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# Application of Probability-based Multi-objective Optimization in Portfolio Investment and Engineering Management Problems

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**Abstract:** Markowitz's approach could not deal with the overall optimization of both minimizing variance of return rate and maximizing return rate simultaneously due to its short of rational method for handling multi-objective optimization. In this article, a hybrid of the probability-based multi-objective optimization with the uniform design for experiments of mixtures is performed to solve the portfolio investment problem of concurrent optimization of both maximizing return rate and minimizing variance of return rate. The probability-based multi-objective optimization is employed to transfer the bi-objective problem of portfolio investment into a mono-objective one with total preferable probability as the goal for the overall optimization of the system in spirit of probability theory, the uniform design for experiments of mixtures is used to perform the subsequent discretization. Project management problem is rather a multi-objective to conduct naturally. The analysis shows the rationality of the hybrid solution.

**Keywords:** engineering management; multi-objective optimization; portfolio investment; probability theory; uniform design of mixtures

## 1 INTRODUCTION

In financial field [1-5], securities investors hope to obtain higher returns from securities investment and minimizing risks at the same time. Generally, it is a very common knowledge that a greater return of security is accompanied with a bigger risk for an investment. In order to reduce risk and pursue high efficiency with low risk, security investors could combine multiple securities according to rational investment ratios, that is, the so-called "portfolio of securities", it aims to obtain the maximum profit with less risk.

The portfolio investment theory of economist Markowitz holds that investors always hope to get as much profit ($E(R)$) as possible under certain risk conditions, or a reduction of risk ($X^{T}CX$) as much as possible under certain rate of return, that is, to minimize the risk when the expected rate of return with $E(R) \geq c$ is met, or to get as much profit under the condition of the established risk $X^{T}CX \leq b$. Furthermore, the expected return rate and variance of return rate are used to evaluate risky securities, and the latter is employed to indicate the risk in Markowitz's algorithm.

Obviously, Markowitz's method can only handle either the minimizing the variance of return rate with setting expected rate of return as a restraint condition, or maximizing the expected return rate and letting variance of return rate as a constraint condition. It is clear that Markowitz's approach could not deal with the overall optimization of both minimizing variance of return rate and maximizing return rate simultaneously due to its short of rational method for handling multi-objective optimization.

Besides, project management problem involves many objectives usually, therefore it is also a multi - objective optimization problem inevitablly. It needs to be implemented and completed with lower project cost, lower resource consumption, control project duration, and ensure project quality [5, 6]. From the point of view of system theory, the multi - objective optimization problem of engineering projects is generally composed of cost subsystem, resource control subsystem, time limit subsystem and quality subsystem, etc. These subsystems are relatively independent, and each completes its own specific functions and operational objectives. On the whole, the subsystems of each project interact and restrict each other, so the overall goal of the project should be fully considered in order to give full play to the overall function of the system. Make the project as a whole optimal, and realize the synergy among the time limit, cost, quality and resource investment, etc.

Recently, the probability-based multi-objective optimization was proposed [7], which is with a brand new concept of "preferable probability" and the assessments for the probability-based multi-objective optimization. As a rationally novel approach, which aims to conduct the overall optimization of the system in spirit of probability theory. The methodology could be used in many fields, including energy planning, programming problem, operation research, financial affairs, etc.

In this paper, a hybrid of the probability - based multi – objective optimization with the uniform design for experiments of mixtures is performed to solve the portfolio investment problem of concurrent optimization of both maximizing return rate and minimizing variance of return rate. Two steps are involved, preliminarily the probability-based multi-objective optimization is used to transfer the bi-objective problem of portfolio investment problem into a mono-objective one by means of preferable probability in spirit of probability theory, subsequently the uniform design for experiments of mixtures is used to perform the successive discretization.

## 2 SECURITIES PORTFOLIO PROBLEM

### 2.1 Fundamental Problem

There are two modes in Markowitz's algorithm for securities portfolio problem, which can be expressed by the following model (a) or (b),
Model (a),

$$\text{Min } f_2 = X^{\mathrm{T}} C X$$
$$\text{s.t. } f_1 = \sum_{i=1}^{n} x_i r_i \geq c \tag{1}$$
$$\sum_{i=1}^{n} x_i = 1,\ i = 1, 2, 3, \ldots, n$$

