Application of toughness limit in assessment of soil compaction identifiers

In geotechnical engineering, the precise evaluation of compaction parameters is essential for quality control assessment. One option is the use of the toughness limit (TL), concisely defined as the water content, at which the behaviour of fine-grained soils evolves from an almost adhesive-plastic to tough-plastic. A database consisting of more than 1000 test results, including the compaction characteristics and Atterberg limits, was compiled to establish correlations between the TL, Atterberg limits, optimum degree of saturation, and compaction properties. The results revealed that the TL has the potential to evaluate many indices and compaction identifiers for different types of soils.

Key words:
Atterberg limits, toughness limit, optimum degree of saturation, compaction characteristics, physical properties

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Satoru Shimobe, Eyyüb Karakan, Alper Sezer

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Procjena pokazatelja zbijenosti tla pomoću granice žilavosti

Evaluacija parametara zbijenosti tla od ključne je važnosti za ocjenu kontrole kakočne u geotehničkom inženjerstvu. Jedna od mogućnosti za to jest primjena granice žilavosti koja se definira kao udio vode pri kojem se ponašanje sitnozrnatog tla mijenja od gotovo ljepljivoplastičnog do žilavoplastičnog. Rezultati provedenih više od 1000 ispitivanja, uključujući ispitivanja svojstva zbijenosti i Atterbergovih granica prikupljeni su u bazu podataka radi utvrđivanja korelacija između granice žilavosti, Atterbergovih granica optimalnog stupnja zasićenosti i svojstva zbijenosti. Rezultati pokazuju da granica žilavosti ima potencijal za procjenu mnogih indeksa i pokazatelja zbijenosti različitih vrsta tala.

Ključne riječi:
Atterbergove granice, granica žilavosti, optimalan stupanj zasićenosti, svojstva zbijenosti, fizička svojstva
1. Introduction

In geotechnical engineering, plasticity is defined as “the ability of cohesive soil to change its mechanical behaviour by changing its water content” [1]. Atterberg limits are the water contents that distinguish certain consistency levels. Plasticity, which influences these levels, is defined as the property of the soil or its ability to deform and permanently retain its shape without fracturing [2, 3]. Clays mostly exhibit plastic behaviour, mineralogy, bonding forces, and work–energy balance, providing an equilibrium of forces among the particles, which are the factors that affect the plasticity. Toughness is defined as the amount of work required per unit volume, which causes a certain deformation level. The water content range, in which the clay can be easily removed determines its toughness, which is also a measure of its workability [4]. A description of toughness levels is provided in BS5930 [5] in terms of the soil condition at its plastic limit. However, ASTM D2488-00 [6] defines this parameter in terms of the pressure required to roll out a thread of soil of diameter 3.2 mm, without the formation of fissures. Plasticity is expressed as the ability of a soil to deform, whereas toughness refers to the effort (energy) required to remove the soil.

In civil engineering applications, toughness affects the efficiency and cost of earthwork construction. Because tougher soils require an elevated level of energy for compaction, an increase in cost is expected. Toughness is also a useful feature for evaluating the elastic behaviour of clay components in earthworks, such as the cores of earth dams, dikes, and embankments. Casagrande [7] first defined toughness as the shear resistance of soil at its plastic limit. Subsequently, Casagrande [8] classified toughness levels by identifying soils from very weak to very tough. Reed [2] expressed toughness as the area under the stress–strain curve. However, he emphasised that only part of the plastic region should be considered. Similarly, Norton [9] and Schwartz [10] showed that the product of the yield stress and maximum deformation is a measure of plasticity, which is also expressed as the amount of work per unit volume. The yield stress is exceeded in the plastic region, and the maximum deformation is dependent on the ductility and strength of the clay; however, these parameters are affected by changes in the shape of the grains. The toughness limit (TL) can be used to quantify the upper limit of the toughness or workability of soil.

