

# H-infinity Variable-Pitch Control for Wind Turbines Based on Takagi-Sugeno Fuzzy Theory

Shengsheng QIN, Sze Song NGU\*, Chaoqun ZHANG, Hui CAI, Yujian CHEN, Ruiqi CHEN, Tingxuan LIU

**Abstract:** When the wind speed is above the rated value, the output power of the wind turbine should be maintained at the rated value in order to prevent the power generation system from overheating. In addition, the natural wind speed will fluctuate randomly in a large range of values, making the traditional control effect not ideal. This paper presents a novel H-infinity ( $H_\infty$ ) pitch control strategy for Wind Turbine Generators (WTGs), which can make the rotor speed and output power constant when the wind speed changes in a large range. In order to shorten response time and reduce overshoot, in the specific solution, the control method combines the  $H_\infty$  theory and the Takagi-Sugeno (T-S) fuzzy theory. Firstly, the linearized models of several operating points were obtained with the T-S fuzzy theory. Then, a robust controller was designed for each linear sub-system based on the  $H_\infty$  control theory. Furthermore, the controllers of the sub-systems were superimposed into a global controller for the entire system through the membership function. Finally, modeling and simulation were carried out in MATLAB/SIMULINK. The simulation results show that when the wind speed changes above the rated speed, the rotor speed can be maintained at the rated value, and the output power also can be maintained at the rated value. Compared with the optimal control, the response speed of this method is faster and the overshoot is smaller. It provides a new idea for the pitch angle control of wind turbine.

**Keywords:** H-infinity ( $H_\infty$ ); pitch control; Takagi-Sugeno (T-S) fuzzy theory; wind speed model; wind turbine generator (WTG)

## 1 INTRODUCTION

Due to energy shortage and environmental pollution, wind power is considered to be the most promising new energy because of its abundancy, renewability and cleanness. In recent years, WTG is developing from fixed pitch to variable pitch. In the high wind speed section, the pitch angle controller starts to function, changing the pitch angle of the fans, and limits the energy captured by the WTG, thereby limiting the fan speed and output power to the rated value [1-4]. In order to achieve a better control, many researchers have done a lot of researches. Zhang et al. [5] adopted the traditional Proportional-Integral-Derivative (PID) control, which is simply structured, independent of any precise control model, and highly stable. However, a PID or Proportional-Integral (PI) controller cannot achieve the desired effect when the problems have the characteristics of strong coupling, nonlinearity, time delay, and wind condition changes. Zheng [6] used single-neuron PID for the control system. Although the control effect was improved, the wind turbine system was a dynamic one while the single neuron is static, so this strategy is not suitable for wind turbines. Based on the artificial bee colony (ABC) algorithm, Yuan et al. [7] conducted an optimization study on the parameters of the pitch controller for large wind turbines. Qinet al. [8] adopted the feedback linearization theory in combination with the  $H_\infty$  control theory, which achieved a better control effect. However, this method has high requirements on the mathematical model of wind turbines and the linearization process is very complicated. Liu et al. [9] designed the pitch controller based on the  $H_2/H_\infty$  hybrid optimization. Namik and Stol [10] used a state-space control strategy to improve the stable output of power. Jaramillo-Lopez F. et al. [11] calculated wind speed based on the neural network online training method, which can achieve approximate actual wind speed but sacrifices more time and space. Jiao et al. [12] designed a variable-pitch controller based on the adaptive continuous neural theory. Ren et al. [13] combined backpropagation (BP) neural network and PID theory, then proposed a BP-PID pitch-controller. Yarmohammadi et al. [14] adopted the Gain Scheduling

(GS) control algorithm to achieve the maximum power tracking of the wind turbine, and effectively adjust the pitch angle of the blades. Colombo et al. [15] designed an efficient sliding mode controller to limit the energy capture of the fan when the fan was operating in the high wind speed section. Narayana M. et al. [16] used adaptive linear prediction to estimate wind speed, but this method needed the right step size or the optimal control would not work. Chen et al. [17] combined an actor-critic structure and a Temporal-Difference (TD) algorithm to design an adaptive dynamic programming (ADP) controller that enabled real-time online learning. In the high wind speed area, the wind energy utilization coefficient was indirectly adjusted by changing the pitch angle, and the stable rotation speed of the rotor was maintained near the rated value. Yang et al. [18] designed a novel L1 adaptive controller, which controlled the fan pitch angle of the wind turbine to achieve stable output of generator speed and power under random wind speeds. Fan et al. [19] combined fuzzy logic control theory with adaptive neuro-fuzzy theory to design a novel nonlinear hybrid control method, to adjust the pitch angle of wind turbines to make the captured wind energy constant. Although the above control algorithms have high precision, they are complex and require very high mathematical model of wind turbine. Especially in the face of variable wind speed, the effect is often poor.

In order to solve the problems above, T-S fuzzy theory can be applied to the controller design to improve the global control effect. Ma et al. [20], Shi & Zhang, [21], Liu et al. [22, 23] and Wang and Li [24] introduced the successful application of T-S fuzzy model in nonlinear system stability control. In the process of controller design for complex nonlinear systems, the combination of T-S fuzzy control method and various control strategies (stochastic control, sliding mode variable structure control,  $H_\infty$  control, etc.) has become a research hotspot of nonlinear system control.

T-S fuzzy control method is a very effective nonlinear control method. It represents a complex nonlinear system as a weighted sum of a set of simple linear subsystems, that is, a complex nonlinear problem is transformed into a linear problem on several small segments. When the system has

some uncertainties, system parameters are unknown or human factors change, the control system can still achieve the desired characteristics. Formally, this approach is similar to the typical piecewise linear model approach to nonlinear systems, and also similar to the approach of some locally linear smooth models to compound through the membership function that describes the global characteristics, but it is still a nonlinear dynamic system in essence. The advantage of this design is that it can not only overcome the traditional control methods that cannot achieve the global accurate control of the dynamic system, but also does not require complex algorithms. Through a large number of linearization points, the global accurate control can be achieved.

The basic goal of the controller design is to keep the output power of the wind turbine constant when the wind speed changes. When the wind speed changes, the pitch angle, the wind energy utilization coefficient are required to make corresponding changes, the rotor speed and the output power should be kept at the rated value. While achieving the basic goal, the controller can respond as fast as possible, and the overshoot is as small as possible. The design process of  $H_\infty$  pitch controller based on T-S fuzzy theory is shown in Fig. 1. Firstly, the nonlinear wind power

system is linearized to linear models at several operating points, every linear model is  $\dot{x} = A_i x + B_{i1} u + B_{i2} v$ . Secondly, the  $H_\infty$  control theory was used to design the pitch controller for every subsystem, the subsystem controller is  $u_i = K_i x$ . Thirdly, the sub-controllers were connected through the membership function to obtain a global pitch controller of the system, the global pitch

controller is  $u = -\sum_{i=1}^l h_i(x) u_i$ . Under the action of the

controller, no matter how the input wind speed changes, the output power of the wind turbine can be stabilized at the rated value. Compared with the exact feedback linearization, the advantage of this method is that the algorithm is simple, which does not have high requirements for the mathematical model. If the number of linearization points is sufficient and the membership function is suitable, the system can approach the original nonlinear system infinitely, which effectively improves the modeling accuracy. In addition, the  $H_\infty$  controller has good robust control performance and is very suitable for time-varying wind power systems.

