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Determination of the steady state response of viscoelastically supported rectangular specially orthotropic plates with varying supported area

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SUMMARY

The influence of the amount of the supported area on the free and forced vibration properties of anisotropic plate is presented. Using the energy based finite difference method, the problem is modelled by a kind of finite difference element. Due to the significiance of the fundamental frequency of the plate, its variation was investigated with respect to the amount of the supported area on the plate, mechanical properties of plate material and translational spring coefficient of supports. The steady state response of viscoelastically supported plates was also investigated numerically for various damping coefficients and amounts of supported areas. Numerical results are obtained to investigate the effect of the ratio of the plate system. In the numerical examples, the natural frequency parameters and steady state responses to a sinusoidally varying force are assessed for the fundamental mode. Results showing effect of supported area ratios of plate indicate that variation of ratio of supported area of plate system is very significiant. Convergence studies are made. Many new results have been presented. Considered problems are solved within the frame work of Kirchhoff-Love hypothesis.

Key words: *Orthotropic plate, viscoelastic support, supported area, damping coefficient, vibration analysis.*

1. INTRODUCTION

It is generally accepted that classical support conditions employed in the analysis of rectangular plate behaviour represent only limiting mathematical conditions. The actual boundary conditions of a real system are mostly not classical, for example in ship plating, machine tables, circuit boards, solar panels, bridge decks, aircraft and marine structures supports generally accepted are elastic. In many branches of modern industry, these panels and plates are fabricated from composite materials. Therefore, the present investigation may be considered to be a problem of the mechanics of elements fabricated from composite materials.

Lot of work has been undertaken for the analysis of a rectangular plate in the case of free and forced vibrations in literature. Extensive investigation has been carried out on the analysis of the free vibration of rectangular plates having classical boundary conditions [1-6] and elastically restrained edges [7-31] have been widely analyzed. Viscoelastically supported plates were studied by several researchers for point supported plate systems. Yamada and co-workers [32] studied free vibrations of elastically point-supported plates and forced vibrations of viscoelastically point-supported isotropic plates. Kocatürk and Altintas [33, 34] extended Yamada's [32] problem in case of anisotropic plates by using finite difference tecnique.

In this paper, plate problems are studied particularly for the case of boundary conditions elastically and viscoelastically restrained against translation. To represent many practical applications on industrial structures supports are located on areas as wide bands parallel to the edges. Line supports on the edges are also investigated as a special case of wide band supports.

A review of the related literature reveals that this problem has not been properly addressed yet. Due to the lack of research work in this area, this paper aims to provide some vibration solutions for plates systems. The accuracy of the results was partially shown by comparing results available from other sources wherever possible.

2. ANALYSIS

Consider a viscoelastically supported loaded plate with side lengths L_X, L_Y and thickness h subjected to a concentrated force as shown in Figure 1. Translational stiffness and damping coefficients were assigned equally per supported area. Supports are obtained by using Kelvin-Voight type point support in every discrete area on finite difference mesh. By choosing different values of translational spring coefficient of supports everybody can obtain free, elastic or translationaly rigid supports while damping coefficient of supports is zero.

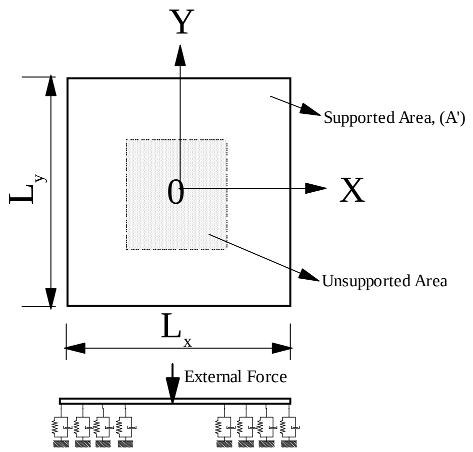

Fig. 1 Viscoelastically supported plate subjected to concentrated force

The elastic symmetry axis of the plate material coincide with the $0X$ and $0Y$ axes. Therefore the plate is specially orthotropic. Given W is the lateral displacement of the mid-surface of the plate corresponding coordinate Z, maximum strain energy of the plate is:

