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Inhomogeneity and forming limits of copper sheets

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SUMMARY

In many calculations of the forming limit diagram (FLD) of sheet metal, based on the M-K theory, the imperfection parameter f, was chosen arbitrarily or for the best fit of calculations with experimental results. The main purpose of this work was to replace fitting inhomogeneity coefficient by a measurable one. In the proposed model it was assumed that the material inhomogeneity was the result of surface roughness and internal defects (voids). It was also assumed that the value of inhomogeneity coefficient changes with the increasing strain. The experiments were carried out for the copper sheets, annealed to produce different material structure. As a result of the experimental work original equations were proposed describing the relationship between inhomogeneity coefficients and effective strain and grain size. These equations were used in the theoretical calculations of the FLD. The theoretical calculations of the strain limits of the tested sheets were based on the associated flow rules, assuming strain hardening and strain softening process.

Key words: *forming limit diagram (FLD), M-K theory, inhomogeneity coefficient, copper sheets.*

1. INTRODUCTION

In stamping operations, where the sheet metal is subjected to biaxial stretching, the occurrence of non-uniform strains within a small region of the sheet results in the formation of local necking leading to fracture. The current interest in understanding sheet metal formability has led to several theoretical analyses of localized necking based on different criteria. These localized necking criteria include: a localized zone along a direction of zero-extension [1], materials imperfections [2] and the presence of a vertex on the yield surface [3].

The Hill's theory [1] predicts that the maximum principal strain ε^* prior to localized necking (i.e. the limit strain) has a magnitude of strain hardening exponent, $\varepsilon^*=n$, at the plane strain and increases to, $\varepsilon^*=(1+r)n$, for the uniaxial tension deformation of sheet exhibiting normal anisotropy with a plastic anisotropy factor r, which is defined as the ratio of width to thickness strain of sheet specimen deformed under uniaxial tension. Hill's theory, however, can not explain the phenomenon of localized necking under biaxial stretching conditions.

Stören and Rice [3] proposed that an instability in the material constitutive equation mirroring effects arising from the discrete nature of crystallographic slip, can give rise to a vertex on the yield surface which in turn can generate a bifurcation in the state of uniform plastic deformation even when the pre-bifurcation deformation pattern contains no in-plane direction of zero-extension; a pre-requisite for local necking is that the vertex becomes sufficiently sharp to locally satisfy the plane conditions. The presence of a vertex on the yield surface, however, experimentally has not been confirmed yet.

The hypothesis that the localized necking in the stretching regime ($\rho=\varepsilon_2/\varepsilon_1>0$) initiates from material imperfections was first proposed by Marciniak and Kuczyñski (M-K) [2]. Originally the M-K analysis assumes the presence of material imperfections in the form of a linear groove, which lies parallel to the ε_2

direction. This hypothesis was extended to the $\rho<0$ regime, when there becomes some orientation ψ with respect to the principal strain. Imposing the same ε_2 inside and outside the groove while proportional straining is maintained outside the groove, M-K have shown that deformation within the groove occurs at a faster rate than the rest of the sheet. The concentration of strain within the groove eventually leads to the plane strain conditions within the groove and localized necking. In many calculations the imperfection parameter, f, which was defined as the ratio of the sheet thickness in the groove to sheet thickness outside the groove, was chosen arbitrarily or for the best fit of calculations with experimental results of the limit strains. The M-K model can be readily adopted to represent the influence of different kinds of material inhomogeneity provided that the assumption that the weak region in the form of a long band is retained [4, 5]. According to the Jonas and Baudelet work [6], in the case of the two forms of external material defects i.e. mechanical damage involving work hardening and geometric faults due to machining variations, machining defects grow from the initiation of flow and their influence on flow localization is in an order of magnitude greater than that of mechanical defects. Chu and Needleman [7] assumed that the week region is a band containing a higher concentration of voids than that in the adjacent areas of the sheet and their analyses of strain localization are based on the constitutive relationships for porous plastic materials. The surface roughness is the second form of machining defects and it was found that the roughness of free surface of the material increases approximately in proportion to the magnitude of effective strain and to grain diameter [8].

