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Outlier Detection with Robust Exact and Fast Least Trimmed Squares Methods in Coordinate Transformation

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ABSTRACT. Different terrestrial reference systems have been defined and used because of some practical and historical events in geodesy domain. The transition from one system to another requires the coordinate transformation. Helmert transformation is the most commonly used model for 2D networks. 2D Helmert transformation are defined by four transformation parameters and two common points in both coordinate systems provides a unique solution. To increase the reliability of the transformation parameters, redundant observations are generally used. In this case, the Least Squares (LS) is the most common method used to obtain the unique solution from redundant observations. However, outliers occur often in dataset and affect severely the results of LS. There are generally two approaches applied for outlier detection: classical outlier tests and robust methods. The most common robust methods are Least Absolute Deviation (L_1), M-estimators, the Total Least Squares (TLS), Generalised M-estimators, the Least Median of Squares (LMS) and the Least Trimmed Squares (LTS). For the solution of the LTS method, there are exact and approximate solutions. In this study, 2D Helmert transformation parameters between ED50 and ITRF coordinates are estimated with the LS method including classical outlier test, exact LTS solution and Fast-LTS solution which is an approximate solution to compare outlier detection performances of the methods.

Keywords: coordinate transformation, outlier detection, robust solution, the least squares, the least trimmed squares, ITRF, ED50, LTS, TLS, GNSS.

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1. Introductıon

Different terrestrial reference systems have been defined and used because of some practical and historical events in geodesy domain (Chang et al. 2017). In Turkey, the European Datum 1950 (ED50) was used as the national coordinate system for various cadastral applications until 2005. After earthquakes causing movements and displacements in Turkey, the Turkish National Fundamental GPS Network (TNFGN) which uses the International Terrestrial Reference Frame 96 (ITRF96) was established in order to make up the shortcomings of the former system (Sisman 2014, Konakoglu et al. 2016). The conversion from ED50 to ITRF96 requires the coordinate transformation between those systems. Helmert (conformal or similarity), affine and projective transformation models are the conventional techniques used for coordinate transformation (Ziggah et al. 2018). The accuracy of the coordinate transformation depends not only on the method and the number of common points in both coordinate systems but also on the accuracy of the common points in both systems. The official coordinates in the target coordinate system are usually well determined and should not be changed. But, using classical transformation approach make official coordinates changed. The problem is resolved by preserving official coordinates by using post transformation corrections based on the classical approach (Ligas and Banasik, 2014, Gargula and Gawronek 2023).

Helmert transformation is the most commonly used model for 2D networks (Öcalan 2019). 2D Helmert transformation are defined by four transformation parameters: two translation along the two axes, scale and rotation angle between the axes of two coordinate systems (Sjöberg 2013). Two common points in both coordinate systems provides a unique solution. However, more than two common points, that is, redundant observations are generally used to increase the reliability of the transformation parameters (Akyilmaz 2007).

The Least Squares (LS) is the most common method used to obtain the most probable solution from all observations, when outliers do not exist (Amiri-Simkooei 2018). Outliers which behave differently from the majority of data occur often in dataset and affect severely the results of LS. Even one outlier could be sufficient for LS to have incorrect results, thus making its breakdown point equals $1/n$ which tends to 0% with increasing n (observation number) (Rousseeuw and Leroy 1987). Therefore, outlier detection is an important issue in geodesy. There are generally two approaches applied for outlier detection: classical outlier tests and robust methods (Rousseeuw and Hubert 2018). If there is one outlier in dataset, the classical outlier tests can successfully detect the outlier. But, they will fail when there is more than one outlier because of the swamping and masking effects of LS, thus making them sensitive to outlier (Sisman 2010, Hekimoglu et al. 2015).