$$U = \frac{D_{XX}}{2}\int_{-\frac{L_X}{2}}^{\frac{L_X}{2}}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}}\left[\left(\frac{\partial^2 W}{\partial X^2}\right)^2 + 2\nu_{YX}\frac{\partial^2 W}{\partial X^2}\frac{\partial^2 W}{\partial Y^2} + e\left(\frac{\partial^2 W}{\partial Y^2}\right)^2 + 4\frac{D_{66}}{D_{XX}}\left(\frac{\partial^2 W}{\partial X^2 \partial Y^2}\right)^2\right] dXdY \tag{1}$$

and maximum kinetic energy of the plate is:

$$T = \int_{-\frac{L_X}{2}}^{\frac{L_X}{2}}\int_{-\frac{L_Y}{2}}^{\frac{L_Y}{2}}\left[\frac{ph\omega^2}{2}\right] WdXdY \tag{2}$$

where D_{XX}, D_{YY} and D_{66} are expressed as follows:

$$D_{XX} = (E_X' h^3)/12, \quad D_{YY} = (E_Y' h^3)/12, \quad D_{66} = (G_{XY} h^3)/12 \tag{3}$$

where G_{XY} is shear modulus. E_X', E_Y' are derived using:

$$\nu_{XY}E_Y = \nu_{XY}E_Y, \quad e = E_Y/E_X, \quad E_X' = E_X/(1-\nu^2_{YX}/e), \quad E_Y' = E_X e/(1-\nu^2_{YX}/e) \tag{4}$$

The additional strain energy and dissipation function per viscoelastic support is:

$$F_s = 1/2\, k'\, W^2_{Si}, \quad D = 1/2\, c'\, (W_{Si})^2 \tag{5}$$

where k' and c' is spring coefficient and damping coefficient per viscoelastic support, E_X, E_Y are Young's moduli in the $0X$ and $0Y$ directions, respectively, and ν_{YX} is the Poisson's ratio for the strain response in the X direction due to an applied stress in the Y direction. The total energy of the whole plate can be found by summing the entire area of plate with supports and external force. The potential energy from external force is:

$$F_e = -F_i W_i \tag{6}$$

where F_i and W_i are external forces and corresponding displacements.

Introducing the following non-dimensional parameters:

$$x = \frac{X}{a}, \quad y = \frac{Y}{b}, \quad \alpha = \frac{a}{b}, \quad \bar{w}(x,y,t) = w(x,y)e^{i\omega t} = W/a, \quad i = \sqrt{-1} \tag{7}$$

the above energy expressions can be written as:

$$U_{m,n} = \frac{D_{xx}}{2}\left[\frac{1}{\alpha\Delta x^4}\left(\bar{w}_{m-1,n} - 2\bar{w}_{m,n} + \bar{w}_{m+1,n}\right)^2 + \frac{2\alpha\nu_{YX}}{\Delta x^2\Delta y^2}\left(\bar{w}_{m-1,n} - 2\bar{w}_{m,n} + \bar{w}_{m+1,n}\right)\left(\bar{w}_{m,n-1} - 2\bar{w}_{m,n} + \bar{w}_{m,n+1}\right) + \frac{4\alpha D_{66}}{D_{XX}\left(4\Delta x^2\Delta y^2\right)^2}\left(\bar{w}_{m-1,n-1} - \bar{w}_{m+1,n-1} - \bar{w}_{m-1,n+1} + \bar{w}_{m+1,n+1}\right) + \frac{e\alpha^3}{\Delta x_2^4}\left(\bar{w}_{m,n-1} - 2\bar{w}_{m,n} + \bar{w}_{m,n+1}\right)^2\right]\Delta x\Delta y$$

$$T_{m,n} = \frac{\rho h a^3 b}{2}\left(\dot{\bar{w}}_{m,n}\right)\Delta x\Delta y, \quad F_s = \frac{1}{2}a^2 k'\,\dot{\bar{w}}^2_{m,n}, \quad D = \frac{1}{2}a^2 c'\left(\dot{\bar{w}}_{m,n}\right)^2, \quad F_e = -aQ\bar{w}_{m,n} \tag{8a-e}$$