Model (b),

$$\text{Max } f_1 = \sum_{i=1}^{n} x_i r_i$$
$$\text{s.t. } f_2^2 = X^{\mathrm{T}} C X \leq b \tag{2}$$
$$\sum_{i=1}^{n} x_i = 1,\ i = 1, 2, 3, \ldots, n$$

In Eqs. (1) and (2), the symbel $n$ represents the number of securities, and the expected retuan rate of each security is indicated by $r_1$, $r_2$, …, $r_n$, respectively; the proportion of the $i^{\text{th}}$ security is $x_i$. $\boldsymbol{C}$ represents the risk matrix of investment, and $X^{\mathrm{T}}CX$ represents the expected value of portfolio risk. Furthermore, $\sigma^2 = X^{\mathrm{T}}CX$ is used to reflect the variance of the return rate of securities portfolio and the risk of investment in principle. While $f_1 = E(R) = \sum_{i=1}^{n} x_i r_i$ is the expected value of the return rate of $n$ kinds of securities invested in a certain period of time; Factor $c$ represents the preset total return rate of portfolio investment; Factor $b$ describes the preset risk of portfolio investment. $\sigma^2$ indicates the deviation of various possible values of return rate from their expected values, that is, the uncertainty of return rate. The standard deviation of portfolio is the square root of variance $\sigma^2$.

In his portfolio investment theory Markowitz adopted either limiting the risk to a certain range to obtain as much profit as possible, or limiting the profit to a certain range to suffer as little risk as possible. This is fully similar to the $\varepsilon$-constraint solution in the of multi-objective optimization problem. Its disadvantage of this approach is that "one objective optimization" is lost in optimization instead of "multi-objective simultaneous optimization", thus the essential and true meaning of "simultaneous optimization of multiple objectives" is uncomplete.

According to Markowitz's practice, the retuan rate function $f_1$ and risk function $f_2$ are introduced, and their expressions are, respectively, as,

$$f_1 = E(R) = \sum_{i=1}^{n} x_i r_i$$
$$\begin{aligned} f_2 = \Big[ & (x_1\sigma_1)^2 + (x_2\sigma_2)^2 + (x_3\sigma_3)^2 + \ldots + (x_n\sigma_n)^2 + \\ & + \beta_{1,2}(x_1\sigma_1)(x_2\sigma_2) + \beta_{1,3}(x_1\sigma_1)(x_3\sigma_3) + \\ & + \beta_{1,4}(x_1\sigma_1)(x_4\sigma_4) + \ldots + \beta_{i,j}(x_i\sigma_i)(x_j\sigma_j) + \ldots + \\ & + \beta_{n-1,n}(x_{n-1}\sigma_{n-1})(x_n\sigma_n) \Big]^{0.5}. \end{aligned} \tag{3}$$

In Eq. (3), $\beta_{i,j}$ is the correlation coefficient between the $i^{\text{th}}$ security and the $j^{\text{th}}$ security.

Furthermore, according to the objective evaluation method of the probability - based multi - objective optimization methodology, $f_1$ belongs to the beneficial objective and $f_2$ belongs to the unbeneficial objective in the assessment.

Therefore, the answer to the "portfolio investment" problem is actually an optimization of bi-objective one. Therefore, the probability-based multi-objective optimization methodology can be used to evaluate it reasonably.

### 2.2 Hybrid of Probability-Based Multi-Objective Optimization Methodology with Uniform Design for Experiments of Mixtures to Solve the Portfolio Investment Problem

**1) Fundamental procedure**

In order to conduct hybrid of the probability - based multi – objective optimization with the uniform design for experiments of mixtures, two steps are involved, the preliminary step is to make conversion of the bi-objective problem of portfolio investment problem into a mono-objective one by means of probability-based multi-objective optimization in spirit of probability theory, the subsequent step is to use the uniform design for experiments of mixtures to perform discretization of successive optimization.

In the portfolio investment problem, the concurrent minimizing variance of return rate and maximizing return rate are two objectives inevitablly, the total preferable probability of this bi - objective problem is the decisive and unique objective function, which needs to be maximizing in a high-dimensional independent variable - space, therefore the complex data treatment might be involved, thus the uniform design for experiments with mixtures (UDEM) can be rationally combined to perform the simplifying treatment of the data processing.

The uniform design for experiments with mixtures (UDEM) was proposed by Fang et al [8], which is on basis of good lattice point (GLP). The method of UDEM could be employed to geneate a set of efficient sampling points for experimental design with the restraint of $x_1 + x_2 + x_3 + \ldots + x_n = 1$ for proportion $x_i$ with total number of $n$ [8], therefore it can be used as a rational sampling method for the portfolio investment problem here to perform the discretization for data processing with simplifying treatment.