Soil compaction identifiers, namely maximum dry density (MDD) and optimum water content (OWC), are used in the quality control assessment of compaction in the field. Unexpected behaviour from the laboratory compaction data should be double-checked, for which the toughness limit is a possible tool. Predictive equations for the compaction identifiers of samples based on the index properties may be another option. In particular, models for fine-grained soils have been proposed for clays and silts by several researchers [11, 12]. Isika and Orden [13] used artificial neural network (ANN) models to estimate the compaction parameters of soils with different grain-size distributions. The literature includes studies focused on establishing correlations between the OWC, MDD, liquid limit, and compaction energy [14, 15]. Several studies have aimed to estimate the compaction parameters using empirical approaches based on the plastic limit [16, 17]. However, the use of liquid limit (LL) or plastic limit (PL) alone may not be sufficient for obtaining reasonable estimates of soil compaction identifiers. For the same LL, the PL may change significantly, causing subsequent changes in the compaction characteristics [18]. A short review of the literature shows that the OWC and MDD of the soils depend on the combined effects of the LL and PL. Therefore, the results from previous research show that equations based on the LL or PL alone cannot be useful for the evaluation of compaction parameters [13, 16, 19-24]. Therefore, the compaction parameters of different soils were analysed under a certain compaction effort, with emphasis on the combined effects of LL and PL. An equation for the toughness limit (TL) of the following form was obtained (in terms of percentage) by Vinod and Pillai [18] using the data reported by Barnes [25]:

Figure 1. Methodology flow-chart
Application of toughness limit in assessment of soil compaction identifiers

\[ TL = PL + 0.42 \times PI \]  

(1)

where plasticity index (PI) is the difference between the LL and PL. Hereafter, we discuss the assumption that Eq. (1) is valid for cohesive soil.

In the first phase of this study, parameters identifying the plasticity and compaction properties of the mixtures were determined based on the results of a series of standard Proctor, fall-cone liquid limit, Casagrande liquid limit, and thread-rolling plastic limit tests \[26, 27\]. The mixtures were combinations of two different types of clay (kaolin and bentonite), two different types of sand (S1 and S2), and silt (M). The results of the 176 tests conducted by the authors were combined with the 928 test results from the literature for certain assessments. This database, which includes more than 1000 test results, was used to establish the relationships among the toughness limit, compaction, and index properties. The methodology used to determine the interdependency between the compaction identifiers and index parameters is shown in Figure 1.

2. Materials and methods

Within the scope of this study, the results of 176 tests were obtained using both the Casagrande and fall-cone tests for 88 clay (bentonite/kaolinite)–sand (S1 and S2) and clay–silt (M) mixtures. Microstructural analyses were performed to determine the characteristics of the sands and clays using the SEM–EDX device at the Ulutem Center. Acceleration voltage of 20 kV was applied on the gold-coated samples.

<table>
<thead>
<tr>
<th>Minerals</th>
<th>B [%]</th>
<th>K [%]</th>
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</thead>
<tbody>
<tr>
<td>SiO₂</td>
<td>72.2</td>
<td>50.7</td>
</tr>
<tr>
<td>Al₂O₃</td>
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</tr>
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</tr>
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</tr>
<tr>
<td>MgO</td>
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<td>0</td>
</tr>
<tr>
<td>Na₂O</td>
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<td>0</td>
</tr>
<tr>
<td>K₂O</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SO₃</td>
<td>0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 1. Chemical analyses of clays provided by manufacturer (ESAN)

Figure 2 shows the images obtained from the scanning electron microscopy (SEM) analyses of the sand (S1, S2), silt (M), and clay (K, B). It is understood that sands are angular, and clays show the expected texture. Moreover, high SiO₂ content was observed in the clays (Table 1). The grain size distributions of the sand and clay are shown in Figure 3. Furthermore, 928 test results were compiled from the literature (Table 2). The characteristics of the data (LL < 50 %) are shown in Figure 4. Analysis of the plasticity chart showed that the majority of the data were clays of low plasticity, and the soil classes according to the unified soil classification system (USCS) were predominantly CL, ML, CH, and MH. In addition, smectite-type clays (LL > 100 %) with very high liquid limit values were also observed in the plasticity chart (Figure 4). In addition, the TL values of various soils were indirectly determined using Eq. (1) based on the Atterberg limits (LL and PL) from the literature and our own data.
3. Results and discussion

The database detailed earlier was used to establish the possible and plausible relationships among the toughness limit, Atterberg limits, activity, and compaction identifiers. The idea was to question the reliability of the toughness limit for predicting the plasticity and compaction characteristics of the different types of soils.

