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Shear influence from the designer's point of view

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SUMMARY

The trend to design orientation in current FE-Applications and the resulting requirements will be discussed in this paper. With this trend in mind the discussion is focused on appropriate finite element selection regarding the type of element and the polynomial degree of the basic shape function. There will be a special look at bending load case and shear influence. The quality of results using different 3D finite elements are compared and tested for practical use. Recommendations for designers concerning these fields are provided. The drawn conclusions are applied to real-life design parts and the advantages of this procedure for design oriented FEA are shown.

Key words: *FEA, 3D finite element, bending, shear, CAD, shape functions.*

1. INTRODUCTION

Today, the analyzing tasks in the companies are distributed in a new way: Simple and repeating FEA tasks are more and more often performed by the designer. The more difficult and complex ones remain to the analysis engineer. This development has been done to shorten the product development process. If the designer analyzes structures by himself, the typical time consuming iteration cycles in the product development process are reduced. Therefore the product development process can be shortened as a whole. Consequently, the performance of FE-Analyses must be also regarded from the designer's point of view [1, 2].

As a result of a questionnaire asking Northern Bavarian FEA-Applicants *90 %* of the companies are performing static analyses (Table 1), Ref. [3]. Therefore there is a high possibility to shorten iteration cycles, if the designer performs static analysis for simple structures by himself.

Table 1 Results of the questionnaire: Frequency of analysis types (Doubles were possible) [3]

Type of Analysis	*Per Cent (%)*
Static	*90*
Thermal	*20*
Contact	*25*
Dynamic	*10*
Molding	*10*
Fluid	*15*
Optimization	*25*
Modal	*25*
Forming	*10*
Material	*10*

Another result of the questionnaire was the CAD-Environment within which FE-Analyses are performed. Table 2 shows that *86 %* of static analyses are performed in a 3D-CAD environment. Already today and even more in the future this 3D-CAD environment will provide 3D FE-Models, even if simplifying is possible. Thus the time for simplifying structures will be more expensive than analysis time. In addition, the direct exchange of models between CAD and FEA will reduce time consuming adjustments. Therefore it will also shorten the product development process. This will force a more frequent use of three dimensional finite elements like tetrahedrons and hexahedrons, which are the main focus of this paper.

Table 2 Results of the questionnaire: CAD-Environment performing static analyses [3]

CAD-Environment	*Per Cent (%)*
2D	14
3D	86

Another main focus of this paper is the bending load case. Regarding real-life engineering problems and linear-static analyses the bending load case is a very common one. Additionally, in most cases this load case is the most critical one, which is responsible for a possible damage of a design part. If there is a superposition of several load cases like pressure, bending or torsion, the bending influence is often the most important one.

2. STATE OF THE ART

2.1 Design oriented FEA

The software companies have already reacted to the above mentioned trend that simple and repeating FEA tasks are more and more performed by designers and the more difficult and complex ones remain to the analysis engineer. They have introduced simplified, defeatured and more design oriented FEA-Programs in addition to their traditional multi-purpose FEA-Software [4, 5]. Similar to the development of CAD-Programs, FEA-Software seems to be also separated into high-end and mid-range software. Using mid-range products for simple and repeating tasks during the design process and high-end products for complex or high sophisticated analysis will be the future [6].

These mid-range FEA-Programs provide different approaches to bringing FEA nearer to designers. One feature is the reduction of parameters needed for FEA-Preprocessing (Figure 1). To reach this feature some of them offer a sequentially controlled menu prompt [4]. Only the parameters, which are relevant for the current step of FEA-Preprocessing, are offered to the user. Thus a rational sequence of FEA-Modelling is implemented.

Another possibility is the integration of design oriented FEA-Programs in CAD-Systems [7]. In addition to an adjusted graphical user interface the exchange of data is easier. Thus FE-Analyses are performed more frequently, the time of iteration cycles is reduced and training courses are minimized.

To reduce the variety of parameters (Figure 1) repeating parts are prepared by analysis engineers. They provide preprocessing procedures for meshing, element selection, applying boundary conditions and solving. Designers can vary certain parameters of the model, usually the parameters for geometry and boundary conditions [5, 8, 9].

