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Preliminary communication

Cassinian Directorial Surfaces Modelling as Geometric Pattern in Designing Architectural Structures

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Abstract: In this paper we propose a method for the genesis of novel geometric surfaces which we named Cassinian directorial surfaces (CDS). They represent spatial geometric locus of points whose product of distances from *k* fixed directrix lines is constant and given by the parameter *S*. The modelling of such Cassinian directorial surfaces is based upon two criteria that are the geometric mean as a proportional harmony and the golden ration i.e. the divine proportion. The three particular spatial dispositions of directors-directrices thus enable the design of three types of Cassinian five-directorial surfaces: CDS1 incorporating directrices which coincide with the sides of a regular pentagon; CDS2 incorporating directrices coinciding with the edges of an upright regular pentagonal prism; and CDS3 incorporating directrices which coincide with the skew diagonals of the sides of an upright regular pentagonal prism. In addition, for each type of the surface we propose possible applications of such surfaces as a geometric pattern in designing architectural structures.

Keywords: architectural structures; Cassinian directorial surfaces; geometric mean; golden ratio; modelling

1 INTRODUCTION

Algebraic curves and surfaces as well as their application in architectural structural forms have come into focus at the turn of the 21st century due to the increased availability of high computational power and accompanying software tools. Even though the generating of such curves and surfaces to a certain extent was possible in the past, nowadays when this process is enriched by integrating multiple design criteria and project demands the contribution to structural, functional aesthetical requirements is certainly enlarged. These criteria coupled with proper mathematical and geometrical analysis and derivation of algebraic curves and surfaces as well as their further modifications by means of various transformations and utilizing a parametric approach, obviously become an inexhaustible source of novel geometric forms applicable in architecture. In such a way one can produce a controllable geometric shape, meaning that each step of the design process can be either reproduced or remodified, if desired.

2 MODELLING METHODS

One of the most difficult aspects of modelling is finding a widely accepted definition of them. Different sources define "modelling" in different ways. Most authors are of the opinion that to model something mathematically, is to "create, apply, refine and validate" repeatedly [16].

Hence, different design workflows and approaches can be derived from mathematical modelling. One of those workflows is depicted in the chapter "Stages of Modelling" [8], where the authors define the division of modelling activities to provide a structure for the rest of their course *An Introduction to Mathematical Modelling*:

"It is helpful to divide up the process of modelling into four broad categories of activity, namely building, studying, testing and use. Although it might be nice to think that modelling projects progress smoothly from building through to use, this is hardly ever the case. In general, defects found at the studying and testing stages are corrected by returning to the building stage. Note that if any changes are made to the model, then the studying and testing stages must be repeated. A pictorial representation of potential routes through the stages of modelling is (shown in Fig. 1a). This process of repeated iteration is typical of modelling projects, and is one of the most useful aspects of modelling in terms of improving our understanding about how the system works" [8].

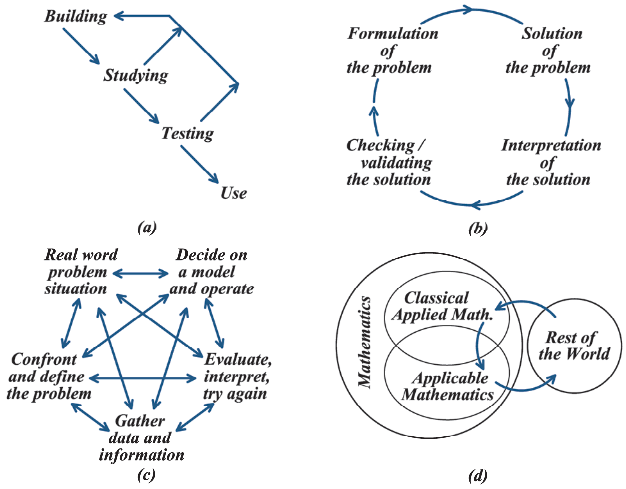


Figure 1 The steps in modelling: (a) Stages of Modelling; (b) Modelling Cycle; (c) Doerr's Nodes of the Modelling Process Visual Maps; (d) Pollak's Venn diagram Visual Map

The whole modelling process should be iterative and integrated. In addition, the process of modelling something is generally done through several phases (studying, use, testing). In the early stages of the modelling process, it is necessary to define the required model parameters, so that changing any parameter yields a specific model. The model is then evaluated in reference to predetermined case-specific criteria after which, if necessary, the parameter values are changed until the fulfilment of the aforementioned criteria.

The mathematical modelling process is a cyclical one, consisting of different steps. After conceptualizing the problem and formulating it, upon reaching the validation step of the process, its cyclical nature can be observed (Fig. 1b). At this stage, a return to the model formulation step of the process is sometimes necessary in order to improve it in reference to its application requirements [16]. The formulation of the model is done by creating or choosing geometrical, graphical, tabular, algebraic or statistical depictions which describe the relationships between the parameters. This four-step cycle can be repeated as many times as needed.

Another approach to mathematical modelling, depicted with the visual map (Fig. 1c), is considerably different from the ones previously mentioned due to its amorphous nature. All nodes are interconnected with one another and there is no right or wrong way to follow them. Doerr, the creator of this approach, describes it as a "nonlinear progression through different phases of the modeling process" [16].

Furthermore, it is important to emphasize that differences between the mathematical world and the real world exist, and that there are different types of applied mathematics within the broader mathematics area (Fig. 1d). The creator of the aforementioned visual map, Pollak, references "classical applied mathematics" and "applicable mathematics" as two types of mathematics, while specifically distinguishing that they are not mutually exclusive [16].

