An inverse multi-period FDH model with undesirable outputs

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Abstract. In some examinations, the changes in performance measures and the efficiency of entities should be addressed over a span of time despite the undesirable outputs and unsatisfactory convexity property. In this study, the performance of processes is analyzed over several periods of time by presenting a multi-period model based on the free disposal hull (FDH) approach, incorporating undesirable outputs. Using the product of availability, performance (efficiency) and quality, the overall equipment effectiveness (OEE) is calculated which shows the level of productivity. After estimating the performance and OEE, an inverse multi-period FDH model is introduced to measure the input changes for the changes made to outputs in different time periods. The proposed approaches are used in the automotive industry to analyze the performance and changes in input measures over multiple periods of time. The results show that the proposed technique is reasonable for assessing the multi-period efficiency, the period and overall OEE of the systems, and the input changes related to several periods while the convexity assumption is violated.

Keywords: data envelopment analysis, free disposal hull (FDH), inverse DEA, multi-period, OEE.

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1. Introduction

Data envelopment analysis (DEA) is a valuable tool for calculating the relative efficiency of decision-making units (DMUs), including multiple inputs and outputs. After determining the efficiency of organizations, necessary decisions are made to modify DMUs. In some cases, however, the efficiency decision-making using is not correct, thus, other factors are used. Productivity is one of these factors, which is obtained by multiplying several factors, including efficiency [15]. The free disposal hull (FDH) is one of the most widely used models in DEA to estimate the relative efficiency of DMUs; in which the convexity principle is not considered.
The incorporation of undesirable data is a significant issue in developing DEA approaches as all available information is not preferred in systems. For example, carbon dioxide is an undesirable output of factories. Overdue loans of banks are another type of undesirable outputs. Different methods have been developed in the DEA literature to handle undesirable measures. For instance, some researchers take undesirable outputs as desirable inputs while considering undesirable inputs as desirable outputs. Interested readers can refer to [13, 30] for more information on the approaches used to address undesirable factors.
The DEA studies also address the estimation of the efficiency of DMUs over a span of time, including several periods. Park and Park [35] proposed a DEA-based approach to evaluate the

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aggregative efficiency of multi-period processes, in which the efficiency of each period can be estimated individually. Kao and Liu [24] presented an approach based on the network DEA model to simultaneously estimate the overall and period efficiency of DMUs. Jablonsky [16] measured the efficiency of Crezh economic departments over four years by applying multi-period DEA models. Razavi Hajiagha et al. [12] analyzed the performance of multi-period production systems using a two-stage DEA model on the basis of Chebyshev inequality bounds. Moreover, some groups (Kordrostami and Jahani Sayyad Noveiri [27] and Liu [31]) evaluated the multi-period efficiency of systems with uncertainty. Jahani Sayyad Noveiri et al. [17] measured multi-period efficiency and changes in the performance of processes in the presence of undesirable outputs. Jahani Sayyad Noveiri and Kordrostami [18] analyzed the multi-period sustainability performance of processes with discrete and bounded measures. Ghobadi [10] estimated performance measures with inter-temporal dependence. Ghobadi et al. [11] also provided inverse dynamic DEA models with fuzzy measures to assess inputs. Foladi et al. [8] assessed some faculties and changes in inputs and outputs using inverse dynamic DEA models. Furthermore, using inverse network dynamic DEA approaches, the sustainability of supply chains was addressed in some studies [22, 23]. Nevertheless, a limited number of studies have estimated multi-period efficiencies and input changes while relaxing the convexity property.

Wei et al. [39] presented an inverse DEA model to assess performance measures. Lertworasirikul et al. [29] considered the inverse DEA approach under the variable returns to scale assumption for a resource allocation problem, contemplating the expansion of some outputs and reduction of others. Ghiyasi and Zhu [9] presented an inverse semi-oriented DEA approach for the cases with negative measures. Hassanzadeh et al. [14] evaluated the sustainability of countries using their inverse DEA models, including input-oriented and output-oriented inverse semi-oriented radial measures [21, 3]. Asadi et al. [1] proposed inverse FDH models from two perspectives: optimistic and pessimistic. The performance measures were also addressed in some inverse DEA studies [19], presenting non-discretionary measures.

Despite the attempts for estimating the performance and the changes of performance measures, no study has assessed the multi-period efficiency of systems with undesirable outputs to manage the changes of inputs in several periods of time while the convexity principle is violated.

Therefore, this study introduces a multi-period FDH model with undesirable outputs to measure the overall and period efficiency of DMUs in the absence of the convexity assumption. Furthermore, its inverse multi-period problem is rendered to estimate inputs for the modification of outputs in multiple periods. The overall equipment effectiveness (OEE) in each period and as a whole is also computed. To illustrate, the role of maintenance in modern manufacturing systems has become more important for companies that take maintenance as a vital element in their business [20]. OEE is a quantitative measure that has been increasingly considered and accepted in production processes to observe and control the efficiency of production equipment. Process improvement in the production environment stems from three factors: availability, productivity, and product quality. OEE was first developed by Nakajima (1988) as a tool to evaluate the improvement through development initiatives and as a proposed repair solution [34]. This approach also reduces operating costs [2]. Simple DEA models are generally applied in the topics addressing OEE to measure efficiency. In this work, however, FDH-based methods are applied as they are more justified [20, 26, 37]. A real dataset of the automotive industry is also used to illustrate the developed approaches.