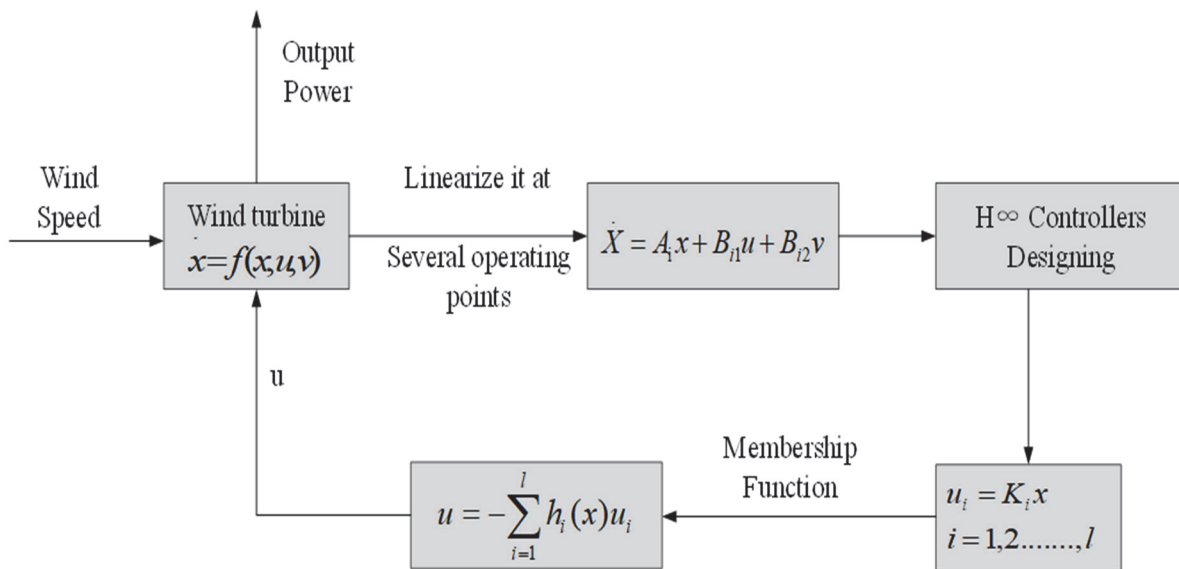


Figure 1 Flowchart of the design

The remainder of this paper is organized as follows. In section 2, the mathematical dynamic model of wind speed and wind turbine is described, the  $H_\infty$  control theory and the T-S theory are presented; in section 3  $H_\infty$  pitch controller based on T-S theory is designed. In section 4, modeling and simulation are carried out in MATLAB/SIMULINK, then the analysis of results is given. In section 5, the final conclusions are drawn.

**2 METHODOLOGY**  
**2.1 Wind Speed Model**

Wind is the driving force for wind turbines to generate electricity, so wind speed directly determines the characteristics of wind turbine output power. To grasp the dynamic characteristics of wind turbines, it is necessary to obtain the wind speed fluctuation data of the wind farm. This paper adopted the four-component wind speed model

that is widely used at home and abroad, where the four components are namely basic wind speed  $\bar{v}$ , gust wind speed  $V_{WG}$ , gradient wind speed  $V_{WR}$  and random wind speed  $V_{WN}$  [25]. These four wind speed models are components of the actual wind speed and are generally not used separately. The wind speed in nature is generally the superposition of some or all of the four wind speed components above. Sometimes, in order to verify the effect of the controller, the gusty wind model or random wind model can be used to test. The dissipated energy of the system is not considered in wind speed modeling here.

- (1) Basic wind speed  $\bar{v}$   
 The basic wind speed model is:

$$\bar{v} = A \cdot \Gamma \left( 1 + \frac{1}{K} \right) \tag{1}$$

where,  $\bar{V}$  is the basic wind (m/s);  $A$  is the scale parameter of the Weibull distribution;  $K$  is the shape parameter of the Weibull distribution.

(2) Gust wind speed  $V_{WG}$

Gusts are used to describe the characteristics of sudden changes in wind speed. Generally, the gust wind speed can be:

$$V_{WG} = \begin{cases} 0 & t < T_{1G} \\ V_{\cos} & T_{1G} \leq t < T_{1G} + T_G \\ 0 & t \geq T_{1G} + T_G \end{cases} \quad (2)$$

where,

$$V_{\cos} = (V_{WGmax} / 2) \{ 1 - \cos 2\pi [ (t / T_G) - (T_{1G} / T_G) ] \};$$

$V_{WG}$  gust wind speed;  $T_G$  the gust period;  $T_{1G}$  the gust start time; and  $V_{WGmax}$  the maximum gust speed.

(3) Gradient wind speed  $V_{WR}$

The mathematical model of the gradient wind speed is:

$$V_{WR} = \begin{cases} 0 & t < T_{1R} \\ V_{ramp} & T_{1R} \leq t < T_{2R} \\ V_{WRmax} & T_{2R} \leq t < T_{2R} + T_R \\ 0 & t \geq T_{2R} + T_R \end{cases} \quad (3)$$

where,  $V_{ramp} = V [ 1 - (t - T_{2R}) / (T_{1R} - T_{2R}) ]_{WRmax}$ ;  $V_{WR}$  the gradient wind speed;  $V_{WRmax}$  the maximum gradient wind speed;  $T_{1R}$  the start time of the gradient wind;  $T_{2R}$  the end time of the gradient wind;  $T_R$  the hold time of the gradient wind.

(4) Random wind speed  $V_{WN}$

The random wind speed model is:

$$V_{WN} = 2 \sum_{i=1}^N [ S_v(w_i) \Delta w ]^{1/2} \cos(w_i t + \varphi_i) \quad (4)$$

where,  $v_i = \left( i - \frac{1}{2} \right) \Delta w$ ;  $S_v(w_i) = \frac{2K_N F^2 |w_i|}{\pi^2 [ 1 + (Fw_i / \mu\pi)^2 ]^{4/3}}$ ;

$N$  is the total number of statistical wind speeds;  $\Delta w$  is the wind speed frequency interval;  $\varphi_i$  a random variable uniformly distributed between  $0 \sim 2\pi$ ;  $K_N$  the surface roughness coefficient;  $F$  the disturbance range; and  $\mu$  the mean wind speed of the relative altitude.

2.2 Nonlinear Model of Wind Turbines

The WTG mainly consists of a wind turbine, a transmission mechanism and a generator, etc. The wind turbine is one of the most important components of a WTG. The schematic diagram of the model is shown in Fig. 2.

In short, the power generation principle of wind turbine is: the blades rotate under the action of wind, turning the wind kinetic energy into the rotational kinetic energy of the wind turbine shaft, and the wind turbine shaft is connected with the generator shaft through the transmission mechanism, turning the rotational kinetic energy into electric energy.