Various visual maps pertaining to mathematical modelling can be used as guidelines for adopting a set of steps for modelling Cassini directorial surfaces (CDS). Alongside the adoption of case-specific modelling steps, it is important for the definition step of our algebraic CDS model (based on classical applied mathematics), to set a specific proportional harmony i.e. to connect the area of classical applied mathematics of surfaces with the area of applicable mathematics of surfaces.

3 CDS MODELLING CRITERIA

In this chapter we analyse two criteria (proportional harmony and divine proportion) which will be incorporated in the modelling of novel geometric surfaces i.e. CDS.

3.1 Proportional Harmony

The achievement of a proportional harmony, a distinct appearance or a particular effect of architectural forms/buildings by means of mathematics dates since the Renaissance. Various proportional systems were used by famous architects of the time, Leon Battista Alberti and Andrea Palladio, such as:

- arithmetic mean (AM),
- geometric mean (GM),
- harmonic mean (HM) and
- the Fibonacci sequence (FS),

being the most basic Nicomachus sequences [9, 11, 19, 21].

Palladio, when defining the height of the ideal room, in his book "The Four Books of Architecture" (1570) the Chapter XXIII - "On the heights of rooms" [12], emphasizes three alternative rules (Fig. 2a):

1) the height of the room (h) equals the half of the length (l) and the width (w) sum, being the AM (Fig. 2b);

2) the height of the room (h) equals the square root of the length (l) and the width (w) of the product, being the GM (Fig. 2c);

3) the height of the room (h) is the harmonic mean (HM) of the length (l) and the width (w), (Fig. 2d).

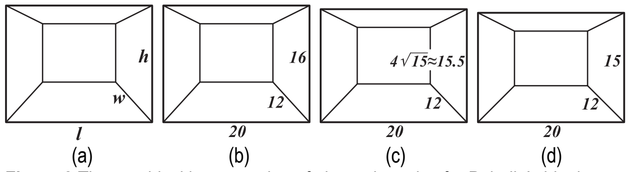


Figure 2 The graphical interpretation of alternative rules for Paladio's ideal room design: (a) general annotation of the room dimensions; (b) arithmetic mean; (c) geometric mean; (d) harmonic mean

Following the rules inherent in the design of Palladio's rooms, Petrović [13] gave a sort of generalization, defining m-focal and k-directorial geometric forms (for m, $k > 1$) which incorporate either AM or GM, as well as a new combinatory derived type of focal-directorial 2D and 3D geometric forms with a constant sum or a constant product of Euclidean distances from both foci and directrices which also incorporate AM or GM, respectively. A shape analysis of focal-directorial 2D geometric forms, which integrate AM is done in the work of Petrović [13].

Alongside the application of Paladio's second alternative rule to ideal room design, another iteration of the proportional harmony can be examined, the golden ratio.

3.2 Divine Proportion

The golden ratio has attracted the attention of both artists and mathematicians/geometricians alike, for centuries. It is also important to note that architects have applied this proportion in their work, considering it the most beautiful proportion, in order to achieve some of the most harmonious forms in architecture [5, 6, 9].

Fehér et al. [2] note in an interesting research work on the regular pentagon: "The occurrence of pentagons in medieval architecture tends to be considered as rare; scholars have suggested that its correct geometrical construction was unknown among artists or architects until the early 16th century", but their paper "provides a series of architectural examples containing pentagonal forms highlighting that its role in the design process of medieval masters was much richer than it is currently considered" [2]. "Half decagonal apses, pentagonal traceries, and even design details were related to geometric methods based on the construction of a regular pentagon", which was also used "in plan design of monumental constructions such as the cathedrals of Reims and Saint Quentin" [2].

"Proportional ratios may be included with architectural qualities that are perceived only indirectly. The presence of the Golden Mean ($\Phi = 1.618$), the ratios 5:3, 8:5, and the $\sqrt{2}$ proportion are found throughout all of architecture, and this topic provides a rich field of study. Nevertheless, no specific mathematical information is communicated to users of a room or façade having the requisite overall proportions, and the effect remains an aesthetic one. What actually occurs is that the use of proportional ratios often also subdivides forms so as to define coincident scales, and this has a strongly positive effect" [15].

Let us emphasize that $\Phi = (1+\sqrt{5})/2 \approx 1.618$.

Given the applicability and the omnipresence of integrated iterative approaches in architecture, one of the most common ways of defining architectural surfaces and curves is through mathematical models.

Generally, mathematical modelling is used to obtain useful information pertaining to a real problem by converting it into a mathematical problem. This is especially beneficial when the acquisition of some model information is not possible or is very expensive to acquire by other means, such as by constructing the structure in question. The goal of this paper is to design novel geometrical forms i.e., to generate their mathematical models according to predefined requirements and to integrate those models as architectural forms.

4 MATHEMATICAL MODEL FOR CDS

In mathematics, a planar geometric locus that satisfies the constraint of having a constant product of distances to m foci is known as a Cassini curve [3, 13, 14]. This curve is an algebraic curve of degree $2m$ (m is number of foci). Cassinian surfaces with m foci (or m poles), the 3-dimensional analogues of Cassinian curves, are the loci of points in space for which the geometric mean of their distances to m foci is a constant [4, 13, 14].