In addition, Fang specially developed uniform design tables and their usage tables for the proper application [9].

According to Fang et al. [8, 9], the concrete steps of uniform design for experiments with mixtures (UDEM) are generally as follows:

**a) Choice of the uniform design table**

Given the number of mixtures $n$, and the number of sampling points $p$, chose the corresponding uniform design table $U^*_p(p^t)$ or $U_p(p^t)$ and its utility table from the uniform design table provided in [8, 9], and the number of columns of the utility table is chosen as $n-1$. Label a mark of the original elements in the uniform design table $\mathrm{U}^*_{\mathrm{p}}(p^t)$ or $U_p(p^t)$ with $\{q_{ik}\}$.

**b) Establishment of a new element $c_{ki}$**

For each $i$, establish its new element $c_{ki}$ according to following rule,

$$c_{ki} = \frac{2q_{ki} - 1}{2p} \tag{4}$$

**c) Establishment of uniform sampling points for the mixtures, $x_{ki}$**

$$\begin{aligned} x_{ki} &= \left(1 - c_{ki}^{\frac{1}{n-i}}\right) \prod_{j=1}^{i-1} c_{kj}^{\frac{1}{n-j}},\ i = 1, \dots, n-1 \\ x_{kn} &= \prod_{j=1}^{n-1} c_{kj}^{\frac{1}{n-j}},\ k = 1, \dots, p \end{aligned} \tag{5}$$

Thus, $\{x_{ki}\}$ derives the corresponding uniform design table $UM_p(p^n)$ of the mixture under the conditions of preset $n$ and $p$.

**2) Example of portfolio investment problem of three securities**

Take the portfolio investment of three securities as the typical example. The specific optimization process is explained in details.

Let the expected return rate of security ***A*** is $r_1 = 14\%$, and the standard deviation of return rate is $\sigma_1 = 6\%$; The expected return rate of ***B*** securities $r_2 = 8\%$, and the standard deviation of return rate is $\sigma_2 = 3\%$; The expected return rate of ***C*** securities is $r_3 = 20\%$, and the standard deviation of return rate is $\sigma_3 = 15\%$. Furthermore, it is assumed that the correlation coefficient between ***A*** and ***B*** securities is $\beta_{1,2} = 0.5$; The correlation coefficient between securities ***A*** and ***C*** is $\beta_{1,3} = -0.2$; The correlation coefficient between securities ***B*** and ***C*** is $\beta_{2,3} = -0.4$. Now we need to make a decision on this portfolio investment.

***Solution.***

In this section, the "portfolio investment" problem is analyzed on intentions of probability - based multi - objective optimization methodology. Now we need to handle the bi-objective simultaneous optimization problem of both minimizing variance of return rate and maximizing return rate.

Let $x_1$, $x_2$, and $x_3$ be the investment percentages of three securities, ***A***, ***B***, and ***C***, respectively. Because of the constraint condition of $x_1 + x_2 + x_3 = 1$, there are in fact two independent variables, nominally $x_1$ and $x_2$.

However, the sampling points of this bi-objective optimization problem are scattered in 3 – dimensional space, therefore, so it has to include at least 19 sampling points with characteristic of "good lattice point" in the efficient zone for the discretization of data processing [7, 10, 11].

According to Fang et al [8, 9], this is a "uniform design for experiments with mixtures" problem due to the restraint condition of the three variables, we could take the uniform table $U^*_{19}(19^7)$ as the initial table to establish a uniform test design table $UM_{19}(19^3)$ with mixtures, which is shown in Tab. 1.

Thus the uniform test table $UM_{19}(19^3)$ with mixtures of Tab. 1 can be estiblished on basis of uniform design table $U^*_{19}(19^7)$. Because the nubmber of variables $n$ equals to 3 here, and $p$ equals to 19 [7, 10, 11], from above rules, $x_{k1} = 1 - c_{k1}^{0.5}$, $x_{k2} = c_{k1}^{0.5} \cdot (1 - c_{k2})$, $x_{k3} = c_{k1}^{0.5} \cdot c_{k2}$ [8, 9].

Furthermore, the values of the return rate function $f_1$ and risk function $f_2$, the values of their partial preferable probability, the total preferable probability and ranking at the sampling points can be obtained, as are shown in Tab. 2.

