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Solving multi level multi-objective linear programming (ML_MOLP) problems with fuzzy parameters (FPs)

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Abstract. Making numerous business decisions in complex decentralized organizations can be presented as a ML_MOLP problem with FPs expressed as triangular fuzzy numbers. This paper presents a methodology that uses several multi-objective programming methods to solve this problem. The Iskander's method was used for defuzzification of the objective functions and constraints of the problem and a multi-objective programming method based on the cooperation among decision makers was used to determine the aspired values of the variables controlled by the decision-makers and to obtain the preferred non-dominated solution of the entire problem. The efficiency of the proposed methodology was tested on an example of production planning in a complex decentralized company.

Keywords: decision-making, fuzzy parameters, multi-level, multi-objective linear programming

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1. Introduction

Decision-making in complex hierarchical business systems is organized in several levels. At each level there is one or more decision-makers (DMs), who may have one or more goals to achieve. Decisions are made hierarchically. DMs of the first (highest) level decide first making strategic decisions. They influence the level of achievement of the goals of DMs at lower levels. Also, the degree of achievement of the goals of DMs at the first level depends on the achievement of the goals of DMs at the lower levels. When decisions are made at the first level, the DMs at the second (lower) level make their decisions. Here, too, the degree of achievement of the goals of the DMs depends on the decisions of the higher level, and their decisions also affect the degree of achievement of the goals of the DMs at the higher level. The decision-making on the successive, lower levels follows the same pattern. Thus, when making decisions, DMs at the first level must consider the goals of the DMs at the second level, DMs at the second level must consider the goals of the DMs at the first and third levels, etc.

If the objective functions and constraints can be represented by linear functions, then we have a ML_MOLP problem. If the parameters of the objective functions and/or constraints are expressed by fuzzy numbers, then it is a ML_MOLP problem with FPs. The analytical

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solution of this problem belongs to the strong NP hard problems [25]. If we start from the assumption that all DMs are interested in the efficient functioning of the entire business system and that they are prepared to cooperate to achieve a compromise solution when making decisions, then the ML_MOLP problem with FPs can be presented as a complex MOLP problem with FPs with several DMs. Since it is a complex MOLP problem in which, in addition, the parameters of the objective functions and/or constraint are expressed by fuzzy numbers, an appropriate methodology should be applied to solve this problem. The applied methodology should be efficient both from the point of view of the DMs and from the point of view of the analyst. It is important for DMs that the methodology used for preparing decisions is simple and understandable so that they have confidence in the solution it provides. Also, it is not acceptable for DMs to give answers to many questions which they find difficult. It is important for the analyst that they do not have to invest great efforts in preparing and solving the model and that the methodology applied provides a preferred solution acceptable to all DMs in a final number of steps, without going in a circle.

Numerous methodologies have been applied to solve the ML_MOLP problem. The proposed methodologies have certain shortcomings both from the point of view of the DM and from the point of view of the analyst. Shih et al. in [24] proposed a new concept to solve decentralized multi-level programming problems and a supervised search procedure (supervised by the top DM) to generate a nondominated satisfactory solution for the problem with fixed parameters in objective functions and constraints. The proposed procedure first forms linear fuzzy membership functions for all objective functions. Based on this, it then forms a linear programming model for obtaining a non-dominated solution, which can later be improved by forming new membership functions and solving a new linear programming problem. The procedure is repeated until a non-dominated solution is obtained, one with which all DMs are satisfied. The proposed procedure does not include the formation of membership functions for control variables, nor does it foresee the determination of a non-dominated solution by decision levels. It requires DMs to know the lower and upper bounds for all objective functions, which is not easy to ensure. Sinha in [26] suggests an improvement on the methodology proposed in [24]. The proposed methodology is also based on fuzzy linear programming where the ML_MOLP problem is solved in stages, by solving a series of linear programming problems with the active participation of DMs. But the procedure is intended to solve the ML_MOLP problem with one objective function at each decision level.

Wang et al. in [27] propose a new method based on the two-stage simplex algorithm with dominance trees for solving ML_MOLP problems in decentralized organizations with fixed parameters in objective functions and constraints. The proposed algorithm, however, provides an optimal solution to the formed model but it does not actively include the DMs in the solution process.

Shih in [23] proposes an interactive approach only to solve bi-level (BL) linear programming problems in a fuzzy environment, while Shi and Xia in [22] have developed an interactive algorithm for solving the BL_MOLP model with one objective function at the upper decision level and two or more objective functions at the lower decision level. Ahlatcioglu and Tiryaki in [1] presented two new fuzzy programming approaches to solve a decentralized two-level linear fractional programming problem with a single DM at the upper level and multiple DMs at the lower level, while Mishra in [14] uses the weighting method to solve the BL linear fractional programming problem.

