PROGRESS OF AUXETIC AND SEMI-AUXETIC MATERIALS

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Abstract:

Auxetic materials with negative Poisson's ratios, display the unique behavior owing to their micro-structure or geometrical build. Auxetic materials resist deformities when subjected to uniaxial, biaxial, and shearing stresses. Cellular based materials are known for their excellent impact and shock energy absorption. When foams are incorporated with auxetic geometry, their impact energy absorption increases. Combination of auxetic materials with conventional materials yield semi-auxetic materials. Ideally, layered structures where facing materials are metallic sheets sandwiching polymeric foam cores are popular. The auxetic materials can be combined with conventional materials to obtain P-N-P and N-P-N semi-auxetic sandwiched structures with diverse potential in several applications like automotive, aerospace, sports equipment, and protective armors.

1 Introduction

The present review encompasses the progression of auxetic metamaterials throughout the past few decades, along with the introduction of novel material models utilized to characterize the behavior of these materials. Technological progress demands the introduction of innovative materials with distinct effects and behavior. Metamaterials are synthetic materials that are not naturally occurring. [1]–[3]. The term derives from the Greek word "meta", meaning beyond. In a more specific sense, "metamaterials," a term introduced by Rodger M. Walser, represent a distinct category of material combinations that exhibit extraordinary properties that are distinct from those exhibited by the constituent materials themselves [4]–[7] . Within the realm of scientific exploration, certain physical properties are commonly assumed to be positive. Yet, it is intriguing to learn that these properties can also exhibit negativity. Negative materials encompass those substances or metamaterials that possess adverse properties. Notable examples of such properties include: negative stiffness [8], negative thermal expansion [9]–[11], negative refractive index [12], negative permittivity [13] and/or negative permeability [13], negative compressibility [14]–[17], hard-sphere molecules with NPR/PPR [18], negative transmissibility of vibrations [19].

The mechanical and thermodynamic models exhibiting auxetic behavior were developed during the 1980s [20]–[23]. Since 1987, when negative Poisson's ratio foams have been developed by Lakes [24], it is a known fact that materials and structures with a negative Poisson's ratio can occur naturally. A wide range of structural 2D and 3D models with negative Poisson's ratio explain auxeticity [25], [26]. These structures encompass various examples, such as the familiar honeycombs (with re-entrant and concave features), chiral and anti-chiral configurations, rotating rigid units like squares, rectangles, and triangles, liquid crystalline polymers, dilating triangles, egg rack structures, sinusoidal ligaments, metamaterials, periodic microstructures such as a square array of circular holes in an elastomeric matrix, and numerous other systems. T-C Lim elucidated analogies among auxetic models based on deformation mechanisms [3], [27]. Numerous researchers have conducted previous investigations into the negative properties of metamaterials. Grima-Cornish et al. [28] explored the role of rotating rigid units in generating negative thermal expansion (NTE) and negative compressibility (NC) in auxetic materials. They proposed a mechanism involving rotating units or quasi-units. Bilski et al. [18] studied 2D multi-body periodic systems composed of binary hard discs, which are circular

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particles of slightly different sizes. Other authors explored various aspects, including bi-phase materials exhibiting negative Poisson's ratio [29], [30], negative thermal expansion coefficient [31], tunable properties ranging from positive to negative [32], [33], and sandwich structures with enhanced indentation resistance [34]. The impact of the half-cylinder elastic body onto the clamped plate (made of material with positive and negative PR) analysis was presented by Strek et al. [34]. The results obtained show a significant influence of Poisson's ratio on contact pressure (indentation issue), especially for extremely negative values close to "-1". The problem of indentation is crucial for the impact behavior of structures. The concept of auxetic materials, which exhibit negative Poisson's ratio (NPR), was introduced by Evans, who coined the term "auxetic" [35]. Poisson's ratio was constrained by two conceptual limits: -1 and 0.5 [24], [36], [37]. Auxetic effect in materials is caused by changes in microstructures as well as geometrical structures adapted to materials- Double arrowhead, star honeycomb, and hexagonal re-entrant, honeycomb, lozenge grids, square grids, sinusoidal ligaments, chiral units, and rotating units (triangle, square, rectangle, tetrahedron), anti-chiral structures, cyclic hexamers, and cyclic trimers [38].