The objective of this paper is to delineate the roles of void growth and surface roughness growth processes on the position and shape of forming limit curve (FLC) of sheet metal. Theoretical calculations will be performed on the basis of the M-K analysis using the flow theory of plasticity in conjunction with different forms of yield function - quadratic Hill's yield function and yield function of porous plastic materials describing the material softening due to void growth. The calculated FLCs will be compared with the experimental results.

2. MATERIALS AND MECHANICAL TESTING

The tests were based on the *1.0 mm* thick copper sheet annealed to produce different grain size. Final grain size has been varied by annealing in four different time periods at the temperature of *923 K*. Average grain diameters (Table 1) have been measured by the linear intercept method.

Table 1. Mechanical properties of the copper sheets

Sheet No.	Average grain diameter	Tensile test			Equibiaxial test	
		Strain hardening exponent	Strain hardening coefficient	Plastic anisotropy ratio	Strain hardening exponent	Strain hardening coefficient
	d, μm	n_t	K_t, MPa	r	n_b	K_b, MPa
Cu-1	48.8	0.48	481	1.06	0.43	505
Cu-2	79.7	0.50	542	1.11	0.44	530
Cu-3	110.7	0.52	500	1.21	0.44	521
Cu-4	138.1	0.53	521	1.22	0.43	518

Tensile samples, *100 mm* in gauge length and *20 mm* wide were prepared from the strips cut at 0°, 45° and 90° to the rolling direction of the sheets. The effective stress - effective strain relation was described using the Hollomon's model. The plastic anisotropy factor r has been determined on the basis of the relationship between the width strain and thickness strain in the whole range of straining using the method proposed by Welch et al. [9].

Since the value of strain-hardening exponent of the copper sheets depends on the strain state conditions, the hydraulic bulge test has been performed using circular die (equibiaxial stretching).

The mechanical properties obtained on the basis of both the tensile and equibiaxial testing (Table 1) show that:

- the mechanical properties of copper sheets obtained in the uniaxial tensile test visibly depend on the grain size,
- the values of the strain hardening exponent obtained in the equibiaxial stretching test n_b are smaller than that of tensile n_t and are almost independent of the grain size,
- the value of the plastic anisotropy factor r increases with the increasing grain size.

In the present investigation the FLDs were determined in-plane stretching test over flat-bottomed rigid punch using the method proposed by Marciniak et al. [10]. Sheet blanks *250 mm* in length and successively narrower widths enabled to produce different strain ratio. A circular grid was imposed on the sheet surface with circle diameter of *2.42 mm*. The driving blanks were prepared from a good quality deep drawing steel sheet. The central hole in the driving blanks was *52 mm* in diameter. The test was continued until a crack was visible and at that moment the test was interrupted. For each specimen the true major ε_1 and minor ε_2 strains were measured on circles adjacent to the crack (visible necking) but not crossing it. It means that the circles include the relatively homogeneously strained area away from the crack and in some cases a part of the sharp neck. The presence of a few cracks on the gauge area of specimens confirmed homogeneous straining.

3. DEVELOPMENT OF SURFACE ROUGHNESS

Evidence of strain inhomogeneity caused by grain anisotropy is available at the sheet surface and its development in incremental stretching can be followed using a stylus instrument. Fukuda et al. [8] measured the roughening which developed at a free surface during stretching of sheets with strain ratio ρ in the range $+1$ to -1 and found that the surface roughness parameter R was close being proportional to the effective strain and grain diameter independently of ρ, thus:

$$R=R^0+Sd\varepsilon_e \tag{1}$$

where R^0 is the value of initial surface roughness, S is a material constant which depends on slip characteristic and d is average grain size.

Figure 1 illustrates the development of surface roughness (Taylor-Hobson Surtronic 3 instrument was used to measure surface roughness) in the tensile tension and biaxial stretching of the copper sheets.