Pramanik and Roy in [21] propose a fuzzy goal programming approach to solve multilevel programming problems with fixed parameters in objective functions and constraints. The proposed procedure is based on the minimization of negative deviation variables and the possible relaxation of decisions at the higher levels. Baky in [2] presents two new algorithms based on the fuzzy linear programming to solve decentralized BL multi-objective programming problems with a single DM at the upper level and multiple DMs at the lower levels, while Baky in [3] proposes two new algorithms to solve ML_MOLP problems with fixed parameters in objective functions and constraints by using the fuzzy goal programming method.

Lachhwani and Poonia in [11] use the fuzzy goal programming method to solve ML fractional programming problems in a large hierarchical decentralized system. Osman et al. in [16] propose a new methodology to solve ML_MOLP problems. The proposed methodology is based on converting the hierarchical system into a scalar optimization problem by finding proper weights using the AHP method, while Emam in [7] presents an interactive algorithm based on using the Charnes and Cooper transformation and ϵ-constraint method to a BL integer multi-objective fractional programming problem.

Lachhwani in [9] proposes a new technique based on the fuzzy goal programming method to solve ML_MOLP problems, while in [10] he proposes a modification of Baky's approach to solve multi-level multi-objective linear fractional programming (ML_MOLFP) problems. The proposed methodology is based on the fuzzy goal programming method.

Chen and Chen in [4] present a two-phase fuzzy goal programming approach to solve ML linear programming problems with fuzzy parameters in objective functions. Osman et al. in [17] propose a new approach for solving the ML multi-objective fractional programming problems with rough intervals of the coefficients in the objective functions. Dalman in [5] presents an interactive fuzzy methodology to solve two-level linear fractional programming problem with a single DM at the upper level and multiple DMs at the lower level. Perić et al. in [18] propose a new fuzzy goal programming methodology to solve BL multi-objective linear programming problems with fixed parameters in objective functions and constraints while Perić et al. in [20] present a methodology for solving ML_MOLFP problems also with fixed parameters.

Kaci and Radjef [12] developed a new method for solving the ML_MOLP problem by first determining the set of all compromise solutions without taking into account the hierarchy of the problem, and then applying a simple criterion to test whether a given compromise solution is preferred or not. In this paper, a new methodology, which includes the application of three operational research methods to solve the ML_MOLP problem with FPs, is proposed. Iskander's approach [8] was applied to defuzify the parameters of the objective functions and constraints of the model, the simplex method was applied to determine the marginal solutions (separate maximization of the objective functions on a given set of constraints), and the MP method proposed in [13] was applied both to solve the MOLP model by decision levels and to solve the overall ML_MOLP of the model to obtain the preferred solution. The main contribution of the paper is in the construction and testing of a new methodology for solving ML_MOLP problems in which the parameters in the objective functions and constraints can be represented as triangular fuzzy numbers. The proposed methodology is easy to use for both analysts and DMs, the problem-solving process takes place through the interaction between the analyst and the DMs, where the DMs provides the information on acceptable value of the objective functions and variables they control, the methodology provides the preferred solution in a finite number of steps, each new iteration leads to improvements in the compromise solution and finally, the applied methods are by their nature simple, so they should be comprehensible to DMs, thus ensuring a high degree of trust in the solutions they provide.

Apart from the introduction, the rest of the paper contains three chapters, conclusion and a list of references. Chapter 2 presents the ML_MOLP model while Chapter 3 discusses the proposed methodology for solving it. In chapter 4, a hypothetical example of the application of the proposed methodology for solving the ML_MOLP problem with fuzzy parameters in a complex business system is presented.

2. ML_MOLP model with fuzzy parameters expressed in the form of triangular fuzzy numbers

Consider a ML_MOLP model with the fuzzy parameters expressed in the form of triangular fuzzy numbers in objective functions and constraints at all levels. Let DMs_l be the decision-makers at level l who control the decision variables $x_l = (x_{l1}, x_{l2}, \ldots, x_{ln_l}) \in \mathbb{R}^{n_l}, l = 1, 2, \ldots, L$ where $x = (x_1, x_2, \ldots, x_L) \in \mathbb{R}^n, n = n_1 + n_2 + \cdots + n_L$. Let us also assume that $\widetilde{f_l}(x), l = 1, \ldots, L$ are the objective function vectors at level l.

The fuzzy coefficients ML_MOLP model can be presented as follows ([3, 20]):

$$\text{Level 1 :} \qquad max(\widetilde{f_{11}}(x), \widetilde{f_{12}}(x), \ldots, \widetilde{f_{1K_1}}(x)) \tag{1}$$

where DMs_1 control variables $x_1 = (x_{11}, x_{12}, \ldots, x_{1n_1})$.

$$\text{Level 2 :} \qquad max(\widetilde{f_{21}}(x), \widetilde{f_{22}}(x), \ldots, \widetilde{f_{2K_2}}(x))$$

where DMs_2 control variables $x_2 = (x_{21}, x_{22}, \ldots, x_{2n_2})$.