The derivative terms were approximated in terms of discrete displacements at gridpoints (see Figure 2) by using the following finite difference operators:

$$\begin{aligned}
\left(\frac{\partial^2\Theta}{\partial x^2}\right)_{m,n} &= \frac{1}{\Delta x^2}\left(\Theta_{m-1,n}-2\Theta_{m,n}+\Theta_{m+1,n}\right)\\
\left(\frac{\partial^2\Theta}{\partial y^2}\right)_{m,n} &= \frac{1}{\Delta y^2}\left(\Theta_{m,n-1}-2\Theta_{m,n}+\Theta_{m,n+1}\right) \qquad (9)\\
\left(\frac{\partial^2\Theta}{\partial x\partial y}\right)_{m,n} &= \frac{1}{4\Delta x\Delta y}\left(\Theta_{m-1,n-1}-\Theta_{m+1,n-1}-\right.\\
&\left.-\Theta_{m-1,n+1}+\Theta_{m+1,n+1}\right)
\end{aligned}$$

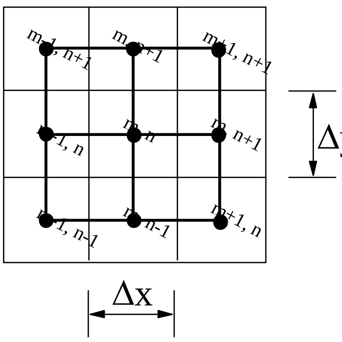

Fig. 2 Typical finite difference mesh on part of a plate

The energy for the whole plate can be found by summing over the entire area of the plate. Thus:

$$\begin{aligned}
U &= \sum_{m=1}^{N}\sum_{n=1}^{N}U_{m,n}, \quad T = \sum_{m=1}^{N}\sum_{n=1}^{N}T_{m,n},\\
F_s &= \sum F_{si}, \quad D = \sum D_{ci}, \quad F_e = \sum F_{ei}
\end{aligned} \qquad (10)$$

where N is taken as the number of the mesh points in each of the two directions in the plate region, $N\times N$ is the total number of the area elements on the plate.

The governing differential equation obtained from the Lagrange's equation is given as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\overline{w}}_{m,n}}\right)-\frac{\partial(T-U)}{\partial \overline{w}_{m,n}}+\frac{\partial D}{\partial \dot{\overline{w}}_{m,n}}+\frac{\partial F_s}{\partial \overline{w}_{m,n}}+\frac{\partial F_e}{\partial \overline{w}_{m,n}}=0 \qquad (11)$$

where is $\overline{w}_{m,n}$ the m, n the discrete displacement and the overdot stands for the partial derivative with respect to time. Introducing the following non-dimensional parameters,

$$\kappa_j=\frac{k_j a^3}{bD_{11}}, \quad \gamma_j=\frac{c_j}{\sqrt{\rho h D_{11}}}, \quad \lambda^2=\frac{\rho h\omega^2 a^4}{D_{11}}, \quad q=\frac{Fa}{D_{11}} \qquad (12)$$

and remembering that $\overline{w}(x_1,x_2,t)=w(x_1,x_2)\,e^{i\omega t}$, which was given in Eq. (7), by using Eq. (11) for the mesh point m, n with Eq. (8a-e) results in the following expression:

$$\begin{aligned}
&\left[-\frac{4}{\alpha\Delta x^4}\left(w_{m+1,n}-2w_{m,n}+w_{m-1,n}\right)-\frac{4\nu_{YX}\alpha}{\Delta x^2\Delta y^2}\left(w_{m,n-1}-2w_{m,n}+w_{m,n+1}\right)-\right.\\
&\left.-\frac{4\nu_{YX}\alpha}{\Delta x^2\Delta y^2}\left(w_{m-1,n}-2w_{m,n}+w_{m+1,n}\right)-\frac{4\alpha^3}{\Delta y^4}\frac{D_{YY}}{D_{XX}}\left(w_{m,n-1}-2w_{m,n}+w_{m,n+1}\right)-\frac{2\lambda^2}{\alpha}w_{m,n}\right]\Delta x\Delta y+\\
&+\left[\frac{2}{\alpha\Delta x^4}\left(w_{m+2,n}-2w_{m+1,n}+w_{m,n}\right)+\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}\left(w_{m+1,n-1}-2w_{m+1,n}+w_{m+1,n+1}\right)\right]\Delta x\Delta y+\\
&+\left[\frac{2}{\alpha\Delta x^4}\left(w_{m,n}-2w_{m-1,n}+w_{m-2,n}\right)+\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}\left(w_{m-1,n-1}-2w_{m-1,n}+w_{m-1,n+1}\right)\right]\Delta x\Delta y+\\
&+\left[\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}\left(w_{m+1,n-1}-2w_{m,n-1}+w_{m-1,n-1}\right)+\frac{2\alpha^3}{\Delta y^4}\frac{D_{YY}}{D_{XX}}\left(w_{m,n-2}-2w_{m,n-1}+w_{m,n}\right)\right]\Delta x\Delta y+\\
&+\left[\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}\left(w_{m+1,n+1}-2w_{m,n+1}+w_{m-1,n+1}\right)+\frac{2\alpha^3}{\Delta y^4}\frac{D_{YY}}{D_{XX}}\left(w_{m,n}-2w_{m,n+1}+w_{m,n+2}\right)\right]\Delta x\Delta y+\\
&+\left[\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}\left(w_{m-2,n-2}-w_{m-2,n}-w_{m,n-2}+w_{m,n}\right)\right]\Delta x\Delta y+\\
&+\left[-\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}\left(w_{m-2,n}-w_{m-2,n+2}-w_{m,n}+w_{m,n+2}\right)\right]\Delta x\Delta y+\\
&+\left[-\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}\left(w_{m,n-2}-w_{m,n}-w_{m+2,n-2}+w_{m+2,n}\right)\right]\Delta x\Delta y+\\
&+\left[\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}\left(w_{m,n}-w_{m,n+2}-w_{m+2,n}+w_{m+2,n+2}\right)\right]\Delta x\Delta y+\\
&+\alpha_S\frac{2(\kappa+i\lambda\gamma)}{\Delta x\Delta y}w_{m,n}=\alpha_Q\frac{q}{\Delta x\Delta y}, \qquad i=\sqrt{-1}
\end{aligned} \qquad (13)$$

where α_S and α_Q are taken values *0, 1* depending on existence of support, load respectively on pivotal point *m*, *n*. One may prefer element and system matrix forms similar to finite element method. In the form of Eq. (13), there is no need for any manipulation to express final form of solution of algebraic equations.

For the whole mesh points, by using Eq. (13), the following set of linear algebraic equations is obtained which can be expressed in the following matrix form:

$$\boldsymbol{Aw} + i\lambda\gamma\boldsymbol{Bw} - \lambda^2\boldsymbol{Cw} = \boldsymbol{q} \qquad (14)$$

where ***A***, ***B*** and ***C***, and are coefficient matrices obtained by using Eq. (13) for all mesh points. For free vibration analysis, when the external force and damping of the supports are zero in Eq. (14), this situation results in a set of linear homogeneous equations that can be expressed in the following matrix form:

$$\boldsymbol{Aw} - \lambda^2\boldsymbol{Cw} = \boldsymbol{0} \qquad (15)$$

Numbering the mesh points is shown in Figure 2. By decreasing the dimensionless mesh widths, the accuracy can be increased.