From a geometric point of view, in plane, a line is dual to a point and vice versa, [13]. Given this, we can define Cassinian surfaces with k directrices. Along with defining these surfaces in a "*Classical Applied Mathematics*" context, we will incorporate proportional harmony (geometric mean and golden ratio) into their definitions i.e., integrate them into an "*Applicable Mathematics in Architecture*" context (see Fig. 1d).

In this section we shall create directorial geometric forms which incorporate GM (i.e. the first criterion in modelling CDS is fulfilled). Such geometric forms are named Cassinian k-directorial surfaces. They represent spatial geometric loci for which the product of distances from k fixed directrix lines is constant. To be more precise, for a point T we say that it fulfils the imposed condition for Cassinian k-directorial surface if the following is satisfied:

$$\textbf{\textit{CDS}}: \; r_1 \cdot r_2 \cdot \ldots \cdot r_i = S^k, \quad S > 0, \quad i = k > 1, \tag{1}$$

where r_1, r_2, …, r_i are Euclidean distances of the point $T(x, y, z)$ from the directrices d_1, d_2, …, d_i respectively, and S is a real positive constant.

Surfaces that are chosen to be generated and afterwards presented are derived over a regular pentagon, i.e. the number of directrices is five (k = 5). The golden ratio (the second criterion in modelling CDS is fulfilled), as is well known, is an intrinsic property of a regular pentagon. Since there are numerous ways to incorporate a regular pentagon as a directorial element in a surface genesis, we shall present three representative ways, differentiated by the space relationships of straight lines: concurrent, parallel and skew lines (Fig. 3a, b, c respectively).

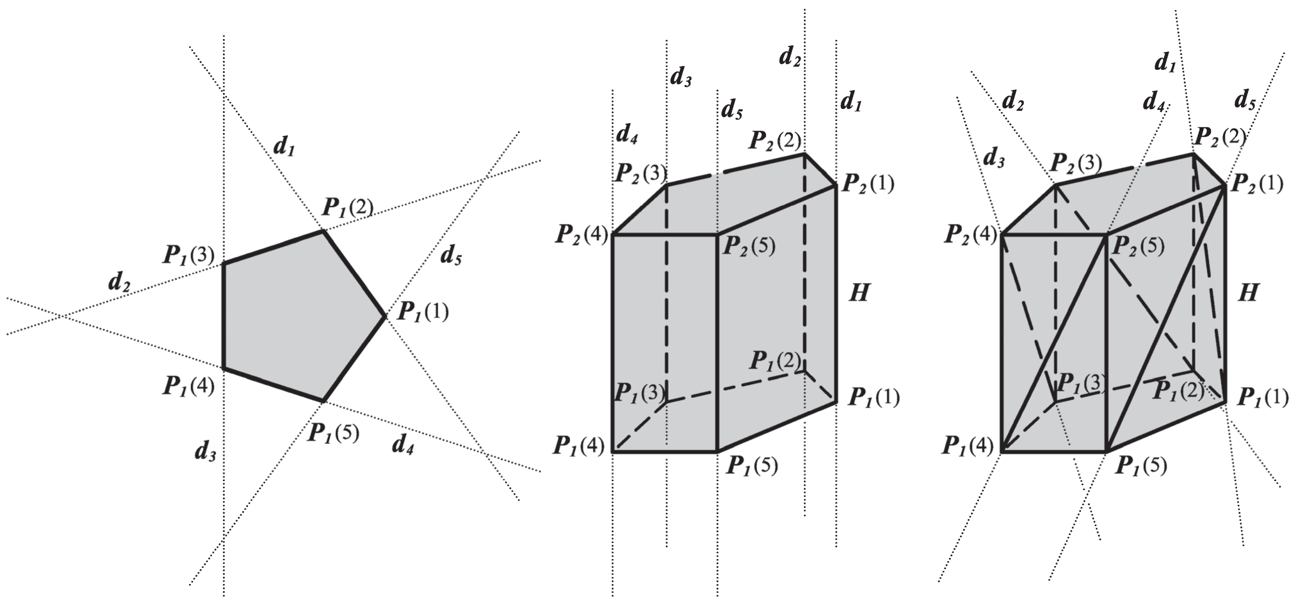


Figure 3 Directorial elements are: (a) sides of pentagon; (b) edges of upright prism with a regular pentagon as a base; (c) or skew diagonals of sides of upright prism with a regular pentagon as a base

The vertices $P_1(i)\left(X_1(i), Y_1(i), Z_1(i)\right)$ of a regular pentagon have the coordinates given by following expressions:

$$P_1(i): \begin{cases} X_1(i) = R\cos(2\pi i / k) \\ Y_1(i) = R\sin(2\pi i / k) \quad i = 1 \ldots k; \quad \text{for } k = 5, \\ Z_1(i) = 0 \end{cases} \tag{2}$$

while the points $P_2(i)\left(X_2(i), Y_2(i), Z_2(i)\right)$ are corresponding vertices of the parallel base of the prism given by following expressions:

$$P_2(i): \begin{cases} X_2(i) = R\cos(2\pi i / k) \\ Y_2(i) = R\sin(2\pi i / k) \quad i = 1 \ldots k; \quad \text{for } k = 5, \\ Z_2(i) = H, \; H > 0 \end{cases} \tag{3}$$

where H is the height of prism and R is the radius of the circumscribed circle of the regular pentagon.

4.1 Cassinian Directorial Surface - Type 1 (CDS 1)

In the first case, we define the directrices as the sides of a regular pentagon (Fig. 3a).