$$\vdots$$

$$\text{Level L :} \qquad max(\widetilde{f_{L1}}(x), \widetilde{f_{L2}}(x), \ldots, \widetilde{f_{LK_L}}(x))$$

where DMs_L control variables $x_L = (x_{L1}, x_{L2}, \ldots, x_{Ln_L})$.
subject to

$$\tilde{A}x \leq \tilde{b}, x \geq 0,$$

where $x, 0 \in \mathbb{R}^n, \tilde{A} \in \mathbb{R}^{m\times n}, \tilde{b} \in \mathbb{R}^m$. The objective functions are defined as $\widetilde{f_{lk}} = \widetilde{c_{lk}}x_1 + \widetilde{c_{lk}}x_2 + \cdots + \widetilde{c_{lk}}x_L, l = 1, 2, \ldots, L, k = 1, 2, \ldots, K_l, \widetilde{c_{lk}} = (\widetilde{c_{lk1}}, \widetilde{c_{lk2}}, \ldots, \widetilde{c_{lkn_l}})$. The component of $\tilde{c}, \tilde{A}$, and $\tilde{b}$ are triangular fuzzy numbers.

In complex decentralized business systems, the objective functions are by their very nature disproportionate, and may even be in conflict. Therefore, the efficient functioning of such systems requires the cooperation of DMs when making important decisions both within decision-making levels and between them. To solve the model (1) it is necessary to apply an appropriate methodology that includes the application of different methods of multicriteria analysis.

3. Methodology to solve ML_MOLP problem with fuzzy parameters

We propose an interactive methodology, which includes the active participation of DMs and the application of several different methods of multi-criteria analysis for solving ML_MOLP problems. The proposed methodology consists of several steps. First, fuzzy parameters in all objective functions and model constraints are defuzzified, the marginal solutions are determined on a given set of constraints, and the pay-off table is formed. Then the preferred non-dominated solution of the MOLP problem by decision-making levels is determined in order to provide DMs with a basis for determining the lower limit and aspirational values of the variables they control. Here DMs also determine the initial aspirational values of the objective functions based on the objective function values from the pay-off table. In the third step, the MOLP model of the entire ML_MOLP problem is formed for solving with the MP method, and the preferred solution is determined in interaction with the DMs.

3.1. Defuzzification of the ML_MOLP model with FPs and the marginal solutions determination

In order to be able to solve the ML_MOLP model with fuzzy parameters expressed in the form of triangular fuzzy numbers, it is necessary to defuzzify all fuzzy parameters in the objective functions and constraints.

A triangular fuzzy number (TFN) is defined as $\tilde{a} = (a_1, a_2, a_3)$, where $a_1 \le a_2 \le a_3$. The triangular-shaped membership function is $\mu_{\tilde{\tilde{a}}}(a;\theta), \theta \in \langle 0, 1]$, where θ is the maximal value of the membership function (when $a = a_2$). Accordingly,

$$\mu_{\tilde{\tilde{a}}}(a;\theta) = \begin{cases} 0 & \text{if} \quad a < a_1, \text{ or } a > a_3 \\ \dfrac{(a-a_1)\theta}{a_2 - a_1} & \text{if} \quad a_1 \le a \le a_2 \\ \theta & \text{if} \quad a = a_2 \\ \dfrac{(a_2 - a)\theta}{a_3 - a_2} & \text{if} \quad a_2 \le a \le a_3 \end{cases} \tag{2}$$

where $\mu_{\tilde{\tilde{a}}}(a;\theta) \in [0, \theta]$.

Problem (1) can be simplified in the form

$$\begin{aligned} max\, F_{lk} = \textstyle\sum_{j=1}^{n} \widetilde{c_{lkj}} x_j \,;\, l = 1, 2, \ldots, L;\, k = 1, 2, \ldots, K_l \\ \text{s.t. } \textstyle\sum_{j=1}^{n} \widetilde{a_{ij}} x_j \le \widetilde{b_i};\, x_j \ge 0;\, j = 1, 2, \ldots, n, \end{aligned} \tag{3}$$

where $\widetilde{c_{lkj}}, \widetilde{a_{ij}}$ and $\widetilde{b_i}$ are $TFNs$ [15].

Therefore, let $\widetilde{c_{lkj}} = (c^1_{lkj}, c^2_{lkj}, c^3_{lkj}), \widetilde{a_i j} = (a^1_{ij}, a^2_{ij}, a^3_{ij}), \widetilde{b_i} = (b^1_i, b^2_i, b^3_i)$ be $TFNs$. By definition of the strict exceedance possibility [6], the possibility that x belongs to a feasible constraint i ($Poss(x \in F_i)$) is defined as follows [8]:

$$Poss(x \in F_i) = \begin{cases} \theta & \text{if} \quad \displaystyle\sum_{j=1}^{n} a^3_{ij} x_j \le b^2_i \\ \delta_i & \text{if} \quad \displaystyle\sum_{j=1}^{n} a^3_{ij} x_j \le b^3_i,\ \sum_{j=1}^{n} a^3_{ij} x_j \ge b^2_i \\ 0 & \text{if} \quad \displaystyle\sum_{j=1}^{n} a^2_{ij} x_j \le b^3_i \end{cases}$$

where

$$\delta_i = \frac{(b^3_i - \sum_{j=1}^{n} a^2_{ij} x_j)\theta}{b^3_i - b^2_i}.$$