The top limitation is the incompressible limit (infinite bulk modulus), which is generally applied to elastomeric materials like rubber, while the lower limit is the infinite shear modulus [39]. The auxetic effect is achieved by bending or hinging the ribs under load in the arrowhead shape of a honeycomb. The chiral honeycomb displays auxetic behavior in the short ligament limit for in-plane uniaxial packing when bent out of a plane, while anticlastic curvature is characteristic of positive Poisson's ratio behavior when bent out of a plane. The cylinder spinning process cancels out during bending, resulting in this loading-dependent Poisson's ratio. The cylinder tension generated by top surface tension is balanced by honeycomb bottom surface compression, which causes the cylinder to rotate in the opposite direction. In pure bending, the honeycomb response is limited to the base hexagonal honeycomb motif, which is thought to result in a positive Poisson's ratio when ligament flexing or hinging is dominant. Chiral honeycombs with the auxetic effect for both in-plane and bending loads are created by arranging the cylinders on the base re-entrant. This permits the auxetic effect to achieve the double-curvature required for dome-shape applications while simultaneously making use of the cylinders and ligaments enhanced through thickness compression and shear resistance characteristics, respectively [40]. α-cristobalite, a naturally occurring form of silica displays Poisson's ratio between –0.5 to –0.16 [41]. Polytetrafluoroethylene (PTFE) was the first auxetic polymer artificially manufactured [42]. The auxeticity arises because of the tensile disfigurement action or micro rotational degree of freedom. One of the recent applications of auxeticity in sports was seen on the sole of Nike's Flyknit® running shoes. When a runner touches the ground with their foot, it stretches, alleviating discomforting pressure points as a result [43]. In the aerospace industry, auxetic materials have been employed for the aircraft nose.

The auxetic materials stiffen due to drag, reducing the turbulence and enhancing flight stability [44]. Auxetics can also serve as protective linings in cargo drops and linings in the Kevlar vest due to their high energy absorption, due to which auxetic fabrics can serve as armor materials [45]. Since nanomaterials have also garnered significant interest, auxeticity in a wide array of nanomaterials such as graphene, metal nanoplates could be a potential amalgamation of novel technologies [46]. Foams are cellular materials created by confining spaces of air or gas in a liquid or solid. Closed-cell and open-cell foams are the two types of solid foams [47]. Closed-cell plastic contains gas-filled pockets that are fully surrounded by solid material, creating isolated compartments. On the other hand, open-cell foam consists of gas pockets that interconnect with each other. Similar to a bath sponge, water can freely permeate throughout the entire structure of open-cell foam, displacing the air within [48]. The pores within open-cell structured foams are interconnected, forming a relatively flexible network. Open-cell foams have the capacity to absorb the surrounding gas. When the open cells are filled with air, they exhibit efficient insulation properties. However, if the open cells become filled with water, their insulation capabilities are reduced. Recent research has focused on investigating the insulation characteristics of open-cell foams. Additionally, it has been discovered that tetraethyl orthosilicate (TEOS)/Wheat gluten bio-foams exhibit comparable insulation properties, closely resembling bio-foams derived from oil-based materials [49], [50]. By default, foams possess a positive Poisson's ratio unless subjected to specific treatments that bestow them with auxetic properties. Closed-cell foams lack interconnected pores. As a result of their unique architecture, closed-cell foams typically exhibit enhanced resistance against compression forces. Due to their denser structure, closed-cell foams require a higher mass of material. To enhance insulation capabilities, closed cells can be filled with gas. In comparison to open-cell foams, closed-cell foams offer superior dimensional tolerance, increased strength, and lower hygroscopicity [51]. In sandwich-structured composite materials, all forms of foam are often utilized as core components.

Conventional foams can be converted into auxetic foams by permanently altering the structure and geometry of the cells, by mechanical, thermal, or chemical treatment [52]. Auxetic foams are a type of foam that possess a negative Poisson's ratio. This means that when subjected to tensile or compressive forces, they resist deformation by actively opposing the applied forces. The production process for auxetic foams involves compressing a foam piece in all three dimensions and then heating it above the softening point of the foam's polymer material within a mold.