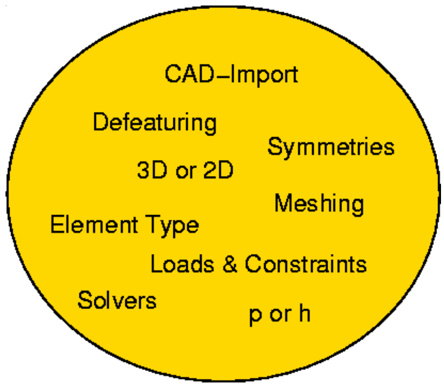

Fig. 1 FE-Preprocessing Parameters

The mid-range FEA-Programs additionally offer a higher level of automation. This is especially helpful for the meshing procedure. But the automatic meshing does not provide high quality meshes every time. They often allow only interpretations according to qualitative aspects and not to quantitative. Especially the analysis of stresses requires adaptive meshing because of local effects, which is only partly offered by this type of programs [6].

2.2 Selection of element type and the appropriate shape function

One of the FE-Preprocessing Parameters mentioned in Figure 1 is the selection of the element type, which is also related to the meshing procedure. Finite elements can be classified by their geometrical shape and their dimension. They can be one-, two- or three-dimensional. In the three-dimensional case, for instance, they can be tetrahedrons, hexahedrons or pentahedrons according to their shape.

The shape functions of a finite element are based on Eq. (1), which expresses the displacement within a finite element with the displacements at the nodes of the element. In this equation u_i are the displacements at the nodes and accordingly N_i are the shape functions:

$$u^{(e)}(x,y) = \sum_{i=1}^{p} u_i^{(e)} N_i^{(e)}(x,y) \tag{1}$$

The main requirement for a shape function is the interpolation between the nodes of the element:

$$N_i^{(e)}\left(x_j^{(e)}, y_j^{(e)}\right) = \begin{cases} 1 & \text{for} \quad j = i \\ 2 & \text{for} \quad j \neq 1 \end{cases} \tag{2}$$

To satisfy this requirement you can use several shape functions. Usually polynomials are preferred. They differ according to the polynomial degree and the completeness:

$$u(x,y)=c_1+c_2x+c_3y+c_4xy \tag{3}$$

$$u(x,y)=c_1+c_2x+c_3y+c_4x\,y+c_5x^2+c_6y^2+c_7x^2y+c_8xy^2 \tag{4}$$

Polynomial in Eq. (3) would be a linear shape function and polynomial in Eq. (4) a quadratic shape function, because it contains also terms of second polynomial degree. Polynomial in Eq. (4) is incomplete, because the term $c_9x^2y^2$ is missed. On the other hand polynomial in Eq. (3) is complete (Element of Lagrange Type) [10, 11, 12].

The element type and the basic shape function influence the result quality [1,13]. Especially looking at bending load cases several authors recommend the usage of finite elements with quadratic shape functions instead of linear shape functions [1,14]. Additionally other authors recommend the usage of hexahedrons instead of tetrahedrons [13].

3. BENDING WITHOUT SIGNIFICANT SHEAR INFLUENCE

The above mentioned points are examined using simple examples. The first one is the beam in Figure 2. This beam is fixed on the left hand side and is loaded with a shear force of *1000 N* on the right hand side. In addition, the beam has got a squared cross-section of *100 mm* $\times$ *100 mm*, a length of *2000 mm* and a resulting moment of inertia of about $8{,}333 * 10^6$ mm^4. The material is steel with the module of elasticity of *206.000* N/mm^2 and a Poisson's Ratio of *0.3*. According to these data the displacement at the right hand side will be *1.524 mm* using Eq. (5). The result is in the range of a kinematic linear problem without a significant shear influence because of a high length to diameter ratio.

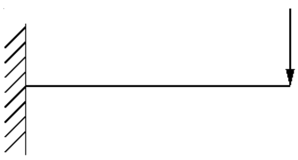

Fig. 2 Example 1: Overhanging Beam loaded with Bending

$$w(x)=\frac{Fx^3}{3EI_b} \tag{5}$$

This analytic solution is compared with a three dimensional elastic continuum. The finite elements used here are tetrahedrons and hexahedrons with linear shape functions (*Tet4*, *Hex8*) as well as quadratic shape functions (*Tet10*, *Hex20*). Table 3 shows the relative error compared with the above mentioned analytic result for each element.