Robust methods are designed to be not sensitive to outliers. Many robust methods are discussed in literature. The most common robust methods are Least Absolute Deviation (L1), M-estimators, the Total Least Squares (TLS) (Golub and van Loan 1980), Generalised M-estimators (Hampel et al. 1986), the Least Median of Squares (LMS) (Rousseeuw 1984) and the Least Trimmed Squares (LTS) (Rousseeuw and Leroy 1987). M-estimators is an important class of robust methods which is defined by an aim function *ρ(v)* of residuals (*v*) to be minimized. Their solution can be realized by iteratively weighted LS (Wieser and Brunner 2001). There are several M-estimator defined by their weight function such as Huber, Hampel, Andrew, Yang I, Yang II, Danish method etc. (Borowski and Banaś 2019). These M-estimators have been applied to various geodetic applications (Yang et al. 2002, Berné Valero and Baselga Moreno 2005, Gökalp et al. 2008, Knight and Wang 2009, Sisman 2010, Třasák and Štroner 2014, Gašinec and Gašincová 2016, Borowski and Banaś 2018, Shin and Oh 2020). However, their breakdown point also equal $1/n$. After all these advancements in robust methods, high breakdown point estimators such as the LMS and the LTS were introduced (Rousseeuw and Leroy 1987). A trimming parameter (h) which determines the size of the subset is used in both methods (Rousseeuw and Leroy 1987, Hekimoglu et al. 2009, Mount et al. 2014) Both methods are also discussed in geodetic applications (Knight and Wang 2009, Yang 2011, Koch et al. 2017). For the solution of the LTS method, there are exact and approximate solutions suggested (Rousseeuw and Leroy 1987, Atkinson and Cheng 1999, Agulló 2001, Rousseeuw and Driessen 2006, Koch et al. 2017, Hofmann et al. 2010).

In this study, 2D Helmert transformation parameters between ED50 and ITRF coordinates are estimated with the LS method including classical outlier test and exact and Fast-LTS solutions to compare outlier detection performance of the methods.

2. Methods

2.1. 2D Helmert Transformation

The Helmert transformation named after Friedrich Robert Helmert (1843–1917) is a geometric transformation method which is often used in geodesy. 2D Helmert transformation is also known as 4-parameter similarity transformation (Fig. 1).

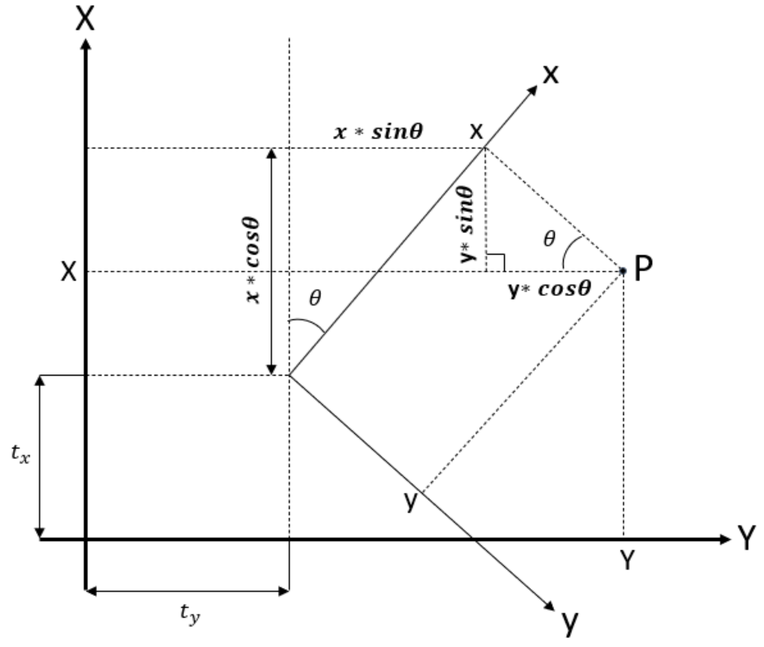

Fig. 1. *2D Helmert Transformation (Öcalan 2019).*

The equation for 2D Helmert transformation between X-Y and x-y which define two planar Cartesian coordinate systems can be expressed as following:

$$X = (S * cos\theta) * x - (S * sin\theta) * y + t_x \tag{1}$$
$$Y = (S * sin\theta) * x + (S * cos\theta) * y + t_y \tag{2}$$

Here, θ rotation angle, S scaling and t_x, t_y translations. Equation (1–2) can be simplified by $a = S * cos\theta$ and $b = S * sin\theta$.

$$X = a * x - b * y + t_x \tag{3}$$
$$Y = b * x + a * y + t_y \tag{4}$$

or in matrix notation

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \tag{5}$$

Here, a, b, t_x, t_y are the four transformation parameters.
The solution of the Eq. (3–4) requires at least two common points in both systems.