The compressed foam is then allowed to cool down to room temperature while still inside the mold, thereby fixing the distorted ribs into their new shape. Through this method, polyester foams have been successfully transformed into foams with a negative Poisson's ratio by applying a compression factor ranging from 1.4 to 4.0 [53]. In the case of reticulated metal foams, no temperature increase is necessary. Instead, these foams undergo permanent geometric distortion in all three axes at ambient temperature to achieve the desired negative Poisson's ratio characteristics [24]. Polyurethane foams are the most often researched foams for a variety of uses. Lakes [24] invented low density, open-cell auxetic foams in 1987 using standard polyester. he creation of these foams involved modifying the microstructure of conventional foams like polyurethane (PU). The auxetic properties were achieved by introducing concave cells within the foam structure. One notable advantage of auxetic PU foams is that they do not undergo microstructural changes when exposed to elevated temperatures or solvents under confined conditions that restrict foam expansion. Enclosing them within a cover helps maintain their volume. The most common method for producing auxetic materials involves converting open-cell foam through a combination of compression and heat treatment techniques.

The resulting auxetic foams exhibit superior characteristics compared to their traditional counterparts. These include enhanced shear resistance, increased hardness, reduced fatigue fracture propagation, improved heat insulation, greater toughness and modulus resilience, and enhanced vibration absorption properties [39]. Auxetic thin-walled tubes endure contortion and find applications in the automotive and aviation sectors, sports equipment such as helmets, shin guards, and racing barriers [54]. In recent times, researchers have undertaken efforts to compare the properties of conventional materials with those of auxetic and semi-auxetic materials. To achieve this, they have focused on modifying the microstructure of readily available foam cores to induce a negative Poisson's ratio. By extensively adjusting the properties and structure of these conventional materials, researchers aimed to establish a meaningful comparison with auxetic materials [55]. Auxetic and semi-auxetic materials are more resistant to a variety of strains and shear stresses than other materials. In 2006, a new branch of auxetic materials, namely "helical auxetics" was invented by Hooks [56]. Two fibers were combined such that, one was the core, and the other wound around it in a helical pattern. Sloan et al. [56] have studied the effect of wrap angles on the Poisson's ratio while Shah et al. [57] have studied the effects of braiding angles of auxetic yarns on the Poisson's ratios. They found that stiff wrapped yarns with low braiding angles, showed better auxeticity at lower strains. Helical auxetic yarns could be braided with Positive Poisson's ratio yarns to obtain hybrid semi-auxetic fabrics [58].

2 Mechanics of Auxetic Materials

This section deals with the approaches by different researchers to define the deformation mechanics of auxetic materials. Several models representing the deformation mechanics of auxetic materials have been developed. Scott [59] assessed the cumulative Poisson's ratio concerning the limiting factors that resisted deformation based on geometric orientation. The author noted that while the Poisson's ratio was inherently positive in the unexcited state, it was found to be negative in the computed cumulative Poisson's ratio when a significant planar extension was applied. Blatz and Ko [60] developed a function that described the relation between Poisson's ratio and the extensions in conventional, moderately auxetic and auxetic stress fields of polyurethane foams as given in equations 1-3, Where $\upsilon(\lambda)$ is the Blatz-Ko Poisson's function, 'μ_0' and 'λ' are the Lame moduli, 'υ_0' is the infinitesimal Poisson's ratio. Eq. (1) relates the lateral and longitudinal stretch, Eq. (2) expresses the Poisson's function, while Eq. (3) shows the infinitesimal Poisson's function.

$$\lambda_1 = \lambda^{-\upsilon_0} \tag{1}$$

$$\upsilon(\lambda) = \frac{1 - \lambda^{-\upsilon_0}}{\lambda - 1} \tag{2}$$

$$\upsilon_0 = \frac{1}{2}\left(\frac{\lambda}{\lambda + \mu_0}\right) \tag{3}$$

Figure 1 shows the variation of the Poisson's function υ(λ) with the stretch function 'λ' for different values of υ_0. When a re-entrant honeycomb is stretched, the matter expands laterally, but when the cells are fully expanded, stretching causes the cells to compress laterally. The relation between Poisson's ratio and stretch as per the modified model by Ciambella et al [61], over the conventional model is shown in Figure 2.