3.1. Basic characteristics of toughness limit (TL) in relation to plasticity and activity

Plasticity is commonly defined based on the Atterberg limits of soil; however, there are alternative identifiers. A description of TL, expressed in Eq. (1), is presented in [18]. Barnes [25] classified the classical range of water content between the LL and PL into plastic (possessing no toughness) and tough-plastic (workable) regions. The TL, which is defined as the water content, is a parameter that divides these two regions [18]; it is defined as the moisture content at zero toughness, as expressed in Eq. (1). The plasticity ratio (R_p) is defined as [28]:

$$R_p = \frac{PL}{LL}$$  \hspace{1cm} (2)

Another parameter identifying the plasticity is the plastic ratio, P_r [28]:

$$P_r = \frac{PI}{PL}$$  \hspace{1cm} (3)

Based on the definitions in Eqs. (2) and (3), the correlation between R_p and P_r can be derived as:

$$R_p = \frac{1}{1 + P_r}$$  \hspace{1cm} (4)

$$P_r = \frac{1}{P_p} - 1$$  \hspace{1cm} (5)

Although these parameters are physically related, they have different definitions. In addition, on the PI-LL plane, the interrelations among these parameters can be expressed as:

$$PI = (1 - R_p) \times LL = \frac{P_r}{1 + P_r} \times LL$$  \hspace{1cm} (6)

Accordingly, the correlations among these parameters are specified in relation to the A-line (PI = tanα (LL-20), tanα = 0.73) on the Casagrande’s plasticity chart as “Equi-R_p (P_r) lines”, as shown in Figure A1 of Appendix. Tangent of plasticity angle related with the gradient of A-line on the Casagrande’s plasticity chart, tanα is calculated as [29]:

$$tan\alpha = \frac{PI}{LL - 20}$$  \hspace{1cm} (7)

Activity (A) is defined as the ratio of the plasticity index to clay fraction, which is the percentage of the material finer than 2 µm [30]:

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Figure 3. Grain size distribution curves of components of mixtures

Figure 4. Plasticity chart including geodatabase

Table 2. Data sources

<table>
<thead>
<tr>
<th>No</th>
<th>Reference</th>
<th>Number of data</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>Authors’ own data</td>
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</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>Spagnoli et al. [40]</td>
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</tr>
<tr>
<td>4</td>
<td>El-Shinawi [39]</td>
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</tr>
<tr>
<td>5</td>
<td>Di Matteo et al. [38]</td>
<td>56</td>
</tr>
<tr>
<td>6</td>
<td>Nini [36]</td>
<td>58</td>
</tr>
<tr>
<td>7</td>
<td>Di Matteo [37]</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>Mishra et al. [69]</td>
<td>26</td>
</tr>
<tr>
<td>9</td>
<td>Ozer [35]</td>
<td>84</td>
</tr>
<tr>
<td>10</td>
<td>Dragoni et al. [34]</td>
<td>60</td>
</tr>
<tr>
<td>11</td>
<td>Orhan et al. [33]</td>
<td>26</td>
</tr>
</tbody>
</table>

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where CF denotes the clay fraction. Figure 5 shows an example of the relationship between the toughness limit and clay fraction (< 2μm), considering the relevance of some parameters defining the plasticity and activity.

Figure 5. Dependency of toughness limit on plasticity identifiers

For silt–clay mixtures, CF has a limited influence on the TL up to 40% [31]. Above 40 %, the rate of increase in TL with an increase in the CF was more evident. A similar trend was observed for the dependence of plastic ratio (P_r) on CF (%). The change in the plasticity ratio (P_r) with CF showed a different trend than that depicted earlier. On the contrary, tangent of plasticity angle (tanα) seems to fall within a narrow range, from 0.64 to 0.48, when the CF is increased from 5 % to 70 %. For these increases, 20 % CF is the limit for the extrema, which changes the aforementioned trends in the opposite direction. Interestingly, activity (A) drastically decreased at a CF of 20 %, followed by a steady trend. This A-CF relationship is analogous to the classification index chart for swelling potential proposed by Seed et al. [32]. According to their chart, the higher the activity and clay fraction, the higher is the swelling potential of the soil.

Most of the relationships stressed earlier were modelled using third-order polynomial models with high coefficients of determination (Figure 5).

Regarding the correlation between the CF and TL, data including the results of the Casagrande and fall-cone tests from over 50 different publications were collected. In addition to the re-evaluation of the results of tests on the sand–kaolinite (S–K), sand–bentonite (S–B), silt–kaolinite (M–K), and silt–bentonite (M–B) mixtures from the study by Karakan and Demir [27], the relationship between TL and CF was also established using the experimental results obtained from the literature (Figure 6). This figure contains information on clays with different plasticity levels and mineralogical properties [33–41]. According to the results, if the CF increased from 4 % to 40 %, the TL value was clustered between 18 % and 48 %. The TL value increased linearly in both S–K and S–B mixtures with an increase in the CF. For reference, the equations obtained for the S–K and S–B mixtures are given in Eqs. (9) (red solid line) and (10) (green solid line). It should be noted that these equations are not derived using all the data in Figure 6; they simply represent the relationship for the S–K and S–B mixtures:

\[
TL = 0.4609 \times CF - 0.8121 \quad \text{(9)}
\]

\[
TL = 1.078 \times CF - 5.1398 \quad \text{(10)}
\]