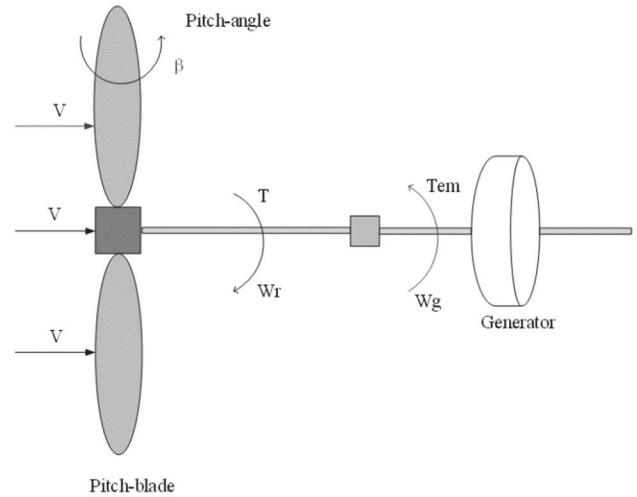


Figure 2 Model of a wind turbine

A wind turbine captures wind energy through the blades and converts it into mechanical energy. The dynamic equation of the fan [8] is:

$$J \dot{\omega}_r = T_{aero} - T_{gen} \quad (5)$$

where,  $J$  is the total inertia of the wind turbine and generator;  $T_{gen}$  the electromagnetic torque of the generator; and  $T_{aero}$  the aerodynamic torque of the wind turbine, which is the driving torque of the whole unit.  $T_{aero}$  can be expressed as:

$$T_{aero} = \frac{\rho \pi R^3 v^2 C_p}{2 \lambda} \quad (6)$$

where,  $v$  the wind speed;  $\rho$  is the density of air;  $R$  the rotor radius;  $\beta$  the pitch angle of fan;  $C_p$  the rotor power coefficient;  $\omega_r$  the rotor speed, and  $\lambda$  the tip-speed ratio, it can be expressed as:  $\lambda = \frac{\omega_r R}{v}$

In this paper, the rotor power coefficient is represented by the characteristic curve of the following formula:

$$C_p = 0.22 \left( \frac{116}{\lambda_i} - 0.4\beta - 5 \right) e^{-12.5/\lambda_i} \quad (7)$$

where,  $\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1}$ .

The output power of the wind turbine is

$$P = \frac{1}{2} \rho \pi R^2 v^3 C_p \quad (8)$$

The pitch-angle actuator is a first-order inertial link:

$$\dot{\beta} = \frac{1}{T_\beta} (\beta_r - \beta) \quad (9)$$

where,  $T_\beta$  is the time constant.

Due to the delay, the tachometer is represented by a first-order inertial link:

$$\dot{\omega}_{rm} = \frac{1}{T_{\omega}}(\omega_r - \omega_{rm}) \tag{10}$$

where,  $\omega_{rm}$  is measured value of rotor speed of the wind turbine; and  $T_{\omega}$  the time constant.

The above Eqs. (5) to (10) constitute the nonlinear model of the wind turbine:

$$\begin{cases} \dot{\beta} = \frac{1}{T_{\beta}}(\beta_r - \beta) \\ \dot{\omega}_r = \frac{1}{J}(T_{aero}(\beta, \omega_r) - T_{gen}) \\ \dot{\omega}_{rm} = \frac{1}{T_{\omega}}(\omega_r - \omega_{rm}) \end{cases} \tag{11}$$

It can be seen from the above discussion that this is a third-order nonlinear system, and the state variables include the pitch angle of the blade ( $\beta$ ), the rotational speed of the rotor ( $\omega_r$ ), and the measured rotational speed of the rotor ( $\omega_{rm}$ ).

### 2.3 Linear Model of Wind Turbines

The nonlinear mathematical model (11) of the WTGs can be written in the form of

$$\dot{X} = f(X) + g_1(X)w + g_2(X)u$$

$$f(X) = \begin{bmatrix} -\frac{1}{T_{\beta}}\beta \\ \frac{1}{J}(T_{aero}(\beta, \omega_r) - T_{gen}) \\ \frac{1}{T_{\omega}}(\omega_r - \omega_{rm}) \end{bmatrix} \tag{12}$$

$$g_1(X) = [0 \ 1 \ 0]^T, g_2(X) = \begin{bmatrix} \frac{1}{T_{\beta}} & 0 & 0 \end{bmatrix}^T, u = \beta_r,$$

$$X = [\beta \ \omega_r \ \omega_{rm}]^T.$$

The system was locally linearized into 6 linear sub-systems which are at  $V_1 = 11$  m/s,  $V_2 = 13$  m/s,  $V_3 = 15$  m/s,  $V_4 = 17$  m/s,  $V_5 = 19$  m/s and  $V_6 = 21$  m/s, respectively. According to the Taylor expansion, the linear model is as follows:

$$\begin{cases} \dot{x} = A_i x(t) + B_{wi} w + B_i u(t) \\ y(t) = Cx(t) \end{cases} \tag{13}$$

where,  $x(t) = [\beta \ \omega_r \ \omega_{rm}]^T$ , representing the state vector;  $u(t) = \beta_r$  is the input of controller;  $y(t)$  the system output. The system matrices are as follows.

$$A_i = \begin{bmatrix} \frac{\partial f_1}{\partial \beta} & \frac{\partial f_1}{\partial \omega_r} & \frac{\partial f_1}{\partial \omega_{rm}} \\ \frac{\partial f_2}{\partial \beta} & \frac{\partial f_2}{\partial \omega_r} & \frac{\partial f_2}{\partial \omega_{rm}} \\ \frac{\partial f_3}{\partial \beta} & \frac{\partial f_3}{\partial \omega_r} & \frac{\partial f_3}{\partial \omega_{rm}} \end{bmatrix} \Big|_{V=V_i}$$

$$B_{wi} = [0 \ 1 \ 0]^T, B_i = \begin{bmatrix} \frac{1}{T_{\beta}} & 0 & 0 \end{bmatrix}^T \text{ and } C = [0 \ 0 \ 1]^T.$$

### 2.4 Control Objectives

Depending on the wind speed, the WTGs will operate in four different regions, as shown in Fig. 3.