The total magnitude of the reaction forces of the supports is given by:

$$\sum_{j=1}^{N} P_i = \sum_{j=1}^{N} \left(k_j + ic_j w\right) aw \qquad (16)$$

and therefore the force transmissibility at the supports is determined by:

$$T_R = \sum P_i / F = \sum \left(\kappa_j + i\gamma_j\lambda\right) w / q \qquad (17)$$

The number of unknown displacements is $(N+2)^2$, where N^2 is the mesh size in the plate region.

3. NUMERICAL RESULTS

The steady state response to a concentrated force acting on an orthotropic square plate, which is viscoelastically supported along all edges is calculated numerically. Viscoelastic supports are given equally per supported area. A brief investigation of the free vibration of an elastically supported plate is necessary for a better understanding of the responses presented in this study. The natural frequencies of the elastically supported plate are determined by calculating the eigenvalues, assuming that the damping parameter of the supports and external force are zero. Poisson ratio υ, is taken *0.3* in all numerical calculations.

In Table 1 the convergence of the fundamental mode is presented for the following E_2/E_1 ratios *0.6*, *0.8*, *1* respectively. Tabulated results in Table 1 were obtained for simple supported plate ($\kappa=\infty$). Frequency parameter is monotonic from below for all E_2/E_1 ratios. Convergence properties are not effected by A_{sup}/A_{plt} ratio and they are not shown here. Due to the lack of the comparable results for different A_{sup}/A_{plt} and E_2/E_1 ratios, only fundamental frequency of simple supported isotropic plate was compared in Table 1. Gorman also studied elastically supported plate but results are given graphically in Ref. [11] .

Table 1 The effect of mesh size on the fundamental frequency (Case of simple support, $\kappa=\infty$)

Mesh size	E_2/E_1=0,6	E_2/E_1=0,8	E_2/E_1=1
10×10	17,057	18,169	19,152
20×20	17,397	18,561	19,588
30×30	17,463	18,637	19,672
40×40	17,485	18,664	19,701
50×50	17,496	18,676	19,716
60×60	17,502	18,683	19,723
70×70	17,505	18,686	19,727
80×80	17,507	18,689	19,730
90×90	17,509	18,691	19,732
100×100	17,510	18,692	19,733
Elasticity Solution			$2x\pi^2$=19,739

Figure 3 shows the frequency parameters λ versus the stiffness parameter κ of the supports for different A_{sup}/A_{plt} ratios. The translational stiffness coefficient κ shows total value of stiffness of translational springs of the whole system. Springs are uniformly distributed per unit area of supported region of plate. In Figure 3, the value of the translational stifness parameter increases, the frequency parameter also increases monotonically for all A_{sup}/A_{plt} ratios. All lines approach zero as their lower limit as expected. It can also be seen that the effect of A_{sup}/A_{plt} ratio on the fundamental frequency of plate is significiant for $\kappa>100$. When κ is smaller than *100*, frequencies of fundamental modes are getting close for all A_{sup}/A_{plt} ratios. The fundamental frequency parameter almost remains the same value for the values of κ are greater than 10^6.

Table 2 depicts eigenvalues of frequency parameter for elastically supported plate for various A_{sup}/A_{plt} ratios.

Table 2 Fundamental frequency parameters of elastically supported plate

	A_{sup}/A_{plt} =0,4	A_{sup}/A_{plt} =0,6	A_{sup}/A_{plt} =0,8	Simple Supported
E_2/E_1=0.6				
κ =1	0,999	0,999	0,999	0,998
κ =10	3,158	3,148	3,128	3,095
κ =100	9,886	9,57	8,993	8,335
κ =1000	27,862	22,389	17,555	14,461
E_2/E_1=0.8				
κ =1	0,999	0,999	0,999	0,998
κ =10	3,16	3,151	3,132	3,104
κ =100	9,903	9,63	9,112	8,493
κ =1000	28,256	22,976	18,167	15,037
E_2/E_1=1				
κ =1	0,999	0,999	0,999	0,998
κ =10	3,16	3,153	3,135	3,110
κ =100	9,913	9,665	9,188	8,603
κ =1000	28,841	23,451	18,677	15,525

In Figure 4 it is seen that transmissibilites are obtained as a function of frequency parameter for sets of A_{sup}/A_{plt} and γ for κ=*1000*. One can note that peak values of transmissibility curves for γ increases while A_{sup}/A_{plt} ratio decreases. This means an effect of damper decreases for A_{sup}/A_{plt} ratio also decreases.