It was observed that the TL values of the S–B mixtures were more than twice the TL values of the K mixtures. The TL values obtained for pure K and B were 45.3 % and 102.7 %, respectively. It was also determined that the clays with high LL values had significantly higher TL values. This study confirms the findings for the kaolin-type clays by Spagnoli et al. [40]: the TL was 60 %. This shows that the behaviour is compatible with the similar clays reported in the literature. For instance, the results analysed by Lupini et al. [42] showed that the TL value increased linearly with an increase in the CF in the bentonite–sand mixtures (Figure 7). The TL value obtained for the 5 % B–95 % S mixture was 12 %, and it was 110 % for the specimen composed of 100 % B. The TL values in this study agree reasonably well...
with those obtained from the S–B mixtures, as shown in Figure 6. The changes in the CF–TL in clays with different mineralogical properties are presented in Figure 8. Using the experimental results of Spagnoli et al. (2018) obtained for clays with different mineralogical properties (Ca-smectite and Na-smectite), the changes in the TL with CF are shown in Figure 8. For the Ca-smectite clay, the CF was 60 %, whereas TL was 100 %. However, for the Na-smectite clay with very high LL (LL> 500 %) values, TL = 230 % was obtained for CF>85 %. Similar to the results obtained by testing 25 types of natural and artificial soils by Wasti and Bezirci [43], 15 types of clays by Yükselen and Kaya [44] and 9 types of clays by Chenari et al. [45], the results presented here show that as the CF increases, a linear increase in TL is observed. Previous studies have shown a distinct relationship between the TL and CF. Na-smectite clays and artificially blended sand–clay mixtures tend to show a steeper increase in TL with an increase in the CF, which boosts the TL levels by 100–250 %. It should be noted that the remaining values followed a gentle trend, constituting approximately 30° of the horizontal direction (Figure 8). The red solid line in the figure represents the trend for silt–clay mixtures [46], and the results are analogous to those obtained by Lupini et al. [42].

The Rp is expressed as the ratio of the plastic limit of the soil to its liquid limit. In this section, the TL–Rp relationship is investigated, which was obtained by arranging the results of the studies conducted by Karakan and Demir [26, 27] with 100 experimental results obtained from the literature. In Figure 9, it can be observed that, in most of the experimental data, the TL value is less than 50, and the corresponding Rp values are clustered between 0.3 and 0.9. Figure 9 includes the data from a study by Lambe and Whitman [47], which summarises the Atterberg limits (LL, PL, PI, and SL) for various clays. Typical clay minerals include...
kaolinite, illite, and montmorillonite. Na⁺, K⁺, Ca²⁺, Mg²⁺, and Fe²⁺ are the exchangeable cations in these clay minerals. For example, the LL of Na-montmorillonite is markedly higher than that of the Ca-montmorillonite. According to these data, the LL values of Ca-montmorillonite and Na-montmorillonite were significantly high, in the ranging 140–710 %, based on the aforementioned cation type. In contrast, the LL values of kaolinite were comparatively low (38–59 %). Based on an analysis of a vast number of test results for natural and artificial clayey soils, Figure 10. reveals that no strong relationship exists between $R_p$ and TL. However, an approximate correlation (Eq. (11)) was established from the data by Lambe and Whitman [47], as indicated by the red solid line (Figure 9):

$$R_p = 0.778 \times e^{-0.006TL}$$  \hspace{1cm} (11)

According to this generalised relationship, the TL values corresponding to $R_p$ values of 0.7 and 0.1 are 25 % and 350 %, respectively. Because the $R_p$ parameter is the ratio of PL over LL, we can rewrite the A-line equation “PI = 0.73 (LL–20)” as “PL = 0.27 LL + 14.6”, dividing the two sides of equation by LL, we obtain an $R_p$ value of 0.27, which is simply a descriptor of the clay–silt discriminator value. We propose this parameter as a boundary value for different behaviours in terms of the plasticity ratio. Divergence from this value (0.27) indicates high plasticity. The $R_p = 0.27$ line obtained by Shimobe and Spagnoli [29] is also shown in Figure 10. The data obtained for bentonite were below this limit. Figure 9 shows that TL is lower for kaolinite and illite, which exhibit higher plasticity ratios. Bentonites have higher TL values corresponding to the smaller plasticity ratios, which is a good indicator of their plasticity. It should be noted that, soils of lower activities are accumulated in a zone limited by TL and $R_p$ in the ranges 0–100 % and 0.2–0.9, respectively. Bentonite, palygorskite, and montmorillonite formed the tail of the relationship, which provided a wide range of TLs and low values of $R_p$ (Figure 10).