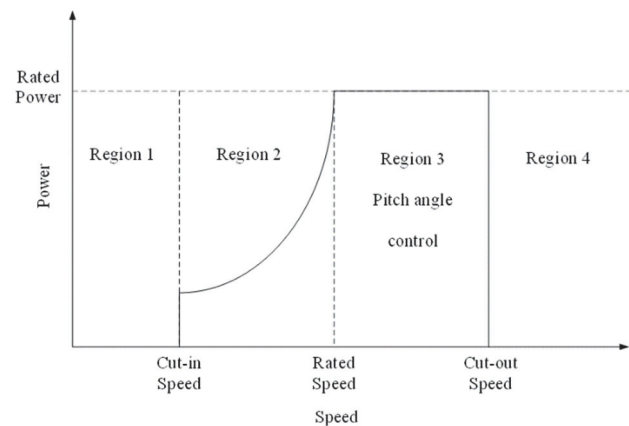


Figure 3 Operating region of the wind turbine

In region 1, the wind speed is lower than the cut-in wind speed, and the airflow does not produce any torque on the blade. The entire blade is actually a damping plate. In the static state, the blade pitch angle is  $90^\circ$ , and the wind turbine does not generate electricity. In region 2, when the speed reaches the cut-in speed, the blade rotates towards  $0^\circ$ , and the airflow produces a certain angle of attack on the blade, and the wind rotor starts to work. In this region, the pitch angle remains unchanged at about  $0^\circ$ , and the control goal is to adjust the generator slip according to the wind speed to make it run at the best tip-speed ratio if possible to achieve the maximum capture of wind energy. In region 3, the speed reaches the rated speed, and the output power of the WTGs also reaches the rated power. With the increase of the wind speed, the power will exceed the rated power, so the specific control objective in this region is to adjust the blade pitch angle according to the wind speed, reduce the wind energy utilization coefficient, limit the wind energy obtained by the wind turbine to a fixed value, and ensure that its output power is maintained near the rated value. In region 4, the speed reaches the cut-out wind speed. For the sake of safety, the wind turbine is shut down. This paper studies region 3, focusing on the design of a pitch controller with superior performance. For the adopted wind turbine in this paper, the cut-in speed is 3 m/s, the rated speed is 11 m/s, the cut-out speed is 21 m/s, and the rated power is 600 kW.

Other parameters are shown in Tab. 1.

**Table 1** Main parameters of the wind turbine model

Parameters	Symbole	Values
Rotor radius	$R$	21.65 m
Rotor inertia	$J$	322000 kg·m <sup>2</sup>
Air density	$\rho$	1.25 kg/m <sup>3</sup>
Time delay	$T_{\omega}$	0.1 s
Cut-in wind speed	$v_{cut-in}$	3 m/s
Rated wind speed	$v_{rated}$	11 m/s
Cut-out wind speed	$v_{cut-out}$	21 m/s
Rated rotor speed	$\omega_{rated}$	4.35 rad/s
Rated output power	$P_{rated}$	600 kW

**2.5 T-S Fuzzy Model**

The T-S fuzzy system model was proposed by Japanese scholars Takagi and Sugeno in 1985. The model is a system composed of a series of IF-THEN fuzzy rules. Its main modeling idea is to construct a set of linear subsystems to describe the local rules in each local region, and connect these linear subsystems to form a global model through membership function. Although the global model is still nonlinear, the local subsystems are linear. This property enables the model to build a bridge between the nonlinear system and the linear system, so that mature linear system theory can be used to solve the nonlinear problems. In the past decades, T-S fuzzy model has been successfully practiced in automatic control, neural network, signal filtering, intelligent robot and big data analysis, which has attracted a large number of scholars to conduct in-depth research in this field.

The central idea is to linearize the nonlinear system at several operating points. The more linearization points, the closer to the original dynamic model, and the higher the modeling accuracy [26]. The H<sub>∞</sub> theory can solve the robust control problem in modeling error systems, with many achievements gained in practical application.

**2.5.1 T-S Controller**

*T-S system.* The formula of the nonlinear system is as follows:

$$\dot{x} = F(x, u) = f(x) + g(x)u \tag{14}$$

The fuzzy formula model of this nonlinear system can be described as:

Rule (i): if  $x_1(t)$  is  $M_1^i$  and if  $x_2(t)$  is  $M_2^i$  and if ... and if  $x_n(t)$  is  $M_n^i$

then  $\dot{x}(t) = A_i x(t) + B_i u(t)$ ,  $i = 1, 2, 3, \dots, l$

Then, the equation of the whole nonlinear system can be expressed as follows:

$$\dot{x} = \sum_{i=1}^l h_i(x) [A_i x(t) + B_i u(t)] \tag{15}$$

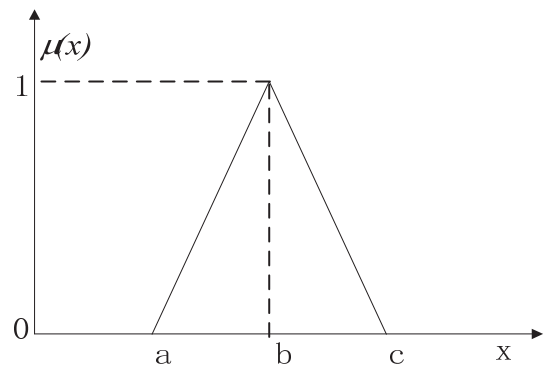
where,  $h_i(x) = \mu^i(x) / \sum_{j=1}^l \mu^j(x)$ ,  $\mu^j(x) = \prod_{j=1}^n \mu_j^i(x)$ , and

$\mu^i(x)$  is the membership function of  $x$  belonging to  $M^i$ , and also represents the applicability of the rule  $i$ . In the fuzzy control system, the methods to select a membership

function mainly include fuzzy statistics, subjective experience and so on. The most commonly used forms of membership functions include normal, triangular and Z-shaped membership functions [27, 28]. Here the triangular membership function is adopted, whose expression is shown in Eq. (16).

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & a < x < b \\ \frac{x-c}{b-c}, & b < x < c \\ 0, & \text{else} \end{cases} \tag{16}$$

The graph of the triangular membership function is shown in Fig. 4.



**Figure 4** Triangular membership function

*Calculation of the T-S Fuzzy Controller.* A random point  $(x_0, u_0)$  near the balance point satisfies  $\dot{x} = F(x_0, u_0) = 0$ . Through Taylor expansion of the

formula,  $\Delta \dot{x} = A \Delta x + B \Delta u$  can be obtained, where,

$$\Delta x = x - x_0, \Delta u = u - u_0, A = \left. \frac{\partial F}{\partial x} \right|_{\substack{x=x_0 \\ u=u_0}}, \text{ and } B = \left. \frac{\partial F}{\partial u} \right|_{\substack{x=x_0 \\ u=u_0}}$$

The state feedback controller of every linear subsystem is obtained by using the H<sub>∞</sub> theory. The parallel distribution compensation method controller is designed as follows:

Rule (i): if  $x_1(t)$  is  $M_1^i$  and if  $x_2(t)$  is  $M_2^i$  and if ... and if  $x_n(t)$  is  $M_n^i$

then  $u_i(t) = -K_i x(t)$

Then, control law of the whole system is obtained through the membership function:

$$u(t) = -\sum_i^l h_i(x) K_i x(t) \tag{17}$$

where,  $h_i(x) = \mu^i(x) / \sum_{j=1}^l \mu^j(x)$ .

**2.5.2 H<sub>∞</sub> Control Theory**

The standard H<sub>∞</sub> control problem is shown in Fig. 5.