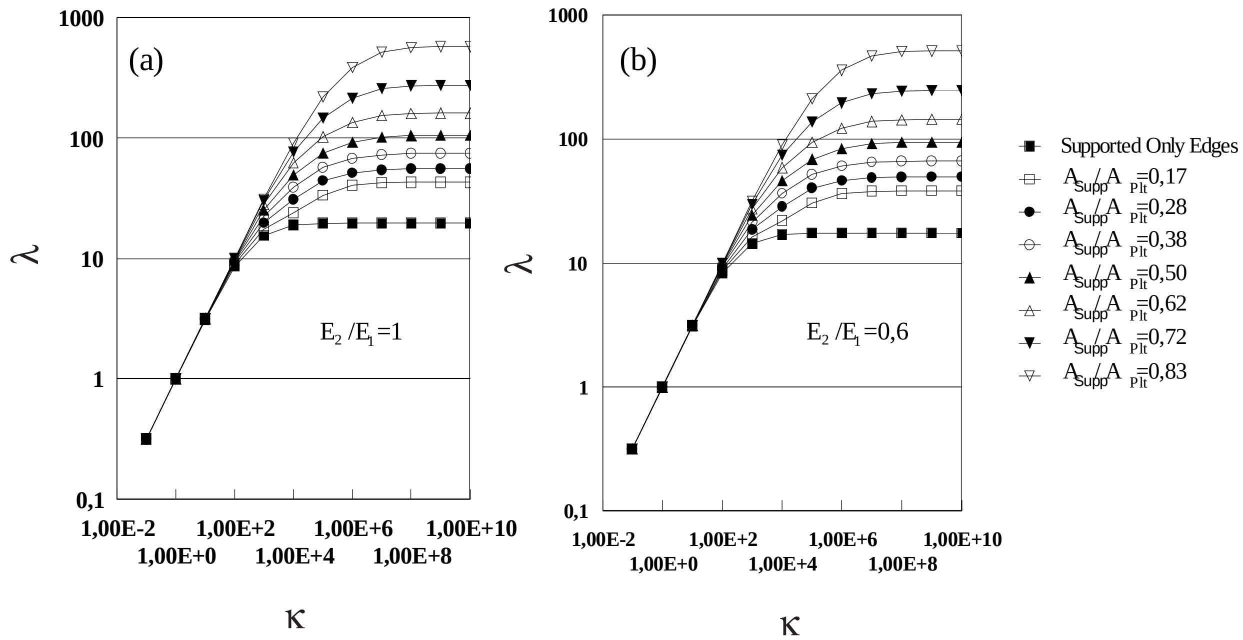

Fig. 3 The effect of supported area and spring stiffness on the non dimensional fundamental frequency