Table 3 Bending results: maximum displacement at the right end of the beam

Type of Element	*Number of Elements*	*Relative Error in %*
Tet4	*18029*	*9,0*
Tet10	*17*	*8,0*
	120	*1,0*
Hex8	*4000*	*1,4*
Hex20	*10*	*1,6*

The result quality and even more the efficiency of the used elements differ a lot. Elements with quadratic shape functions (*Tet10*, *Hex20*) have got a higher result quality and a higher efficiency than elements with linear shape functions (*Tet4*, *Hex8*). For nearly the same result quality *18029 Tet4* elements are needed but only *17 Tet10* elements, thus the analysis time will be much higher for *Tet4* elements. Looking at hexahedrons it is the same: *4000 Hex8* elements provide a similar result quality as *10 Hex20* ones. Comparing the element type hexahedrons are more efficient than tetrahedrons: *120 Tet10* to only *10 Hex20*. *Tet4* elements provide a poor result quality, even using large numbers of them.

If the same structure is loaded with traction instead of bending (Figure 3), the results are completely different (Table 4). The results have got a high quality, independent of the element type and the used shape function. *Tet4* and *Hex8* elements are more efficient in this case, because of a shorter analysis time.

Fig. 3 Example 2: Overhanging Beam loaded with Traction

Table 4 Traction results: maximum displacement at the right end of the beam

Type of Element	*Number of Elements*	*Relative Error in %*
Tet4	*240*	*-0,4*
Tet10	*240*	*-0,2*
Hex8	*10*	*-0,4*
Hex20	*10*	*-0,3*

To get accurate results and reasonable analysis times, the element type and even more the shape function are important parameters. The appropriate shape function depends on the current load case: quadratic ones for bending and linear ones for tension. The influence of the element type is less important. Thus it should be better selected according to geometric aspects.

This conclusion is checked with a two dimensional structure by a computer algebra system. At first the structure is loaded with traction then with bending.

Figure 4 shows both load cases together. The structure is analyzed by the computer algebra system MuPAD [16].

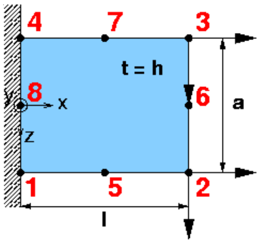

Fig. 4 Example 3: 2D-Model loaded with bending and traction

The computer algebra system offers a parametrized analysis of the problem. The resulting formula can be compared with the analytic formula as in Eq. (5). The condition for the analytic formula is that the length to diameter ratio is more than five. In the examples 1 and 2 the ratio is *20*. According to this, the results of the computer algebra system are simplified because of $l>>a$. After this simplification the results consist of a constant factor multiplied with a stiffness term and a function of ϑ, which means the Poisson Ratio (Table 5). The lower order element (PlaneStress4) provides only a linear dependency between the displacement and the length l. However, the higher order element (PlaneStress8) is able to describe a cubic proportion between the displacement and the length, which is necessary to fulfil the requirements of the analytical approach, mentioned in the last row of Table 5.

Table 5 MuPAD-Results: 2D-model loaded with bending and traction

Type of Element	Traction	Bending
PlaneStress4	$\frac{Fl}{Eah}\cdot f(\vartheta)$	$8\cdot\frac{Fl}{Eah}\cdot(1+\vartheta)$
PlaneStress8	$\frac{Fl}{Eah}\cdot f(\vartheta)$	$3\cdot\frac{Fl^3}{Ea^3h}\cdot f(\vartheta)$
Analytic	$\frac{Fl}{Eah}$	$4\cdot\frac{Fl^3}{Ea^3h}$

The results of this approach show again that elements with linear shape functions cannot be used for an analysis, which consists of a main bending influence. Vice versa it is not necessary to use elements with quadratic shape functions for traction load cases. It consumes just more time for modelling and analysis.

