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Subject review

Application of Hyperbolic Paraboloid in Architectural Design

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Abstract: Hyperbolic paraboloid is a kind of ruled space surface with beautiful shape. It is often used in architectural design and can achieve a free and flexible appearance effect. Due to the complexity of curved surfaces, many architects do not know how to navigate them. The main purpose of this article is to explore how to use hyperbolic paraboloids in architectural design. Firstly, the formation principle of hyperbolic paraboloid is analyzed from a mathematical perspective. Then, through investigating examples, it expounds its application in architectural design. Hyperbolic paraboloids are mainly used in building roofs, especially in large span buildings. There are three uses of hyperbolic paraboloids in roofs, corresponding to three different architectural shapes.The first is to cut a hyperbolic paraboloid vertically with four planes, and the contour projection is a rectangle or parallelogram. The second is to cut the hyperbolic paraboloid vertically and horizontally with four planes, and the contour projection is a curved quadrilateral. The third is to cut hyperbolic paraboloid with elliptic surface, and the contour projection is an ellipse. Finally, the conclusion is drawn on how to flexibly use hyperbolic paraboloids in architectural design, and what are the advantages and disadvantages of hyperbolic paraboloids, which has important reference value for architects to carry out related designs.

Keywords: architectural design; hyperbolic paraboloid; roof; ruled surface

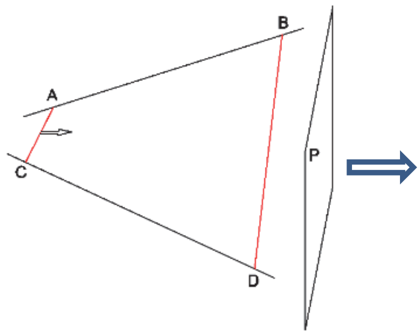
1 INTRODUCTION

Curved surface elements are often used in architectural design, which gives people a sense of liveliness, flexibility, stretch and elegance. Surfaces used in architecture include ruled surfaces and non-ruled surfaces. Ruled surface refers to the surface that can be obtained by moving or rotating a straight line along a certain track, such as cylindrical surface, conical surface, hyperbolic paraboloid, univalent hyperboloid, tangent surface, helicoid, etc. Non ruled surface refers to a surface that cannot be formed by a straight line moving along a certain trajectory, such as spherical surface, ellipsoid, paraboloid, hyperboloid, etc. Among them, ruled curve is more favorable for structural layout, so it is more used in architecture. Hyperbolic paraboloid is a ruled surface especially loved by architects [1].

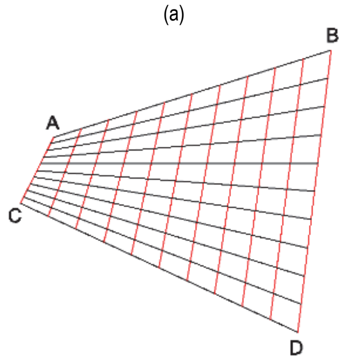
Hyperbolic paraboloid, also known as saddle surface, is a ruled surface with double clusters of straight generatrix. The formation of hyperbolic paraboloid can be imagined as setting two different plane straight lines and a third straight line moves parallel to a specific plane through a point on the two different plane straight lines. The moving track of this straight line is hyperbolic paraboloid [2]. As shown in Fig. 1a, *AB* and *CD* are straight lines of different planes, *AC* is parallel to plane *P*, *AC* moves in the direction of plane P on the two straight lines and is always parallel to plane P, then the formed surface is a hyperbolic paraboloid, as shown in Fig. 1b [3]. Another cluster of straight generatrix is added in Fig. 1b, that is, the straight generatrix perpendicular to the common vertical lines of *AB* and *CD*. Change the observation angle to obtain the figure shown in Fig. 1c. and the upper contour envelope is a parabola. Hyperbolic paraboloid is mainly used for roof in buildings and is often used in Long-Span Public buildings, such as gymnasium, Convention and Exhibition Center, Grand Theater, railway station [4, 5], etc.

