of the K-anti-blocking coefficient vector is relatively large, which needs to be calculated by computer. For the anti-blocking coefficient vector of each small area in the urban road traffic network, a database can be established by computer and the calculation results can be input into the database. It can be used by relevant departments to query and use the data of the K-anti-blocking coefficient of the traffic network in the future to face some sudden traffic congestion problems and reduce the loss caused by traffic congestion. It can also provide good suggestions for the traffic management of the transportation department and the planning department of the urban road traffic network.

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周荣虎 葛琴

城市道路防堵能力评价指标设计与计算

摘要

针对路网交通评价指标中缺乏防堵能力指标的问题,本文提出了衡量城市道路交通 网防堵能力的防堵能力指标: K-抗堵塞系数,该系数可用于衡量城市道路交通网络 中任意一对起点和终点之间的最短路径。当最短路径的当前边被堵塞后,从最短路 径的当前节点继续选择最短路径;如果最短路径的当前边缘再次被阻断,则继续选 择,直到选择到终点的最短路径。在无法恢复的拥堵情况下,分析了任何起点-目的 地、一条路径和整个交通网络上的抗堵塞系数向量的特性,并给出了抗堵塞系数的 算法和算法特定。最后,以某城市的交通网络为例进行了分析。

关键词

城市道路; 交通网络; K-抗堵塞系数; 抗堵能力