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DOUBLE STRIPPING CALCULATION OF THE $^{9}Be\,(^{6}He,\,^{4}He)\,^{11}Be$ REACTION

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Abstract: The total cross section in a double stripping picture of the $^{9}Be\,(^{6}He,\,^{4}He)^{11}Be$ reaction is calculated assuming that the first two levels of the final nucleus take part in the reaction. The harmonic oscillator functions are used for two nuclei. The calculation is performed in the plane wave Born approximation using the Gaussian form of the interaction Hamiltonian. Good agreement with the experimental result is obtained.

1. *Introduction*

The reaction $^{9}Be\,(^{6}He,\,^{4}He)\,^{11}Be$ represents one of the few processes by which the unstable ^{11}Be isotope can be obtained[1,2]. The total cross-section for this reaction was measured by using the 8 MeV ^{6}He beam produced in the $^{9}Be\,(n,\,\alpha)\,^{6}He$ reaction and the value of 11 ± 4 mb was obtained[3].

In the present paper this reaction is treated in the framework of a double stripping process theory[4] on the basis of the following assumptions:

1) A direct reaction mechanism was assumed, i. e. two neutrons are captured in their relative S state ($l = 0$). Then, the following selection rules are valid

$$\begin{aligned} &\vec{J}_f = \vec{J}_i + \vec{J}; \quad \vec{J} = \vec{\Lambda} + \vec{S}; \quad \vec{\Lambda} = \vec{l}_1 + \vec{l}_2 = \vec{l} + \vec{L}; \\ &\vec{S} = \vec{s}_1 + \vec{s}_2; \quad S = 0; \quad l = 0; \quad \Pi_i \Pi_f = (-)^J = (-)^L, \end{aligned} \tag{1}$$

where J_i and J_f are the total angular momenta of the considered states of ^{9}Be and ^{11}Be nuclei, respectively, S, Λ and J the spin, the angular momentum and the total angular momentum of the nucleon pair that undergoes the transition, and l and L their relative and centre-of-mass angular momenta, respectively. Other symbols have their usual meanings.

2) The ground state ($J_f = 1/2^+$) and the first excited state ($E = 0.319$ MeV.; $J_f = 1/2^-$) of the final nucleus were considered as being dominant in the reaction cross section amplitude. Other states have not been taken into account because their energies are considerably higher and their quantum numbers are not yet determined.

3) In the framework of the plane wave Born approximation it was supposed that the reaction proceeded via an interaction of the Gaussian type, and a sort of cluster model wave function was used for the initial and the final nucleus. The Coulomb forces were neglected.

2. *Calculation of the reaction amplitude and the cross section*

The reaction amplitude is defined as

$$A_{i\to f} = \langle\, {}^4\mathrm{He},\ {}^{11}\mathrm{Be},\ e^{i\vec{k}_f\vec{r}_f} \mid V_{\mathrm{int}} \mid {}^6\mathrm{He},\ {}^9\mathrm{Be},\ e^{i\vec{k}_i\vec{r}_i} \rangle, \tag{2}$$

where $\vec{r}_i$ and $\vec{r}_f$ are the vectors connecting the centres of mass of the nuclei in the incoming and the outgoing channels respectively, $\vec{k}_i$ and $\vec{k}_f$ the corresponding wave vectors and

$$V_{\mathrm{int}} = \sum_{j=1}^{4} [V_{1j}\, e^{-\alpha^2(\vec{r}_1-\vec{\xi}_j)^2} + V_{2j}\, e^{-\alpha^2(\vec{r}_2-\vec{\xi}_j)^2}],$$

$$V_{1j} = V_{2j} = V_0 = 45\ \mathrm{MeV};\quad \alpha^2 = 0.27\ \mathrm{fm}^{-2}. \tag{3}$$

$\vec{r}_1$, $\vec{r}_2$ and $\vec{\xi}_j$ are the position vectors of the particles »1« and »2« (two captured neutrons) and of the α-particle nucleons, respectively.

For the ground state of ^{6}He the Dawson-Walecka[5] model was used

$$|\, {}^6\mathrm{He} \rangle = a \,|\, (P_{3/2})^2,\ \mathrm{O} \rangle + b \,|\, (P_{1/2})^2,\ \mathrm{O} \rangle, \tag{4}$$

with $a = 0.886$ and $b = 0.464$. The separation energy is $E_{P_{1/2}} - E_{P_{3/2}} = 1$ MeV.

The ^{11}Be wave function is clusterized. The corresponding state defined by the total angular momentum J_f and its projection M_f can be written as

$$|\, J_f M_f \rangle = \sum_{j_1 j_2 J M M_i} S(j_1 j_2 J;\ J_i J_f)\, C^{J_f M_f}_{J M\, J_i M_i}\, |\, JM \rangle\, |\, J_i M_i \rangle, \tag{5}$$

where $S(j_1 j_2 J;\ J_i J_f)$ is the spectroscopic factor dependent on the particular structure of ^{9}Be and ^{11}Be. $|\, J\, M \rangle$ is the state of two neutrons which undergo the transition and $|\, J_i\, M_i \rangle$ is the state of the initial nucleus.