There are many well-known buildings in the world that use hyperbolic parabolic roofs, and their structural forms are different. The Xochimilco Restaurant in Mexico (1958) and the Aloysius Church in the United States (2009) are constructed with a thin concrete shell structure. The indoor bicycle gymnasium of London Olympic (2011) and Qatar Russell Stadium in Qatar (2021) adopt cable net structure. The Bird's Nest Stadium in Beijing (2008) adopts steel structure. The main stage of Chengdu Open Air Music Square (2019) adopts cable net and membrane structure. Currently, there are still many experts and scholars studying the application of hyperbolic paraboloids in architecture and how to organically combine architecture and structure.

There are three main types of hyperbolic paraboloid roofs [6].



(a)



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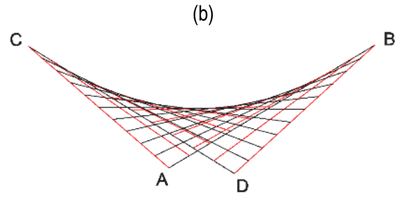


(c)
Figure 1 Formation of hyperbolic paraboloid

2 THE FIRST ROOF OF HYPERBOLIC PARABOLOID

The horizontal projection of the roof is rectangular, rhombic or parallelogram. Two clusters of the straight generatrix are parallel to edges of the rectangle, diamond or parallelogram as shown in Fig. 2. The plane of the lower building can correspond to the shape of the roof, rectangular, rhombic or parallelogram. Of course, other plane shapes can also be adopted according to functional or artistic requirements. The lower part of the building shown in Fig. 3 adopts a pyramid shape, and the upper roof adopts a rectangular hyperbolic paraboloid roof [7].

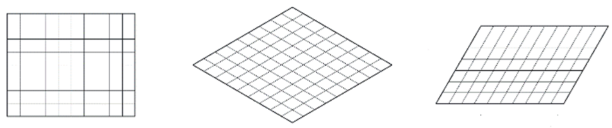


Figure 2 Horizontal projection of roof

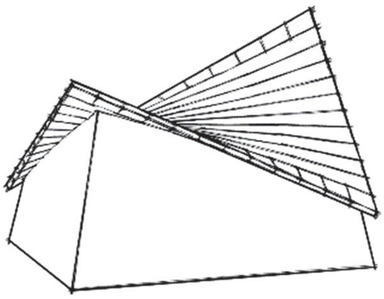


Figure 3 Schematic diagram of hyperbolic paraboloid roof (I)

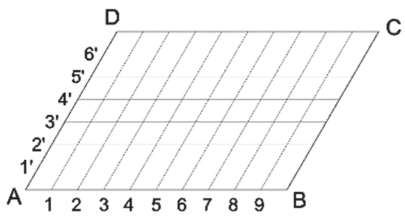


Figure 4 Coordinate positioning of hyperbolic paraboloid roof

The coordinate and elevation positioning of this roof is relatively easy. When making an architectural scheme, first fix the coordinates and elevations of the four corners, then divide the two adjacent edges into several equal parts, and make parallel lines of the adjacent edges through the equal points. They are straight lines, so it is easy to calculate the coordinates and elevations of each node. As shown in Fig. 4, the horizontal projection of hyperbolic paraboloid roof is shown. Using the method of elevation projection, first determine the coordinate s and elevations of four corner points *A*, *B*, *C* and *D*, then make equidistant parallel lines, and calculate the coordinate and elevation of each intersection point in turn. The structural engineer can arrange the roof structure according to this positioning drawing. The roof can be a steel structure or a reinforced concrete structure. If a steel structure is used, it is usually a steel frame or suspension structure, and steel beams or suspension cables can be arranged in both directions along a straight line. If a reinforced concrete structure is used, it is usually a thin shell structure. The coordinates and elevation values of each feature point on the roof are calculated, the templates are fixed, and then the concrete is poured to complete the process. Figs. 5 and 6 are examples of this type of hyperbolic parabolic building roof [8]. Fig. 5 shows Xinghai Concert Hall of Guangzhou, with a reinforced concrete shell roof. Fig. 6 shows San Vicente de Paul Chapel, designed by renowned architect Felix Candela. Its roof consists of three hyperbolic parabolic thin shells of reinforced concrete. The hyperbolic paraboloid structure is subjected to reasonable forces, and the material is mainly subjected to axial tensile and compressive stresses, which can fully utilize the mechanical properties of the material. So the roof can be made very thin and the materials are saved.