2.1 Stress concentration, fracture, and damage in auxetic materials

Localized increase in stress due to restrictions such as limited cross-sectional areas lead to a stress concentration. As a result, when the stress proceeds beyond the ultimate strength of the material, materials fail via fracture, i.e., crack propagation. The stress concentration factor (SCF), the ratio of maximum stress to nominal stress is an important criterion to optimize safe designs [62].

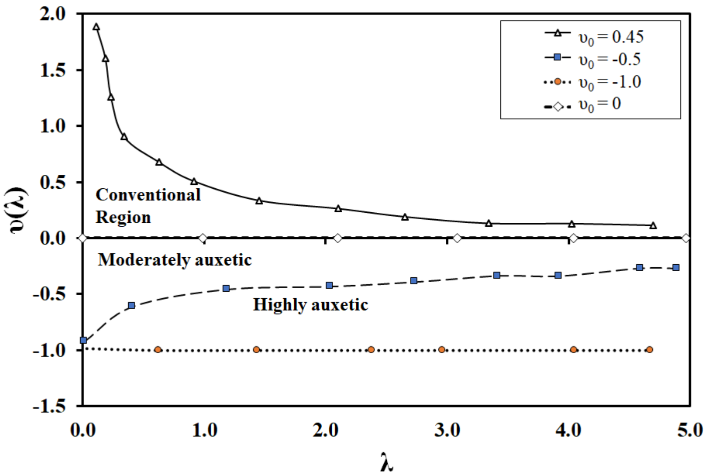

Figure 1. Poisson's function variation for conventional and auxetic regions [61].

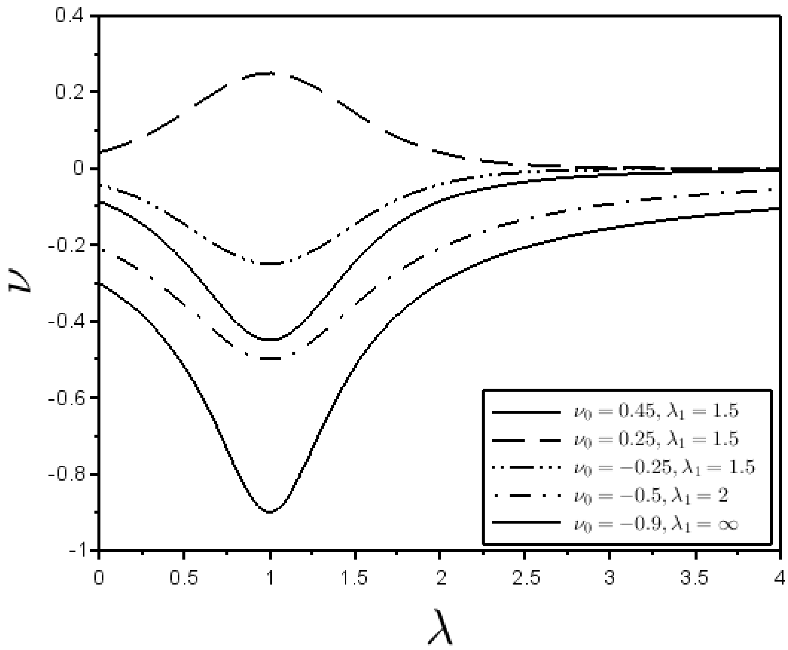

Figure 2. Variation of 'υ' with 'λ' as per Ciambella-Saccomandi model [61].