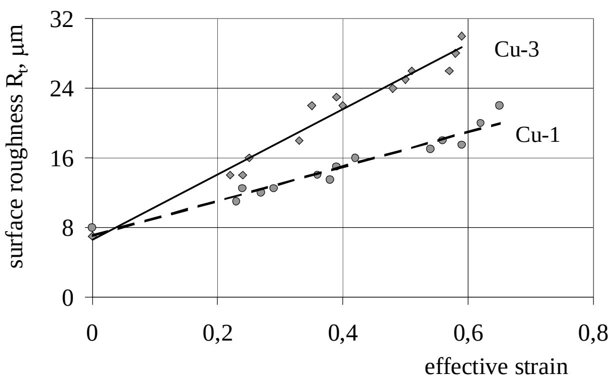

Fig. 1 The value of surface roughness parameter R_t as a function of effective strain, after incremental straining of copper sheets in the tensile tension test and biaxial stretching

The relationship between the R_t value and effective strain was linear in the whole range of straining. The results obtained in the tensile test were independent of specimen orientation and were in a good agreement with the results obtained from the surface profile measurement on the specimen after biaxial stretching. The value of coefficient S was grain size dependent and because of this it could not be treated as a material constant. In the case of copper sheets with various grain size, the value of the S was in the range of $S=0.691\div0.428$. The relationship between surface roughness and grain size could be better described using modified equation in the form of:

$$R=R^0+kd^{0.5}\varepsilon_e \tag{2}$$

In the case of copper sheets the k-value, in the range of $k=4.58\pm0.15\ \mu m^{0.5}$, could be treated as independent grain size.

4. VOID GROWTH DURING METAL FORMING

It is well established that during cold plastic working the ductile fracture occurs by nucleation, growth and coalescence of voids. It is very important to notice that voids can be nucleated in the early stage of plastic deformation. Even, in the case of sheets, damage due to cold rolling is often present previous to any plastic test. Several experimental methods are used to characterize damage. Some methods can assess absolute value of damage while others can only measure the increase of damage relative to the initial stage. In present investigation the voids growth during different forming operations will be determined in the microscopic observation and relative density change measurement.

An optical microscope has been used to the observation on polished surfaces performed both at the sheet plane and perpendicular to the sheet surface. No chemical etching or electropolishing has been used in order to prevent enlargement of voids. Different material defects i.e. voids and particles, have been observed in the form of dark fields with different shape and size. The material defects were counted and measured. The mean defect (dark field) diameter d_d was determined by the intercept method. Since it was difficult to discern the nature of defects - if they were voids or particles - the increase in the volume fraction of voids caused by the sheet straining was calculated as:

$$f_\upsilon = f_d^{\,t} - f_d^{\,0} \tag{3}$$

were $f_d^{\,t}$ and $f_d^{\,0}$ are volume fractions of defects for deformed and undeformed sheet respectively.

In such a way the results of microscopic observations could be compared with the results of relative density change measurement:

$$f_\upsilon \approx -\Delta\gamma/\gamma_0 \tag{4}$$

The presence of the internal voids causes some changes in the density of the material. The relative density change was determined using the Ratcliff's technique, with an undeformed sample as dummy.

The experimental results of microscopic observation of volume fraction of voids and relative density change measurement have shown that these two methods of investigation are complementary to one another. The increase in volume fraction of voids was strongly influenced by the grain size of deformed sheets.

Presentation of the experimental results in the half-logarithmic coordinate (Figure 2) illustrates linear relationship between $ln\ f_d^{\,t}$ and effective strain. The relationship between volume fraction of material defects and the effective strain and grain size can be expressed as:

$$f_t^{\,t} = f_d^{\,0} exp(\lambda\varepsilon_e d) \tag{5}$$

where coefficient λ could be treated as a material constant and in the case of the copper sheet amounts $\lambda=21.5\ mm^{-1}$.

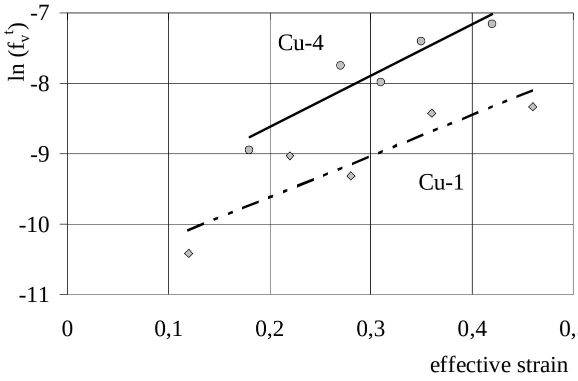

Fig. 2 Volume fraction of material defects as a function of effective strain of copper sheets deformed in uniaxial tension and equibiaxial stretching