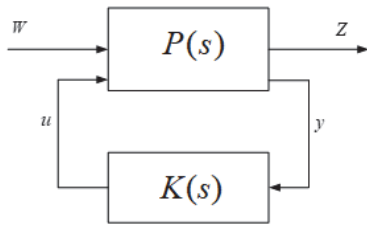


Figure 5 Standard H $\infty$  control problem

$P(s)$  is a linear time-invariant system described by the following state space:

$$\begin{cases} \dot{x} = Ax + B_1\omega + B_2u \\ z = C_1x + D_{11}\omega + D_{12}u \\ y = C_2x + D_{21}\omega + D_{22}u \end{cases} \quad (18)$$

where,  $x \in R^n$  is the state vector;  $u \in R^m$  the control input;  $y \in R^p$  the measurement output;  $z \in R^r$  the modulated output;  $\omega \in R^q$  the external disturbance;  $A, B_1, B_2, C_1, D_{11}, D_{12}, C_2, D_{21}, D_{22}$  the constant matrices; and  $A \in R^{n \times n}$ ,  $B_1 \in R^{n \times q}$ ,  $B_2 \in R^{n \times m}$ ,  $C_1 \in R^{r \times n}$ ,  $D_{11} \in R^{r \times q}$ ,  $D_{12} \in R^{r \times m}$ ,  $C_2 \in R^{p \times n}$ ,  $D_{21} \in R^{p \times q}$  and  $D_{22} \in R^{p \times m}$ . From input signal  $\omega, u$  to output signal  $z$ , the object represented by the transfer function  $K(s)$  of  $y$  is the augmented controlled object, which includes the actual controlled object and the weighting function set to describe the indicators, etc.; and  $K(s)$  is the control law [29-32].

The purpose of the H $\infty$  control problem is to design the controller  $u(s) = K(s)y(s)$ , to make the closed-loop system as shown in Fig. 5 internally stable, and to minimize the H $\infty$  norm of the transfer function  $T_{zw}(s)$  from  $\omega$  to  $z$ , that is,  $T_{zw}(s)_\infty$ .

If the system has state feedback, that is  $C_2 = I, D_{21} = D_{22} = 0$ , as long as the system satisfies the following conditions  $-(A, B_2)$  can be stable and detectable and  $D_{11} = 0$ ;  $D_{12}^T [C_1 \ D_{12}] = [0 \ I]$ , the system must be able to introduce a state feedback H $\infty$  controller  $u = Kx$  to make the closed-loop system asymptotically stable, and the transfer function  $T_{zw}(s)$  of this closed-loop system must satisfy:

$$\|T_{zw}(s)\|_\infty = \left\| (C_1 + D_{12}K) [sI - (A + B_2K)]^{-1} B_1 + D_{11} \right\|_\infty < \gamma$$

In the above formula,  $\gamma$  is a positive number as small as possible to reduce the adverse effect of disturbances on the system. The  $P$  matrix satisfies the following Riccati equation:

$$A^T P + PA - P(B_2 B_2^T - \gamma^{-2} B_1 B_1^T)P + C_1^T C_1 = 0 \quad (19)$$

Therefore, the feedback control of this system is obtained as follows:

$$u = -Kx = -B_2^T P x \quad (20)$$

### 3 DESIGN OF AN H $\infty$ CONTROLLER BASED ON T-S FUZZY THEORY

The main parameters of the WTGs used are shown in Tab. 1.

The T-S fuzzy model is a powerful tool for dealing with nonlinear systems. With an appropriate membership function selected, according to the H $\infty$  control theory, the 6 pitch angle controllers ( $u_1, u_2, u_3, u_4, u_5, u_6$ ,  $u_i = K_i x$ ) for the six linear subsystems were designed by Eqs. (15) and (16), respectively.

According to the fuzzy rule:

If  $V_1 = 11$  m/s and  $\beta = -2.6^\circ$ , then  $u_1 = K_1 x$ ;

If  $V_2 = 13$  m/s and  $\beta = 7.9^\circ$ , then  $u_2 = K_2 x$ ;

If  $V_3 = 15$  m/s and  $\beta = 14.6^\circ$ , then  $u_3 = K_3 x$ ;

If  $V_4 = 17$  m/s and  $\beta = 19.5^\circ$ , then  $u_4 = K_4 x$ ;

If  $V_5 = 19$  m/s and  $\beta = 23.3^\circ$ , then  $u_5 = K_5 x$ ;

If  $V_6 = 21$  m/s and  $\beta = 26.2^\circ$ , then  $u_6 = K_6 x$ ;

This paper selects the triangular membership function:

When  $V_1 = 11$  m/s, the most suitable pitch angle is  $-2.6^\circ$ ,

$$\mu_1 = \begin{cases} \frac{7.9 - \beta}{7.9} & -2.6^\circ \leq \beta \leq 7.9^\circ \\ 0 & \text{else} \end{cases}$$

When  $V_2 = 13$  m/s, the most suitable pitch angle is about  $7.9^\circ$ ,

$$\mu_2 = \begin{cases} \frac{\beta}{7.9} & 0^\circ \leq \beta \leq 7.9^\circ \\ \frac{14.6 - \beta}{14.6 - 7.9} & 7.9^\circ \leq \beta \leq 14.6^\circ \\ 0 & \text{else} \end{cases}$$

When  $V_3 = 15$  m/s, the most suitable pitch angle is about  $14.6^\circ$ ,

$$\mu_3 = \begin{cases} \frac{\beta - 7.9}{14.6 - 7.9} & 7.9^\circ \leq \beta \leq 14.6^\circ \\ \frac{19.5 - \beta}{19.5 - 14.6} & 14.6^\circ \leq \beta \leq 19.5^\circ \\ 0 & \text{else} \end{cases}$$

When  $V_4 = 17$  m/s, the most suitable pitch angle is about  $19.5^\circ$ ,

$$\mu_4 = \begin{cases} \frac{\beta - 14.6}{19.5 - 14.6} & 14.6^\circ \leq \beta \leq 19.5^\circ \\ \frac{23.2 - \beta}{23.2 - 19.5} & 19.5^\circ \leq \beta \leq 23.2^\circ \\ 0 & \text{else} \end{cases}$$

When  $V_5 = 19$  m/s, the most suitable pitch angle is about  $23.2^\circ$ ,

$$\mu_5 = \begin{cases} \frac{\beta - 19.5}{23.2 - 19.5} & 19.5^\circ \leq \beta \leq 23.2^\circ \\ \frac{26.2 - \beta}{26.2 - 23.2} & 23.2^\circ \leq \beta \leq 26.2^\circ \\ 0 & \text{else} \end{cases}$$

When  $V_6 = 21$  m/s, the most suitable pitch angle is about  $26.2^\circ$ ,

$$\mu_6 = \begin{cases} \frac{\beta - 23.2}{26.2 - 23.2} & 23.2^\circ \leq \beta \leq 26.2^\circ \\ 0 & \text{else} \end{cases}$$

Finally, the pitch angle controllers of each linear subsystem are superimposed through the membership function to form the controller of the entire system:

$$u = \frac{\mu_1 u_1 + \mu_2 u_2 + \mu_3 u_3 + \mu_4 u_4 + \mu_5 u_5 + \mu_6 u_6}{\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6}$$

At this point, the design of the  $H_\infty$  controller based on the T-S theory is completed. If the designed controller has good performance, the output power of the wind turbine can be stabilized at the rated value in a short time, and the overshoot is small enough. If the controller performance is poor, the output power of the wind turbine cannot be maintained at the rated value, or the overshoot is too large, which takes a long time to stabilize.

