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A Novel Method for Solving Multi-objective Shortest Path Problem in Respect of Probability Theory

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Abstract: Transportation process or activity can be considered as a multi-objective problem reasonably. However, it is difficult to obtain an absolute shortest path with optimizing the multiple objectives at the same time by means of Pareto approach. In this paper, a novel method for solving multi-objective shortest path problem in respect of probability theory is developed, which aims to get the rational solution of multi-objective shortest path problem. Analogically, each objective of the shortest path problem is taken as an individual event, thus the concurrent optimization of many objectives equals to the joint event of simultaneous occurrence of the multiple events, and therefore the simultaneous optimization of multiple objectives can be solved on basis of probability theory rationally. The partial favorable probability of each objective of every scheme (routine) is evaluated according to the actual preference degree of the utility indicator of the objective. Moreover, the product of all partial favorable probabilities of the utility of objective of each scheme (routine) casts the total favorable probability of the corresponding scheme (routine), which results in the decisively unique indicator of the scheme (routine) in the multi-objective shortest path problem in the point of view of system theory. Thus, the optimum solution of the multi-objective shortest path problem is the scheme (routine) with highest total favorable probability. Finally, an application example is given to illuminate the approach.

Keywords: concurrent optimization; favorable probability; multi-objective; probability theory; shortest path

1 INTRODUCTION

The shortest path was a classic problem in network optimization. A shortest path problem with single objective has been well solved by Dijkstra's algorithm in 1950s and Floyd's algorithm in 1960s [1]. However in 1990s, some new problems have raised owing to the new development of intelligent technology, communication technology, and information science. These new problems concrete the new aspect of the shortest path problem with multiple objectives, which makes the study of the shortest path problem active again [2-7].

Usually, the minimization of one objective, such as cost, or transportation time, etc., is considered as the general problem of shortest path. However, since the route selection of the transportation network often needs to consider multiple objectives at the same time, such as cost, time, risk, safety, etc., different objectives need to be compromised in the solution processing simultaneously.

The classification and generalization of multi-objective shortest path problem were stated by Current et al. [8]. The main methods for solving multi-objective shortest path problem can be classified into three types:

1) Utility function method
2) Interactive method
3) Production method.

The utility function method requires prior preference information of the decision maker to determine the corresponding utility function and the interactive method uses preference information in the entire problem solving process. The production method could only directly give the set of Pareto optimization or set of approximate optimization solution, it mainly includes dynamic programming method, Pareto labeling method and Pareto ranking method, etc. [9].

For the multi-objective shortest path algorithm, the usual processing method is to linearly weight different objectives, or convert some objectives into constraints. As to the linear weighting method, the determination of its weight is very problematic. For the constrained shortest path problem, it has been proved to be NP-hard. In fact, above methods not only deviate from the original intention of multi-objective actually, but also consume large time and space, or even unsolved when the scale of the problem is large [10, 11].

The dual-objective shortest path problem is a common situation in the multi-objective shortest path issues. In order to solve the dual-objective shortest path problem, J. Current, C. Revelle, J. Cohon [12] and J. Coutinaho-Rodrigues, J. Climcao, J. Current [13] studied the general dual-objective shortest path algorithm and proposed an interactive dual-objective algorithm. In the shortest path problem with dual objectives, it is often necessary to obtain an effective path and then select it. Hansen [14], Climaco and Martins [15] got certain achievement on the acquisition of effective paths for dual objectives.

In this paper, a novel method for solving multi-objective shortest path problem is developed in respect of probability theory. It takes the each objective of the shortest path problem as an individual event analogically, the concurrent optimization of many objectives thus equals to the joint event of simultaneous occurrence of the multiple events. The partial favorable probability of each objective (event) of every scheme (routine) and the total favorable probability of each scheme (routine) are evaluated as the uniquely decisive indexes of the scheme (routine) in the multi-objective shortest path problem, which is in the viewpoint of system theory. Examples are given to illuminate the approach in detail.

2 A NOVEL PROBABILITY THEORY - BASED METHOD

2.1 Assessment of Favorable Probability of the Probability-Based Method

The recently proposed probability-based multi-objective optimization (PMOO) is in the viewpoint of system theory [16, 17], each objective can be analogically taken as an individual event, and the concurrent optimization of many objectives thus equals to the joint event of simultaneous occurrence of multiple events in the respect of probability theory. Therefore, the problem of concurrent optimization of many objectives is thus converted into a joint probabilistic problem of simultaneous occurrence of multiple events analogically. Furthermore, the preference degree of utility of each objective (event) of a scheme is transferred into its partial favorable probability. The concurrent optimization of multiple objectives is described by the integral (overall) event, its total (overall) favorable probability is thus rationally the product of all the partial favorable probabilities of all individual events of the scheme, which is in the respect of probability theory. In the evaluation, the utility of the objective is preliminarily classified as unbeneficial or beneficial type in accordance with the characteristic of the preference or role in the assessment. The methodology of PMOO is shown in Fig. 1.