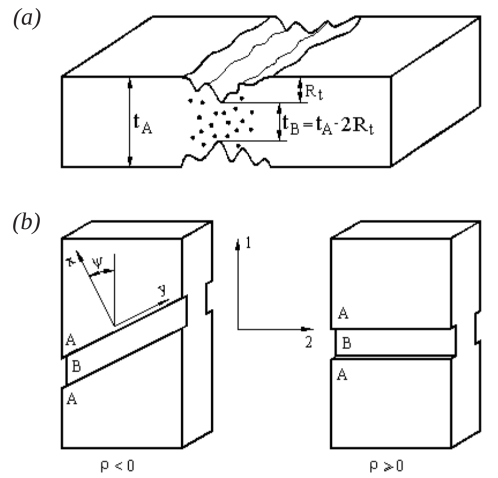

Fig. 3 Schematic representation of the material model: (a) sheet element with geometric imperfections caused by surface roughness and internal defects concentrated in weak band B; (b) orientation of the weak band B according to principal strain

5. COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL FLC'S

According to the original M-K theory a sheet element was divided into two parts - region A with no material defects and weak region B that is initially inclined at an angle ψ_0 (Figure 3). The material in the band *B* is softened due to the presence of surface dimples and internal voids. It is assumed that sheet metal characterize isotropic hardening and Hollomon type effective stress - effective strain relation $\sigma = K\varepsilon^n$. To describe the biaxial behaviour of the material in region *A* the quadratic Hill's yield function was used. Assuming normal anisotropy, for this yield criterion, the effective stress and effective strain, for plane strain condition ($\sigma_3 = 0$) are given by [11]:

$$\sigma_e = \left[\frac{3}{2}\frac{r+1}{r+2}\left(1 - \frac{2r}{1+r}\alpha + \alpha^2 \right) \right]^{0.5} \sigma_1 \qquad (6)$$

$$d\varepsilon_e = \left[\frac{2}{3}\frac{(2+r)(1+r)}{1+2r}\left(1 + \frac{2r}{1+r}\rho + \rho^2 \right) \right]^{0.5} d\varepsilon_1 \qquad (7)$$

where:

$$\alpha = \frac{\sigma_2}{\sigma_1},\ \alpha = \frac{d\varepsilon_2}{d\varepsilon_1}.$$

To describe the plastic flow of the material in weak region *B* the yield function for porous media was used in its general form given by:

$$\sigma_e^2 = \beta_1 \sigma_p^2 + \beta_2 \sigma_m^2 \qquad (8)$$

where:

σ_e – effective stress,
σ_p – flow stress of the matrix material,
σ_m – mean stress,
β_1, β_2 – functions of the volume fraction of voids f_v in the material.

If we assume the normality rule proposed by Drucker, the associated flow rules with above mentioned form of yield criterion can be written as follows:

$$\frac{d\varepsilon_x}{(a+\beta)\sigma_x - (1-\beta)\sigma_y} = \frac{d\varepsilon_y}{(a+\beta)\sigma_y - (1-\beta)\sigma_x} = \frac{r}{2(2r+1)}\frac{d\varepsilon_{xy}}{\tau_{xy}} = \frac{3r}{2(2+r)}\frac{d\varepsilon_e}{\sigma_e} \qquad (9)$$

where: $a = (1+r)/r$; $\beta = 2r/3(2+r)\beta_1$.

It is assumed that for a material without internal defects Eq. (9) reduces to the convectional equation. That means that the stress and strain states in region *A* could be described using the same equations as for region *B*, assuming $f_v = 0$. The stress and strain fields in the regions *A* and *B* are interrelated by:
– the equilibrium conditions:

$$\sigma_x = \frac{t_A}{t_B}(\sigma_1 \cos^2\psi + \sigma_2 \sin^2\psi) \qquad (10)$$

$$\tau_{xy} = \frac{t_A}{t_B}(\sigma_1 - \sigma_2)\sin\psi \cos\psi \qquad (11)$$

– the kinematic constrains:

$$d\varepsilon_y = d\varepsilon_1 \sin^2\psi + d\varepsilon_2 \cos^2\psi \qquad (12)$$

$$d\varepsilon_{2A} = d\varepsilon_{2B} \qquad (13)$$

The kinematic relations, Eqs. (12) and (13), are a consequence of the condition, that the strain rate components should not be discontinuous at the passage from region *A* to region *B*.