The outliers are indicated by the red dotted enclosing line (hereafter, similarly). With the trend line based on the data from Lambe and Whitman [47], as shown in Figure 11, the $P_r$–TL variation of the soils obtained from the experimental results of Karakan and Demir [26, 27] and data from the literature are shown in the red circle. As can be seen from the experimental results, TL varied in the range 10–60 %, and $P_r$ was limited in the range 0–2. In contrast, the bentonites had significantly high $P_r$ values, in the range 3–14 (Figure 12). In addition, an exponential relationship was obtained between the $P_r$ and TL, as shown in Eq. (12) (red solid lines in Figures 11 and 12).
Similar comments can be made regarding the dependency of \( P_r \) on TL; however, TL increases exponentially with increasing \( P_r \) values. Illite, kaolinite, and palygorskite data were encountered above or below the curve owing to their \( P_r \) whereas montmorillonite with a high \( P_r \) was located above this curve. Figure 13 shows the variation in the TL and tangent of the plasticity angle (\( \tan \alpha \) in Eq. (7)). When the TL was approximately 16%, \( \tan \alpha \) was calculated as 5. A rapid decrease was observed with an increase in the TL. It then shows a distribution around the constant \( \tan \alpha = 0.73 \) suggested by Shimobe and Spagnoli [48] for increasing TL values (TL = 50–350%). In addition, the TL values obtained for bentonite were higher than those obtained for kaolinite and illite. Regarding the vast amount of data obtained in the literature, \( \tan \alpha \) shows a distinct trend with TL values of up to 25%; however, this portion is composed of a lower amount of data. Beyond this value of TL, the \( \tan \alpha \) values are scattered in the range 0.4–1, and an average value of 0.73 (which is simply the slope of the A-line in the plasticity chart) is a reasonable value for assessment. Nevertheless, judging from several parameters representing the soil plasticity thus far, it can be inferred that the TL values for natural common soils are less than approximately 50–60%.

### 3.2. Applicability of TL for prediction of compaction characteristics

Many studies have proposed methods to estimate the OWC and MDD values, which are functions of the Atterberg limits (Table 3). Gurtug and Sridharan [16] revealed the effect of compaction energy on the compaction properties of clays. They conducted a series of standard and modified Proctor tests to determine the compaction characteristics of fine-grained soils of different mineralogies and origins (i.e. kaolinite, bentonite, Tuzla clay, Akdeniz clay, and Degirmenlik clay). By applying 22 tests, the equations OWC = 0.92 · PL and MDD = 0.92 · \( \rho_{d,PL} \) (where \( \rho_{d,PL} \) the dry density at PL: \( \rho_{d,PL} = G_s \rho_w/(1+G_s PL/100) \)) for the standard compaction effort were obtained. Sridharan and Nagaraj [49] conducted various experimental studies at the standard Proctor energy level to determine the index properties that correlated well with the compaction characteristics of clays. The authors asserted that the correlations between the PL and compaction characteristics were considerably better than those established with the LL or PI. The equations proposed by Sridharan and Nagaraj [49] were consistent with those proposed by Gurtug and Sridharan [15]. Sivrikaya [17] obtained equations for the PL, MDD, and OWC to estimate the standard compaction properties based on the published results of the experiments on clays from different regions.
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**Table 3. Summary of empirical equations proposed by various researchers (under standard Proctor effort)**

<table>
<thead>
<tr>
<th>Study</th>
<th>Empirical Equations for OWC and MDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gurtug and Sridharan [16]</td>
<td>OWC = 0.92PL MDD = 0.92ρ_d-PL</td>
</tr>
<tr>
<td>Sridharan and Nagaraj [49]</td>
<td>OWC = 0.92PL MDD = 21.459 - 0.23PL</td>
</tr>
<tr>
<td>Sivrikaya [17]</td>
<td>OWC = 0.942PL MDD = 21.97 - 0.2538PL</td>
</tr>
<tr>
<td>Günaydin [22]</td>
<td>OWC = 0.323LL_c + 0.563PL MDD = 0.78LL - 0.62PL</td>
</tr>
<tr>
<td>Đjoković et al. [50]</td>
<td>OWC = 4.18 + 0.16LL + 0.323PL MDD = 0.214 - 0.078LL - 0.05PL</td>
</tr>
<tr>
<td>Pillai and Vinod [51]</td>
<td>OWC = 0.172LL + 0.563PL MDD = 3.142ρ_d-LL + 7.4ρ_d-PL</td>
</tr>
<tr>
<td>Vinod and Pillai [18]</td>
<td>OWC = 0.61STL MDD = 1.134ρ_d-STL</td>
</tr>
<tr>
<td>Pillai and Vinod [24]</td>
<td>OWC = 0.623TL MDD = 1.15ρ_d-TL</td>
</tr>
</tbody>
</table>

ρ_d-LL, ρ_d-PL and ρ_d-TL stand for dry densities at PL, LL and TL, respectively.