The meanings of the variables and factors in Fig. 1 are as following:

P_{ij} expresses the partial favorable probability of the j^{th} performance utility indicator of the i^{th} candidate scheme, X_{ij}; n reflects the total number of the candidate scheme; m shows the total number of the performance (objective); $\overline{X_j}$ indicates the arithmetic value of the j^{th} performance utility indicator; $X_{j\max}$ and $X_{j\min}$ express the maximum and minimum values of the j^{th} performance utility indicator, respectively; α_j and β_j represent the normalized factors of the j^{th} performance utility indicator X_{ij} in beneficial status and unbeneficial status, individually; the beneficial status or unbeneficial status of the j^{th} performance utility indicator X_{ij} is specified according to its particular preference or role in the problem; P_i is the total (overall) favorable probability of the i^{th} candidate scheme [16, 17].

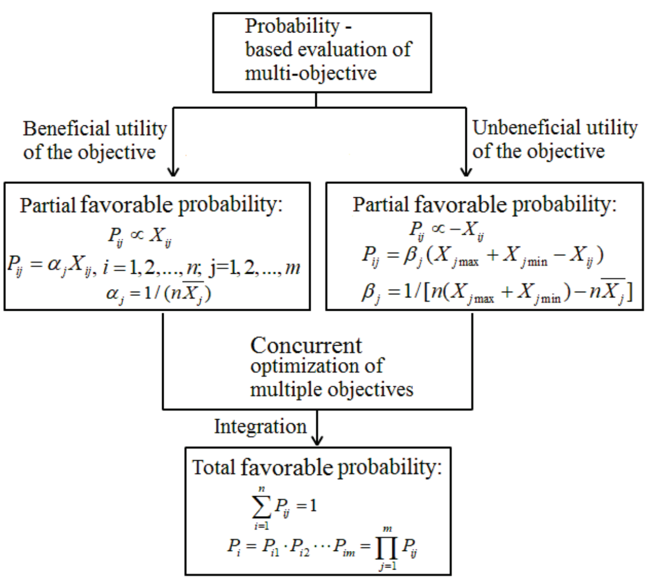

Figure 1 Evaluation of the PMOO methodology

The probability-based multi-objective optimization is a brand new methodology, which has the potential applications in many fields concerning multi - objectives, such as material selection, mechanical design, programming problem, operation research, engineering design, etc.

2.2 Assessment Procedure of Concurrent Optimization of Shortest Path Problem with Many Objectives in Respect of Probability Theory

1) Assessment of objective and event

As to the multi-objective shortest path problem, the cost, time, risk, safety, etc., are different objectives, of which each objective can be analogically taken as individual event. Thus, the multi-objective shortest path problem is transferred into the joint probabilistic problem of simultaneous occurrence of multiple events equivalently. The accumulated cost, time, risk, safety, etc., in each scheme (route) are accounted according to the actual interval of each scheme (route) individually.

2) Assessment of favorable probability

The partial favorable probabilities of cost, time, risk, safety, etc., are assessed according to their specific function or characteristic of the preference for every scheme analogically. Finally, the overall (total) favorable probability of the integral event is the product of all the partial favorable probabilities of all events of each scheme (route) in the respect of probability theory, it thus completes the concurrent optimization of multi-objective, and provides the decisively unique indicator of each scheme (routine) in the shortest path problem with many objectives.

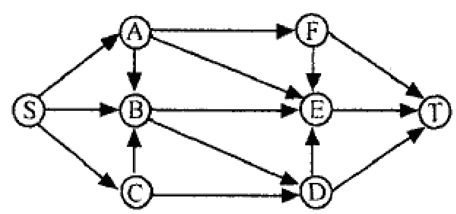

Figure 2 Transportation Network

3) **Solution of the multi-objective shortest path problem**

At last, the optimal solution of the multi-objective shortest path problem is the specific scheme (routine), which is with highest total favorable probability.

3 APPLICATION IN IMPORTANT GOODS TRANSPORTATION SHORTEST PATH PROBLEM

The important goods transportation path problem is a significant thing; the distance, the cost, and accident rate are taken as the evaluating objectives [18].

The transportation network is shown in Fig. 2. Tab. 1 shows the basic values of each interval path [18]. The current problem is to find the multi-objective shortest path of this transportation for important goods.

As can be seen from Fig. 2, the transportation process is from the start point S to destination T. After every round of the k shortest path searching, the algorithm produces a multi-objective shortest path or alternative path by the intersection

[18], which is shown in Tab. 2 for the 13 routes and accounted from the data in Tab. 1 and Fig. 2.