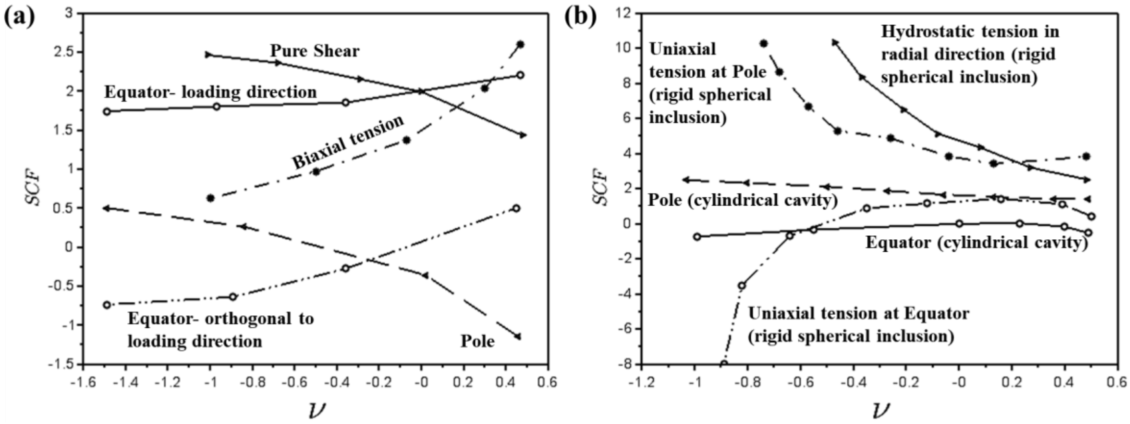

Figure 3. Depiction of stress concentration factors (SCF) around: (a) spherical cavities under the action of tensile forces and pure shear in two of the three mutually perpendicular axes (b) cylindrical cavity and rigid spherical inclusion [26].

The maximum stress is computed by the elasticity theory and the nominal stress by the uniform stress distribution. Stress concentration areas can be avoided by rapid alterations in the geometry by imparting suitable pits, grooves, fillets, chamfers, and notches [54], [63]–[65]. Roderick Lakes discovered that negative Poisson's ratios tend to lower the SCF in some cavity conditions while raising the same in others [52]. In comparison to spherical cavities, ellipsoidal cavities led to higher SCFs. Sadowsky et al [66], [67] provided exact solutions for the SCF around an ellipsoid cavity in infinitely elastic solids. Chiang [50], [68] studied the generalized stress concentrations arising from tri-axial ellipsoidal cavities using the equivalent inclusion method. Two cavity types- prolate ellipsoidal cavity and an oblate ellipsoidal cavity were analyzed. The stress concentration factors around these special variants of cavities were found to be within the limits of $-1 \leq \upsilon \leq -0.5$. Lim [69], [70] evaluated the auxetic effect on the SCFs of rods with hyperbolic groove and large thin plates with circular holes and rigid inclusions (as shown in Figure 3). Apart from the SCF, six damage variables were identified by Voyiadjis and Kattan [71] - damage equivalent stress, von Mises accumulated plastic strain, elastic strain energy, porosity in terms of cavities relative volume, the radius of cavities, and relative area of micro-cracks and cavities in a plane.

The damage equivalent stress was later computed using the laws and concepts of thermodynamics, and it was found that it is directly impacted by the material's Poisson's ratio. The von Mises equivalent stress and hydrostatic stress are the main key metrics dictating material behavior. It was also found that as the Poisson's ratio of the material became more negative, the damage criterion hinged on hydrostatic stress instead of hinging on the von Mises equivalent stress [72], [73]. Shear deformation is recognized to be more prominent in the crosswise loading of thick beams and plates than thin ones, resulting in higher transverse deflection than predicted by traditional beam and plate theories. From the studies carried out by multiple researchers, it was found that as the negative nature of Poisson's ratio increased, it led to alleviated shear deformations, thereby suggesting that geometrically thick beams and plates could be compared to mechanically thin beams and plates, respectively, if the Poisson's ratio is sufficiently negative [74], [75]. Studies were conducted regarding buckling of columns that possess similar directional and orientational properties. It was observed that the negative nature of Poisson's ratio leads to a rise in the buckling load [76]. It was also established that the buckling strength in a column made of materials with negative Poisson's ratio was bettered when crosswise shear deformation was diminished [26], [77], [78]. In the studies carried out on the vibration of thick plates displaying similar orientational properties, it was computed that as the plate material's Poisson's ratio increased in negativity, the Mindlin-to-Kirchhoff natural frequency ratio increased at a diminishing rate [79].