2.2. The Least Squares and Classical Outlier Tests

Geodetic observations usually include errors. Therefore, redundant observations ($n > u$, where n observation number and u unknown number) are made in geodetic applications. In this case, LS method is usually used for the estimation. The mathematical model generally used for LS in geodetic applications is the linear Gauss-Markov model given as (Klein et al. 2017):

$$V = Ax - l \tag{6}$$

Here, V, A, x and l are the residual vector of the observations, design matrix, the vector of the unknowns and the remains vector, respectively. The objective function minimizes the residuals of the observations (Okwuashi and Asuquo 2014):

$$Min \sum_{i=1}^{n} P_i v_i^2 \tag{7}$$

Here, P is the weights of the observations which define the stochastic relationship of the observations. In order to obtain the estimated vector of the unknowns $\hat{x}$, normal equation (8) is derived with respect to x and equalized to zero,

$$A^T P A \hat{x} - A^T P l = 0 \tag{8}$$

then $\hat{x}$ is estimated by

$$\hat{x} = (A^T P A)^{-1} A^T P l \tag{9}$$

LS is the best linear unbiased estimation if the observations have only random errors (Akyilmaz 2007). However, an outlier detection is needed because geodetic observations usually contain various errors. Outliers can be detected and removed from dataset by analysing the residuals obtained from the LS solution statistically (Yetkin 2013). Data-Snooping (DS) (Baarda 1968), Pope test (Pope 1976) and t test (Koch 1999) are the classical outlier tests used frequently in geodetic applications. These tests use the cofactor matrix of the residuals Q_{VV} and variance factors (*a priori variance* σ_0^2*, a posteriori variance* m_0^2 *and a posteriori variance* m_{0i}^2 *in the Data-Snooping, Pope* *test and t test, respectively*) (Berber and Hekimoglu 2003). The variance factors and Q_{VV} can be calculated as follow:

$$\sigma_0^2 = \frac{\varepsilon^T P \varepsilon}{n} \tag{10}$$

$$m_0^2 = \frac{V^T P V}{n-u} \tag{11}$$

$$m_{0i}^2 = [(v^T P v) - \frac{v_i^2}{q_{vv_i}}]/(f-1) \tag{12}$$

$$Q_{VV} = P^{-1} - A(A^T P A)^{-1} A^T \tag{13}$$

Here, $f = n - u$ the degree of freedom and ε errors. Test values are calculated and compared with the critical value obtained from distribution table. The test values and critical values can be calculated as in Table 1 for classical outlier tests.

Table 1. *Test and critical values of the classical outlier tests (Sisman 2010).*

Tests	Test Value	Critical Value
Data-Snooping	$W_i = \frac{\lvert v_i \rvert}{\sigma_0 \sqrt{Q_{V_i V_i}}}$	$N_{(1-\alpha_0/2)}$
Pope test	$T_i = \frac{\lvert v_i \rvert}{S_0 \sqrt{Q_{V_i V_i}}}$	$\tau_{f,(1-\alpha_0/2)}$
t test	$t_i = \frac{\lvert v_i \rvert}{m_{0i} \sqrt{Q_{V_i V_i}}}$	$t_{f-1,(1-\alpha_0/2)}$

In Table 1, α_0 the significance level, N normal, τ tau and t student distribution.

These tests were initially developed for only one outlier. But, they can be applied iteratively, detecting one outlier after another for multiple outliers (Lehmann and Lösler 2016).

2.3. Breakdown Point

The breakdown point is an important issue for robustness and it was first introduced by Hampel (1971) for location functionals. Donoho and Huber (1983) later introduced a simple-finite version of the breakdown point. It is the main criterion for the robustness of an estimator, which is the smallest fraction of outliers that can lead the estimator incorrect results (Mount et al. 2014).

The breakdown point of some robust methods (L_1 method and M-estimators) is equal to $1/n$ like LS. Siegel (1982) introduced the first high breakdown point estimator, the repeated median which has a 50% breakdown point. Later, another high breakdown point estimator, LMS with 50% breakdown point was introduced. But, the LMS has a poor asymptotic efficiency. Finally, the LTS was introduced to repair this situation (Rousseeuw and Leroy 1987).

2.4. The Least Trimmed Squares

The LTS which is a high breakdown point estimator is quite similar to the LS method. The difference of these methods is that all observation are not used in the LTS contrary to the LS method (Rousseeuw and Leroy 1987). The objective function of the LTS is given as

$$Min \sum_{i=1}^{h} P_i v_i^2 \tag{14}$$

Here, h represents the number of the observation which are included in the parameter estimation (Koch et al. 2017). The best robustness is achieved when trimming parameter $h = (n + u + 1)/2$ $(h \leq n)$ (Rousseeuw and Leroy 1987). For the solution of the LTS, there are exact and approximate approaches suggested by (Rousseeuw and Leroy 1987, Atkinson and Cheng 1999, Agulló 2001, Rousseeuw and Driessen 2006, Hofmann et al. 2010, Koch et al. 2017). The workflow of Exact LTS solution is given in Fig. 2.