#### 4 SIMULATION AND RESULTS ANALYSIS

The working principle of T-S fuzzy controller is: when the wind speed changes, the pitch angle controller starts to work, making the pitch angle change with the change of the wind speed. The change of the pitch angle brings about the change of the wind energy utilization coefficient, that is, adjusts the wind energy captured by the wind turbine, and maintains the rotating speed and output power of the wind turbine at the rated value. The  $H_\infty$  controller based on the T-S theory was substituted into the original WTGs, and then the model was built and simulated in MATLAB/SIMULINK. Different wind speed models were used in the simulation, with their response curves shown below.

##### 4.1 Response Curves of a WTG at Combined Wind Speeds

When the wind speed is the superposition of several components, its mathematical model is:

$$V = \bar{V} + V_{WG} + V_{WR} + V_{WN}$$

At this wind speed, the response curve of the WTG is shown in Fig. 6.

When the adopted wind speed model is the superposition of multiple components, it can be seen from the curves in Fig. 6a and Fig. 6b that the pitch angle changes with the change of wind speed under the action of the controller. In short, the pitch angle increases with the increase of wind speed, and decreases with the decrease of wind speed. It can be seen from Fig. 6c that the rotor power efficiency also changes accordingly. Fig. 6d and Fig. 6e show that the rotor speed and output-power can be stabilized near the rated value in, it indicates that the controller completes the desired control objectives.

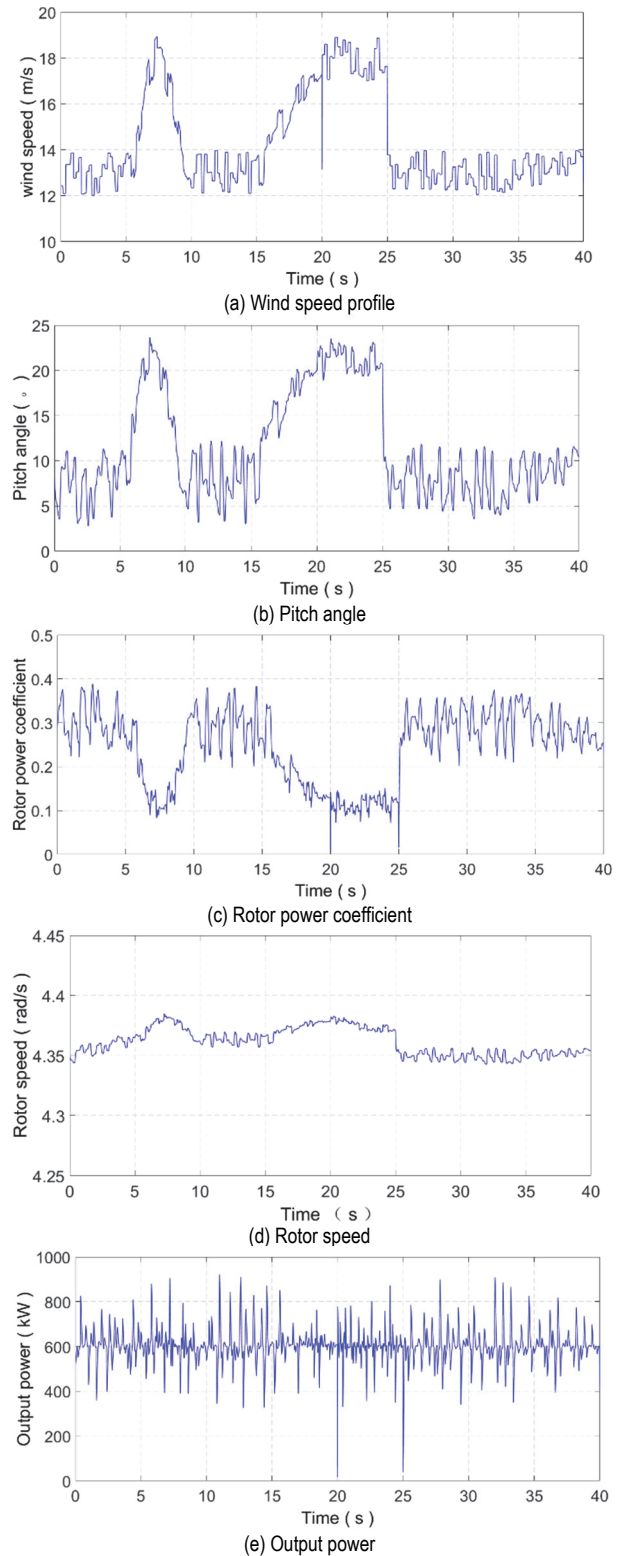
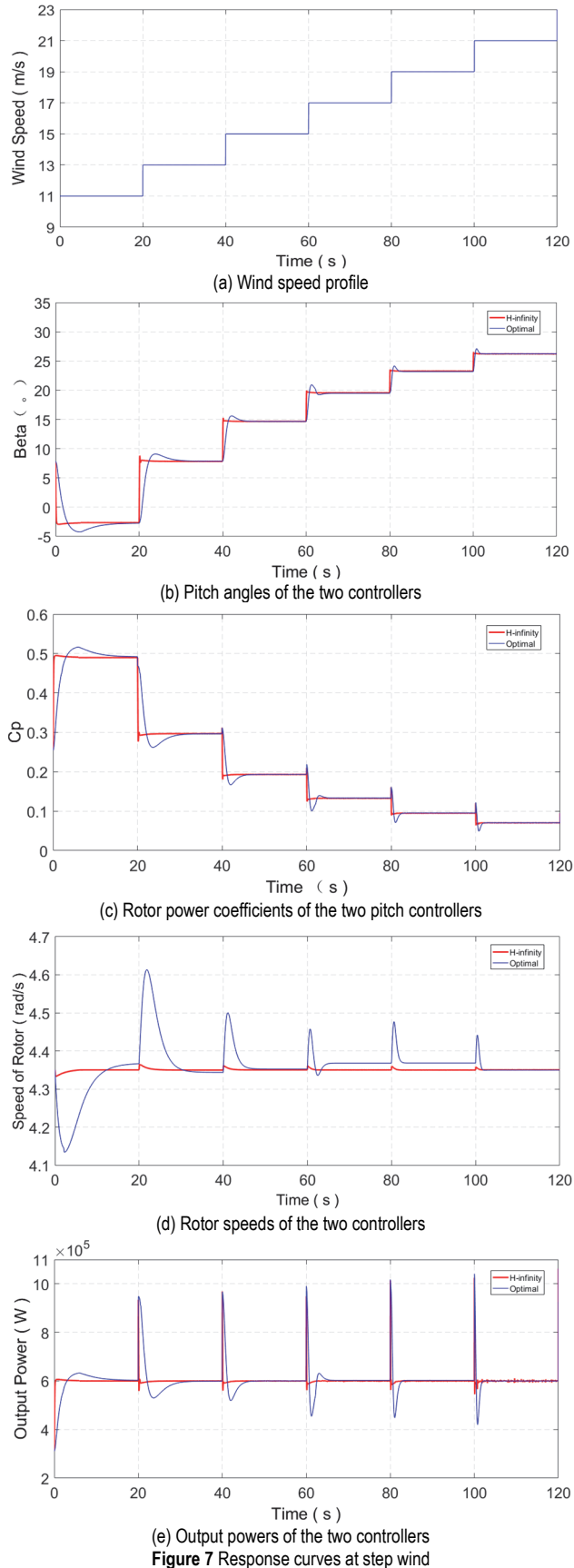


Figure 6 Response curves at combined wind speed

## 4.2 Response Curves of a WTG at Step Wind Speed

Fig. 7 shows the response curves of a WTG when the wind speed changes stepwise from 11 m/s to 21 m/s.