3 Semi Auxetic Materials

There are two broad categories of semi-auxetic materials. The first category essentially includes those that have non-uniform, non-directional or no specific orientational properties whatsoever. It incorporates materials that behave conventionally in one plane but display auxetic nature in another, and hence, they are termed directional semi-auxetic materials [80]. The other category includes those containing two materials with opposing Poisson's ratios. These materials may vary in composition and structure, gradually over volume, with dissimilar Poisson's ratios that are positioned opposite to one another at either ends. Hence, they are known as positional semi-auxetic solids. When the subjected loads are in a direction normal to the plane, a Poisson-Shear material exhibits substantial in-plane shear strain. To achieve this phenomenon, such material should exhibit Poisson's ratios that are similar in scalar terms but of opposite signs to one another at the ends, or vice versa in both planes. When normal strain is imposed in the third direction and material has a positive Poisson's ratio in one plane (Ex: 2–3 plane) and a negative Poisson's ratio in another plane (Ex: 1–3 plane), substantial 1–2 plane shearing occurs. A Poisson-Shear material may well be generated schematically by combining conventional and auxetic characteristics in opposite directions.

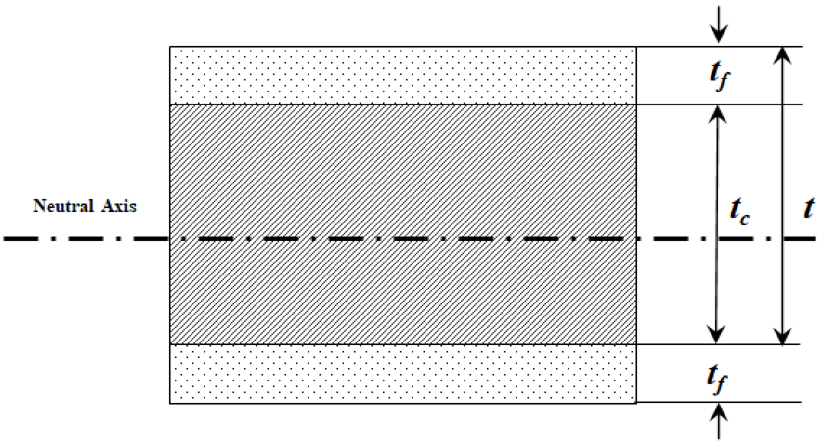

Figure 4. Three layered sandwich plate with auxetic core and non-auxetic face sheets (PNP configuration)[82].

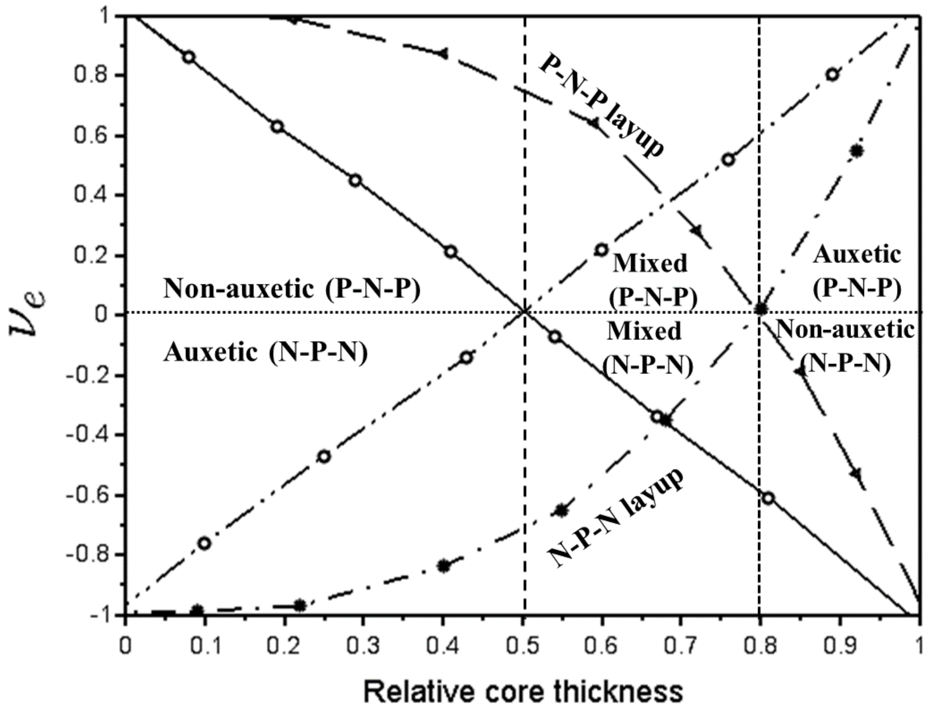

Figure 5. Equivalent Poisson's ratio vs core thickness in PNP setup and NPN setup, subjected to axial stretching (dotted) and bending (bold) [82].