Also, according to $TFNs$, the possibility that the objective function F_{lk} is equal to any value of f_{lk} can be presented as

$$Poss(F_{lk} = f_{lk}) = \begin{cases} \dfrac{(f_{lk} - \sum_{j=1}^{n} c^1_{lkj} x_j)\theta}{\sum_{j=1}^{n} c^2_{lkj} x_j - \sum_{j=1}^{n} c^1_{lkj} x_j)} & \text{if} \quad \displaystyle\sum_{j=1}^{n} c^1_{lkj} x_j \le f_{lk} \le \sum_{j=1}^{n} c^2_{lkj} x_j \\ \theta & \text{if} \quad f_{lk} = \displaystyle\sum_{j=1}^{n} c^2_{lkj} x_j \\ \dfrac{(\sum_{j=1}^{n} c^3_{lkj} x_j - f_{lk})\theta}{\sum_{j=1}^{n} c^3_{lkj} x_j - \sum_{j=1}^{n} c^2_{lkj} x_j)} & \text{if} \quad \displaystyle\sum_{j=1}^{n} c^2_{lkj} x_j \le f_{lk} \le \sum_{j=1}^{n} c^3_{lkj} x_j \end{cases}$$

Therefore, using the α - level approach for any given value of θ, in which the value of α is determined by the DM, the equivalent *crisp problem* can be presented as follows [8]:

$$\begin{aligned} & max \textstyle\sum_{j=1}^{n} \big(c_{lkj}^3 - \frac{\alpha}{\theta} c_{lkj}^3 + \frac{\alpha}{\theta} c_{lkj}^2\big) x_j,\ l = 1, 2, \ldots, L;\ k = 1, 2, \ldots, K_l \\ & \text{s.t. } \textstyle\sum_{j=1}^{n} \big(\frac{\theta}{\alpha} a_{ij}^2 - a_{ij}^2 + a_{ij}^3\big) x_j \le \big(\frac{\theta}{\alpha} - 1\big) b_i^3 + b_i^2,\ i = 1, 2, \ldots, m \\ & \sum_{j=1}^{n} a_{ij}^2 x_j \le b_i^3\,,\ i = 1, 2, \ldots, m \\ & x_j \ge 0\,,\ j = 1, 2, \ldots, n. \end{aligned} \tag{4}$$

Model (4) is the MOLP problem. By individual maximization of the objective functions of the model (4) on the given set of constraints, we obtain marginal (optimal) solutions for the given values of α and θ. Then we form the pay-off table for the objective functions of the model (4).

3.2. Determining the lower bound and the aspirational value of the variables controlled by DMs at the decision making levels

To help DMs determine the lower bound and the aspirational value of the variables they control, we propose applying the MP method for MOLP problem solving [13]. Before applying the MP method to determine the preferred solution by decision levels, DMs should determine the initial aspirational levels of their objective functions (d_{lk}). The marginal (optimal) values achieved on a given set of constraints (f_{lk}^*) can be used as initial aspirational levels of the objective functions. The interactive procedure of the MP method ensures the achievement of the preferred nondominated solution in a finite number of steps [13].

In the first step of applying the MP method to determine the preferred nondominated solution by decision levels, the following LP model is solved for each decision level individually:

$$max\, \lambda_l,\ l = 1, 2, \ldots, L \tag{5}$$

subject to

$$\begin{aligned} & \textstyle\sum_{j=1}^{n} \big(c_{lkj}^3 - \frac{\alpha}{\theta} c_{lkj}^3 + \frac{\alpha}{\theta} c_{lkj}^2\big) x_j \ge \lambda d_{lk},\ k = 1, 2, \ldots, K_l \\ & \sum_{j=1}^{n} \Big(\frac{\theta}{\alpha} a_{ij}^2 - a_{ij}^2 + a_{ij}^3\Big) x_j \le \Big(\frac{\theta}{\alpha} - 1\Big) b_i^3 + b_i^2,\ i = 1, 2, \ldots, m \\ & \sum_{j=1}^{n} a_{ij}^2 x_j \le b_i^3\,,\ i = 1, 2, \ldots, m \\ & \lambda_l \ge 0,\ x_j \ge 0,\ j = 1, 2, \ldots, n. \end{aligned}$$

After that, the method calculates the degree of realization of the DM's initial aspirations:

$$\lambda_{lk} = \frac{f_{lk}(x^*)}{d_{lk}},\ k = 1, 2, \ldots, K_l \tag{6}$$

If all DMs are satisfied with the achieved optimal solution of the model (5), the solution procedure in this step of solving the ML_MOLP problem is stopped, and the DMs, based on the optimal values of the variables of the model (5), determine the lower limits and aspirational values of the variables they control. Otherwise, model (5) is solved again with new aspirational values of the objective functions so that DMs with a high coefficient of realization of their aspirations reduce their aspirations in order to increase the coefficient of realization of aspirations of dissatisfied DMs. The procedure continues until a solution is obtained that will ensure a satisfactory level of realization of the aspirations of all DMs.