The inversion of the $1\, P_{1/2}$ and $2\, S_{1/2}$ one-particle levels characterizes the ^{11}Be nucleus. This was experimentally proved by Hinds *et al.*[2] from the angular distribution measurement of the ^{9}Be(t, p) ^{11}Be, reaction, and theoretically explained by Talmi and Unna[6] as an effect of the configuration mixing under the action of residual forces.

The selection rules (1) allow the calculation of the spectroscopic factor with the above assignment of the quantum numbers. (Of course, another »clusterization« would be chosen but the problem become technically difficult to be solved.) For the case $J_i = 3/2^-$ and $J_f = 1/2^+$ (ground state) only one value of the angular momentum is possible, $J = L = 1$, and the two-particle states differ in the quantum number M. These states are defined by $|\ 1\ M\ (1\, P_{3/2},\ 2\, S_{1/2}) \rangle$. Similarly, for the first excited state $J_f = 1/2^-$ the $J = L = 2$ value of the angular momentum is allowed, and the two-particle state is then $|\ 2\ M\ (1\, P_{3/2},\ 1\, P_{1/2}) \rangle$. In both cases the spectroscopic factor is equal to 1.

The reaction amplitude (2) is calculated by the Talmi coefficient technique using the harmonic oscillator function for the radial part of the matrix element. The angular distribution is given by the expression

$$\left(\frac{d\sigma}{d\Omega}\right)_{i\to f} = \frac{\mu_i \mu_f}{(2\pi \hbar^2)^2} \frac{k_f}{k_i} \frac{f^2}{(2J_i + 1)} \cdot \sum_{M_i M_f} |A_{i\to f}|^2, \tag{6}$$

where μ_i and μ_f are the nuclear reduced masses before and after the reaction and f^2 is the statistical factor due to the antisymmetrization

$$f^2 = \binom{\tilde{N}+2}{2}\binom{\tilde{n}+2}{2}, \tag{7}$$

$\tilde{N}$ and $\tilde{n}$ representing the number of neutrons in the ^{9}Be and in the ^{4}He nucleus, respectively.

Somewhat complicated calculations give the following expressions for the differential cross sections connected with the ground state and the first excited state

$$\frac{d\sigma}{d\Omega}(3/2^- \to 1/2^+) = 1.3891 \cdot 10^4 \frac{\mu_i \mu_f}{(2\pi \hbar^2)^2} \frac{k_f}{k_i} \left[\frac{V_0 R_0^2}{\alpha^3 \chi_1} F_1(R_0)(a\sqrt{2} + b)\right]^2 D_1,$$
$$\frac{d\sigma}{d\Omega}(3/2^- \to 1/2^-) = 0.3473 \cdot 10^4 \frac{\mu_i \mu_f}{(2\pi \hbar^2)^2} \frac{k_f}{k_i} \left[\frac{V_0 R_0^2}{\alpha^3 \chi_2} F_2(R_0)(a\sqrt{2} + b)\right]^2 D_2, \tag{8}$$

where $F_1(R_0)$ and $F_2(R_0)$ are given combinations of integrals depending on the centre-of-mass coordinate around the value of the interaction radius R_0,

$$F_1(R_0) = 0.4564\, R_{10}(R_0)\, R_{11}(R_0) - 0.1178\, R_{10}(R_0)\, R_{00}(R_0),$$
$$F_2(R_0) = R_{10}(R_0)\, R_{02}(R_0). \tag{9}$$

R_{NL} is defined as

$$R_{NL} = N_{NL}\, e^{-\frac{1}{2}\left(\frac{R}{b}\right)^2} \left(\frac{R}{b}\right)^2 \sum_p \binom{N}{p} (-2)^p \frac{(2L+1)!!}{(2L+2p+1)!!} \left(\frac{R}{b}\right)^{2p}, \tag{10}$$

with the harmonic oscillator parameter $b = \left(\frac{\hbar}{2m_N \omega}\right)^{1/2}$. χ depends on the separation energy of two neutrons in the ^{11}Be nucleus, and is defined by

$$\chi^2 = \frac{36}{11} \frac{m_N (B(2n) - E_{exc})}{\hbar^2} \tag{11}$$

The functions D_1 and D_2 depend on the momentum transfer of the reaction

$$D_1 = \left[j_1(Q R_0)\, e^{-\frac{q^2}{4\alpha^2}\left(1 + \frac{3}{32}\frac{\alpha^2}{\beta^2}\right)}\right]^2,$$
$$D_2 = \left[j_2(Q R_0)\, e^{-\frac{q^2}{4\alpha^2}\left(1 + \frac{3}{32}\frac{\alpha^2}{\beta^2}\right)}\right]^2, \tag{12}$$

$j_1(Q R_0)$ and $j_2(Q R_0)$ are the spherical Bessel functions,

$$\vec{q} = \frac{3}{2}\vec{k}_i - \vec{k}_f; \quad \vec{Q} = \vec{k}_i - \frac{9}{11}\vec{k}_f,$$

and β is the parameter of the internal wave function of the α-particle ($\beta^2 = 0.16\, \text{fm}^{-2}$ for which the Gaussian form is used.