The vertices $P_1(i)\left(X_1(i), Y_1(i), Z_1(i)\right)$ of a regular pentagon have the coordinates given by Eq. (2) where *R* is the radius of the circumscribed circle of the regular pentagon. The equations of distances from a point *T*(*x*, *y*, *z*) on CDS 1, to a directrix d_i i.e., to a line $L\left(P_1(i),\ P_1(i+1)\right)$, where $P_1(k+1) \equiv P_1(1)$ are defined in the MAPLE program package based on the following pseudo-code:

```
Input(S);
k: = 5;  # Number of directrices
R: = 1;  # Unit circle radius
prod: = 1;   # Product initial value
with(geom3d):   # Library
for i from 1 to k do
    L(i): = 'L':   # Type of object (Directorial Line)
    point(T, x, y, z):  # Point determination
    line (L(i), [X1(i)+(X1(i)−X1(i+1))· t,
    Y1(i)+(Y1(i)−Y1(i+1))· t, Z1(i) + (Z1(i)−Z1(i+1))· t],
    t): # Line determination
    rd(i): = distance(T, L(i)):# Distance between T and L
    prod: = prod · rd(i):  # Product calculation
end do:
implicitplot3d (CDS1= prod − S^k);
```

A surface that can be generated according to the Eq. (1) is of Cassini type and its form varies with respect to the arbitrary constant *S*. Particularly when $S = S_1$, where S_1 equals the radius *r* of the circle inscribed into the regular pentagon, a unique surface shape (CDS 1) is obtained (shown in Fig. 4b). Let us emphasize that

$$r = cos(\pi/5) = (1+\sqrt{5})/4 = \Phi/2, \qquad \text{for } R = 1. \qquad (4)$$

The variation of *S*, either $S < S_1$ or $S > S_1$, produces a multitude of various surfaces (Fig. 4a and Fig. 4c).

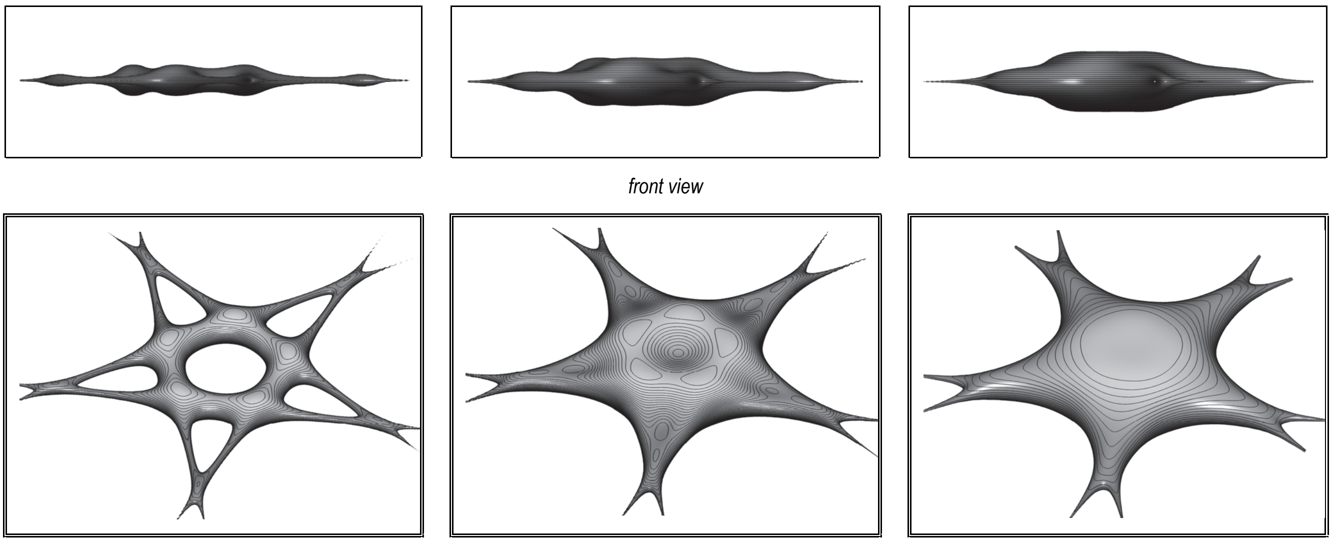
4.2 Cassinian Directorial Surface - Type 2 (CDS 2)

In the second case, the five directrices of the Cassinian directorial surface are the edges of the upright prism over a regular pentagon (Fig. 3b).

The vertices $P_1(i)\left(X_1(i), Y_1(i), Z_1(i)\right)$ of a regular pentagon, being the base of the upright prism, have the coordinates given by Eq. (2), while the vertices $P_2(i)\left(X_2(i), Y_2(i), Z_2(i)\right)$ of the upper pentagonal base of the upright prism have the coordinates given by Eq. (3); where *R* is the radius of the circumscribed circle of the regular pentagon. The equations of distances from a point *T*(*x*, *y*, *z*) on CDS 2, to a directrix d_i i.e., to a line $L\left(P_1(i), P_2(i)\right)$ are defined in the MAPLE program package based on the following pseudo-code:

```
Input(S);
k: = 5;  # Number of directrices
R: = 1;  # Unit circle radius
prod: = 1;   # Product initial value
with(geom3d):   # Library
 for i from 1 to k do
    L(i): = 'L'   # Type of object (Directorial Line)
    point(T, x, y, z):  # Point determination
    line (L(i), [X1(i) + (X1(i)−X2(i)) · t, Y1(i) +
    (Y1(i)−Y2(i)) · t, Z1(i) + (Z1(i)−Z2(i)) · t], t):     # Line
    determination
    rd(i): = distance(T, L(i)): # Distance between T and L
    prod :=  prod · rd(i):      # Product calculation
  end do:
implicitplot3d (CDS 2 = prod − S^k);
```