3.3. Applying the MP method to determine the preferred solution of the ML_MOLP problem

To determine the preferred non-dominated solution of the ML_MOLP problem by applying the MP method for the given θ and α, the following LP problem is solved:

$$max\,\lambda, \tag{7}$$

subject to

$$\begin{gathered}
\sum_{j=1}^{n}\left(c_{lkj}^{3}-\tfrac{\alpha}{\theta}c_{lkj}^{3}+\tfrac{\alpha}{\theta}c_{lkj}^{2}\right)x_j \geq \lambda d_{lk},\ l=1,2,\ldots,L,\ k=1,2,\ldots,K_l \\
x_j \geq \lambda d_j, j=1,2,\ldots,n \\
\sum_{j=1}^{n}\left(\tfrac{\theta}{\alpha}a_{ij}^{2}-a_{ij}^{2}+a_{ij}^{3}\right)x_j \leq \left(\tfrac{\theta}{\alpha}-1\right)b_i^3+b_i^2,\ i=1,2,\ldots,m \\
\sum_{j=1}^{n}a_{ij}^{2}x_j \leq b_i^3\,,\ i=1,2,\ldots,m \\
\lambda \geq 0, x_j \geq 0\,,\ j=1,2,\ldots,n.
\end{gathered}$$

For the initial aspirational levels of the objective functions (d_lk), we suggest using the preferred values of the objective functions obtained by solving model (5), while the aspirational levels of the variables were determined in the previous phase of problem solving.

After that, the degree of realization of the DM's initial aspirations is calculated:

$$\lambda_{lk}=\frac{f_{lk}(x^{*})}{d_{lk}},\ k=1,2,\ldots,K_l \tag{8}$$

The calculated λ_{lk} values indicate DMs with a high degree of realization of their aspirations, who should reduce their aspirations in the case that some DMs are not satisfied with the achieved level of their aspirations, so that in the next phase of solving the model (5) the degree of realization of the aspirations of dissatisfied DMs will increase.

3.4. Algorithm for solving the ML_MOLP problem

An algorithm for solving the ML_MOLP problem with fuzzy parameters in the form of triangular fuzzy numbers is presented here:
Step 1. Define the parameters θ and α.
Step 2. Determine marginal solutions of model (4) by separately maximizing all objective functions on a given set of constraints and form the pay-off table.
Step 3. Determine the initial aspiration levels of the objective functions by decision levels.
Step 4. Determine the preferred solutions of model (5) by decision levels.
Step 5. Define the aspirational values of the variables controlled by the DMs.
Step 6. Define the aspirational values of the objective functions to solve the model (7).
Step 7. Form and solve the model (7).
Step 8. Stop if the DMs are satisfied with the obtained solution. Otherwise, return to step 5.

4. Practical example

Consider a decentralized company with three decision making levels. Each decision level has two DMs, and each DM has one objective function. Maximization of the total production and total investment in the development of the company are the goals at level 1. Maximization of total net profits of production units of the company are the goals of the DMs at level 2. Maximization of the total inventory and total investment in the promotion are the goals of the DMs at level 3. The company produces six products, three (P_1, P_2, P_3) in the department

1, and three (P_4, P_5, P_6) in the department 2. 1,000,000 m.u. is intended for research and development of the company with an annual cost of 10%. The minimum production of each product should be 500 pieces. To ensure continuity of delivery, the total stock should be at least 10% of the total production, while the stock of each product should be between 50 and 800 pieces. The investment in promotion must be between 0.5 and 5% of the total gross profit of the company and must be at least 1,000 and at most 15,000 m.u. for each product. The minimum total production must be 14,000 pieces. Table 1 shows the manufacturing data expressed in the form of triangular fuzzy numbers. Perić et al. [19] solve a similar example but with data that are assumed to be fixed.

Data per unit	Products					
	P_1	P_2	P_3	P_4	P_5	P_6
Machines, department 1 (h) Capacity: (12000, 13000, 13800)	(1.2, 2, 3)	(0.5, 1, 1.2)	(2, 3, 3.5)			
Machines, department 2 (h) Capacity: (10800, 12000, 13500)				(0.4, 1, 1.5)	(1.2, 2, 2.4)	(0.6, 1, 1.8)
Sales price per unit (monetary units)	(1200, 1400, 1580)	(1080, 1200, 1350)	(1220, 1300, 1480)	(760, 900, 980)	(1020, 1100, 1150)	(620, 700, 880)
Gross profit per unit (monetary units)	(88, 100, 118)	(105, 120, 134)	(60, 80, 99)	(120, 150, 175)	(135, 200, 350)	(170, 180, 188)
Inventory cost per unit (monetary units)	(7, 8, 10)	(5, 6, 7)	(8, 10, 11)	(9, 10, 13)	(4, 6, 7)	(6, 8, 11)

Table 1: *Data for the practical application*

Let $x_1, x_2, x_3, x_4, x_5, x_6$ is the quantity of products $P_1, P_2, P_3, P_4, P_5, P_6$ respectively; x_7, x_8 is the capital invested in the departments 1, 2 respectively; $x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}$ is the quantity of stock products $P_1, P_2, P_3, P_4, P_5, P_6$ respectively, $x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}$ is the quantity of investment in promotion of products $P_1, P_2, P_3, P_4, P_5, P_6$ respectively.