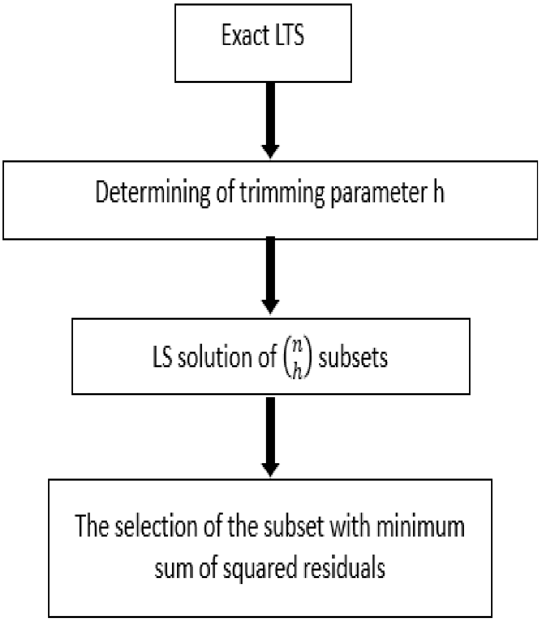

Fig. 2. *The workflow of exact LTS solution.*

The problem of Exact LTS solution given is its computation because it requires a full search through $\binom{n}{h}$ subsets to find the subset whose LS solution has the minimum sum of the squared residuals $[VV]$ (Dilmaç and Şişman 2023, Fig. 2). A full search through $\binom{n}{h}$ subsets is not possible unless the observation size is small (Li 2005).

For the approximate solution, the Fast-LTS algorithm introduced by Rousseeuw and Driessen 2006) is discussed. The Fast-LTS algorithm consists of several parts like creating initial subset, C-steps and selective iteration. The size of initial subset is decided by the unknown number u. There are many possibility for creating u-element initial subset from n observations. Therefore, a certain number of u-element initial subset (for example, 500) out of n observations can be drawn randomly. Then, $\hat{x}$ (estimated unknown vector) of each 500 initial subsets and their corresponding residuals for all n observation are calculated.