4. BENDING WITH SIGNIFICANT SHEAR INFLUENCE

If the length of the beam in Figure 1 is shortened to *200 mm*, the ratio l/d is *2* instead of *20*. In the example 4, shear will have got a significant influence. The resulting displacement at $x=l$ can be separated in the displacement resulting from bending w_B and resulting from shear w_S [17]:

$$w(l)=w_B(l)+w_S(l)=\frac{Fl^3}{3EI}+\frac{Fl}{GA_s}=\frac{Fl^3}{3EI}\left(1+\frac{3EI}{GA_sl^2}\right) \quad (6)$$

The difference to Eq. (5) is expressed by the last term of Eq. (6). This term can be changed using the relation between modulus of elasticity and modulus of shear, which is $G=E/(2(1+\upsilon))$. For the squared cross section of the example 4, A_s will be expressed in this way: $A_s=kA=5/6*a^2$. The moment of inertia is $I=a^4/12$ for this cross section. In addition, the used material is steel with a Poisson's Ratio of $\upsilon=0.3$:

$$\frac{3EI}{GA_sl^2}=\frac{3}{5}\cdot\frac{a^2}{l^2}\cdot(1+\upsilon)=0.78\cdot\frac{a^2}{l^2} \quad (7)$$

Using Eq. (7), the shear influence compared to bending is *0.2 %* for example 1. In contrast to this, the shear influence of example 4 is about *20 %*.

Table 6 shows the results of example 4 regarding different finite elements and shape functions. Again, quadratic shape functions (*Tet10*, *Hex20*) are more efficient than linear shape functions (*Tet4*, *Hex8*), and hexahedrons are more efficient than tetrahedrons, because less elements are needed and thus analysis and modelling time is shortened. But even when the number of *Hex20* elements increased, a relative error of more than *2 %* remains.

Table 6 Example 4: maximum displacements at the right end of a short beam

Type of Element	Number of Elements	Relative Error in %
Tet4	82944	-2,7
Tet10	384	-2,9
Hex8	3456	-3,0
Hex20	128	-2,8
	1024	-2,3

Looking at the two dimensional example 5, the difference of quadratic and cubic shape functions are checked. Table 7 shows some results out of analysis with the free FE-System *Z88*, which implements plane stress elements with quadratic and cubic shape functions [15]. The difference between these shape functions is small.

Table 7 Example 5: maximum displacements at the right end of a two dimensional short beam

Type of Element	Polynomial Degree	Number of Elements	Relative error in %	
			middle	surface
7	quadratic	32	-1,8	-0,5
11	cubic	32	-1,6	-0,3

All the mentioned results are compared with the analytical approach of Eq. (6). But this approach is also incomplete. After the deformation, the cross sections are not plane any more, which is an assumption of the analytical approach [17]. The curvature of the cross sections (Figure 5) is not considered in the mentioned analytical approach.

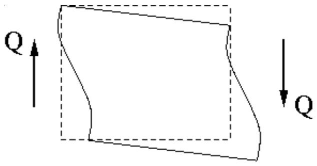

Fig. 5 Cross sections after deformation of a beam [17]

5. CONCLUSION AND EXAMPLES

Design oriented FEA will be higher integrated and thus closer to the product development process. That means too that inexperienced users like designers work with sophisticated FE-Systems. Thus new requirements arise for these systems.

On the one hand, other, simplified types of FE-Systems could be a solution. But on the other hand, some support considering procedures and connections will be necessary. An example for these could be the above mentioned connection with polynomial degree of shape functions and main load case of the design part: If the bending load case is the main one, elements based on quadratic shape functions should be selected to get a reasonable result quality or analysis time. If there is a tension load case, elements with linear shape functions should be selected.

Looking at the important bending load case, two other questions come into mind: Is there any difference to the rules above, if there is a significant shear influence in the design part? Could reduced integration help to get high quality results in combination with less analysis time?

But the above mentioned rules are also correct for significant shear influence. Even so, quadratic shape functions should be used regarding a bending load case. It is a different matter with the influence of integration order: The problem of this method for the designer is that he cannot control or estimate the results in a reasonable manner. Moreover, reduced integration has got some other impacts like hourglass-modes, which provide difficulties even for analysis engineers.

All the results presented above are based on displacement, not on tension. Of course, tensions are very important for the dimensioning of design parts. But in many cases, the main criteria for dimensioning are rather displacements than tensions. The problem is often not the displacement of the design part, but the displacement of the part, which is manufactured by the design part. These displacements directly influence the product quality, which is usually a very important aspect.

One example for this kind of design part is the press frame in Figure 6. In the figure, the tool, which moves up and down in the middle of the frame is missing. Moving downward, it forms a sheet metal, which is situated on the bed of the press.