The ML-MOLP model with the fuzzy coefficients is presented as:

Level 1

$max\, f_{11} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$ (9)

$max\, f_{12} = x_7 + x_8$

Level 2

control variables x_7 and x_8

$max\, f_{21} = (88, 100, 118)x_1 + (105, 120, 134)x_2 + (60, 80, 99)x_3 - 0,1x_7 - (7, 8, 9)x_9 -$
$(5, 6, 7)x_{10} - (9, 10, 13)x_{11} - x_{15} - x_{16} - x_{17}$

$max\, f_{22} = (120, 150, 175)x_4 + (135, 200, 350)x_5 + (170, 180, 188)x_6 - 0,1x_8 - (9, 10, 13)x_{12} -$
$(4, 6, 7)x_{13} - (6, 8, 11)x_{14} - x_{18} - x_{19} - x_{20}$

control variables $x_1, x_2, x_3, x_4, x_5, x_6$

Level 3 $max\, f_{31} = x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}, max\, f_{32} = x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20}$

control variables $x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}$

subject to

$(1.2, 2, 3)x_1 + (0.5, 1, 1.2)x_2 + (2, 3, 3.5)x_3 \leq (12000, 13000, 13800), (0.4, 1, 1.5)x_4 + (1.2, 2, 2.4)x_5$
$+(0.6, 1, 1.8)x_6 \leq (10800, 12000, 13500), x_7 + x_8 \leq 1000000, x_7 - x_8 = 0, x_1, x_2, x_3, x_4, x_5, x_6 \geq$
$500, x_9 \geq 0.1x_1, x_{10} \geq 0.1x_2, x_{11} \geq 0.1x_3, x_{12} \geq 0.1x_4, x_{13} \geq 0.1x_5, x_{14} \geq 0.1x_6, 50 \leq x_9, x_{10},$

$x_{11}, x_{12}, x_{13}, x_{14} \leq 800, x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \leq 0.15((88, 100, 118)x_1 + (105, 120, 134)x_2 + (60, 80, 99)x_3 + (120, 150, 175)x_4 + (135, 200, 350)x_5 + (170, 180, 188)x_6), x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20} \leq 15000,\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 14000, x_7, x_8 \geq 0, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20} \geq 1000$

In the first step the decision makers with the help of the analyst define parameters θ and α. In our example, we assumed that $\theta = 1$ and $\alpha = 0.8$.

In second step we defuzzify the fuzzy parameters of the model (9), and form the following ML_MOLP model:

Level 1

$max\, f_{11} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$ (10)

$max\, f_{12} = x_7 + x_8$

control variables x_7 and x_8

Level 2

$max\, f_{21} = 103.6x_1 + 122.8x_2 + 83.8x_3 - 0.1x_7 - 8.2x_9 - 6.2x_{10} - 10.6x_{11} - x_{15} - x_{16} - x_{17}$

$max\, f_{22} = 155x_4 + 230x_5 + 181.6x_6 - 0.1x_8 - 10.6x_{12} - 6.2x_{13} - 8.6x_{14} - x_{18} - x_{19} - x_{20}$

control variables $x_1, x_2, x_3, x_4, x_5, x_6$

Level 3

$max\, f_{31} = x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}, max\, f_{32} = x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20}$

control variables $x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}$

subject to

$$S = \left\{ \begin{array}{l} x_1, x_2, ..., x_{20} :\ 3.5x_1 + 1.45x_2 + 4.25x_3 \leq 17250,\ 1.75x_4 + 2.9x_5 + 2.05x_6 \leq 17250,\ x_7 + x_8 \leq \\ 1000000,\ x_7 - x_8 = 0,\ x_1, x_2, x_3, x_4, x_5,\ x_6 \geq 500, x_9 \geq 0.1x_1,\ x_{10} \geq 0.1x_2,\ x_{11} \geq 0.1x_3,\ x_{12} \geq \\ 0.1x_4,\ x_{13} \geq 0.1x_5,\ x_{14} \geq 0.1x_6,\ 50 \leq x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \leq 800,\ x_{15} + x_{16} + x_{17} + x_{18} + \\ x_{19} + x_{20} \leq 0.05(143x_1 + 164x_2 + 119x_3 + 212.5x_4 + 400x_5 + 233x_6),\ x_{15} + x_{16} + x_{17} + x_{18} + \\ x_{19} + x_{20} \geq 0.005(143x_1 + 164x_2 + 119x_3 + 212.5x_4 + 400x_5 + 233x_6),\ x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, \\ x_{20} \leq 15000,\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 14000,\ x_7, x_8 \geq 0, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20} \geq 1000.) \end{array} \right\}$$

The marginal solutions of model (10) obtained by separately maximizing the objective functions on a given set of constraints are presented in Table 2, while the pay-off values of the objective functions for the obtained marginal solutions are presented in table 3.