Based on the data from another study by the same authors, the empirical equations listed in Table 3 were proposed to predict the compaction characteristics under the standard Proctor energy using the TL [24]. The authors tested natural and commercially available soils, along with locally available river sand. Three natural soils, namely, Cochin marine clay, Kuttanad clay, and Thonakkal clay, as well as kaolinite and bentonite, were tested to obtain relevant parameters. Several equations were developed by performing multiple linear regression analysis to estimate the compaction properties of fine-grained soils, considering all the 493 data points reported in the literature so far. The results of the regression analysis are shown in the last three equations in Table 3 (in the last three equations, especially for the OWC and MDD predictive models, the parameters TL and ρ_d-PL were utilised, respectively), where ρ_d-PL is the dry density (DD) at TL and ρ_d-LL = Gsρ_w/(1 + GsTL/100). This table summarises the empirical equations proposed by the various researchers.

Hereafter, we use the aforementioned equations by Pillai and Vinod [24] against the discussion of the compaction parameters. Using the standard Proctor test results reported in the literature and considering a vast amount of data (approximately 500), the variation in OWC with TL is shown in Figure 14. It was observed that the data in the OWC-TL relationship, based on the data from the literature and results of tests on the S–K and M–K mixtures, were compatible with the one (black solid line) proposed by [24]. The equation in Figure 14 is new (red solid line) and establishes the relationship between the OWC and TL based on the authors’ own test results for sand–silt–clay mixtures.
More than 350 data points concerning the modified Proctor compaction test results were used to show the variation in the OWC with TL, as shown in Figure 15. The results of the tests on the fine-grained (natural and artificial), organic, gravel, and sandy soils were used to establish this relationship. As shown, the equation (black solid line) proposed based on the standard Proctor energy by Pillai and Vinod [24] is not a descriptor of the data trend. The TL varied 15–60 % when the modified Proctor energy was applied, whereas the OWC values clustered at 8–27 %, as indicated by the red circle. Furthermore, the bentonite mixtures are highly outliers (data group is surrounded by red dotted line), as well as the case in Figure 10.

Figures 16 and 17 show the relationships between the MDD and TL generated using both the standard Proctor test results and modified Proctor test results, respectively. In the samples compacted under the standard Proctor energy, the MDD generally complies with the equation (black solid line) proposed by Pillai and Vinod [24]. However, considering both the standard and modified Proctor test results shown in Figures 16 and 17, a different behaviour was observed in the bentonites with high plasticity. Analysing Figure 17 based on data from the literature, it is evident that the TL-MDD relationship varies depending on the soil type. In particular, mixtures of bentonite showed higher TL and lower MDD values [52]; however, road subgrade soils and data from the soils exposed to modified Proctor tests (e.g. [53]) had lower TL and higher MDD values (Figure 17). It should be noted that, the data from Katte et al. [53] is obtained by testing the road subgrade soils from Cameroon, Africa. The samples were clayey lateritic gravel and were mostly in the A-2–7 class, in accordance with the AASHTO (American Association of State Highway and Transportation Officials) classification system. The gravel, sand, and fine contents (silt...
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and clay) were in the ranges 47.2–86.5 %, 2.9–13.7 %, 10.5–38.2 %, respectively. In addition, the Atterberg limits of the soils were in the range 43.6–92.6 % for the LL, 26.5–62.1 % for the PL, and 13.6–44.3 % for the PI. Compaction characteristics are also provided in the database. The OWC, MDD, and California bearing ratio (CBR) were obtained as 9.6–16.5 %, 1.910–2.328 g/cm$^3$, and 14.2–49.5 %, respectively. Therefore, the results indicate that the samples are suitable pavement materials. Owing to the different properties of the materials, the TL values were remarkably high (33.7–74.9 %) despite the significantly high MDD values.