Overall, the contribution of this research can be described in four aspects:

- Introducing a multi-period FDH model with undesirable outputs to measure the overall and period efficiency of the DMUs,
- Computing the OEE in each period and as a whole by considering the FDH approach,
- Estimating multi-period inputs for the perturbation of outputs in multiple periods by applying the proposed inverse multi-period FDH technique, and
- Clarifying the developed approaches using a real dataset of the automotive industry.

The rest of this paper is organized as follows: prerequisites are described in Section 2. Section 3 is devoted to introducing a multi-period FDH model with undesirable outputs to assess the multi-period efficiency under non-convex technology. After dealing with period and overall OEEs, an inverse multi-period FDH approach is provided for estimating inputs. A real case study of the automotive industry is presented in Section 4 for further clarification. Finally, the conclusions and recommendations are presented in Section 5.

2. Preliminaries

In this section, the essential prerequisites and models, containing FDH, multi-period DEA, and inverse DEA are reviewed.

2.1. Free disposal hull

The production possibility set (PPS) in DEA is constructed using several principles, one of which is convexity. According to this principle, a convex combination of DMUs also belongs to the PPS. This principle is not, however, satisfied in many real-world situations, so it is ignored. An alternative PPS is, thus, created by relaxing this principle. Deprins et al. presented the FDH model that does not include the convexity assumption [6]. This model was then extended by Lovell et al. [32]. An appealing characteristic of the FDH model is that it submits only one of the existing efficient DMUs as a reference unit for each inefficient DMU. Consequently, the FDH reference set is more compatible with many practical applications. By considering the observations (X_j, Y_j) $j = 1, 2, ..., n$, the PPS of the FDH model under constant returns to scale (CRS) is as follows [25]:

$$T = \Big\{(X, Y) \ : \ \sum_{j=1}^{n} \lambda_j \omega X_j \le X, \quad \sum_{j=1}^{n} \lambda_j \omega Y_j \ge Y, \quad \omega \in \mathbb{R}_+, \lambda_j \in \{0,1\}, \sum_{j=1}^{n} \lambda_j = 1\Big\}$$

where $X_j \in \mathbb{R}^m_+$ are inputs, $Y_j \in \mathbb{R}^s_+$ present outputs for DMU_j and ω denotes a non-negative real variable that shows a scaling factor allowing for a particular scaling of observations spanning the frontier [4]. Moreover, $\lambda_j, (j = 1, ..., n)$ are intensity variables. By considering the technology T, the input-oriented FDH model can be expressed by:

$$\begin{aligned} & min \ \ \theta && (1) \\ & s.t. \ \ \sum_{j=1}^{n} \lambda_j \omega x_{ij} \le \theta x_{io}, \quad i = 1, 2, ..., m, \\ & \sum_{j=1}^{n} \lambda_j \omega y_{rj} \ge y_{ro}, \quad r = 1, 2, ..., s, \\ & \lambda_j \in \{0, 1\}, \quad j = 1, 2, ..., n, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \omega \ge 0. \end{aligned}$$

Problem (1) is a mixed-integer nonlinear programming problem that can be transformed into the following mixed-integer linear programming problem using the big-M method [36]:

$$\begin{aligned} & min \;\; \theta \\ & s.t. \;\; \sum_{j=1}^{n} \Lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1,2,...,m, \\ & \qquad \sum_{j=1}^{n} \Lambda_j y_{rj} \geq y_{ro}, \quad r = 1,2,...,s, \\ & \qquad 0 \leq \Lambda_j \leq M\lambda_j, \quad j = 1,2,....,n, \\ & \qquad \sum_{j=1}^{n} \lambda_j = 1, \\ & \qquad \lambda_j \in \{0,1\}, \quad j = 1,2,....,n, \end{aligned} \tag{2}$$

where M is a sufficiently large number and $\lambda_j \omega = \Lambda_j$.

2.2. Multi-period DEA

This subsection discusses multi-period DEA models. Suppose we have n DMUs, DMU$_j$ ($j = 1,2...,n$), each with m inputs x_{ij}^t ($i = 1,2,...,m,\ j = 1,2,...,n,\ t = 1,2,...,T$) and s outputs y_{rj}^t ($r = 1,2,...,s,\ j = 1,2,...,n,\ t = 1,2,...,T$). We also have information about these units in T different periods ($t = 1,2,...,T$). One strategy is to separately calculate the efficiency in each period, another strategy involves considering each period an independent unit and determine the efficiency in all periods, thus, the number of units will be nT. An alternative strategy is to combine the inputs and outputs of different periods into one DMU and measure the efficiency of these new DMUs. This study applies an approach presented by Jablonsky [16] which takes DMUs with all their periods as a unit encompassing different periods as follows:

$$\begin{aligned} & Max \;\; \frac{\sum_{t=1}^{T} \theta_o^t}{T} \\ & s.t. \;\; \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq x_{io}^t, \qquad i = 1,2,...,m, t = 1,2,...,T, \\ & \qquad \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq \theta_o^t y_{ro}^t, \qquad r = 1,2,...,s, t = 1,2,...,T, \\ & \qquad \lambda_j^t \geq 0, j = 1,2,...,n, t = 1,2,...,T. \end{aligned} \tag{3}$$

The overall efficiency is identified as the mean of the period efficiencies. The multi-period process o is called overall efficient if and only if $\frac{\sum_{t=1}^{T} \theta_o^{*t}}{T} = 1$. This means that it is efficient in each period. The optimal value of θ_o^{*t} shows the efficiency of DMU_o in period t that $\theta_o^{*t} \geq 1$. If $\theta_o^{*t} = 1$, DMU_o is efficient in period t. Otherwise, it is called period inefficient.