From the simulation in the figures above, we can see that when wind speed varies over a wide range (11 m/s to 21 m/s), the pitch angle of the fans also increases, the wind energy utilization coefficient decreases, the rotor speed of the WTG can be maintained at 4.35 rad/s, and the output power of the WTG can be maintained at a rated power of 600 kW. In addition, taking gusty conditions in the 40 s as an example, the overshoot of rotational speed under the optimal controller is 3.4%, and the response time reaches about 7 seconds. However, under the H infinity controller, the overshoot of speed is only 0.3%, and the response time is only about 2 seconds. It can be seen that the controller designed in this paper is obviously superior to the optimal controller in terms of response speed and overshoot.

## 5 CONCLUSION

A control strategy combining the T-S fuzzy model theory and the  $H_\infty$  theory was proposed to design a pitch controller for WTGs in this paper. The goal of the proposed controller is to keep the rotor speed and the output power of the WTGs constant above the rated wind speed. The main conclusions can be drawn as follows:

(1) The controller is designed based on T-S fuzzy model theory, which can realize stable control in the whole wind speed range. The simulation results in Fig. 7 show that when the wind speed changes in a large range, the rotor speed and output power can be kept at the rated value.

(2) As a kind of robust control,  $H_\infty$  control theory has excellent performance in dealing with uncertain parameters or unknown disturbances. The simulation results in Figure 7 show that the control effect of the  $H_\infty$  controller is obviously better than that of the optimal controller. The  $H_\infty$  controller proposed in this paper has fast response speed and small overshoot in all aspects.

According to the two conclusions above, under the action of the controller, the output power of the wind turbine can be maintained near the rating value regardless of how the wind speed changes, and the response time is shorter and the overshoot is smaller, indicating that the control performance is good.

In the future, it is an inevitable trend to apply intelligent control algorithm to the pitch angle control of wind turbine. At the same time, the combination of intelligent algorithm and global control has improved the working stability of wind turbine, reduced the operating loss of the unit, extended the service life of the wind turbine, and saved the cost of operation. However, this study did not take into account the power grid connected operation of wind turbines, the voltage and current phase issues concerned by grid connected operation, and only considered the wind turbine itself, which has certain limitations. The wind turbine and the grid should be considered as a whole in subsequent studies. In addition, since the operation stage of wind turbines includes startup stage, underpower operation stage and rated power output stage of wind turbines above rated wind speed, but there are switching processes of different operation stages in the actual operation process of wind turbines. Therefore, to ensure the safe and stable operation of the system, it is necessary to design a whole control system.



## Acknowledgements

Thank you to University Malaysia Sarawak, UNIMAS and Yancheng Institute of Technology for funding this research work.