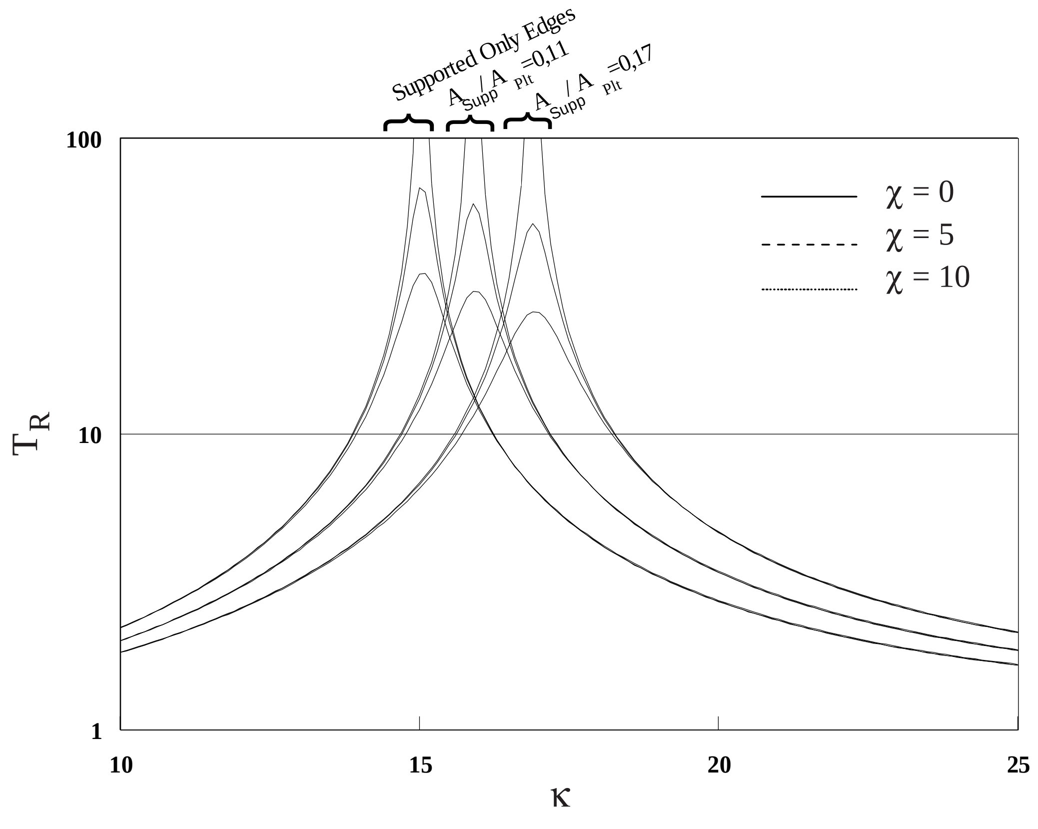

Fig. 4 Force transmissibilities (E_2/E_1= 0.8)

Figure 5 shows that the peak values of force transmissibilities occur for variation of damping parameter. When damping parameter increases transmissibilities decrease rapidly where the minimum peak values occur. In Figure 5 plotted for κ*=100* minimum peak values occur, smaller magnitudes of γ than in Figure 5 are plotted for κ*=1000*. When κ increases some lines for example line "*p*" and line "*d*" in Figure 5 do not have minimum peak values in the range of fundamental mode.