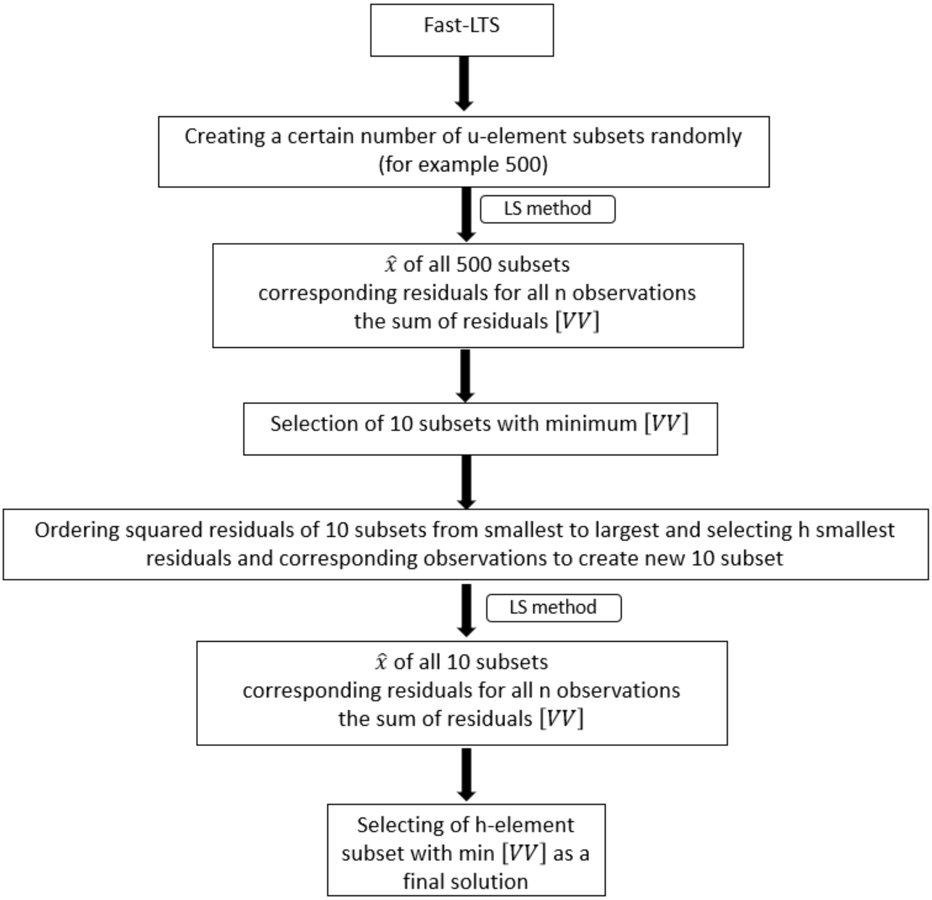

Fig. 3. *The workflow of the Fast-LTS solution.*

Then, 10 subsets with the minimum sum of the squared residuals $[VV]$ are selected for the next step. The squared residuals of these 10 subsets are ordered from smallest to largest. The corresponding observations of the h smallest residuals of each 10 subsets are used for the new $\hat{x}$ and their corresponding residuals for all n observations. This iteration is called C-step and is continued until the consecutive sum of the squared residuals $[VV]$ of subsets converge. However, Rousseeuw and Driessen (2006) stated that taking just two-C steps can achieve global optimum in practice. The workflow of the Fast-LTS algorithm is given in Fig. 3.

3. Case Study

For the comparative analysis, the network was established in Havza, the South side of Samsun, Turkey. In this network the 36 points coordinates (North, East, Orthometric and Ellipsoidal Height) were known in both ED50 and ITRF96 coordinate systems. The total area of this network is about 682 km^2 (Fig. 4).

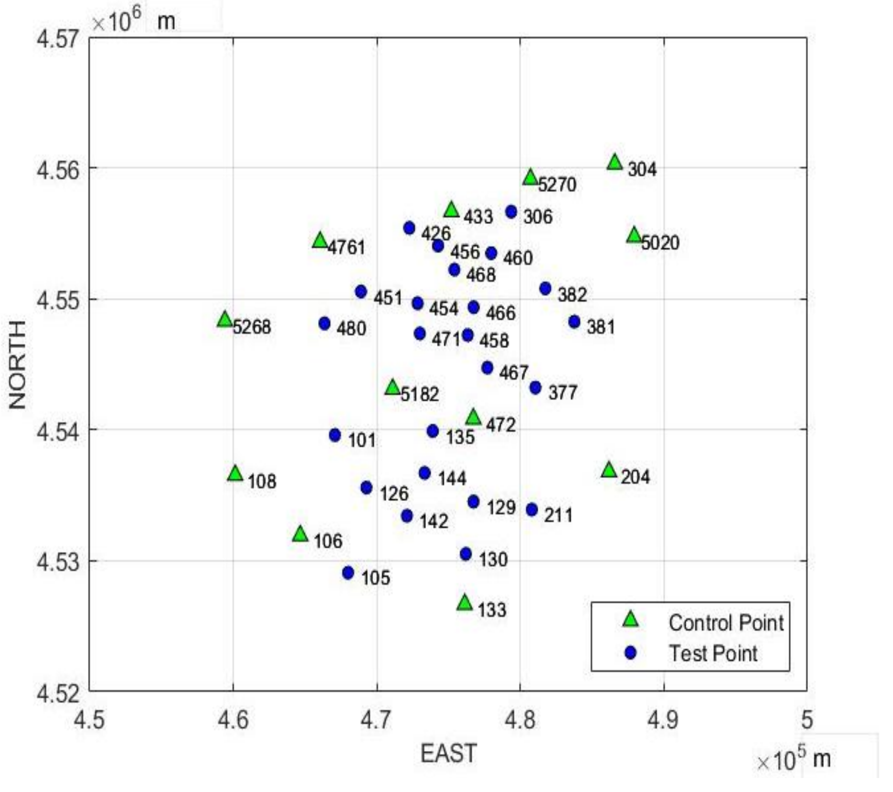

Fig. 4. *The 36-points network.*

Firstly, ED50 (x-y) to ITRF96 (X-Y) point coordinates were calculated for all points. Then, the 2D Helmert transformation parameters from ED50 (x-y) to ITRF96 (X-Y) was estimated by 12 control points (CP) for LS, exact LTS and Fast-LTS solution. The other 24 points were used as test point (TP) to check the validation of the estimated transformation parameters that they are whether close to the original coordinates or not. The ED50 coordinates of 24 test points were converted to ITRF96 system by using the estimated transformation parameters. Then, obtained ITRF96 coordinates of TD were compared with the original ITRF96 coordinates. The linear Gauss-Markov model is same for the methods.