The press frame consists of two dimensioning problems related to displacements. The frame should be very stiff to minimize displacements in all Cartesian coordinates. At first, the displacements in the direction of the moving tool are important. If there are too large displacements in this direction, the measures of the formed sheet metal are not within the correct range. The product quality requirements cannot be satisfied. Secondly, the displacements in the other two Cartesian coordinates are important: If the porters of the press frame buckle too much, the tool gets stuck. This can cause a breakdown of the whole press and probably of the whole assembly line.

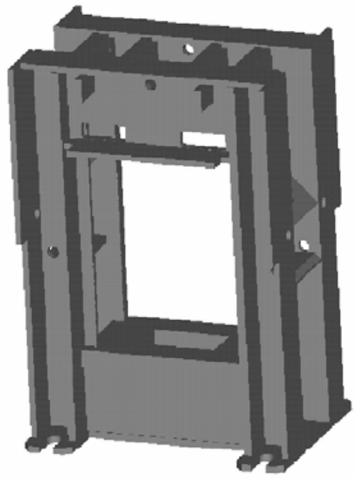

Fig. 6 Example 6: Press frame

The main load is the pressure of the tool during the forming of the sheet metal. This load causes a bending load case resulting in a displacement of the porters, the cross head and the bed. According to the bending load case and the above mentioned connections, a designer should use elements with quadratic shape functions, independent of shear influence. Additionally, he should use elements with full integration. The designer wants to mesh the part automatically to reduce modelling time. Thus, he uses *Tet10* elements instead of *Hex20*.

Table 8 shows the FEA results compared with experimental ones regarding the different element types. *Tet10* elements provide a higher result quality and a faster convergence. Therefore, quadratic shape functions are more reliable and efficient, if bending is the main load case.

Table 8 Example 6: results of experiment and FEA looking at the direction of the moving tool

Type of Element	*Number of Elements*	*Displacement in μm*	*Relative Error in %*
Tet4	*13609*	*184*	*-19,3*
Tet10	*13609*	*211*	*-7,5*
Tet4	*27352*	*205*	*-10,1*
Tet10	*27352*	*213*	*-6,6*
Experiment		*228*	*-*

The connecting rod of example 7 is loaded by pressure and traction (Figure 7). Therefore, traction is the main load case and the designer should use elements with linear shape functions for his FE-Analysis according to the considerations above.

Fig. 7 Example 7: Connecting rod

Due to automatic meshing the designer selects *Tet4* elements, which is a reasonable choice looking at the results presented by Table 9. To minimize analysis and modelling time by reasonable result quality, *Tet4* elements have got advantages in contrast to *Tet10* elements. The difference between experimental and analytical results is caused by an incomplete geometry representation. The change of the cross section measures over the length can be neglected. This logarithmic influence is very small, because the reduction of the cross section plane is about *5 %*.

Table 9 Example 7: results of experiment and FEA looking at the axial direction of the connecting rod

Type of Element	***Axial Displacement in μm***	***Relative Error in %***	
		experimental	*analytic*
Tet4	*29,60*	*-8,6*	*-1,2*
Tet10	*29,65*	*-8,5*	*-1,0*
experimental	*32,4*	-	-
analytic	*29,955*	-	-

Thus, the proposed procedures for the inexperienced users are correct, useful and practice-oriented.

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UTJECAJ SILE POSMIKA SA STANOVIŠTA PROJEKTANATA

SAŽETAK

U ovom radu opisuju se smjernice projektiranja u tekućim FE-Aplikacijama kao i zahtjevi koji iz toga proizlaze. Imajući na umu ovaj trend, opis se usredotočuje na primjereni odabir konačnog elementa s obzirom na tip elementa kao i na stupanj polinoma osnovne funkcije oblika. Posebno se osvrće na slučaj savijanja i na utjecaj posmika. Valjanost rezultata, koristeći različite 3D konačne elemente, uspoređuje se i testira za praktičnu uporabu. Daju se i preporuke projektantima koji se bave ovim problemima. Zaključci se primjenjuju na projekte iz stvarnog života. Iznose se i prednosti ove metode za FEA orijentirane projekte.

Ključne riječi: *FEA, 3D konačni element, savijanje, posmik, CAD, bazne funkcije.*