Figure 5 Example of hyperbolic paraboloid roof (I)

Figure 6 Example of hyperbolic paraboloid roof (II)

The four sides of this roof are straight lines. It has simple structure, easy production of components and parts, and low construction difficulty. The roof has a warped surface with a simple and generous appearance. The building plane is relatively regular, and the internal space is easy to use. But the height changes greatly, with two diagonals higher and the other two diagonals lower. It is suitable for museums, exhibition halls, music halls, etc.

3 THE SECOND ROOF OF HYPERBOLIC PARABOLOID

The hyperbolic paraboloid shape of this roof is shown in Fig. 7. The horizontal projection of the roof is close to a rectangle, with a slight inward depression in the middle. The straight line on the roof is one of the clusters of straight generatrix of hyperbolic paraboloid. For the convenience of analysis, the hyperbolic paraboloid is put into a rectangular coordinate systema as shown in Fig. 8 [9].

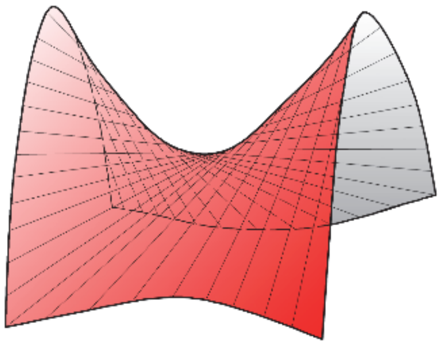


Figure 7 The second kind hyperbolic paraboloid for roof

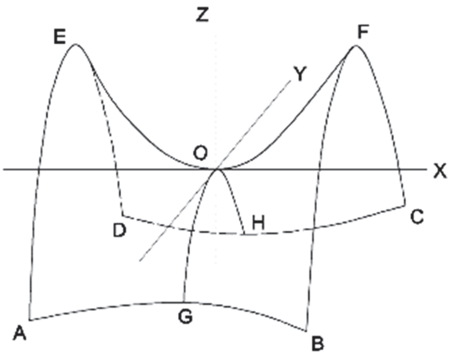


Figure 8 Spatial analysis of hyperbolic paraboloid

The mathematical equation of the hyperbolic paraboloid is:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z \quad (a,\ b>0) \tag{1}$$

(Note: x, y, and z are unknowns representing the coordinates of points, while a and b are constants).

It can be seen from Eq. (1) that when the surface is cut with a plane parallel to the *YOZ* coordinate plane, the intersection line is a parabola with the opening downward. The apex of the parabola is located on the ridge of the hyperbolic paraboloid, that is, the curve *EOF*. All parabolic openings are the same size. The larger the value of b, the larger the opening of the parabola. Curve *BFC* is one of the parabolas. When the plane parallel to the *XOZ* coordinate plane is used to cut the surface, the intersection line is a parabola with the opening upward. The apex of the parabola is located on the curve *GOH*, another ridge of the hyperbolic paraboloid. All parabolic openings are the same size. The larger the value of a, the larger the opening of the parabola [10, 11]. The ridge *EOF* is the highest parabola. Therefore, the surface contains two-way paraboloids. The surface can be regarded as formed by the movement of parabola *BFC* along parabola *EOF* (the vertex and direction remain unchanged), or it can be regarded as formed by the movement of parabola *EOF* along parabola *GOH* (the vertex and direction remain unchanged). If the surface is cut with a plane parallel to the *XOY* coordinate plane, the resulting intersection line is a pair of hyperbolas. As shown in Fig. 8, curve *AGB* and curve *DHC* [12].