A unique surface CDS 2 (Fig. 5b) is obtained for $S = S_1$, where S_1 is equal to the radius *R* of the circumscribed circle of the regular pentagon ($R = 1$); while multitude of surfaces can be obtained when *S* is either less or greater than S_1. In Fig. 5 (cases: *a* and *c*) we present two surfaces for different values of *S*: $S < S_1$ and $S > S_1$. Note that each surface is developable.



front view

Axonometry

(a) $S < S_1$ (b) $S = S_1$ (c) $S > S_1$

Figure 4 Cassinian directorial surfaces (CDS 1)

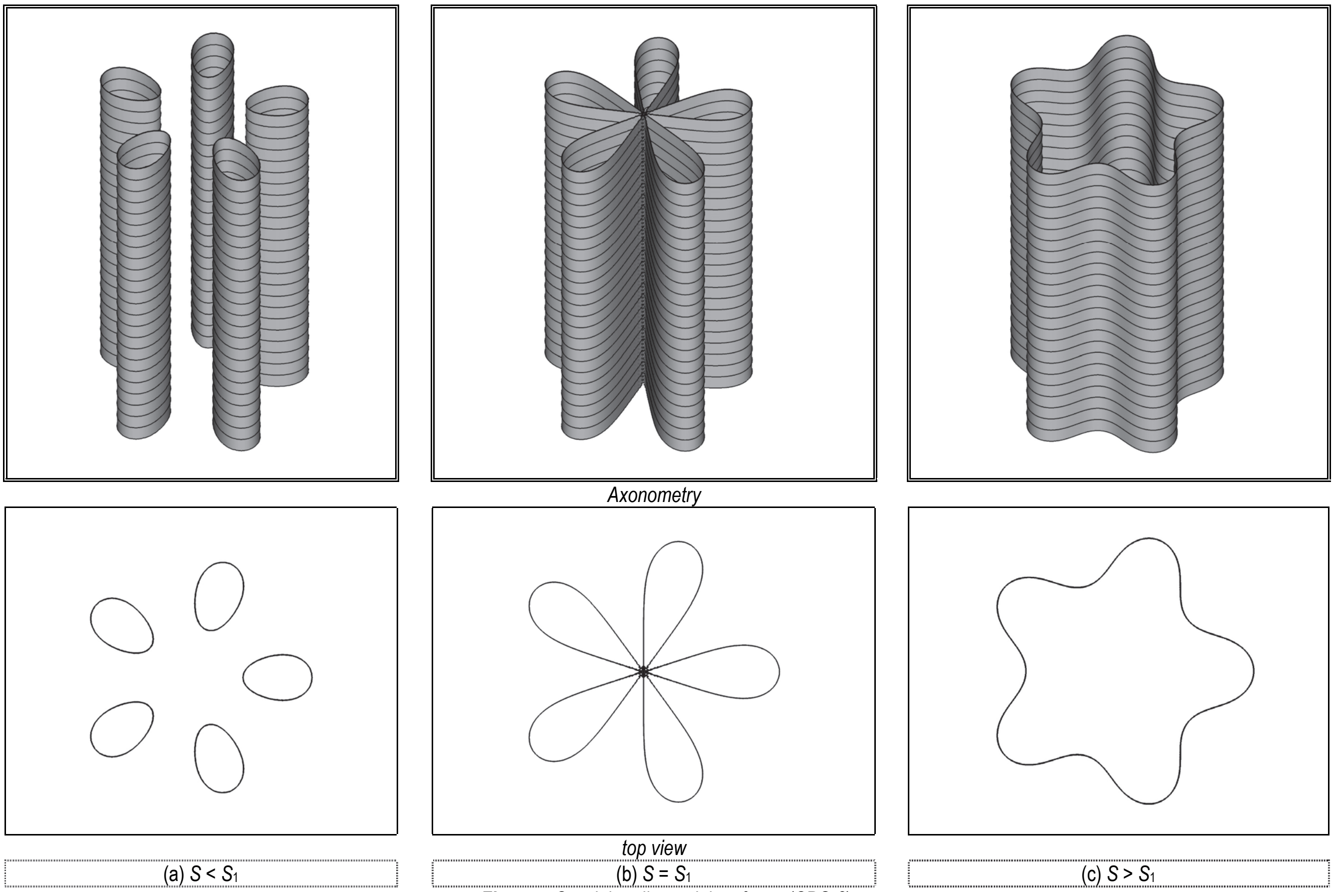


Figure 5 Cassinian directorial surfaces (CDS 2)

4.3 Cassinian Directorial Surface - Type 3 (CDS 3)

The third type of 3D geometric form genesis is created using the skew diagonals of the sides of an upright regular pentagonal prism (Fig. 3c).