Fig. 1 represents return rate vs risk at discrete sampling points. The results indicate that the 5th and 7th discrete sampling points exhibit the maximum total preferable probability, therefore they could be taken as the optimal solution of this portfolio problem.

As to the 5th sampling point, the corresponding investment ratio is $x_1' = 0.5133$, $x_2' = 0.3714$, $x_3' = 0.1153$, which results in the return rate of 12.46% and the risk of 3.13%.

While for the 7th sampling point, its investment ratio is at $x_1'' = 0.4151$, $x_2'' = 0.5079$, $x_3'' = 0.0770$, and the corresponding return rate is 11.41% with the risk of 2.42%.

**Table 1** Uniform experimental table with mixtures of $UM_{19}(19^3)$ based on $U^*_{19}(19^7)$

| No. | $x_{10}$ | $x_{20}$ | $c_1$ | $c_2$ | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 9 | 0.0263 | 0.4474 | 0.8378 | 0.0896 | 0.0726 |
| 2 | 2 | 18 | 0.0789 | 0.9211 | 0.7190 | 0.0222 | 0.2588 |
| 3 | 3 | 7 | 0.1316 | 0.3421 | 0.6373 | 0.2386 | 0.1241 |
| 4 | 4 | 16 | 0.1842 | 0.8158 | 0.5708 | 0.0791 | 0.3501 |
| **5** | **5** | **5** | **0.2368** | **0.2368** | **0.5133** | **0.3714** | **0.1153** |
| 6 | 6 | 14 | 0.2895 | 0.7105 | 0.4620 | 0.1557 | 0.3823 |
| **7** | **7** | **3** | **0.3421** | **0.1316** | **0.4151** | **0.5079** | **0.0770** |
| 8 | 8 | 12 | 0.3947 | 0.6053 | 0.3717 | 0.2480 | 0.3803 |
| 9 | 9 | 1 | 0.4474 | 0.0263 | 0.3311 | 0.6513 | 0.0176 |
| 10 | 10 | 10 | 0.5 | 0.5 | 0.2929 | 0.3536 | 0.3536 |
| 11 | 11 | 19 | 0.5526 | 0.9737 | 0.2566 | 0.0196 | 0.7238 |
| 12 | 12 | 8 | 0.6053 | 0.3947 | 0.2220 | 0.4709 | 0.3071 |
| 13 | 13 | 17 | 0.6579 | 0.8684 | 0.1889 | 0.1067 | 0.7044 |
| 14 | 14 | 6 | 0.7105 | 0.2895 | 0.1571 | 0.5989 | 0.2440 |
| 15 | 15 | 15 | 0.7636 | 0.7632 | 0.1264 | 0.2069 | 0.6667 |
| 16 | 16 | 4 | 0.8158 | 0.1842 | 0.0968 | 0.7368 | 0.1664 |
| 17 | 17 | 13 | 0.8684 | 0.6579 | 0.0681 | 0.3188 | 0.6131 |
| 18 | 18 | 2 | 0.9211 | 0.0789 | 0.0403 | 0.8839 | 0.0758 |
| 19 | 19 | 11 | 0.9737 | 0.5526 | 0.0132 | 0.4414 | 0.5453 |