2.3. Inverse DEA

Unlike traditional optimization problems, which calculate optimal decisions concerning goals and constraints, inverse optimization takes decisions as inputs and sets goals and/or constraints that optimise those decisions optimally or precisely. Inverse DEA is a branch of inverse optimization in which changes are made to inputs (outputs) and appropriate outputs (inputs) are sought for those changes.

In this stage, the inverse DEA approach presented by Wei et al. [39] is briefly reviewed. Consider the CCR (Charnes, Cooper and Rhodes) model (4) originally initiated by Charnes et al. [5]:

$$
\begin{aligned}
min \;\; & \theta \\
s.t. \;\; & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, 2, ..., m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, 2, ..., s, \\
& \lambda_j \geq 0, \quad j = 1, 2, ..., n,
\end{aligned} \tag{4}
$$

in which x_{ij} $(i = 1, 2, ..., m, \; j = 1, 2, ..., n)$ shows the i-th input related to the j-th DMU, and y_{rj} $(r = 1, 2, ..., S, \; j = 1, 2, ..., n)$ is the r-th output of the j-th unit. Suppose that the optimal value of model (4) is shown by θ^* and the values are added to Y_o $(\beta_o = Y_o + \Delta Y)$. The purpose is to estimate the new input values α that are greater than X_o so that (α, β) has the same efficiency as θ^*. Thus, the following model can be computed:

$$
\begin{aligned}
min \;\; & (\alpha_{1o}, \alpha_{2o}, ..., \alpha_{mo}) \\
s.t. \;\; & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta^* \alpha_{io}, \quad i = 1, 2, ..., m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \beta_{ro}, \quad r = 1, 2, ..., s, \\
& x_{io} \leq \alpha_{io}, \quad i = 1, 2, ..., m, \\
& \lambda_j \geq 0, \quad j = 1, 2, ..., n.
\end{aligned} \tag{5}
$$

The weighted sum method can be applied to solve problem (5) [33]. An important point in the inverse DEA model is that the efficiency of the other DMUs remains constant after adding the new DMU. In other words, these new DMUs do not alter the efficiency frontier, and the efficiency of these new DMUs is the same as that of the previous DMUs.
In the next section, the discussed models are extended and integrated to analyze the multi-period performance and input changes where the convexity principle is not held.

3. Inverse multi-period FDH model

In this section, the approach presented in [16] is extended to analyze the performance of multi-period processes in the absence of the convexity property, whereas there are undesirable outputs. Suppose there is no connection between periods; also each period has independent inputs and outputs. x^t_{ij} $(j = 1, 2, ..., n, \; i = 1, 2, ..., m)$ are the inputs of DMU j for the period t, y^t_{kj} $(j = 1, 2, ..., n, \; k = 1, 2, ..., K)$ are the desirable outputs of DMU j related to the period t and u^t_{sj} $(j = 1, 2, ..., n, \; s = 1, 2, ..., S)$ show the undesirable outputs of DMU j for the period t. Moreover, it is assumed that the efficiency of DMUs with undesirable outputs should be addressed in T periods, $t = 1, 2, ..., T$. Thus, the following multi-period FDH model can be provided to analyze the performance of multi-period systems under nonconvexity:

$$
\begin{aligned}
& Max\ \frac{\sum_{t=1}^{T}\theta_o^t}{T} && && (6)\\
& s.t.\ \sum_{j=1}^{n}\omega^t\lambda_j^t x_{ij}^t \le x_{io}^t, && i=1,2,...,m, t=1,2,...,T,\\
& \sum_{j=1}^{n}\omega^t\lambda_j^t y_{kj}^t \ge \theta_o^t y_{ko}^t, && k=1,2,...,K, t=1,2,...,T,\\
& \sum_{j=1}^{n}\omega^t\lambda_j^t u_{sj}^t \le u_{so}^t, && s=1,2,...,S, t=1,2,...,T,\\
& \sum_{j=1}^{n}\lambda_j^t = 1, && t=1,2,...,T,\\
& \lambda_j^t \in \{0,1\},\\
& \omega^t \ge 0.
\end{aligned}
$$

Based on [28, 7], the strong disposability property is used to incorporate undesirable outputs. Here, $\omega^t \ge 0$ is because the CRS has been considered and it makes every coefficient of a unit belonging to the PPS. The CRS is contemplated as the scale size of the entity does not affect the efficiency.