Table 1 Basic values of each interval path

Interval path	Objective		
	Distance (km)	Cost (¥RMB)	Accident rate (%)
SA	52	120	0.20
SB	48	100	0.38
SC	45	84	0.15
AB	42	30	0.40
CB	29	70	0.60
AF	40	60	0.50
AE	35	50	0.45
BE	38	90	0.10
BD	23	30	0.80
CD	42	30	0.20
FE	60	75	0.70
DE	31	50	0.35
FT	48	30	0.45
ET	40	80	0.50
DT	50	200	0.40

Table 2 Selected alternative paths and their target values

No.	Route	Distance, *d* (km)	Cost, *c* (¥RMB)	Accident rate, *a* (%)
1	S-B-D-T	121	330	1.58
2	S-B-E-T	126	270	0.98
3	S-A-E-T	127	250	1.15
4	S-C-D-T	137	314	0.75
5	S-A-F-T	140	210	1.15
6	S-B-D-E-T	142	260	2.03
7	S-C-B-D-T	147	384	1.95
8	S-C-B-E-T	152	324	0.9
9	S-C-D-E-T	158	244	1.20
10	S-C-B-D-E-T	168	314	2.40
11	S-A-B-E-T	172	320	1.20
12	S-A-B-D-E-T	188	310	2.25
13	S-A -F-E-T	192	335	1.90

1) Without considering weight factor

Here in this section, lets' study the transportation path problem under condition of each objective with equal importance, i.e., without comparable weight factor.

In this transportation problem, the objectives (events), i.e., the distance, the cost, and accident rate are all unbeneficial type indexes for each scheme in the assessment. The evaluations of partial favorable probabilities and the total favorable probabilities P_i for each of the five possible transport options can be conducted according to Fig. 1.

Table 3 Assessments of partial favorable probabilities and the total favorable probabilities for each scheme from S to T

No.	P_d	P_a	P_c	$P_i \times 10^4$	Rank
1	0.0915	0.0684	0.0730	4.5699	7
2	0.0891	0.0840	0.1009	7.5499	2
3	0.0886	0.0892	0.0930	7.3485	3
4	0.0838	0.0726	0.1116	6.7917	4
5	0.0824	0.0996	0.0930	7.6297	1
6	0.0815	0.0866	0.0521	3.6733	9
7	0.0791	0.0544	0.0558	2.4022	10
8	0.0767	0.0700	0.1046	5.6166	6
9	0.0738	0.0907	0.0907	6.0748	5
10	0.0691	0.0726	0.0349	1.7486	13
11	0.0672	0.0710	0.0907	4.3262	8
12	0.0596	0.0736	0.0418	1.8347	12
13	0.0576	0.0672	0.0581	2.2495	11

Tab. 3 presents the partial favorable probabilities and the total favorable probabilities P_i enlarged by 10^4 for each of the five possible transport options from S to T. Tab. 3 shows that scheme No. 5 S-A-F-T exhibits the highest total favorable probability, which might be taken as our optimum route.

2) Considering weight factor

If there is weight factor to reflect the comparable importance of the objective, the weight factor can be taken as the exponent of partial favorable probability in the product of individual partial favorable probability for the total favorable probability assessment [16], i.e., $P_i = P_{i1}^{w1} \cdot P_{i2}^{w2} \cdots P_{im}^{wm}$, in which w_j expresses the weight factor of the j^{th} objective.

Here, for this problem, let us assume the weight factors for the distance, the cost, and accident rate are 0.1, 0.4 and 0.5, respectively, thus the evaluation can be conducted accordingly. Tab. 4 gives the partial favorable probabilities and the total favorable probabilities enlarged by 10^2 for each of the five possible transport options from S to T with the weight factors of 0.1, 0.4 and 0.5, individually. Tab. 4 shows that scheme No. 5 (S-A-F-T) displays the highest total favorable probability P_i luckily, which might be selected as our optimum route.

Table 4 Assessments of partial favorable probabilities and the total favorable probabilities of each scheme from S to T with weight factors (weighted 0.1, 0.4, 0.5)

No.	P_d	P_c	P_a	$P_i \times 10^2$	Rank
1	0.0915	0.0684	0.0730	7.2761	8
2	0.0891	0.0840	0.1009	9.2600	2
3	0.0886	0.0892	0.0930	9.1006	4
4	0.0838	0.0726	0.1116	9.1306	3
5	0.0824	0.0996	0.0930	9.4413	1
6	0.0815	0.0866	0.0521	6.6740	9
7	0.0791	0.0544	0.0558	5.7209	11
8	0.0767	0.0700	0.1046	8.6357	6
9	0.0738	0.0907	0.0907	8.8850	5
10	0.0691	0.0726	0.0349	5.0062	13
11	0.0672	0.0710	0.0907	7.9803	7
12	0.0596	0.0736	0.0418	5.4340	12
13	0.0576	0.0672	0.0581	6.1522	10

4 CONCLUSION

From above analysis and discussion, a probability-based shortest path approach is proposed. In the evaluation, each objective is taken as an individual event analogically; the favorable probability of each objective is evaluated according to its preference degree of the corresponding objective (event) individually; the concurrent optimization of many objectives equals to the simultaneous occurrence of the multiple events with their favorable probabilities, which thus completes the concurrent optimization of multiple objectives shortest path rationally; under condition of weight factor existence, the weight factor can be taken as the exponent of partial favorable probability in the product of individual partial favorable probability for the total favorable probability assessment.

Conflict Statement

There is no conflict of interest.

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