Eq. (1) is equivalently transformed to get:

$$\begin{cases} \frac{x}{a}+\frac{y}{b}=2\lambda \\ \left(\frac{x}{a}-\frac{y}{b}\right)\lambda=z \end{cases} \quad (\lambda \in R) \tag{2}$$

(Note: x, y, and z are unknowns representing the coordinates of points, while a and b are constants. λ is a variable, and R represents real number).

Eq. (2) represents a curved surface formed by the cluster of spatial straight lines, and it can be seen that the hyperbolic paraboloid is a ruled surface.

Eq. (1) can also be transformed by a similar equivalent to get:

$$\begin{cases} \frac{x}{a}-\frac{y}{b}=2\mu \\ \left(\frac{x}{a}+\frac{y}{b}\right)\mu=z \end{cases} \quad (\lambda \in R) \tag{3}$$

(Note: x, y, and z are unknowns representing the coordinates of points, while a and b are constants. μ, λ is a variable, and R represents real number).

Obviously, Eq. (3) expresses another group of spatial line clusters of hyperbolic paraboloids [13].

For practical engineering, designer usually designs the preliminary building plan first, then covers the hyperbolic paraboloid top, and then fine-tunes the two to make them match. Assuming that the plan shape of the building is shown in Fig. 9a, the hyperbolic paraboloid shown in Fig. 8 is covered on it, and the coordinate system is still used. The designer can draw the parabola *BFC* according to the modeling ratio, and then the coordinates of each point on the parabola *BFC* have been determined. Substitute the coordinate values of point *F* and point *B* into Eq. (1) to obtain the values of *a* and *b*. The horizontal projection of the parabola *BFC* is divided into several segments, as shown in Fig. 9c. Take one of the equally divided points, such as 4 point, and substitute its coordinate value into Eq. (2) to obtain the value of λ, and set it as λ_1. When λ is a fixed value, Eq. (2) expresses a certain plane (vertical direction). Assuming that the *Z* coordinate value of point *B* is Z_1, the equation of the original base hyperbola *AGB* is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z_1$. Combining it with Eq. (2), we get:

$$\begin{cases} \frac{x}{a}+\frac{y}{b}=2\lambda_1 \\ \left(\frac{x}{a}-\frac{y}{b}\right)\lambda_1=z \\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=2z_1 \end{cases} \tag{4}$$

(Note: x, y, and z are unknowns representing the coordinates of points, while a, b, λ_1 and z_1 are constants. R represents real number).

The coordinate value of the 4' point at the other end of the straight generatrix (located on the hyperbola) can be solved by Eq. (4). The connecting line between 4 and 4' is a straight generatrix on the curved surface. The other straight generatrixes on the curved surface can be obtained by the same method. The horizontal projections of all the straight generatrixes in this group are parallel. Fig. 9b shows a three-dimensional schematic diagram of the distribution of straight generatrixes. Similarly, another set of straight generatrixes on the surface can be obtained [14]. Draw both sets of straight generatrixes to obtain the position diagram of the straight generatrix of hyperbolic paraboloid as shown in Fig. 9d. With the coordinates of the two ends of the line, the length of the line and the elevation of each point on it can be obtained [15].

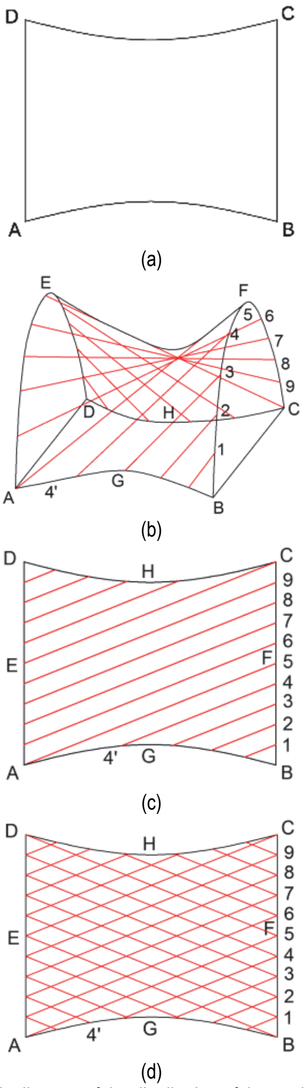


Figure 9 Schematic diagram of the distribution of the straight generatrix of a hyperbolic paraboloid