Model (6) is a nonlinear programming problem that includes integers. This model can be converted to a mixed integer linear technique with a change in the variables. Therefore, as long as $\Lambda_j^t = \omega^t\lambda_j^t$, model (6) can be substituted by model (7):

$$
\begin{aligned}
& OE^* = Max\ \frac{\sum_{t=1}^{T}\theta_o^t}{T} && && (7)\\
& s.t.\ \sum_{j=1}^{n}\Lambda_j^t x_{ij}^t \le x_{io}^t, && i=1,2,...,m, t=1,2,...,T,\\
& \sum_{j=1}^{n}\Lambda_j^t y_{kj}^t \ge \theta_o^t y_{ko}^t, && k=1,2,...,K, t=1,2,...,T,\\
& \sum_{j=1}^{n}\Lambda_j^t u_{sj}^t \le u_{so}^t, && s=1,2,...,S, t=1,2,...,T,\\
& \sum_{j=1}^{n}\lambda_j^t = 1, && t=1,2,...,T,\\
& 0 \le \Lambda_j^t \le M\lambda_j^t,\\
& \lambda_j^t \in \{0,1\},
\end{aligned}
$$

Definition 1. The unit under consideration, DMU_o, is called overall FDH-efficient if and only if $OE^* = 1$. In model (7), that means it is FDH efficient in each period. Also, it is FDH-efficient in period t if and only if the optimal value θ_o^{*t} is equal to one. Otherwise, it is FDH-inefficient in period t.

The overall OEE coefficient is then obtained using the following equation:

$$OEE_o = \sum_{t=1}^{T}\frac{1}{\theta_o^t}\times Q_o^t \times Ava_o^t, \tag{8}$$

in which $\frac{1}{\theta_o^t}$ is the efficiency of DMU_o in period t, Q_o^t denotes the quality of DMU_o in period t and Ava_o^t shows the availability of DMU_o in period t. Also, the period OEE coefficient can be estimated using the following expression:

$$OEE_o^t = \frac{1}{\theta_o^t} \times Q_o^t \times Ava_o^t, \tag{9}$$

In this stage, an inverse multi-period FDH model is proposed to estimate inputs of multiple periods for the changes in outputs related to several periods when the overall and period efficiency values achieved using model (7) are preserved. Assume β_{ko}^t are the permuted data of y_{ko}^t ($\beta_{ko}^t = y_{ko}^t + \delta_{ko}^t$, and $\delta_{ko}^t \geq 0$) and α_{io}^t are input projections for the period t. In this way, the following multi-objective problem is proposed:

$$\begin{aligned}
& Min\ (\alpha_{1o}^1, \alpha_{2o}^1, ..., \alpha_{mo}^1, ..., \alpha_{1o}^T, \alpha_{2o}^T, ..., \alpha_{mo}^T) && \qquad (10)\\
& s.t.\ \sum_{j=1}^{n} \Lambda_j^t x_{ij}^t \leq \alpha_{io}^t, && i = 1, 2, ..., m, t = 1, 2, ..., T,\\
& \sum_{j=1}^{n} \Lambda_j^t y_{kj}^t \geq \theta_o^{*t} \beta_{ko}^t, && k = 1, 2, ..., K, t = 1, 2, ..., T,\\
& \sum_{j=1}^{n} \Lambda_j^t u_{sj}^t \leq u_{so}^t, && s = 1, 2, ..., S, t = 1, 2, ..., T,\\
& \sum_{j=1}^{n} \lambda_j^t = 1, && t = 1, 2, ..., T,\\
& \lambda_j^t \in \{0, 1\},\\
& \alpha_{io}^t \geq x_{io}^t,\\
& 0 \leq \Lambda_j^t \leq M \lambda_j^t,
\end{aligned}$$

θ_o^{*t} can be obtained by computing model (7). The weighted sum method was employed to solve the above multi-objective programming problem. For further illustration, by solving model (10), the minimum multi-period inputs no less than the current levels are found for the changes of multi-period outputs where the multi-period efficiency levels are preserved.
In the following section, the proposed models are demonstrated using a real application in the automotive industry.

4. A case study of the automotive industry

Regarding its strong ties with other enterprises, the automotive industry has a critical importance for nations. Moreover, the automotive industry of Iran is one of the most active industries. Consequently, the efficacy analysis of this sector is significant for policy makers and managers of to make appropriate decisions. Therefore, a case study of Iran's automotive industry is provided in this section, and the suggested models in this research are applied to address the performance and changes of multi-period inputs. The dataset is related to a pump company with a strategic position in the parts manufacturing industry. Actually, a pump company plays a crucial role in the parts manufacturing industry, especially in the automotive sector. Pump companies are responsible for designing, manufacturing, and supplying various types of pumps. The company, which includes 33 machines, has set its main mission to produce and supply various types of automotive engine parts to supply the domestic market and automotive parts. Inputs, desirable outputs, and undesirable outputs are taken as follows:

Inputs
Total stop time (x_1): Total stop time refers to the time that the manufacturing process was intended to be running but was not due to unplanned (e.g. breakdowns) or planned (e.g. changeovers) stops.
Total maintenance time (x_2): Total maintenance time is defined as the time devoted to the maintenance of machines or equipment. It includes all maintenance activities, such as inspections, repairs, replacements, and preventive measures that are performed throughout a specific period.
Average time between failures (x_3): It describes the expected time between failures and stoppages.
Average time between repairs and recovery (x_4): The average time spent to repair a system or piece of equipment and restore it to full functionality after a breakdown.
Desirable outputs
Total time available per month (y_1): The time that a machine is available for production or operation within a given month.
Operating time (y_2): Operating time is defined as the total time that the equipment has been operating since its first commissioning.
Product quality (y_3): Quality considers defects (including parts that need rework). A quality score of 100% implies no defects (only good parts are being produced).
Total number of production parts (y_4): All products are produced.
Planned production quantity (y_5):Planned production quantity is the number scheduled to be produced.
Machine life (y_6): The number of years a device can operate.
Availability (y_7): Availability refers to the percentage of time that the machine is capable of operating, without being affected by any downtime or unplanned interruptions. An availability score of 100% means that the process is always running during planned production time.
Percentage of healthy parts (y_8): Percentage of healthy parts is the ratio of healthy parts to total parts.
Undesirable outputs
Number of machine breakdowns (u_1): The total number of breakdowns in machines.
Total number of defective parts (u_2): The total number of defective parts produced.
The dataset is related to 33 machines as DMUs with four inputs, eight desirable outputs, and two undesirable outputs that are examined in 6 months (periods) of 2021. Multi-period efficiency is achieved when a DMU can produce the desired level of outputs considering the minimum amount of inputs and undesirable outputs. By incorporating these inputs into the DEA analysis, the performance of machines is evaluated on the basis of their ability to minimize stop time and maintenance time, as well as their track record in terms of average time between failures and repairs. Desirable outputs refer to the desired or intended results or produced goods by a DMU. The listed desirable outputs are relevant in evaluating the performance of machines in the automotive industry, indicating productivity and use. However, DEA also considers the concept of undesirable outputs, which are the outputs that a firm wants to minimize. Including these undesirable outputs in the evaluation can help identify machines that produce fewer breakdowns and defective parts, which is desirable in the automotive industry. The DEA is aimed at evaluating the relative efficiency of different DMUs by comparing their inputs to outputs. The ideal scenario is to achieve a high level of efficiency, where DMUs utilize their inputs effectively to maximize desirable outputs while minimizing undesirable ones. By identifying inefficient DMUs, DEA provides insights to improve resource allocation, decision-making processes, and overall performance within an organization. The related statistical data can be found in Table 1. For a better understanding of the data and their relationships, the correlations between the inputs and outputs are listed in Table 2.

Period	Statistical Index	Inputs				Desirable outputs								Undesirable outputs	
		x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	u_1	u_2
1	Mean	277.30	268.79	5586.64	97.79	14970.91	9190.61	99.48	3805.27	3803.03	10.00	61.09	99.47	279.12	242.42
	Var	25551.22	89705.48	6007476.68	3504.61	2757627.27	3599788.68	4.01	4694015.45	4733896.78	0	59.48	4.00	45765.05	45010.56
	Min	24.00	1.00	1284.00	17.00	12960.00	6290.00	88.70	550.00	500.00	10.00	48.53	88.70	1.00	1.00
	Max	900.00	1234.00	10684.00	230.00	19440.00	14560.00	100.00	9175.00	9200.00	10.00	78.70	100.00	615.00	586.00
2	Mean	1548.27	1690.82	17557.97	270.42	29847.27	20478.06	99.55	8868.06	8766.67	10.00	68.47	98.35	118.03	87.67
	Var	1091463.89	933861.40	138222082.47	19204.13	36582545.45	23547861.93	1.01	23234766.93	24249010.42	0	54.01	51.66	13047.53	8377.17
	Min	153.00	176.00	41.00	49.00	25920.00	12960.00	95.48	969.00	950.00	10.00	50.00	58.70	1.00	1.00
	Max	4147.00	3430.00	35816.00	558.00	38880.00	31650.00	100.00	21736.00	21700.00	10.00	81.40	100.00	298.00	272.00
3	Mean	1740.00	1366.70	11233.67	218.21	21403.64	14218.85	99.13	8160.03	8154.55	10.00	252.31	99.13	334.48	259.73
	Var	5726609.00	2940063.47	75501144.29	14096.05	9145636.36	7327936.88	8.40	17455590.16	17413650.57	0	1140068.19	8.44	102032.20	69826.14
	Min	17.00	53.00	70.00	26.00	19440.00	11520.00	83.54	1160.00	1200.00	10.00	57.27	83.50	2.00	1.00
	Max	8833.00	6632.00	32218.00	497.00	25920.00	19452.00	100.00	18142.00	18100.00	10.00	6200.00	100.00	916.00	789.00
4	Mean	878.12	815.36	13642.82	110.06	25920.00	18739.09	99.09	9399.52	9237.88	10.00	72.30	99.08	271.58	301.15
	Var	914513.67	1004636.86	76976027.78	7039.68	0	1291038.21	3.77	27249982.38	22790160.98	0	19.22	3.78	61561.06	71930.07
	Min	26.00	3.00	803.00	21.00	25920.00	15920.00	92.61	2121.00	2100.00	10.00	61.42	92.60	2.00	1.00
	Max	4860.00	5503.00	28567.00	502.00	25920.00	20440.00	100.00	26086.00	21700.00	10.00	78.86	100.00	740.00	694.00
5	Mean	715.36	720.48	2297.39	789.67	21403.64	14452.42	98.74	7325.30	7209.09	10.00	67.30	98.74	345.61	420.33
	Var	327607.93	321847.76	5354161.37	4202628.85	9145636.36	7502589.00	10.41	16219413.59	14188977.27	0	40.72	10.39	148419.93	155308.42
	Min	4.00	27.00	324.00	33.00	19440.00	9040.00	84.56	1201.00	1200.00	10.00	46.50	84.60	2.00	1.00
	Max	1918.00	1908.00	9659.00	9972.00	25920.00	19320.00	100.00	17938.00	18000.00	10.00	79.84	100.00	1159.00	1130.00
6	Mean	346.70	429.91	11088.55	187.36	21403.64	15057.15	99.54	6714.76	6710.61	10.00	70.30	99.54	192.85	138.27
	Var	59752.53	87781.21	35076425.57	13443.11	9145636.36	6215331.51	1.45	11683751.13	11651212.12	0	30.82	1.47	18019.82	18186.77
	Min	41.00	24.00	136.00	1.00	19440.00	12000.00	93.94	1257.00	1250.00	10.00	61.73	93.90	10.00	1.00
	Max	869.00	1123.00	22598.00	414.00	25920.00	20130.00	100.00	15563.00	15500.00	10.00	80.04	100.00	387.00	387.00