Figures 18 and 19 show the variation in both the standard Proctor and modified Proctor test results between the optimum degree of saturation (ODS) at the MDD-OWC and TL, respectively. Moreover, the reference lines (red and black solid lines) determined based on the equations of Pillai and Vinod [24] are depicted together with the test results. The experimental results show that the ODS varies 85–95 % for high TL values in both the S–B and M–B mixtures. For low plasticity soils (20 %≤TL≤40 %) in the literature, the ODS values at standard Proctor energy showed a scattered behaviour at 60–110 %. From Figure 12, it can be seen that at the same ODS, the TL values are largely affected by the clay mineralogy, and the majority of the data are not limited by the ODS limits of 85–95 % [e.g. (54)]. Therefore, a unique correlation could not be obtained between the TL and ODS. In Figure 19, the plot of ODS against the TL is analysed based on the 246 data points when the TL values vary 20–50 %, while the corresponding ODS values are in the range 60–100 %. In Figures 12 and A7 in the Appendix, the data showing ODS > 100 % are theoretically impossible. This may be because of the reliability of the three different test results for specific gravity ($G_s$), water content ($w$), wet density ($\rho_t$), and dry density ($\rho_d$) in calculating the degree of saturation ($S_r$). The dry density and saturation level can be formulated as:

$$\rho_s = \rho_d \frac{1}{1 + \frac{w}{100}}$$  \hspace{1cm} (13)$$

Thereby, ODS can be calculated as:

$$ODS = \frac{OWC \times G_s \times MDD}{G_s \times \rho_d - MDD}$$  \hspace{1cm} (15)$$

where $\rho_w$ is the density of water (=1.0 g/cm$^3$). In contrast, Spagnoli and Shimobe [48] suggested that significantly low ODS values were encountered in the literature (ODS < 40 %). The tested soils were mostly clayey sand, silt, sandy clay, and silty clay. Therefore, there was a difference between the soil types. This may lead to the question of the reliability of using different tests to obtain specific gravity ($G_s$), which is then used to calculate the degree of saturation ($S_r$).
A comparison of the reported and predicted values of the OWC for the standard Proctor compaction test results on a wide range of soils worldwide is shown in Figure 20.a, based on Vinod and Pillai’s approach \[24\]. It is clear that the \(\text{OWC}_{\text{measured}}\) satisfactorily correlates with \(\text{OWC}_{\text{predicted}}\) within absolute error of \(\pm 5\%\), except for some data (e.g., bentonite mixtures). In addition, we examined the applicability of previous equations proposed for specialised soils using the experimental data obtained from the same soil-sampling regions. Here, as the sampled soils and related previously proposed equations, Iraqi and USA soils \[55, 56\], and equations from Pillai and Vinod \[24\] and Al-Khafaji \[57\] were selected. A comparison of the prediction based on the TL with the previous empirical equations for specialised soils is also generally satisfactory, as can also be seen from the \(\text{OWC}_{\text{measured}} - \text{OWC}_{\text{predicted}}\) plot shown in 20.b. It is seen from these figures that the difference between the \(\text{OWC}_{\text{measured}}\) and \(\text{OWC}_{\text{predicted}}\) is approximately \(\pm 5\%\) in absolute error within the range of \(10\% < \text{OWC}_{\text{measured}} < 40\%\), except for the data in part (e.g., bentonite mixtures and data from Hussain and Atalar) \[58\].

Figures 21.a and 21.b show plots of the predicted versus measured values of MDD. It can be observed that the TL-based predictions for MDD are viable, as are the cases of OWC shown in Figure 20. The difference between \(\text{MDD}_{\text{measured}}\) and \(\text{MDD}_{\text{predicted}}\) is only \(\pm 0.2 \text{ g/cm}^3\) in absolute error within a range of \(1.2 \text{ g/cm}^3 < \text{MDD}_{\text{measured}} < 2.2 \text{ g/cm}^3\). In previous research on MDD predictions (empirical formulations, e.g. \[55, 59, 60\]), the absolute error in the current study was roughly similar to those in the literature. The data obtained by testing bentonite and bentonite mixtures \[52\] showed that \(\text{MDD}_{\text{measured}} > \text{MDD}_{\text{predicted}}\). Data from Setiawan \[61\], partially
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3.3. Effect of compaction energy levels (standard and modified Proctor compaction)

The ratio of MDD_{MP} and MDD_{SP} is defined as [62, 63]:

$$\kappa = \frac{\text{MDD}_{MP}}{\text{MDD}_{SP}}$$  \hspace{1cm} (16)

where $\kappa$ denotes a dimensionless parameter. Figure 22.a shows the variation in $\kappa$ with TL. As the TL values increase, the corresponding measured $\kappa$ values seem to converge substantially to a constant value, and the $\kappa$ parameter seems to be in the range 1.00–1.20.