When the structure is arranged, according to the structure form, beams or tie rods can be arranged along the direction of the straight generatrix. This reduces the number of curved components and simplifies the component form. Graphical method can also be used to draw the straight generatrix cluster on the hyperbolic paraboloid, that is, the hyperbolic paraboloid model is first built with computer graphics software, and then the straight generatrix is drawn on the surface of the model. The length, angle and elevation of each point can be directly measured in the software. Fig. 10 shows a schematic diagram of the shape of a building with a hyperbolic paraboloid roof [16]. For this type of roof, the cross-section parabola can also be obtained according to the hyperbolic paraboloid equation, and the parabolic beams or steel frames can be evenly arranged along the longitudinal direction. Then, steel tie rods or cables are arranged between the beams or steel frames to form a support structure of the roof. The shape is also a hyperbolic paraboloid, as shown in Fig. 11. The steel mesh can also be arranged according to the shape of the hyperbolic paraboloid, and concrete is cast outside to form a reinforced concrete shell structure [17].

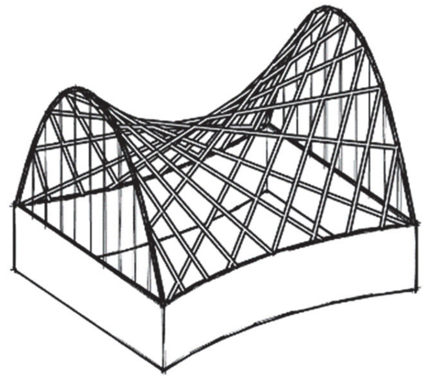


Figure 10 Schematic diagram of hyperbolic paraboloid roof (II)

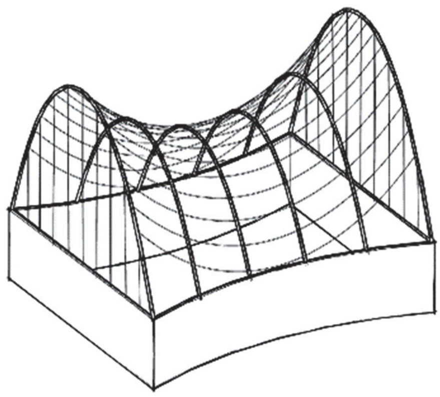


Figure 11 Schematic diagram of hyperbolic paraboloid roof (III)

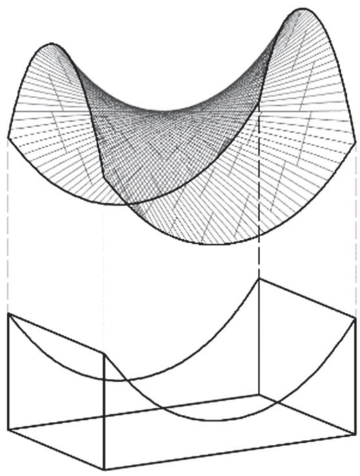


Figure 12 Schematic diagram of hyperbolic paraboloid roof when the building plane is rectangular

If you want the lower plane shape to be rectangular, a pair of bottom edges of the upper hyperbolic paraboloid roof need to be slightly adjusted from the original pair of hyperbolas to a pair of parabolas with upward openings, and its bottom is no longer horizontal [18]. A pair of parabolic notches of the same size will also be left on the upper part of the lower wall to coincide with the upper roof as shown in Fig. 12. In fact, this roof can be regarded as the diamond roof in Fig. 2b, which is obtained by cutting four corners with four vertical planes, and the horizontal projection after cutting is rectangular. As shown in Fig. 13. The dotted line in the figure shows four vertical sections. After cutting, the rectangular part in the center is the hyperbolic paraboloid roof in Fig. 12. Then using the horizontal section plane passing through four corners to cut once, the hyperbolic paraboloid roof in Fig. 10 or Fig. 11 is obtained [19].