Table 1: *Descriptive statistics of data*

	X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	U_1	U_2
X_1	1.0000	0.7917	-0.0984	0.2315	0.1674	0.1039	-0.1302	0.1972	0.1853	0.0353	-0.2110	-0.1085	0.1142	-0.0326
X_2	0.7917	1.0000	-0.0064	0.4364	0.0183	-0.0442	-0.0759	0.1389	0.1258	-0.0357	-0.3008	-0.0572	0.0320	-0.0355
X_3	-0.0984	-0.0064	1.0000	0.2523	-0.2211	-0.1897	0.2687	-0.2624	-0.2548	-0.0848	-0.0681	0.2536	-0.1146	0.1662
X_4	0.2315	0.4364	0.2523	1.0000	-0.2550	-0.3475	0.1969	0.1102	0.1014	-0.1419	-0.3670	0.1731	0.0340	0.1154
Y_1	0.1674	0.0183	-0.2211	-0.2550	1.0000	0.7280	-0.3892	0.0149	0.0142	0.0436	0.1160	-0.3615	0.0108	-0.2321
Y_1	0.1039	-0.0442	-0.1897	-0.3475	0.7280	1.0000	-0.3693	-0.0263	-0.0104	0.0484	0.5614	-0.3582	0.0269	-0.3780
Y_1	-0.1302	-0.0759	0.2687	0.1969	-0.3892	-0.3693	1.0000	0.0752	0.0766	-0.0287	-0.0915	0.8500	-0.3116	0.3510
Y_2	0.1972	0.1389	-0.2624	0.1102	0.0149	-0.0263	0.0752	1.0000	0.9947	-0.0437	-0.0850	0.0861	-0.0029	0.0002
Y_3	0.1853	0.1258	-0.2548	0.1014	0.0142	-0.0104	0.0766	0.9947	1.0000	-0.0448	-0.0570	0.1045	-0.0105	-0.0036
Y_4	0.0353	-0.0357	-0.0848	-0.1419	0.0436	0.0484	-0.0287	-0.0437	-0.0448	1.0000	0.0012	-0.0311	0.0114	-0.0955
Y_5	-0.2110	-0.3008	-0.0681	-0.3670	0.1160	0.5614	-0.0915	-0.0850	-0.0570	0.0012	1.0000	-0.0863	-0.0057	-0.1336
Y_6	-0.1085	-0.0572	0.2536	0.1731	-0.3615	-0.3582	0.8500	0.0861	0.1045	-0.0311	-0.0863	1.0000	-0.2612	0.3428
U_1	0.1142	0.0320	-0.1146	0.0340	0.0108	0.0269	-0.3116	-0.0029	-0.0105	0.0114	-0.0057	-0.2612	1.0000	0.0317
U_2	-0.0326	-0.0355	0.1662	0.1154	-0.2321	-0.3780	0.3510	0.0002	-0.0036	-0.0955	-0.1336	0.3428	0.0317	1.0000

Table 2: *Correlation between data.*

Notice that non-convex DEA models, including FDH, offer a more flexible and accurate approach for efficiency analysis, accommodating the complexities and relationships in real-world production systems. As convex combinations of observations may not exist in the technology, FDH-based approaches have been applied. Accordingly, model (7) is used to measure the relative efficiency of multi-period systems. The outcomes are given in Table 3. The efficiency of each period is expressed separately, and the overall efficiency is shown in column 8. As the model is output-oriented, the inverse values of optimal objective are expressed as overall and period efficiencies. As seen, only DMU 18 is inefficient in period 1. For periods 2-6, approximately 24%, 18%, 9%, 12%, and 27% of DMUs are inefficient. Furthermore, 15 DMUs are totally efficient with the score of one. DMU 29 with an efficiency score of 0.53 is the most inefficient unit, generally. Furthermore, it is determined to be inefficient in periods 2, 3, 4, and 6. Policymakers of inefficient units should revisit their performance and make more attempts to improve efficiency by taking appropriate decisions.