Variables	max f_{11}	max f_{12}	max f_{21}	max f_{22}	max f_{31}	max f_{32}
x_1	1007.14	3686.2	1007.14	3686.2	3686.2	3686.2
x_2	8000	1533.31	8000	1533.31	1533.31	1533.32
x_3	500	500	500	500	500	500
x_4	8000	500	3492.86	500	500	500
x_5	500	500	500	500	500	500
x_6	878.05	7280.49	500	7280.49	7280.49	7280.49
x_7	0	0	0	0	0	0
x_8	0	1000000	0	0	0	0
x_9	800	800	100.71	800	800	800
x_{10}	800	800	800	800	800	800
x_{11}	800	800	50	800	800	800
x_{12}	800	800	800	50	800	800
x_{13}	800	800	800	50	800	800
x_{14}	800	800	800	728.05	800	800
x_{15}	1000	1000	1000	1000	1000	15000
x_{16}	1000	1000	1000	1000	1000	15000
x_{17}	1000	1000	1000	9203.47	1000	15000

x_{18}	1000	1000	1000	1000	1000	15000
x_{19}	1000	1000	1000	1000	1000	15000
x_{20}	1000	9203.47	7871.27	1000	9203.47	15000

Table 2: *The marginal solutions of model (10)*

	max f_{11}	max f_{12}	max f_{21}	max f_{22}	max f_{31}	max f_{32}
max f_{11}	18885.19	0	1105640	1479034	4800	18100
max f_{12}	14000	1000000	598080.8	1383114	4800	14203.47
max f_{21}	14000	0	1119324	717002	3350.71	12871.27
max f_{22}	14000	0	580877.3	1504536	3228.05	14203.47
max f_{31}	14000	0	598080.8	1483114	4800	14203.47
max f_{32}	14000	0	547082	1449317	4800	90000

Table 3: *The pay-off values of the objective functions for the obtained marginal solutions*

As the initial aspirational value of the objective function for determining the aspirational value of the variables controlled by the DMs by decision-making levels their optimal values are taken from the pay-off table.

To determine the aspirational levels of the variables controlled by the DMs, model (5) is solved for each decision level. The following model was solved for the first decision making level:

$$max\,\lambda_1 \tag{11}$$

subject to
$S_1 = \{S \cup \{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 18885.19\lambda_1,\ x_7 + x_8 \geq 1000000\lambda_1,\ \lambda_1 \geq 0\}\}$
The following solution was obtained:
$x_1 = 1007, x_2 = 8000, x_3 = 500, x_4 = 8000, x_5 = 500, x_6 = 878, x_7 = 500000, x_8 = 500000, x_9 = 800, x_{10} = 800, x_{11} = 800, x_{12} = 800, x_{13} = 800, x_{14} = 800, x_{15} = 1000, x_{16} = 1000, x_{17} = 1000, x_{18} = 1000, x_{19} = 1000, x_{20} = 13100, f_{11} = 18885.2, f_{12} = 100000, f_{21} = 1055640, f_{22} = 1429035, f_{31} = 4800, f_{32} = 18100.$
The DMs have chosen $x_7 = 500000$ and $x_8 = 500000$.
The following LP model was solved for decision level 2:

$$max\,\lambda_2 \tag{12}$$

subject to
$S_2 =$

$$\left\{ S \cup \left\{ \begin{array}{l} 103.6x_1 + 122.8x_2 + 83.8x_3 - 0,1x_7 - 8.2x_9 - 6.2x_{10} - 10.6x_{11} - x_{15} - x_{16} - x_{17} \geq \\ 1119324_2, 155x_4 + 230x_5 + 181.6x_6 - 0,1x_8 - 10.6x_{12} - 6.2x_{13} - 8.6x_{14} - x_{18} \\ -x_{19} - x_{20} \geq 1504536\lambda_2, \lambda_2 \geq 0) \end{array} \right\} \right\}$$

The following solution was obtained:
$x_1 = 1007, x_2 = 8000, x_3 = 500, x_4 = 500, x_5 = 500, x_6 = 7280, x_7 = 0, x_8 = 0, x_9 = 101, x_{10} = 800, x_{11} = 50, x_{12} = 50, x_{13} = 50, x_{14} = 728, x_{15} = 1000, x_{16} = 1000, x_{17} = 5945, x_{18} = 1000, x_{19} = 1000, x_{20} = 7646, f_{11} = 17788, f_{12} = 0, f_{21} = 1114377, f_{22} = 1497886, f_{31} = 1779, f_{32} = 17591.$
The DMs have chosen $x_1 = 1007, x_2 = 8000, x_3 = 500, x_4 = 500, x_5 = 500, x_6 = 7280.$
For decision level 3 the following LP model has been solved:

$$max\,\lambda_3 \tag{13}$$

subject to
$S_3 =$
$$\left\{ S \cup \begin{cases} x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} \geq 4800\lambda_3,\ x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \\ \geq 90000\lambda_3, \lambda_3 \geq 0 \end{cases} \right\}$$