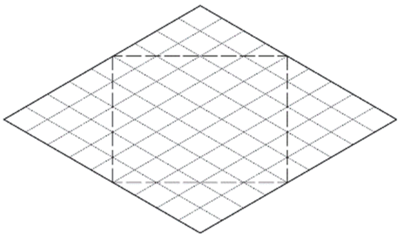


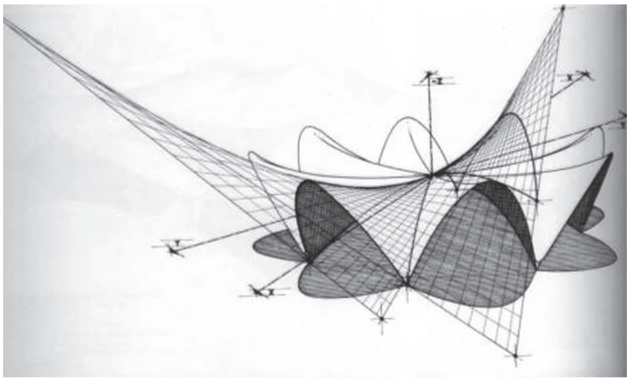
Figure 13 Schematic diagram of horizontal projection of hyperbolic paraboloid roof when vertical sectioning

There are many cases in engineering about this kind of roof. Fig. 14 shows the Palmyra church designed by Candela, which is a typical building of hyperbolic paraboloid roof. This kind of roof can also be cut and combined to form a more complex building of hyperbolic paraboloid roof [20].

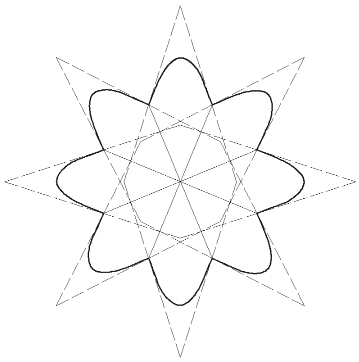
Figure 14 Example of hyperbolic paraboloid roof (III)

Figure 15 Example of hyperbolic paraboloid roof (IV)

As shown in Fig. 15, Xochimilco Restaurant, also designed by Candela, is a compound hyperbolic paraboloid roof building composed of four hyperbolic paraboloids. Fig. 16 is a schematic diagram of the combination and cutting of the roof of the Xochimilc restaurant. (a) Is a three-dimensional schematic diagram, and (b) is a planar projection schematic diagram. The roof is composed of four equal hyperbolic paraboloids, forming eight intersecting lines on the surface. The plane like 8 petals, and the ends of the petals are cut into a parabolic shape. In the Fig. 16b the thick solid line represents the outer contour, while the thin solid line represents the intersecting lines between hyperbolic paraboloids.



(a)



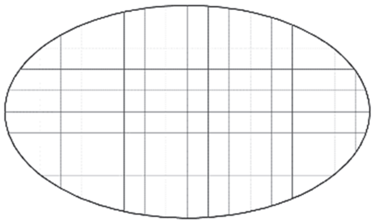
(b)

Figure 16 Schematic diagram of combination and cutting of hyperbolic paraboloid

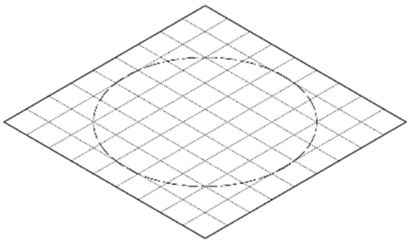
The four sides of this roof are curved. Its structure is relatively complex, the production of components and fittings is not easy, and the construction is difficult. The roof is shaped like a saddle, with a stunning shape and strong visual impact. The plane of the building is close to a rectangle, and its shape is relatively regular. The internal space varies greatly, with high in the middle and low around. It is suitable for stations, exhibition halls, gymnasiums, etc.