For fine-grained soils, Farooq et al. [64] showed that $\kappa$ was in the range 1.07–1.08. On the contrary, the ratio of OWC_{MP} to OWC_{SP} is defined by parameter $\beta$ as [63]:

$$\beta = \frac{\text{OWC}_{MP}}{\text{OWC}_{SP}}$$  \hspace{1cm} (17)

Figure 22.b shows the dependence of dimensionless parameter $\beta$ on TL. The plot is highly scattered and a rough trend cannot be established, unlike that shown in Figure 22.a. In addition, OWC predictions based on the Atterberg limits generally have lower accuracy than those based on the MDD. This was attributed to the presence and effect of fines (silt and clay). It should be emphasised that $\beta$ ranges from 0.6 and 1.0. Farooq et al. [64] presented $\beta = 0.80$–0.83 for 105 fine-grained soils. In addition, for reference, the trend lines for these dimensionless parameters $\kappa$ and $\beta$ are combined based on the TL approach, and several previous research results [16–17, 65–68] are depicted together in Figures 22.a and 22.b. As a result, it can be observed that the $\beta$–TL

Figure 21. Comparison of predicted and measured values of MDD in standard Proctor compaction test results: b) comparison with previously proposed equations for Iraqi and USA soils

Figure 22. a) Ratio of maximum dry density ($\kappa$)–TL relationship; b) ratio of optimum water content ($\beta$)–TL relationship
relationships for the OWC are more scattered than the $\kappa$–TL plots for the MDD. Moreover, although the general trend for the $\kappa$–values based on the combined correlations increases with an increase in the TL, regarding the change in $\beta$-value with varying TL, clear trends cannot be found whether it is constant or decreases with both the experiments and suggested equations. However, at present, the $\kappa$–TL correlation based on the results from Sivrikaya [17] fits comparatively well over the entire range of the experimental results. Finally, natural fine-grained soils with a TL less than approximately 50–60 % sufficiently need to be considered for practical use in field compaction applications.

4. Conclusions

In this study, a vast amount of data were used to analyse the dependence of compaction and index properties on the TL. The following conclusions were drawn from the analysis of the results:

- This study considered a database consisting of more than 1000 data points, including recent laboratory tests on sand/silt and kaolinite/bentonite mixtures, to provide an overview of the correlations between the toughness limit and the clay fraction, plasticity ratio, plastic ratio, tangent of the plasticity angle, optimum water content, and maximum dry density.

- A distinct correlation exists among the TL, optimum water content, and maximum dry density, considering the varying compaction energies. One exception is the specialised soil data (e.g. bentonite mixtures). Determination of the Atterberg limits is necessary for the prediction of the TL, PL, and LL, to subsequently assess the MDD and OWC. Scatter plots and the corresponding empirical relationships can also be used for verification because the application of the Proctor tests, particularly in highly plastic sand-clay and silt-clay mixtures, can be problematic. Nevertheless, the plasticity of the fine-grained part could partially prevent the transfer of the potential energy of the hammer to the soil. Therefore, Proctor tests may not always be applicable; we may need empirical approaches to predict the abovementioned parameters, namely the TL, PL, LL, MDD, and OWC. In this regard, the equations obtained in the current study have potential; however, this should be verified using the data obtained from the future studies.

- An exponential trend describes the dependence of the plasticity and plastic ratios on the TL. An exponential trend follows the clay mineralogy characteristics over the higher TL values (up to 350 %) regarding the correlations between the plasticity ratio and/or plastic ratio with the TL.

- The optimum degree of saturation was correlated with the TL considering the well-known reference optimum degree of saturation lines. In addition, the curve shape in the toughness limit and optimum degree of saturation relationships did not change with the varying Proctor energies.

- According to experimental test results from the literature, the difference between the measured and TL-based predicted values for the optimum water contents and maximum dry density was approximately ± 5 % and ± 0.2 g/cm³ except for the specialised data in part (e.g., bentonite and laterite mixtures), respectively.

- Considering the experimental evidence, even if the TL values of bentonite and kaolinite are very close to each other, clays with higher plastic limits have a lower maximum dry density and higher optimum water content than those with lower plastic limits. It should be noted that for natural fine-grained soils, the variation in most of the abovementioned parameters is highly scattered for TL values less than 50–60 %. Because the minimum CBR value in the field was 2 %, the corresponding TL value was 50–60 % for the pavement design. Therefore, in practice, soils with the TL higher than 50–60 % are considered unsuitable for subgrade construction.

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