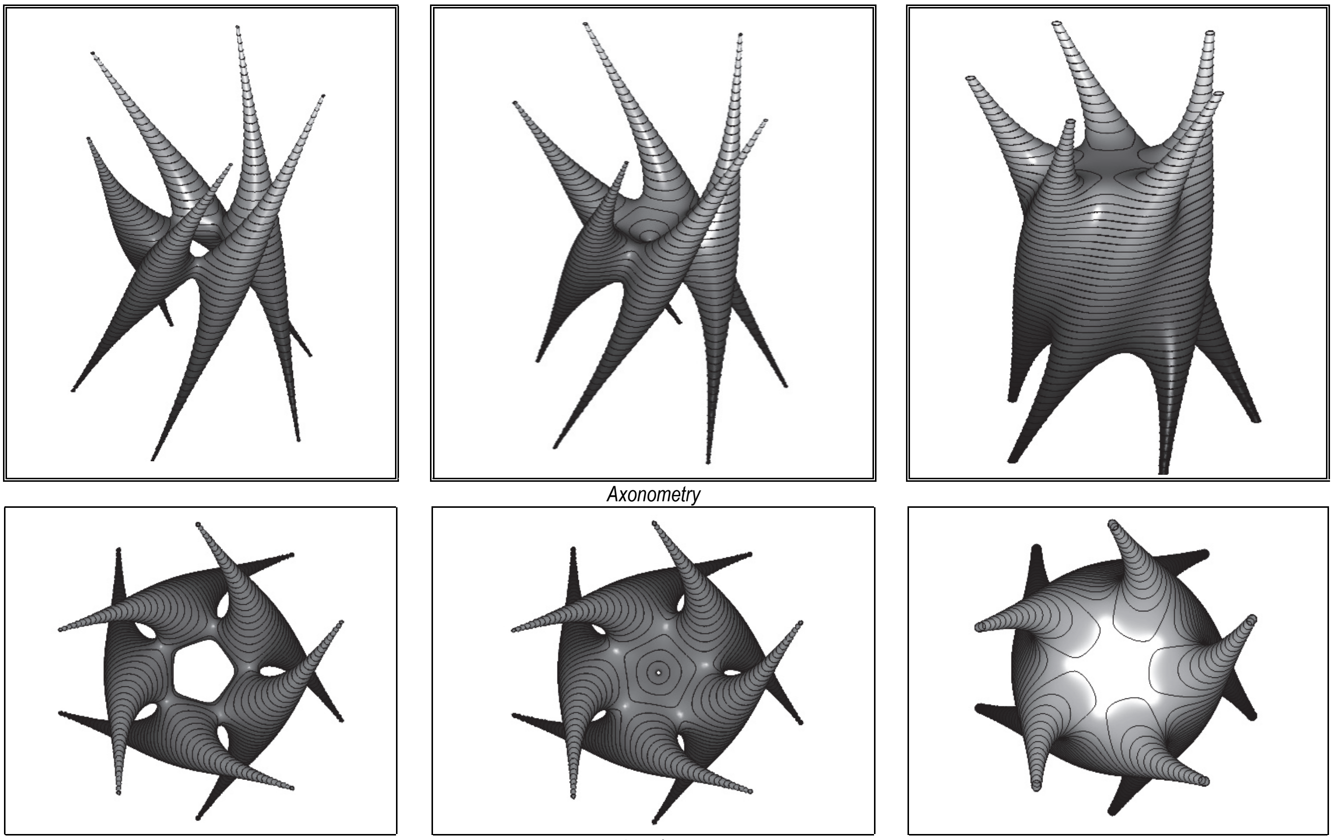
The vertices $P_1(i)(X_1(i), Y_1(i), Z_1(i))$ of a regular pentagon, being the base of the upright prism, have the coordinates given by Eq. (2), while the vertices $P_2(i)(X_2(i), Y_2(i), Z_2(i))$ of the upper pentagonal base of the upright prism have the coordinates given by Eq. (3); where R is the radius of the circumscribed circle of the regular pentagon. The equations of distances from a point $T(x, y, z)$ on CDS 3, to a directrix d_i i.e., to a line $L(P_1(i), P_2(i+1))$ where $P_2(k+1) \equiv P_2(1)$, are defined in the MAPLE program package based on the following pseudo-code:

```
Input(S);
k: = 5;   # Number of directrices
R: = 1;  # Unit circle radius
prod: = 1;    # Product initial value
with(geom3d):    # Library
for i from 1 to k do
     L(i): = 'L':    # Type of object (Directorial Line)
     point(T, x, y, z):  # Point determination
     line
     (L(i),[X1(i)+(X1(i)−X2(i+1))·t,Y1(i)+(Y1(i)−Y2(i+1))·t,
     Z1(i)+(Z1(i)−Z2(i+1))·t], t):    # Line determination
     rd(i): = distance(T, L(i)):# Distance between T and L
     prod: =  prod · rd(i):       # Product calculation
end do:
implicitplot3d (CDS 3 = prod − S^k);
```

The created surfaces are therefore Cassinian five-directorial surface (CDS 3). Depending on the value of S, we obtain a unique shape (Fig. 6b) when $S = S_1$, and S_1 equals the radius r of the circle inscribed into the regular pentagon (see Eq. (4)); while otherwise ($S < S_1$ or $S > S_1$), a multitude of surfaces can be created. A representative surface is shown for each case (Fig. 6a and Fig. 6c).

The three, previously created, pseudo-codes fully define mathematical models of the desired geometric surfaces CDS 1, CDS 2 and CDS 3.