From the geometry of the material element (Figure 3) and initial inclination of the band *B* the current inclination ψ of the band is found to be:

$$tg\psi = tg\psi_0 \exp(\varepsilon_1 - \varepsilon_2) \qquad (14)$$

It may be noted that for $\psi_0 = 0$, i.e. for biaxial stretching, from Eq. (14) becomes $\psi = 0$ and Eqs. (10)

and (12) reduces to the equations in the original M-K analysis, see Ref. [2].

The solution to the M-K problem was achieved in a straight-forward incremental numerical procedure of calculations. In our calculations of the FLDs we have used no fitting parameters to describe the inhomogeneity of the material but we have based on the experimentally obtained Eqs. (2), (3) and (5). In the beginning of our calculations we have tested different forms of yield functions of porous plastic materials i.e. that proposed by Shima-Oyane [12]:

$$\Phi_{S-O} = \frac{\sigma_e^2}{\sigma_p^2} + 6.2\, f_\upsilon^{1.028}\left(\frac{\sigma_m^2}{\sigma_p^2}\right) - (1 - f_\upsilon)^5 = 0 \qquad (15)$$

– Tvergaard [13]:

$$\Phi_T = \frac{\sigma_e^2}{\sigma_p^2} + 3 f_\upsilon \cosh\left(\frac{\sigma_m}{\sigma_p}\right) - (1 - 2.25 f_\upsilon^2) = 0 \qquad (16)$$

– Spitzig et al. [14]:

$$\Phi_{S-S-R} = \frac{\sigma_e^2}{\sigma_p^2} + 2 f_\upsilon^m \cosh\left(\frac{3}{2} m \frac{\sigma_m}{\sigma_p}\right) - 1 - f_\upsilon^{2m} = 0 \qquad (17)$$

where $m=(2+n)/3$ is a material constant, and
– Melander [15]:

$$\Phi_M = \frac{\sigma_e^2}{\sigma_p^2} - B_0 + B_1\left(\frac{\sigma_m}{\sigma_p}\right) - B_2\left(\frac{\sigma_m}{\sigma_p}\right)^2 = 0 \qquad (18)$$

where: $B_0=1.06-3.45 f_v$, $B_1=-0.38+0.032/(f_v+0.05)$, $B_2=-0.09-0.3 f_v$.

The comparison between experimentally determined FLC of the copper sheet and the results of calculations has shown that:

– the best agreement between experimental and calculated values of the limit strains in the whole range of strain ratio $-0.5<\rho<1.0$ was obtained when using the yield function given by Eq. (15) proposed by Shima and Oyane,
– application of the yield function in the forms of Eqs. (16) and (17) underestimate the value of limit strains in the range of deformation near to plane strain,
– application of the yield function in the form of Eq. (18) proposed by Melander overestimates the position of FLC in the whole range of analyzed strain ratio.

The best fit between experimental and calculated limit strains when using the yield function in the form proposed by Shima and Oyane could be the result of the fact that the value of the coefficients of Eq. (15) were determined experimentally on the basis of sintered copper while in the Eqs. (16) and (17) there are still some fitting constants.

Using the above mentioned remarks the FLCs of all the materials tested were calculated and compared with experimental results (Figure 4). The correlation between experimental and calculated results seemed to be satisfactory.

The position of a FLC is often identified with a value of strain-hardening exponent, i.e. it is suggested that the value of the limit strains increases with increasing n-value. The present investigation has shown that in the case of copper sheets the increase in n-value with the grain size increasing is accompanied with the limit strains decreasing.