$$\underbrace{\begin{bmatrix} V_X \\ V_Y \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} x & -y & 1 & 0 \\ y & x & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{x} - \underbrace{\begin{bmatrix} X \\ Y \end{bmatrix}}_{l} \tag{15}$$

Only difference between the LS method and LTS methods is the size of design matrix (A) and the remains vector (l) because of trimming parameter (h). Matlab programming language was used for the solution of the methods.

4. Results

Firstly, the LS method was applied. In LS method, all of the CPs were used for the estimation of transformation parameters. Then, t test was applied iteratively to detect outliers. As a result, 433 and 472 were detected as outlier and removed from estimation (Table 2).

Table 2. *The results of iterative LS method for outlier detection.*

Parameter	**Iterative LS method**		
	(12 CPs)	**(11 CPs)**	**(10 CPs)**
a	1.000461	1.000030	0.999996
b	$-4.7 * 10^{-6}$	$3.4 * 10^{-5}$	$8.7 * 10^{-6}$
$c\ (m)$	−2276.666	−306.022	−161.730
$d\ (m)$	−218.445	−188.714	−58.921
$[VV]$	9407.784	421.676	0.698
$m_0(\pm m)$	21.688	4.840	0.209
Max. Test Value	20.122	60.337	2.120
Critical Value	2.845	2.878	2.921
Outlier No	433	472	–
$[VV]\ of\ TP$	1271.055	106.133	1.390

Then, the Exact and Fast-LTS methods were applied, 4 different h values (11, 10, 9 and 8 which was calculated by $(n + u + 1)/2)$) were used. For h=11, exact and Fast-LTS methods found the same results. They removed 433 from estimation just as iterative LS method (Table 3).

Table 3. *The results of Exact and Fast-LTS methods for h=11.*

Parameter	Exact and Fast LTS (h=11)
a	1.000030
b	$3.4 * 10^{-5}$
$c\ (m)$	−306.022
$d\ (m)$	−188.714
$[VV]$	421.676
$m_0(\pm m)$	4.840
Removed Observations	433
$[VV]\ of\ TP$	106.133

For h=10, Exact and Fast-LTS methods found again the same results. They removed 433, 472 from estimation just as iterative LS solution (Table 4).

Table 4. *The results of Exact and Fast-LTS methods for h=10.*

Parameter	Exact and Fast LTS (h=10)
a	0.999996
b	$8.7 * 10^{-6}$
$c\ (m)$	−161.730
$d\ (m)$	−58.921
$[VV]$	0.698
$m_0(\pm m)$	0.209
Removed Observations	433, 472
$[VV]\ of\ TP$	1.390

For h=9, Exact and Fast-LTS methods found different observations. While the exact LTS found 108, the Fast-LTS found 4761. Although exact LTS had lower $[VV]\ of\ CP$, it didn't go same for the $[VV]\ of\ TP$ (Table 5).

Table 5. *The results of Exact and Fast-LTS methods for h=9.*

Parameter	Exact LTS (h=9)	Fast-LTS (h=9)
a	0.999992	0.999998
b	$9.6 * 10^{-6}$	$9.2 * 10^{-6}$
$c\ (m)$	−144.869	−170.814
$d\ (m)$	−61.289	−62.125
$[VV]$	0.461	0.550
$m_0(\pm m)$	0.182	0.198
Removed Observations	433, 472, 108	433, 472, 4761
$[VV]\ of\ TP$	1.336	1.242

For h=8, Fast-LTS methods had the smallest $[VV]\,of\,TP$. While the exact LTS found 106, the Fast-LTS found 108 in addition to the previous results. Three of removed observation (433, 472, 108) are common for exact and Fast-LTS methods (Table 6).