The following solution was obtained:
$x_1 = 500, x_2 = 4720, x_3 = 500, x_4 = 500, x_5 = 500, x_6 = 7280, x_7 = 500000, x_8 = 500000, x_9 = 800, x_{10} = 800, x_{11} = 800, x_{12} = 800, x_{13} = 800, x_{14} = 800, x_{15} = 15000, x_{16} = 15000, x_{17} = 15000, x_{18} = 15000, x_{19} = 15000, x_{20} = 15000, f_{11} = 14000, f_{12} = 1000000, f_{21} = 558316, f_{22} = 1399228, f_{31} = 4800, f_{32} = 90000.$
The DMs have chosen
$x_9 = 800, x_{10} = 800, x_{11} = 800, x_{12} = 800, x_{13} = 800, x_{14} = 800, x_{15} = 15000, x_{16} = 15000, x_{17} = 15000, x_{18} = 15000, x_{19} = 15000, x_{20} = 15000.$
Therefore, the aspirational values of the objective functions (obtained by solving models (11), (12) and (13)) are:
$d_{11} = 18885, d_{12} = 1000000, d_{21} = 1114377, d_{22} = 1497886, d_{31} = 4800, d_{32} = 90000.$
To determine the preferred solution of the model (9) the following LP model has been solved:

$$max\, \lambda \tag{14}$$

subject to
$S_4 =$
$$\left\{ S \cup \begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 18885\lambda, x_7 + x_8 \geq 1000000\lambda, 103.6x_1 + 122.8x_2 + \\ 83.8x_3 - 0,1x_7 - 8.2x_9 - 6.2x_{10} - 10.6x_{11} - x_{15} - x_{16} - x_{17} \geq 1114377\lambda, \\ 155x_4 + 230x_5 + 181.6x_6 - 0,1x_8 - 10.6x_{12} - 6.2x_{13} - 8.6x_{14} - x_{18} - x_{19} - x_{20} \geq \\ 1497886\lambda, x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} \geq 4800\lambda, x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + \\ x_{20} \geq 90000\lambda, x_1 \geq 1007\lambda, x_2 \geq 8000\lambda, x_3 \geq 500\lambda, x_4 \geq 500\lambda, x_5 \geq 500\lambda, \\ x_6 \geq 7280\lambda, x_7 \geq 500000\lambda, x_8 \geq 500000\lambda, x_9 \geq 800\lambda, x_{10} \geq 800\lambda, x_{11} \geq \\ 800\lambda, x_{12} \geq 800\lambda, x_{13} \geq 800\lambda, x_{14} \geq 800\lambda, x_{15} \geq 15000\lambda, x_{16} \geq 15000\lambda, \\ x_{17} \geq 15000\lambda, x_{18} \geq 15000\lambda, x_{19} \geq 15000\lambda, x_{20} \geq 15000\lambda, \lambda \geq 0) \end{cases} \right\}$$

The following solution was obtained:
$x_1 = 1007, x_2 = 8000, x_3 = 500, x_4 = 500, x_5 = 500, x_6 = 7093, x_7 = 457017, x_8 = 457017, x_9 = 731, x_{10} = 800, x_{11} = 731, x_{12} = 731, x_{13} = 731, x_{14} = 731, x_{15} = 13710, x_{16} = 13710, x_{17} = 13710, x_{18} = 15000, x_{19} = 11132, x_{20} = 15000, f_{11} = 17600(93.2\% of f^*_{11}), f_{12} = 914034(91.4\% of f^*_{12}), f_{21} = 1023091(91.4\% \text{ of } f^*_{21}), f_{22} = 1375188(91.4\% \text{ of } f^*_{22}), f_{31} = 4455(92.8\% \text{ of } f^*_{31}), f_{32} = 82262(91.4\% \text{ of } f^*_{32}).$

The obtained solution gives the objective functions approximately equal percentages of the realization of aspirational values. It can be improved if one of the DMs would not be satisfied with the realized value of his objective function by solving model (12) again but with reduced values of the aspirational levels of the objective functions and/or variables of the satisfied DMs.

5. Conclusion

This paper proposes a methodology for solving the ML_MOLP problem with fuzzy parameters. The proposed methodology was tested on the example of decision-making on production, investments, stock size and investments in promotion in an assumed complex production system with three levels of decision-making and two DMs at each level. The proposed methodology assumes that all DMs are aware that business success of the entire business system depends on the level of achievement of the goals of all DMs of that business system and they are ready for cooperation and compromise in order to improve the efficiency of the entire business system. The main contribution of the paper is in the construction and testing of a new methodology for solving ML_MOLP problems in which the parameters in the objective functions and constraints can be represented as triangular fuzzy numbers. Future research could test the possibility of applying the proposed methodology when making decisions in concrete business systems with a decentralized decision-making system.

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