Therefore, in Chapter 4, we have defined and graphically depicted novel algebraic surfaces, which change their geometrical form whith respect to the locations of the five directrices and the values of the parameter S. Three distinct surface classes were created in accordance with the locations of their five directrices (CDS 1, CDS 2, CDS 3). Furthermore, we have also shown three different forms in each class of the aforementioned surfaces based on changes made to the parameter S (Fig. 4, Fig. 5, Fig. 6). If the value of the parameter S corresponds to the golden mean i.e., it equals the radius r of the circle inscribed into the regular pentagon (see Eq. (4)), the obtained algebraic surfaces CDS 1 and CDS 3 have a singular point. The cylindrical surface (CDS 2) has a singular line for the value of the S parameter being equal to the radius of the circle circumscribed about the regular pentagon ($R = 1$).

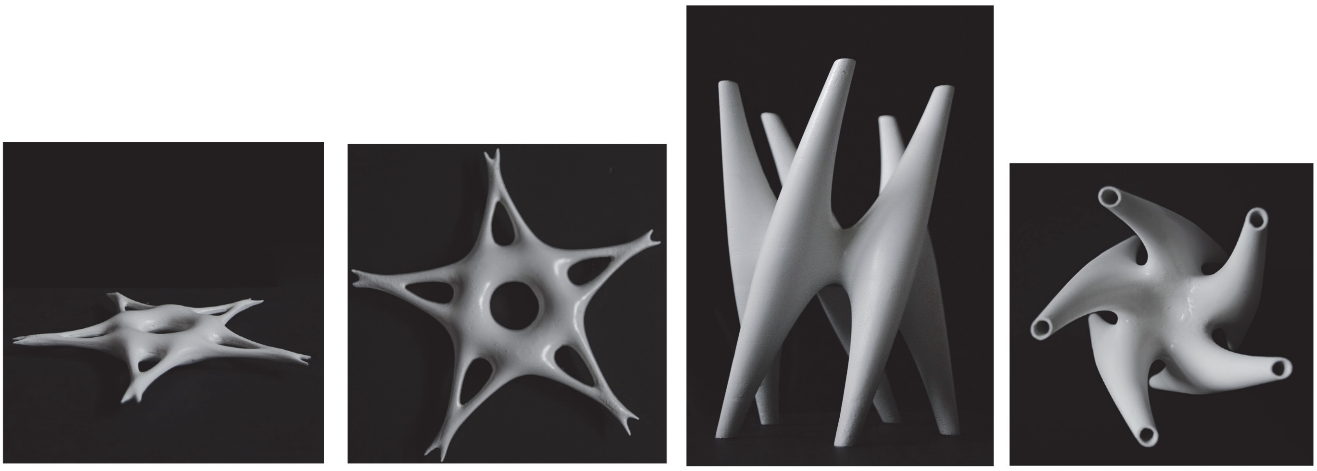


(a) $S < S_1$ (b) $S = S_1$ (c) $S > S_1$

Figure 6 Cassinian directorial surfaces (CDS 3)

The variation of the form of CDS, apart from the desired number of directrices, is possible by mere change of the constant *S*, as a product of distances from *k* fixed directrices. Note that in the third case of skew directrices, the height of the prism *H* (see Eq. (3)) also affects the form of CDS 3.

Today, thanks to modern technologies and various materials used in 3D printing, objects very close to ideal mathematical surfaces can be produced, [1, 7, 10, 17, 18, 20]. Let us emphasize that some of the *CDS* (Fig. 7) 3D models were presented at the exhibitions Designing complexity and NEWNESS, [20].



(a) (b)

Figure 7 These 3D printed models of surfaces CDS 1(a) and CDS 3(b) were presented at exhibitions in Novi Sad and New York

5 CDS AS A PATTERN FOR ARCHITECTURAL DESIGN MODEL

The multiplication or the combining of the generated surfaces, either in the whole or its particular parts, may be a generator for obtaining new complex geometric forms, such as, for instance, cupolae, curtain walls self-support structures, landscape architecture objects, urban furniture and equipment, interior elements.

Amongst numerous real-world criteria, such as functionality, structural stability, sustainability, performance etc., it is necessary to formulate the design criteria as parameters for the mathematical modelling approach. These criteria can be formulated differently based on the designer or user preferences and are hence prone to various interpretations. In the cases shown in previous examples, lines are used as linear actuators or guiding factors, coupled with numerical parameters, in order to produce geometric depictions of the mathematical modelling workflow. Lines can indicate flow, movement direction, verticality, force propagation, form and similar concepts in order to formulate the main geometrical body of the design. Numerical parameters are present to allow for smaller variations in the overall look and shape, without changing the bulk of the main geometrical shape or form.

In the sequel we present examples of some self-support structures.

5.1 CDS 1 as a Pattern for the First Design Model

In the first example shown in Fig. 3a, the concurrent lines i.e. the sides of the pentagon are positioned in a desired plane. Following the notion that lines indicate flow and directional movement, the type of form or shape to derive from this can mostly be used for architectural typologies related to large scale movement in a single plane without significant vertical exploration. These typology indicators mostly depict airports, where a large number of people are directed to a couple of floors while their movement on a single floor can be prolonged related to various checkpoints that need to be passed. The linear flow shown by the lines generates the bulk of the form, while numerical parameters allow for the inclusion of atrium like openings to allow for more lighting, larger quantity of floors and/or more diverse directional movement to exist (Fig. 8).