Table 6. *The results of Exact and Fast-LTS methods for h=8.*

Parameter	Exact LTS (h=8)	Fast-LTS (h=8)
a	0.999989	0.999994
b	$9.1 * 10^{-6}$	$1.1 * 10^{-5}$
$c\ (m)$	-128.251	-153.453
$d\ (m)$	-57.196	-68.509
$[VV]$	0.319	0.321
$m_0(\pm m)$	0.163	0.164
Removed Observations	433, 472, 108, 106	433, 472, 4761, 108
$[VV]\ of\ TP$	1.586	1.153

5. Discussion and Conclusion

In this study, 2D Helmert transformation parameters from ED50 (x-y) to ITRF96 (X-Y) was estimated by iterative LS method, Exact LTS and Fast-LTS methods to compare the outlier detection performances of these methods by using a real dataset. In LS method, the outlier detection process were performed iteratively according to t test. As a result, 433 and 472 points were detected as outliers. Because iterative LS method found two points as outliers, h was taken 10 for the LTS methods to compare the methods. The results of LTS methods are the same as iterative LS method because the errors which 433 and 472 contain are too large (Tables 2 and 4). When the observations contain errors that are very close to random error, LS method generally show masking and swamping effects. But, it is not the case in this study. As a result, three methods found the same points as an outlier.

To compare exact LTS and Fast-LTS both each other, in addition to 10, 3 different h values (11, 9 and 8) were taken. The Exact and Fast-LTS methods mostly found the same subset especially when the h was large, that is, size of the subset is close to original size (12 points) (Tables 3 and 4). However, it can be said from Tables 5 and 6 that the methods tend to find different subsets when the size of the subset moves away from the original size. But, it must be said that the differences of the subsets are generally not different from each other, that is, they generally produce the same subset.

As a result, the Exact and Fast-LTS method could be used for the outlier detection in geodetic applications like georeferencing, surface fitting etc. according to authors.

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References

Akyilmaz, O. (2007): Total least squares solution of coordinate transformation, Survey Review, 39 (303), 68–80.

Amiri-Simkooei, A. R. (2018): On the use of two L1 norm minimization methods in geodetic networks, Earth Observation and Geomatics Engineering, 2 (1), 1–8.

Baarda, W. (1968): A Testing Procedure for Use in Geodetic Networks, Publications on Geodesy, 2 (5), 97.

Berber, M., Hekimoglu, S. (2003): What is the reliability of Conventional Outlier Detection and Robust Estimation in trilateration networks? Survey Review, 37 (290), 308–318.

Borowski, Ł., Banaś, M. (2018): Application of Robust Estimation in Polynomial Modelling, Proceedings – 2018 Baltic Geodetic Congress, BGC-Geomatics 2018, 62–66.

Borowski, Ł., Banaś, M. (2019): The Best Robust Estimation Method to Determine Local Surface, Baltic Journal of Modern Computing, 7 (4), 525–540.

Chang, G., Xu, T., Wang, Q. (2017): Error analysis of the 3D similarity coordinate transformation, GPS Solutions, 21 (3), 963–971.

Golub, G. H., van Loan, C. F. (1980): An Analysis of the Total Least Squares Problem, SIAM Journal on Numerical Analysis, 17 (6), 883–893.

Hampel, F. R. (1971): A General Qualitative Definition of Robustness, The Annals of Mathematical Statistics, 42 (6), 1887–1896.

Hofmann, M., Gatu, C., Kontoghiorghes, E. J. (2010): An exact least trimmed squares algorithm for a range of coverage values, Journal of Computational and Graphical Statistics, 19 (1), 191–204.

Klein, I., Matsuoka, M. T., Guzatto, M. P., Nievinski, F. G. (2017): An approach to identify multiple outliers based on sequential likelihood ratio tests, Survey Review, 49 (357), 449–457.

Knight, N. L., Wang, J. (2009): A comparison of outlier detection procedures and robust estimation methods in GPS positioning, Journal of Navigation, 62 (4), 699–709.

Koch, I., Veronez, M. R., da Silva, R. M., Klein, I., Matsuoka, M. T., Gonzaga, L., Larocca, A. P. C. (2017): Least trimmed squares estimator with redundancy constraint for outlier detection in GNSS networks, Expert Systems with Applications, 88, 230–237.

Lehmann, R., Lösler, M. (2016): Multiple Outlier Detection: Hypothesis Tests versus Model Selection by Information Criteria, Journal of Surveying Engineering, 142 (4), 04016017.

Li, L. M. (2005): An algorithm for computing exact least-trimmed squares estimate of simple linear regression with constraints, Computational Statistics and Data Analysis, 48 (4), 717–734.

Mount, D. M., Netanyahu, N. S., Piatko, C. D., Silverman, R., Wu, A. Y. (2014): On the least trimmed squares estimator, Algorithmica, 69 (1), 148–183.