Table 3 also presents the multi-period efficiency of the CCR model for comparison with the efficiency of the multi-period FDH model. The Malmquist index (MI) was also determined for different periods under examination, and their average is presented in the last column of Table 3. Despite the widespread use of the Malmquist DEA index [38], this approach suffers from some drawbacks. Specifically, an aggregated efficiency value of multiple periods is not considered in this approach [35]. As can be seen, the efficiency scores achieved from the proposed approach are different from the multi-period CCR and the Malmquist DEA index in some DMUs.

For a more accurate assessment of DMUs, overall and period OEEs are measured using the statements (8) and (9) as presented in Table 4. The product quality of the machines in different periods and the availability time are also provided in Appendix A.

The obtained OEE coefficients are shown in Table 4.

DMU	Efficiency								
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Total	Multi-period CCR	MI average
1	1	1	1	1	1	1	1	1	1.23
2	1	0.55	1	1	1	1	0.88	0.80	0.97
3	1	1	1	0.68	1	1	0.93	0.93	0.91
4	1	0.86	1	1	1	0.43	0.80	0.75	0.95
5	1	1	1	1	1	1	1	1	0.94
6	1	1	1	1	1	1	1	1	1.19
7	1	1	1	1	1	1	1	0.93	1.01
8	1	1	1	1	1	1	1	1	1.20
9	1	1	0.38	1	1	1	0.79	0.79	0.82
10	1	1	0.30	1	1	1	0.72	0.69	0.73
11	1	1	1	1	1	1	1	1	1.01
12	1	1	1	1	1	1	1	0.95	0.96
13	1	1	1	1	1	1	1	0.89	1.09
14	1	1	1	1	1	1	1	1	1.13
15	1	0.98	1	1	1	1	1	1	1.15
16	1	1	0.37	1	1	1	0.78	0.63	0.81
17	1	1	1	1	1	1	1	1	1.30
18	0.95	1	1	0.86	1	0.85	0.94	0.94	0.89
19	1	0.72	1	1	1	1	0.94	0.90	0.98
20	1	0.60	0.67	1	0.77	1	0.80	0.78	0.82
21	1	1	1	1	1	1	1	0.98	1.03
22	1	1	1	1	1	0.72	0.94	0.94	1.08
23	1	1	1	1	1	0.96	0.99	0.95	1.04
24	1	1	1	1	1	1	1	1	1.09
25	1	1	1	1	1	1	1	1	1.12
26	1	1	1	1	1	0.66	0.92	0.78	0.92
27	1	1	1	1	0.86	1	0.97	0.92	0.91
28	1	1	0.47	1	0.90	0.51	0.73	0.63	0.78
29	1	0.66	0.49	0.35	1	0.35	0.53	0.42	0.53
30	1	1	1	1	0.58	1	0.89	0.85	0.81
31	1	0.71	1	1	1	0.91	0.92	0.92	1.02
32	1	0.98	1	1	1	0.65	0.92	0.80	0.91
33	1	1	1	1	1	1	1	1	1.30

Table 3: *Efficiency scores.*

DMU	OEE						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Total
1	5760.00	7301.83	6173.00	7315.00	6420.00	7387.00	40356.83
2	5493.00	2747.25	6420.00	7315.00	6411.65	7469.00	35855.91
3	7360.00	7285.86	7202.00	4178.23	7984.00	6461.00	40471.09
4	5867.00	5507.76	8092.21	7639.00	6298.61	3059.83	36464.41
5	7485.51	6637.03	7285.10	6814.35	6294.29	7199.99	41716.27
6	6452.04	7206.19	7197.54	7072.77	5908.21	6565.47	40402.22
7	7139.33	6270.69	6215.00	7080.30	6602.81	6630.34	39938.47
8	5937.42	6624.72	7351.53	7075.45	7148.68	7308.97	41446.76
9	5280.00	7315.00	3153.26	6743.33	6173.00	6420.00	35084.58
10	6667.00	7701.00	2501.52	7701.00	6420.00	7418.00	38408.52
11	7865.28	6508.28	5727.00	6634.86	5975.22	6945.08	39655.71
12	5787.00	6350.37	5627.25	6985.44	6739.00	7482.84	38971.90
13	4840.38	6734.23	6359.63	7546.13	6889.58	6743.63	39113.59
14	5707.43	8096.86	6998.87	6495.67	7339.95	7733.38	42372.16
15	5493.00	6990.20	5918.30	7060.00	6402.67	6173.00	38037.16
16	7253.00	6504.84	2647.79	7884.42	4648.14	6173.00	35111.19
17	6636.00	7073.00	5806.00	6620.00	6890.00	6427.00	39452.00
18	5231.43	6929.00	6155.72	5481.03	6173.00	6383.76	36353.94
19	5760.00	4263.31	6170.53	7099.00	6924.40	7490.00	37707.24
20	5120.00	3012.05	3950.67	7701.00	4938.46	6173.00	30895.18
21	6343.60	7182.13	7389.91	7616.14	7235.62	6751.10	42518.50
22	5120.00	7511.24	6173.00	6233.52	6934.00	4473.19	36444.95
23	6280.00	7315.00	6156.95	7315.00	7508.50	7181.73	41757.18
24	5120.00	7309.88	6173.00	7515.00	6934.00	6955.00	40006.88
25	5567.00	7701.00	6164.98	7506.73	6420.00	7984.00	41343.71
26	5763.00	6494.40	5926.00	7562.00	7536.11	4687.42	37968.93
27	6133.00	7315.00	6168.06	7785.00	5410.34	7572.00	40383.41
28	5520.00	6998.29	3307.48	7840.00	6413.51	3613.71	33692.99
29	5867.00	4026.49	3264.04	2559.51	7160.00	2386.11	25263.15
30	6480.00	7711.03	6185.74	7125.01	3732.99	7510.00	38744.76
31	6347.00	4071.63	6173.00	7315.00	6955.00	5836.36	36697.99
32	6347.00	6353.92	6173.00	6739.28	6955.00	4034.64	36602.84
33	6133.00	7083.00	6165.59	7330.00	5662.53	8004.00	40378.12