4 THE THIRD ROOF OF HYPERBOLIC PARABOLOID

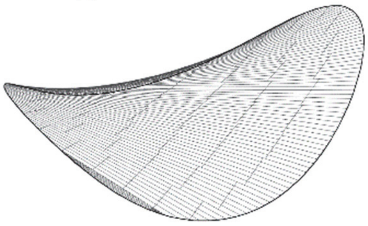
The horizontal projection of the roof is oval, as shown in Fig. 17a. The roof can be regarded as an ellipse dug from the middle of the diamond hyperbolic paraboloid in Fig. 2b, as shown in Fig. 17b. The curvature of the roof is usually not too large and relatively flat. Its stereoscopic view is shown in Fig. 17c. This hyperbolic paraboloid roof is often used in gymnasium, theater, concert hall and other performance buildings. The roof contour is oval, and the plane of the lower building is also oval [21].



(a) Horizontal projection of the roof



(b) The formation of the roof



(c) Roof axonometric drawing

Figure 17 Schematic diagram of oval hyperbolic paraboloid roof

The roof structure is usually a steel structure. When the structure is arranged, one is to arrange steel beams, steel rods or steel cables along the direction of the straight generatrix, so that the structural members of the roof are all straight lines. The effect is shown in Fig. 18 [22]. The other is to arrange steel grids or steel cables along the long axis and short axis of the ellipse respectively. The shape of the structural member is a parabola, which opens upward along the long axis and opens downward along the short axis. The horizontal projection of the parabola is parallel to the long axis and the short axis of the ellipse respectively, the effect is shown in Fig. 19 [23]. The first design is suitable for both rigid members and flexible cable networks. When using steel beams, the span should not be too large, otherwise the cross-section is too large and uneconomical. Because they are all linear components, the shape of the roof is easy to control. The second design is only applicable to tensioned cable net structures, which adjust the tension angle of each cable to give the roof a hyperbolic parabolic shape. This method requires high construction accuracy, but relatively few materials are used. Both structural forms of roofs are mainly covered with lightweight roof panels.

No matter what kind of structural arrangement, the first thing to do is to determine the frame of the roof. Often starting from a plane, the oval roof is placed in the center of a rhombus hyperbolic paraboloid [24]. According to the design shape, determine the elevation of the four corners of the rhombus. Then draw a number of equidistant straight generatrixes, find the intersection of the straight generatrix and the ellipse, calculate the elevation of each intersection point, and connect each point with a smooth curve, then the outline of the oval roof frame is determined [25]. As shown in Fig. 20. The internal structural members can also be arranged immediately after the diamond and ellipse are determined. Find the intersection of the rod and the ellipse, so as to determine the space profile of the ellipse. This process is usually simulated by computer, and the results will be more accurate. All members or cable nets are supported on the frame, so the strength and stability of the frame are very important, which affects the overall safety and stability of the roof structure. Elliptical roof frames are mostly made of steel trusses, supported by columns at the bottom. These structural technologies have matured and fully meet the safety requirements of the upper structure. Fig. 21 shows the indoor cycling Hall of the 2012 London Olympic Games, its oval roof is a hyperbolic paraboloid. For larger open-air stadiums, if the roof also adopts this curved surface, an oval hole can be dug in the middle. There are many similar buildings at home and abroad, such as Russell stadium in Qatar and bird's Nest Stadium in Beijing [26].