## 6 REFERENCES

- [1] Ye, H. Z. (2015). *Wind Turbine Control Technology*. Mechanical Industry Press, Beijing.
- [2] Toriki, M. B., Asy'ari, M. K., & Musyafa A. (2021). Enhanced performance of PMSG in WECS using MPPT-fuzzy sliding mode control. *Journal Européen des Systèmes Automatisés*, 54(1), 85-96. <https://doi.org/10.18280/jesa.540110>
- [3] Yuan, T. Z., Li, H., & Jia, D. (2022). Modeling and Control Strategy of Wind-Solar Hydrogen Storage Coupled Power Generation System. *Journal of Intelligent Systems and Control*, 1(1), 18-34. <https://doi.org/10.56578/jisc010103>
- [4] Benabbas, A. Zaidi, E., & Abdessemed, R. (2022). Sliding mode control of a wind power system based on a self-excited asynchronous generator. *Journal Européen des Systèmes Automatisés*, 55(1), 131-137. <https://doi.org/10.18280/jesa.550114>
- [5] Zhang, L., Jin, S. T., & Nie, S. L. (2006). Simulation research on large wind turbine control based on PID method. *China Science and Technology Information*, 22, 34-36. <https://doi.org/10.3969/j.issn.1001-8972.2006.22.007>
- [6] Zheng, Y. (2012). Independent pitch control of wind turbine based on neuron PID. *Water Resources and Power*, 30(2), 151-154.
- [7] Yuan, C. Y., Li, J., Chen, J. Y., & Xu, Q. (2019). Research on variable pitch ABC-PID control of large wind turbines. *Acta Energetica Solaris Sinica*, 40(10), 3002-3008. <https://doi.org/10.19912/j.0254-0096.2019.10.039>
- [8] Qin, S. S., Hu G. W., Gu, C. L., & Li, D. (2012). Constant power  $H_\infty$  robust control of wind power generation system. *Control Theory & Applications*, 29(5), 617-622. <https://doi.org/10.7641/j.issn.1000-8152.2012.5.ccta100816>
- [9] Liu, Y. M., Zhu, J. S., Yao, X. J., Ma, K. C., Wang, X. D., & Guo, Q. D. (2015). Research on robust control technology of variable pitch for large wind turbines based on  $H_2/H_\infty$  hybrid optimization. *Acta Energetica Solaris Sinica*, 36(3), 714-719. <https://doi.org/10.3969/j.issn.0254-0096.2015.03.032>
- [10] Namik, H. & Stol, K. (2010). Individual blade pitch control of floating offshore wind turbines. *Wind Energy: An International Journal for Progress and Applications in Wind Power Conversion Technology*, 13(1), 74-85. <https://doi.org/10.1002/we.332>
- [11] Jaramillo-Lopez F, Kenne G, & Lamnabhi-Lagarigue F. (2016). A novel online training neural network-based algorithm for wind speed estimation and adaptive control of PMSG wind turbine system for maximum power extraction. *Renewable Energy*, 86, 38-48. <https://doi.org/10.1016/j.renene.2015.07.071>
- [12] Jiao, X., Meng, W., Yang, Q., Fu, L., & Chen, Q. (2019). Adaptive continuous neural pitch angle control for variable-speed wind turbines. *Asian Journal of Control*, 21(4), 1966-1979. <https://doi.org/10.1002/asjc.1963>
- [13] Ren, H., Hou, B., Zhou, G., Shen, L., Wei, C., & Li, Q. (2020). Variable pitch active disturbance rejection control of wind turbines based on BP neural network PID. *IEEE Access*, 8, 71782-71797. <https://doi.org/10.1109/ACCESS.2020.2987912>
- [14] Yarmohammadi, M. J., Sadeghzadeh, A., & Taghizadeh, M. (2020). Gain-scheduled control of wind turbine exploiting inexact wind speed measurement for full operating range. *Renewable Energy*, 149, 890-901. <https://doi.org/10.1016/j.renene.2019.09.148>
- [15] Colombo, L., Corradini, M. L., Ippoliti, G., & Orlando, G. (2020). Pitch angle control of a wind turbine operating above the rated wind speed: A sliding mode control approach. *ISA Transactions*, 96, 95-102. <https://doi.org/10.1016/j.isatra.2019.07.002>
- [16] Narayana, M., Sunderland, M., Putrus, G., & Conlon, M. F. (2017). Adaptive linear prediction for optimal control of wind turbine. *Renewable Energy*, 113, 895-906. <https://doi.org/10.1016/j.renene.2017.06.041>
- [17] Chen, P., Han, D., Tan, F., & Wang, J. (2020). Reinforcement-based robust variable pitch control of wind turbines. *IEEE Access*, 8, 20493-20502. <https://doi.org/10.1109/ACCESS.2020.2968853>
- [18] Yang, Q., Jiao, X., Luo, Q., Chen, Q., & Sun, Y. (2020).  $L_1$  adaptive pitch angle controller of wind energy conversion systems. *ISA transactions*, 103, 28-36. <https://doi.org/10.1016/j.isatra.2020.04.001>
- [19] Fan, Y. J., Xu, H. T., & He, Z. Y. (2022). Smoothing the output power of a wind energy conversion system using a hybrid nonlinear pitch angle controller. *Energy Exploration & Exploitation*, 40(2), 539-553. <https://doi.org/10.1177/01445987211041779>
- [20] Ma, X., Wong, P. K., Zhao, J., & Xie, Z. (2019). Cornering stability control for vehicles with active front steering system using TS fuzzy based sliding mode control strategy. *Mechanical Systems and Signal Processing*, 125, 347-364. <https://doi.org/10.1016/j.ymsp.2018.05.059>
- [21] Shi, J. & Zhang, Q. (2019). Dynamic sliding-mode control for T-S fuzzy singular time-delay systems with  $H_\infty$  Performance. *IEEE Access*, 7, 115388-115399. <https://doi.org/10.1109/ACCESS.2019.2935456>
- [22] Liu, J., Yin, T., Xie, X., Tian, E., & Fei, S. (2019). Event-triggered state estimation for T-S fuzzy neural networks with stochastic cyber-attacks. *International Journal of Fuzzy Systems*, 21(2), 532-544. <https://doi.org/10.1007/s40815-018-0590-4>
- [23] Liu, J., Cui, Y., Song, H., Zhang, X., & Qu, Y. (2021). Stability analysis of TS fuzzy-model-based coupled control systems with nonlinear TS fuzzy control and its application. *Neural Computing and Applications*, 33(22), 15481-15493. <https://doi.org/10.1007/s00521-021-06170-9>
- [24] Wang, P. & Li, N. (2019). Stable controller design for T-S fuzzy control systems with piecewise multi-linear interpolations into membership functions. *International Journal of Fuzzy Systems*, 21, 1585-1596. <https://doi.org/10.1007/s40815-019-00665-3>
- [25] Wu, X. G., Zhang, X. C., Yin Y. H., & Dai, H. Z. (1998). Mathematical model for dynamic stability analysis of asynchronous wind power generation system and its application. *Grid Technology*, 22(6), 68-72.
- [26] Sun, Z. X. (2021). *Intelligent Control*. Beijing, Tsinghua University Press.
- [27] Qin, S., Ngu, S., & Zeng, T. (2022). Optimal constant power control of wind turbine generators based on Takagi-Sugeno fuzzy model. *Alexandria Engineering Journal*, 61(8), 5977-5982. <https://doi.org/10.1016/j.aej.2021.11.024>
- [28] Fang, J. S., Tsai, J. S. H., Yan, J. J., & Guo, S. M. (2021). Adaptive H-infinity SMC-based model reference tracker for uncertain nonlinear systems with input nonlinearity. *International Journal of Control, Automation and Systems*, 19(4), 1560-1569. <https://doi.org/10.1007/s12555-019-0967-7>
- [29] Tan, S., Yang, J., Khajepour, A., Zhao, X., & Yu, W. (2021). H-Infinity shifting control in a dual-speed transmission for electric vehicle. *International Journal of Automotive Technology*, 22, 155-164. <https://doi.org/10.1007/s12239-021-0016-4>
- [30] Mu, Y., Zhang, H., Su, H., & Wang, Y. (2021). Robust normalization and  $H_\infty$  stabilization for uncertain Takagi-Sugeno fuzzy singular systems with time-delays. *Applied Mathematics and Computation*, 388, 125534.

<https://doi.org/10.1016/j.amc.2020.125534>

- [31] Du, Z. B. & Hu, S. S. (2017). Fuzzy hybrid  $H_2/H_\infty$  sampled-data control for nonlinear systems. *Control and Decision*, 32(5), 930-934.
- [32] Anagie, G. A., Hassen, A. A., & Sintie, Y. T. (2021). Performance investigation of small wind turbine installed over a pick up vehicle to charge an electric vehicle battery. *Journal Européen des Systèmes Automatisés*, 54(5), 783-788. <https://doi.org/10.18280/jesa.540514>

**Contact information:**

**Shengsheng QIN**, Lecturer  
Department of Electrical and Electronic Engineering, Faculty of Engineering,  
University Malaysia Sarawak, Sarawak 94300, Malaysia  
School of Electrical Engineering,  
Yancheng Institute of Technology,  
Yancheng 224051, China  
E-mail: qss@ycit.cn

**Size Song NGU**, Doctor, Senior Lecturer  
(Corresponding author)  
Department of Electrical and Electronic Engineering, Faculty of Engineering,  
University Malaysia Sarawak,  
Sarawak 94300, Malaysia  
E-mail: ssngu@unimas.my

**Chaoqun ZHANG**  
School of Electrical Engineering,  
Yancheng Institute of Technology,  
Yancheng 224051, China  
E-mail: zcq1593572022@126.com

**Hui CAI**, Associate professor  
School of Electrical Engineering,  
Yancheng Institute of Technology,  
Yancheng 224051, China  
E-mail: caihui@ycit.edu.cn

**Yujian CHEN**  
School of Electrical Engineering,  
Yancheng Institute of Technology,  
Yancheng 224051, China  
E-mail: chen1jj@126.com

**Ruiqi CHEN**  
School of Electrical Engineering,  
Yancheng Institute of Technology,  
Yancheng 224051, China  
E-mail: 17205108381@163.com

**Tingxuan LIU**  
School of Electrical Engineering,  
Yancheng Institute of Technology,  
Yancheng 224051, China  
E-mail: njltx123@126.com