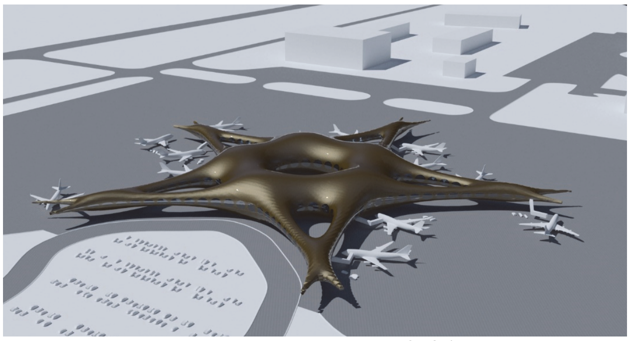


Figure 8 Airport shaped as a CDS 1

When designing this surface the following conditions were satisfied. In order to obtain five boarding gates, with accompanying condition to ensure the sufficient amount of light, the fine adjustment of the parameter S provides the control of ratio between the closed space and atriums.

5.2 CDS 2 as a Pattern for the Second Design Model

As for the parallel lines, depicted in Fig. 3b, i.e. the vertical edges of the upright prism, their application in architecture can mostly be applied to structural performance issues, vertical movement and the sense of verticality. The vertical lines indicate the sense of propagation of force and allow for a sleek and elegant framework for architectural typologies related to skyscrapers. The existence of numerical parameters allows for the exploration of various forms, exploring for example a group form concept of residential buildings, or a skyscraper with a joined central area for mixed content or an enhanced load bearing propagation model (Fig. 9). Furthermore, single tower-like forms can excellently serve as elevator shafts in order to ground the entire notion of verticality with adequate movement direction.

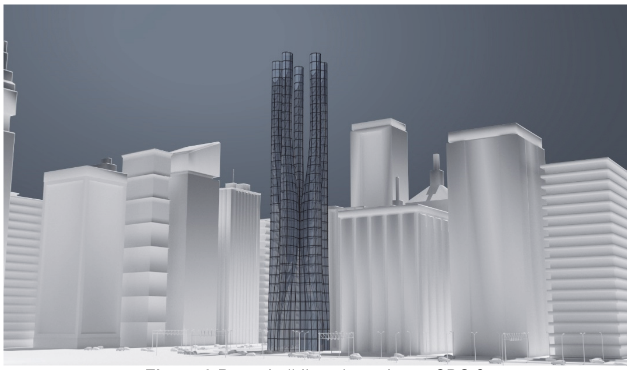


Figure 9 Penta building shaped as a CDS 2

The surface in Fig. 9 consists of a central part, with five towers that fluidly grow up out of it. The ratio between the floor plan and the height of the central part is 2:3, and the ration between the heights between the central part and each of the towers is 3:5. Both ratios reflect the Fibonacci sequence, as well as the design of the tower facade, with a raster being a rectangular grid consisting of 8×34 panels at each tower.

5.3 CDS 3 as a Pattern for the Third Design Model

Finally, the skewed lines depicted in Fig. 3c i.e. the skewed diagonals of upright prism sides are different than the previous examples. They are not set in a ground plane, but are also not depictions of verticality, given that skewed lines as shown in this manner, cannot be proper load bearing models. However, as with any architectural project, the concept of fluidity and twist is introduced as a novel property that can be used for architectural typologies that strive for being iconic, noticed, but at the same time look sturdy and provide a sense of balance.

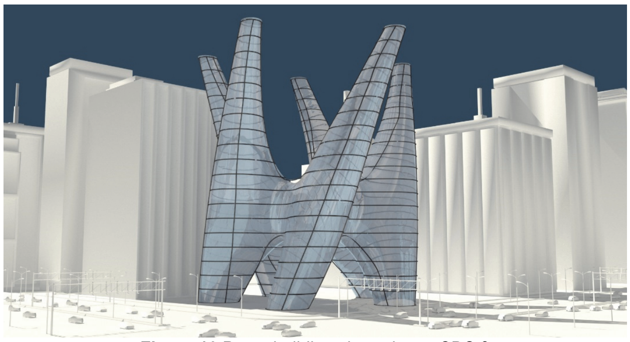


Figure 10 Penta building shaped as a CDS 3

As can be seen in Fig. 10, the general geometrical shape that can be produced houses a joined volume for mixed content, while the twisted notion can be seen in the towers which can allocate different resources, departments and present an overall categorization of work in a single building. On the other side, the entire structure can be flipped upside down, having a large canopied open space area, coupled with an open roof terrace and balanced on twisted mega-columns that can produce the sense of wonder and structural ingenuity. The numerical parameters can aid in the formulation of a sleeker design or a more bulk joined interior, given the scope of work involved and structural limitations pertaining to load bearing.

This particular surface incorporates harmonic proportions between dimensions of appropriate elements.

6 CONCLUSION

In this paper, starting from two criteria (the geometric mean and the golden ratio), we modelled a novel class of geometric surfaces which we named Cassinian five-directorial surfaces. The spatial disposition of the directors-directrices (either concurrent, parallel or skew) enabled three different types of such surfaces. We also proposed their possible application in architectural space design. The considerable factor that nominates such directorial surfaces as a fruitful generator of various geometric forms is the possibility of choice of not only the number of directrices, their spatial disposition but of the value of a particular parameter S. Therefore, the control of form variation becomes achievable giving opportunity to obtain the preferred result. Some of these forms for a certain range of the parameter S are stable and therefore suitable for easy assembling and transportation from one place to another. Furthermore, segments of these surfaces may be used as a geometric pattern for modelling and designing novel architectural forms.

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