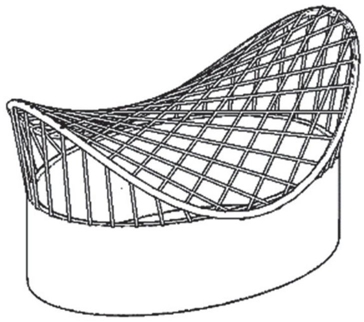


Figure 18 Arranging structure along the straight generatrix

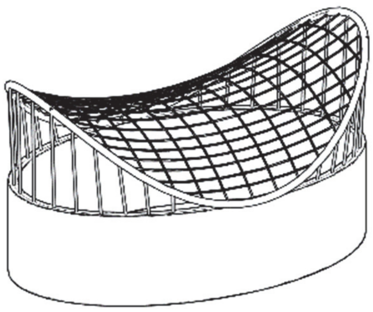


Figure 19 Arranging structure along the long and short axis

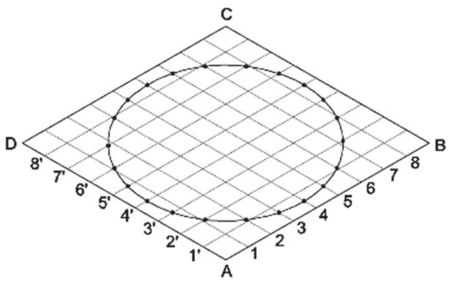


Figure 20 Elevation diagram of each point of elliptical roof contour

Figure 21 Example of hyperbolic paraboloid roof (V)

This roof has a continuous curve around it. The structure is relatively complex, the production of components and fittings is difficult, and the construction is also difficult. The roof is oval saddle shaped, with a gentle surface and a beautiful appearance. The plane of a building is usually oval in shape and has a large area. The internal height changes little. It is particularly suitable for stadium and gymnasium buildings.

5 CHARACTERISTICS OF ROOF OF HYPERBOLIC PARABOLOID

Hyperbolic paraboloid is a ruled surface, and its structural components can use straight lines when used on roofs, simplifying structural design and construction techniques, and reducing construction costs. When using steel structures, the members can be prefabricated first and assembled on site. When using concrete structures, the hyperbolic parabolic roof is similar to an arch shell structure. Its main stress is compressive stress, which can fully utilize the mechanical properties of the material. The roof can often be made very thin - called thin shell structures. Concrete may not even be equipped with load-bearing steel bars, but only with connecting steel bars or steel wire mesh. As shown in Fig. 15, the concrete roof of Xochimilco restaurant is only equipped with thin steel wires. The thinnest part of the roof is only 4cm, and the thickest part is only 12 cm, greatly saving materials and reducing costs.

Compared with hyperbolic paraboloid roofs, non straight curved roofs cannot form straight components on their surfaces, which greatly increases the difficulty of structural design and construction, and also increases their cost. Traditional flat or sloping roofs have a smaller span than hyperbolic parabolic roofs. They are not suitable for large space buildings, and their outward appearance is not visually impactful. In addition, the thickness of flat or sloping roofs of reinforced concrete is often large, usually not less than 12 cm, which increases the load, wastes materials, and increases the cost.

The disadvantage of hyperbolic paraboloid roofs is that the positioning of components must be accurate during construction, which requires a high level of skill from workers. Especially for concrete thin shell structures, they must be cast on-site, the supporting formwork must be very precise, and workers must strictly control the concrete thickness and surface flatness at each point. For hyperbolic paraboloid buildings of steel structure, their components are mainly produced in factories, and on-site installation is fast. Construction operations are less affected by weather. For hyperbolic paraboloid buildings of concrete structure, the construction process is entirely carried out on the construction site and is greatly affected by the weather.

6 CONCLUSIONS

The hyperbolic paraboloid is a ruled space surface, which is widely used in various buildings, especially in large-span buildings. If used properly, it will have a strong visual impact and create a rich and varied shape. In terms of structure, hyperbolic paraboloid components have more reasonable stress, large span, material saving and low cost. The common structural forms include reinforced concrete shell structure, steel grid structure, cable structure, curtain structure and so on. However, many architects do not understand the mathematical principle of hyperbolic paraboloid and they cannot use it or use it unreasonably in architectural design. Through this paper, architectural designers will better understand the characteristics and application of hyperbolic paraboloid, make more use of this modeling element in architectural design, broaden design ideas, enrich design techniques and improve the level of architectural design.

The several types of hyperbolic paraboloids described in this paper are the most basic and simplest applications in architecture. There are still many changes in practical engineering, such as cutting, rotation, combination of multiple hyperbolic paraboloids, etc., which can obtain endless changes in appearance modeling. However, as long as architects master the mathematical principles of hyperbolic paraboloids and are familiar with their basic application in architecture, more complex application problems will be easily solved.

Acknowledgements

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