Since it was interesting which of the two material inhomogeneity components i.e. surface roughness R_t and volume fraction of voids f_v played more important role in the strain localization process and thus effected the value of limit strains, the calculation has been carried out using the following material parameters:
– strain hardening exponent $n=0.50$
– strain hardening coefficient $K=500$ MPa
– plastic anisotropy factor $r=1.2$
– grain diameters $d=16\ \mu m$ and $144\ \mu m$
– sheet thickness $t=1.0$ mm
– surface roughness parameters $R^0=1.8\ \mu m$, $k=4.6\ \mu m^{0.5}$

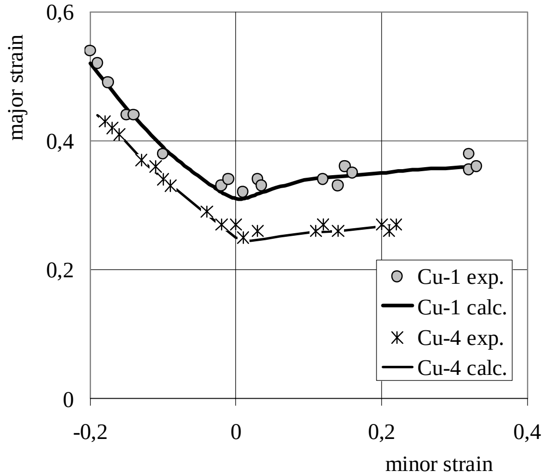

Fig. 4 Comparison between experimentally determined limit strains and calculated FLC of fine grained (Cu-1) and coarse grained (Cu-4) copper sheets

The results of calculation (Table 2) of the limit strain ε^* at plane strain state ($\rho=0$) illustrate that:
– the material inhomogeneity caused by the surface roughness growth is more important in the strain localization process than the inhomogeneity caused by the voids growth,
– the influence of the volume fraction of voids on the FLD of fine-grained material can be neglected, especially when compared with the influence of the surface roughness growth process,

Table 2. Effect of material inhomogeneity parameters on the limit strain at plane strain

Type of sheet material	*The value of inhomogeneity parameters*	*Limit strain ε^**
Fine grained	$f_v \neq 0,\ R_t \neq 0$	0.32
	$f_v = 0,\ R_t \neq 0$	0.33
	$f_v \neq 0,\ R_t = 0$	0.44
Coarse grained	$f_v \neq 0,\ R_t \neq 0$	0.22
	$f_v = 0,\ R_t \neq 0$	0.25
	$f_v \neq 0,\ R_t = 0$	0.31

6. CONCLUSIONS

The forming limit diagram of copper sheets can be determined theoretically based on the mechanical properties obtained in the tensile and equibiaxial testing including the tests which enable to describe the geometric inhomogeneity of the material. The largest grain size the material possesses the smaller limit strains can be achieved due to more intensive surface roughness growth and internal voids growth, especially in the range of biaxial stretching. The beneficial effect of the increasing in the strain-hardening exponent with the grain size increasing is decreased by the growth of material inhomogeneity. The growth of surface roughness seems to be more important factor which affected the level of the FLC than the inhomogeneity component caused by void growth, especially in the case of fine grained materials.

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NEHOMOGENOST I OGRANIČENJA PRI OBLIKOVANJU BAKRENIH PLOČA

SAŽETAK

U mnogim izračunima dijagrama ograničenja oblikovanja metalnih ploča (FLD), koji se temelji na M-K teoriji, parametar nesavršenosti f odabran je proizvoljno da bi najbolje odgovarao izračunima s eksperimentalnim rezultatima. Glavni cilj ovog rada bio je da se zamijeni koeficijent nehomogenosti onim mjerljivim. Predloženi model pretpostavlja da je nehomogenost materijala nastala uslijed neravne površine i unutarnjih nedostataka (šupljina). Pretpostavilo se da se vrijednosti koeficijenata nehomogenosti mijenjaju povećanjem deformacije. Napravljeni su eksperimenti za bakrene ploče koje su se zagrijavale da se postigne različita struktura materijala. Kao rezultat eksperimentiranja predložene su originalne jednadžbe koje opisuju odnos između koeficijenata nehomogenosti i stvarne deformacije i veličine zrna. Te jednadžbe koristile su se u teorijskim izračunima FLD. Teorijski izračuni granica naprezanja testiranih ploča zasnivaju se na pridruženim pravilima tečenja uzimajući u obzir proces deformacijskog očvršćivanja i omekšavanja.

***Ključne riječi**: dijagram oblikovanja metalnih ploča (FLD), M-K teorija, koeficijent nehomogenosti, bakrene ploče.*