3. *Results and discussion*

The crucial point of this calculation is the choice of the parameters incorporated in different parts of the evalution of the expression (2). We have tried to be consistent with the values commonly used for the direct reaction processes performed on light nuclei.

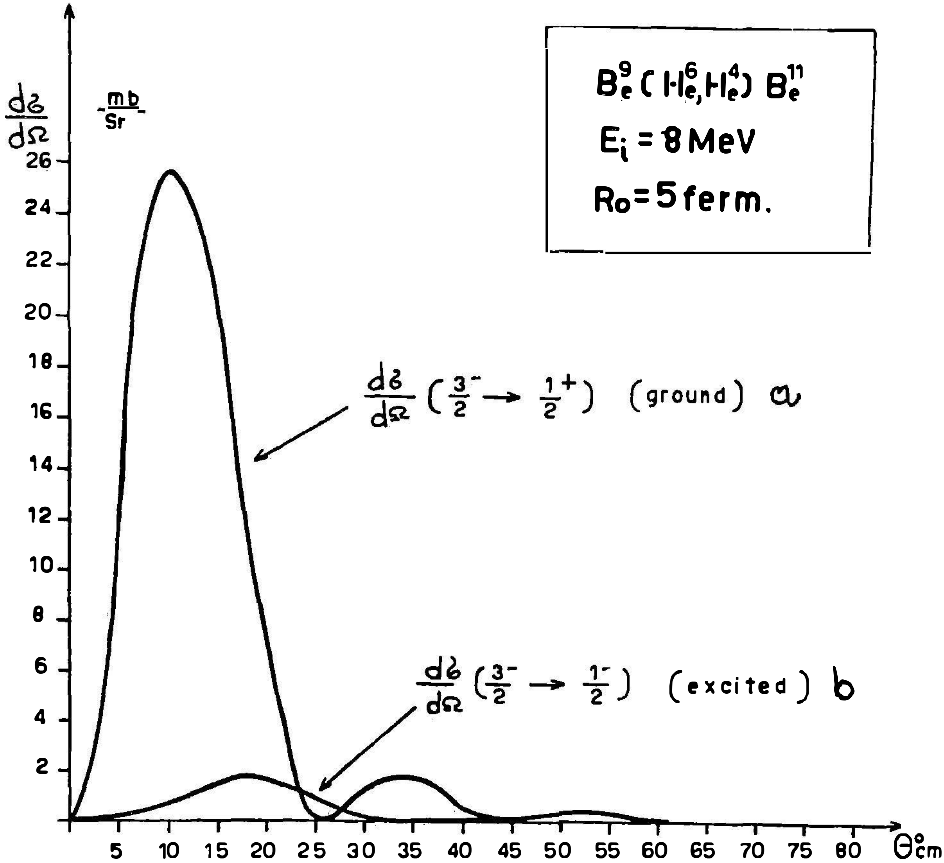

Fig 1 The calculated angular distribution for the ^{9}Be (^{6}He, ^{4}He)^{11}Be reaction; curve a) for the (3/2$^-$ → 1/2$^+$) transition, curve b) for the (3/2$^-$ → 1/2$^-$) transition.

For the two transitions in question the differential cross section is calculated with $\hbar\omega = 10.5$ MeV and $R_0 = 5$fm. The results are shown in Fig. 1. The obtained total cross sections are

$$\sigma_1 (3/2^- \rightarrow 1/2^+) = 8.46 \text{ mb},$$
$$\sigma_2 (3/2^- \rightarrow 1/2^-) = 0.82 \text{ mb},$$

and their sum σ_{tot} is equal to 9.28 mb, which is in fair agreement with the experimental results.

This is a more realistic picture than that in which only the first excited state contributes to the reaction cross section. As reported previously[7] it would require a drastic change of the harmonic oscillator parameter.

It is important to note that here, as in a many types of nuclear reactions, the ambiguities in the choice of the numerical values of the parameters do not allow a decisive answer about the form for the wave function of $1p$ shell nuclei.

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O REAKCIJI ^{9}Be (^{6}He, ^{4}He) ^{11}Be

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Sadržaj

Izračunat je totalni udarni presek za reakciju ^{9}Be (^{6}He, ^{4}He) ^{11}Be u modelu »double stripping«, uz pretpostavku da prva dva nivoa konačne jezgre učestvuju u reakciji. Za obe jezgre upotrebljene su funkcije harmoničkog oscilatora. Račun je izveden u Bornovoj aproksimaciji sa ravnim valovima uz upotrebu Gausovog oblika Hamiltonijana međudjelovanja. Rezultat se podudara sa izvršenim merenjima.