**Table 2** Evaluation results of $f_1$, $f_2$, preferable probability and ranking at discrete sampling point

| No. | $f_1$ | $f_2$ | $P_{f1}$ | $P_{f2}$ | $P_t \times 10^3$ | Rank |
|---|---|---|---|---|---|---|
| 1 | 0.1390 | 0.0500 | 0.0526 | 0.0562 | 2.9553 | 8 |
| 2 | 0.1542 | 0.0557 | 0.0584 | 0.0516 | 3.0148 | 5 |
| 3 | 0.1331 | 0.0391 | 0.0504 | 0.0648 | 3.2670 | 3 |
| 4 | 0.1563 | 0.0584 | 0.0592 | 0.0494 | 2.9248 | 10 |
| **5** | **0.1246** | **0.0313** | **0.0472** | **0.0710** | **3.3518** | **1** |
| 6 | 0.1536 | 0.0585 | 0.0582 | 0.0494 | 2.8715 | 13 |
| **7** | **0.1141** | **0.0242** | **0.0432** | **0.0767** | **3.3151** | **2** |
| 8 | 0.1479 | 0.0559 | 0.0560 | 0.0515 | 2.8839 | 11 |
| 9 | 0.1020 | 0.0199 | 0.0386 | 0.0801 | 3.0952 | 4 |
| 10 | 0.14 | 0.0510 | 0.0530 | 0.0554 | 2.9357 | 9 |
| 11 | 0.1823 | 0.1007 | 0.0690 | 0.0158 | 1.0911 | 19 |
| 12 | 0.1302 | 0.0444 | 0.0493 | 0.0606 | 2.9873 | 7 |
| 13 | 0.1759 | 0.0974 | 0.0666 | 0.0184 | 1.2271 | 18 |
| 14 | 0.1187 | 0.0370 | 0.0450 | 0.0665 | 2.9897 | 6 |
| 15 | 0.1676 | 0.0919 | 0.0635 | 0.0228 | 1.4444 | 17 |
| 16 | 0.1058 | 0.0303 | 0.0401 | 0.0718 | 2.8775 | 12 |
| 17 | 0.1577 | 0.0847 | 0.0597 | 0.0285 | 1.7024 | 16 |
| 18 | 0.0915 | 0.0274 | 0.0347 | 0.0741 | 2.5677 | 14 |
| 19 | 0.1462 | 0.0761 | 0.0554 | 0.0354 | 1.9596 | 15 |

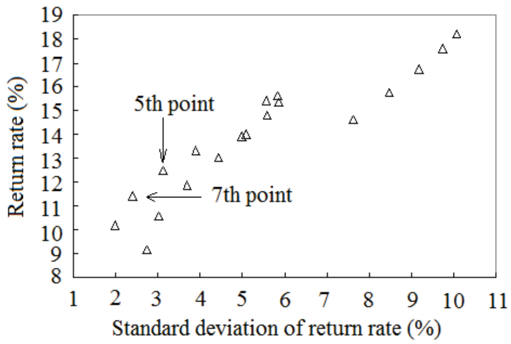


**Figure 1** Return rate vs risk at discrete sampling points

## 3 MULTI-OBJECTIVE OPTIMIZATION OF PROJECTS

In this section, the probability-based multi-objective optimization method is used to study the collaborative optimization of engineering projects rationally.

In the example of engineering project, the selection involves six new schemes and an original plan; the duration, resource consumption and total cost of each scheme are as the objectives to be optimized, see Tab. 3 [6]. Moreover, the weight factors of duration, resource consumption and total cost are 0.23, 0.07 and 0.70, respectively.

According to the meaning of the attributes, the duration, resource consumption and total cost are all unbeneficial indicators. Tab. 4 shows the evaluation results of this project.

The evaluation results show that scheme No. 4 is with the highest overall preferable probability, so it can be selected as the optimal scheme for this problem.

**Table 3** Six schemes and original plan of the project, and attribute values

| Scheme | Duration, A (day) | Resource consumption factor, *B* | Total cost, *C* (10K RMB) |
|---|---|---|---|
| 1 | 24 | 3.942 | 55.8 |
| 2 | 21 | 2.916 | 55.5 |
| 3 | 20 | 3.048 | 55.4 |
| 4 | 18 | 1.883 | 55.9 |
| 5 | 16 | 1.928 | 59.4 |
| 6 | 14 | 1.886 | 64.4 |
| Original | 26 | 4.876 | 57.0 |

**Table 4** Evaluation results of engineering projects

| Scheme | Partial preferable probability | | | Overall preferable probability | Rank |
|---|---|---|---|---|---|
| | $P_A$ | $P_B$ | $P_C$ | $P_t$×10 | |
| 1 | 0.1135 | 0.1050 | 0.1471 | 1.3532 | 6 |
| 2 | 0.1348 | 0.1432 | 0.1477 | 1.4434 | 4 |
| 3 | 0.1418 | 0.1383 | 0.1480 | 1.4585 | 3 |
| 4 | 0.1560 | 0.1817 | 0.1468 | 1.5113 | 1 |
| 5 | 0.1702 | 0.1800 | 0.1388 | 1.4813 | 2 |
| 6 | 0.1844 | 0.1816 | 0.1273 | 1.4211 | 5 |
| Original | 0.0993 | 0.0702 | 0.1443 | 1.2590 | 7 |

## 4 CONCLUSION

In this paper, the hybrid of the probability - based multi – objective optimization with the uniform design for experiments of mixtures is performed to solve the multi-objective optimization problems of portfolio investment and project management properly. The results show the rationality of the hybrid, which has bright future in applications in many fields.

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