Unlike completely traditional and fully auxetic materials, a plane-reliant semi-auxetic matter shows both auxetic and conventional activity in two orthogonal planes and conventional activity in the third plane, mutually perpendicular to the former two planes. Stretching along 1-axis leads to a longitudinal increase in dimensions along 2-axis but constriction along 3-axis, whereas stretching along with 2-axis causes leads to a longitudinal increase in dimensions along 1-axis but a decrease in longitudinal dimensions along 3-axis. Tensile loading in 3-axis, as with typical materials, causes constriction in both axes 2 and 3. The partially auxetic materials thus display a restricted auxetic behaviour both along translational and rotational terms [25], [83].

Table 1. Effective Poisson's ratio of different laminates [83].

Relative Thickness	Core Loading Modes	P-N-P layup	N-P-N layup
$\frac{t_c}{t} = \frac{1}{2}$	Axial	0	0
	Pure Bending	Positive	Negative
$\frac{t_c}{t} = \left(\frac{1}{2}\right)^{1/3}$	Axial	Negative	Positive
	Pure Bending	0	0
$\frac{1}{2} < \frac{t_c}{t} < \left(\frac{1}{2}\right)^{1/3}$	Axial	Negative	Positive
	Pure Bending	Positive	Negative

Plane-dependent, semi-auxetic materials demonstrate auxetic behaviour during compressive stress in the three mutually perpendicular directions. However, two out of three available mutually perpendicular planes, of a three-dimensional structure display positive Poisson's ratio under tensile loading. The scalar value of the Poisson's ratio is substantially governed by both the un-deformed geometry and the degree of strain in the case of strain-reliant semi-auxetic materials [83]. Considering the laminate comprising of three layers depicted in Figure 5, a symmetrical laminate may be made with a lay-up by using isotropic materials with opposing Poisson's ratio for the face-sheet and core sections. As a result, two broad categories can be distinguished as - positive-negative-positive (PNP) lay-up, in which the face-sheets and core have positive and negative Poisson's ratios, and the negative-positive-negative (NPN) lay-up, in which the face-sheets and core have negative and positive Poisson's ratios. The three-layered sandwich plate with consistently aligned laminates has a thickness '*t*' embodying face-sheets and core with corresponding thicknesses of t_f and t_c respectively (Figure 4). Table 1 compares the effective Poisson's ratios of different combinations. The presence of interfacial tensile forces between the thin plates is inherent in nearly all synthesized laminates, and Poisson's ratio discrepancy is not an abnormality either. In Figure 5, the results of opposing Poisson's ratio signs are represented diagrammatically.

The conventional laminar theory assumed that all of the synthesized layers are subjected to plane stress, ignoring any out-of-plane forces. This assumption stands true in areas outside of the free edges. However, shear forces between adjoining layers can occur at the margins. As a result, using a suitable bonding substance is critical for avoiding inter-laminar delamination. It has been computed that sideways increment in dimensions has been observed, along with an anti-clastic shape being induced in the P-N-P laminates under varying loading conditions, whereas sideways decrement in dimensions has been observed, along with a synclastic shape being induced in the PNP laminates under different loading conditions. Hence the concept of laminates can be used to sandwich multiple plates of different materials with varying values of PR to obtain the required equivalent Poisson's ratios in the assembled unit [84].

4 Conclusion

The review covers the advances in auxetic and semi-auxetic materials for the past two decades. Auxetic materials as stand-alone materials have popularly found applications in sports, shielding armors, medical applications, automotive and aerospace sectors. The progress in the combination technologies of auxetic materials with positive Poisson's ratio materials to obtain hybrid configurations delivering a diverse range of mechanical properties have been covered. While sandwich structures use alternate layers of auxetic and positive Poisson's ratio materials, braiding technologies combine auxetic yarns with regular yarns to deliver hybrid, blended semi-auxetic fabrics. There is still a scope for the improvement in the mechanical properties by 3D braiding of auxetic fabrics using either regular yarns or auxetic yarns. When extended to composites, the popular matrix materials (with υ >0.3) used must not interfere with the performance of the auxetic fabrics, hence technologies lowering the Poisson's ratio of matrix material by thermal or chemical methods would be necessary.

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