Table 4: *OEE coefficients.*

According to Table 4, generally, DMU 29 has the worst situation while DMU 21 has the best situation. Interestingly, the quality of DMU 21 is worse than DMU 29 in all periods. However, because OEE is made of several factors, it makes DMU 21 better than DMU 29. Another attractive issue is the situation of DMU 29, which has been determined as the most overall inefficient unit based on model (7) and has the least overall productivity level utilizing the expression (8). Moreover, considering the period OEE level of DMUs as the productivity measure in periods, DMU 11, DMU 14, DMU 4, DMU 16, DMU 3, and DMU 33 have the best conditions in comparison with other DMUs in periods 1-6, respectively. DMUs 13, 2, 10, 29, 30, and 29 also had the worst situations in these periods. Thus, more attention should be paid by the managers of these DMUs.

After efficacy assessment using the multi-period FDH model and examination of the OEEs, model (10) is computed to estimate inputs in several periods. To solve the inverse multi-period FDH in model (10), the weighted sum method is used assuming equal weights. We add 10 percent to the desirable output values and then measure the input changes employing model (10). As can be seen, there are four inputs in each period, resulting in 24 new inputs. Input changes related to the first period are given in Table 5, and those of other periods are presented in Appendix B. Accordingly, the multi-period inputs increased or remained unchanged for this increase of outputs. Considering DMU 28, 10% increase of desirable outputs led to the increment of only the second input i.e. total maintenance time with change of 29.87 in period 1. Also, in periods 2 and 5, all inputs of DMU 28 expanded, while no input change can be seen in the period 3. The fourth (average time between repairs) and the first (total stop time) inputs of DMU 28 increased in periods 4 and 6, respectively. Similarly, the multi-period input changes of other DMUs can be investigated.

DMU	$\alpha_{1o}^{1}-x_{1o}^{1}$	$\alpha_{2o}^{1}-x_{2o}^{1}$	$\alpha_{3o}^{1}-x_{3o}^{1}$	$\alpha_{4o}^{1}-x_{4o}^{1}$
1	221.54	43.73	0	48.35
2	27	1.90	654.90	10
3	30.50	18.90	427.80	2
4	2.97	99.87	0	0
5	55.70	77.60	291.20	11
6	90	90.70	128.40	8.20
7	48.40	18.90	232.30	8
8	23.10	36	390.40	6.80
9	49.10	23.90	480.40	1.70
10	14.43	112.98	0	6.90
11	3	14.20	1019.90	2.90
12	11.90	34.50	335.50	11.70
13	23.10	123.40	514	23
14	26.50	17.50	150.40	5.20
15	0	23.87	0	0
16	44.80	92.90	137.40	20.70
17	144.95	79.27	58.37	10.35
18	0	25	0	197
19	0	93.82	0	4.80
20	254	529	684	217
21	15	11.30	419.90	7.40
22	0	122.87	0	1.21
23	0	142.87	0	9.21
24	229.54	87.73	0	0
25	0	128.87	0	5.21
26	35.38	0	0	44.20
27	18	39.70	920	17.90
28	0	29.87	0	0
29	0	55.87	0	13.21
30	327.54	0	0	61.35
31	81.03	28.70	0	55.41
32	0	62.18	0	7
33	159.62	55.26	0	0

Table 5: *Input changes related to period 1.*

5. Conclusion

A multi-period FDH model with undesirable outputs was presented in this study to analyze the multi-period efficiency of processes while violating the convexity assumption. Along with the product of availability, performance (efficiency), and quality for different periods, the overall and period OEE coefficients were also appraised. Then, the inverse multi-period FDH model was proposed to assess multi-period inputs for the modifications of desirable outputs over several periods of time. The introduced approaches were utilized in a case study of the automotive industry. The results showed that the applicability of the models for analyzing the multi-period efficiency of systems and the multi-period input changes while the convexity assumption is not held. The methods were presented in such a way that there is no connection between periods. The extension of the proposed models dealing with interconnections of periods and in the presence of imprecise measures could be an interesting topic for further analyses. Furthermore, inverse dynamic network FDH models can be developed to assess inputs (outputs) in dynamic network processes while the convexity assumption is violated. It should be mentioned that this is a primary study to estimate the input measures of multiple periods with an inverse multi-period FDH model with undesirable outputs. Further investigations are recommended to more discriminate the performance of periods.

6. Appendix

Appendix materials are available at: 10.6084/m9.figshare.24850746.

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