Okwuashi, O., Asuquo, I. (2014): Basics of Least Squares Adjustment Computation in Surveying, International Journal of Science and Research (IJSR), 3 (8), 1988–1993.

Pope, A. J. (1976): The statistics of residuals and the detection of outliers, NOAA Technical Report NOS 65 NGS 1.

Rousseeuw, P. J. (1984): Least median of squares regression, Journal of the American Statistical Association, 79 (388), 871–880.

Rousseeuw, P. J., Hubert, M. (2018): Anomaly detection by robust statistics, Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery, 8 (2), 1–14.

Siegel, A. F. (1982): Robust regression using repeated medians, Biometrika, 69 (1), 242–244.

Sisman, Y. (2010): Outlier measurement analysis with the robust estimation, Scientific Research and Essays, 5 (7), 668–678.

Sisman, Y. (2014): Coordinate transformation of cadastral maps using different adjustment methods, Journal of the Chinese Institute of Engineers, Transactions of the Chinese Institute of Engineers, Series A, 37 (7), 869–882.

Sisman, Y., Dilaver, A., Bektas, S. (2012): Outlier detection in 3D coordinate transformation with fuzzy logic, Acta Montanistica Slovaca, 17 (1), 1–8.

Sjöberg, L. E. (2013): Closed-form and iterative weighted least squares solutions of Helmert transformation parameters, Journal of Geodetic Science, 3 (1), 7–11.

Třasák, P., Štroner, M. (2014): Outlier detection efficiency in the high precision geodetic network adjustment, Acta Geodaetica et Geophysica, 49 (2), 161–175.

Wieser, A., Brunner, F. K. (2001): Robust estimation applied to correlated GPS phase observations, In: Carosio, A., Kutterer, H. (eds.), Proceedings of the first international symposium on robust statistics and fuzzy techniques in geodesy and GIS, ETH Zurich, 193–198.

Yang, Y., Song, L., Xu, T. (2002): Robust estimator for correlated observations based on bifactor equivalent weights, Journal of Geodesy, 76 (6), 353–358.

Yetkin, M. (2013): A review of outlier detection procedures used in Surveying Engineering, 2nd International Balkans Conference on Challenges of Civil Engineering, May, 23–25.

Ziggah, Y. Y., Issaka, Y., Laari, P. B., Hui, Z. (2018): 2D Cadastral Coordinate Transformation using extreme learning machine technique, Geodesy and Cartography, 67 (2), 321–343.

Otkrivanje grubih pogrešaka uz pomoć robusnih točnih i brzih metoda regresije najmanjih kvadrata u transformaciji koordinata

SAŽETAK. Različiti terestrički referentni sustavi definirani su i korišteni zbog nekih praktičnih i povijesnih događaja u domeni geodezije. Prijelaz iz jednog sustava u drugi zahtijeva transformaciju koordinata. Helmertova transformacija najčešće je korištena kao model za 2D mreže. 2D Helmertova transformacija definirana je s četiri transformacijska parametra, a dvije zajedničke točke u oba koordinatna sustava daju jedinstveno rješenje. Kako bi se povećala pouzdanost parametara transformacije, obično se koriste redundantna opažanja. U ovom slučaju, najmanji kvadrati (LS) najčešća su metoda koja se koristi za dobivanje jedinstvenog rješenja iz redundantnih opažanja. Međutim, grube pogreške često se pojavljuju u skupu podataka i ozbiljno utječu na rezultate najmanjih kvadrata. Općenito se primjenjuju dva pristupa za otkrivanje grubih pogrešaka: klasično ispitivanje grubih pogrešaka i robusne metode. Najčešće robusne metode su najmanje apsolutno odstupanje (L_1), M-procjenitelji, ukupni najmanji kvadrati (TLS), generalizirani M-procjenitelji, najmanji medijan kvadrata (LMS) i metoda regresije najmanjih kvadrata (LTS). Za rješavanje metode regresije najmanjih kvadrata postoje točna i približna rješenja. U ovoj studiji, parametri 2D Helmertove transformacije između ED50 i ITRF koordinata procjenjuju se LS metodom uključujući klasična ispitivanja grubih pogrešaka, točna LTS rješenja i brza LTS rješenja što je približno rješenje za usporedbu učinkovitosti metoda u otkrivanju grubih pogrešaka.

Ključne riječi: *transformacija koordinata, otkrivanje grubih pogrešaka, robusno rješenje, najmanji kvadrati, regresija najmanjih kvadrata, ITRF, ED50, LTS, TLS, GNSS.*

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