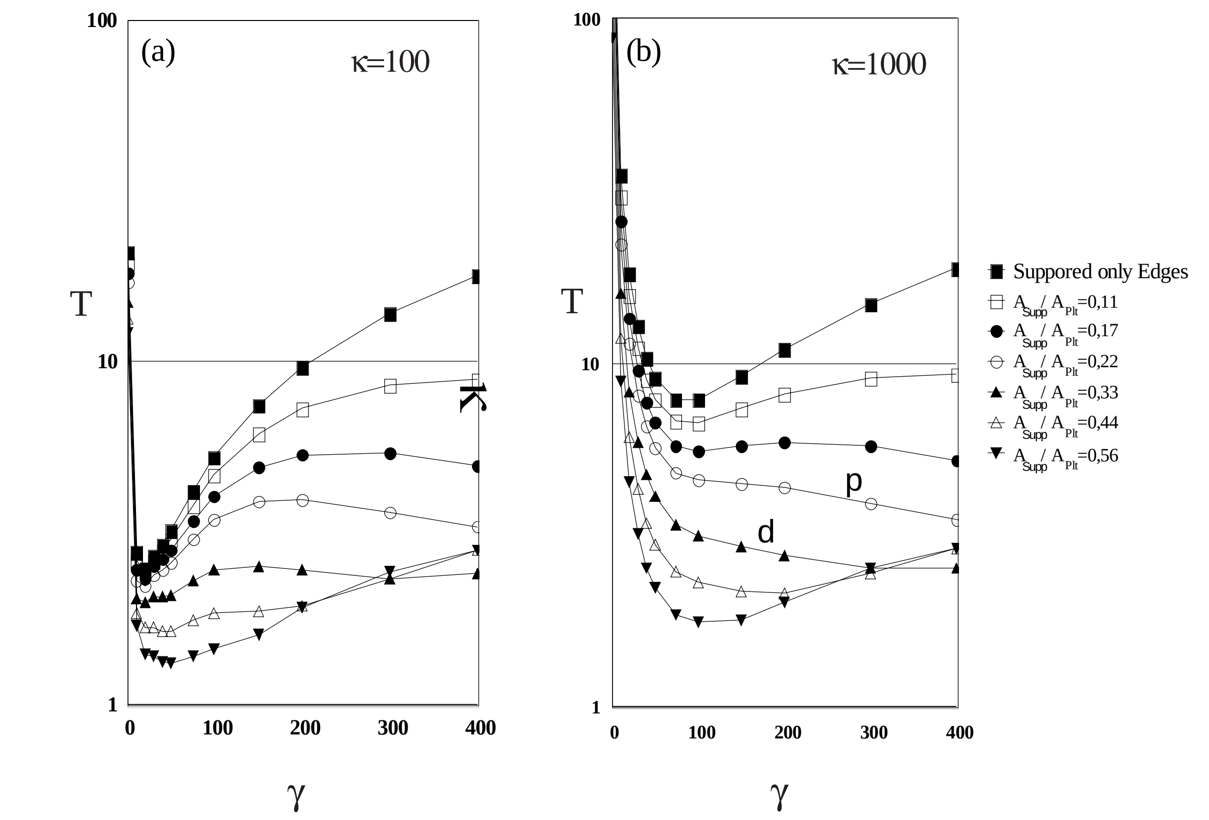

Fig. 5 Force transmissibilities

Table 3 The frequencies at which the peak values of the force transmissibilities occur

			A_{sup}/A_{plt} =0,8		A_{sup}/A_{plt} =0,7		A_{sup}/A_{plt} =0,6	
			T_R	λ	T_R	λ	T_R	λ
κ =100	E_2/E_1=0.6	γ =5	4,229	8,838	3,699	9,175	3,264	9,417
		γ =10	2,606	9,461	2,291	9,803	2,025	9,867
		γ =50	3,029	17,482	2,480	19,719	1,946	22,130
		γ =100	4,464	19,431	3,114	22,636	2,214	26,929
κ =10	E_2/E_1=0.6	γ =5	1,288	2,676	1,271	2,606	1,258	2,550
		γ =10	1,344	15,508	1,314	18,206	1,091	2,662
		γ =50	2,833	17,966	2,326	20,199	1,830	22,652
		γ =100	4,387	19,553	3,056	22,725	2,169	26,962
κ =100	E_2/E_1=0.8	γ =5	3,996	8,957	3,517	9,251	3,132	9,450
		γ =10	2,476	9,599	2,186	9,840	1,949	9,802
		γ =50	2,853	17,875	2,359	19,941	1,867	21,916
		γ =100	4,092	20,569	2,944	24,004	2,143	28,601
κ =10	E_2/E_1=0.8	γ =5	1,279	2,636	1,264	2,577	1,253	2,530
		γ =10	1,384	17,335	1,098	3,077	1,088	2,470
		γ =50	2,655	18,291	2,203	20,279	1,749	22,149
		γ =100	4,016	20,717	2,889	24,131	2,102	28,687

Lines plotted for γ and changes of frequency parameter are not shown in Figure 5. Table 3 depicts frequency parameters and transmissibility values where the peak values occur for the set of κ, γ, E_2/E_1. Frequency parameter tends to be close to the natural frequency of the plate for lower values of γ.

4. CONCLUSIONS

This paper presents, to the authors' knowledge, the first known vibration analysis of viscoelastically supported anisotrpic plate with varying supported area. The model can be used to simulate the actual boundary conditions of the plates. A simple numerical method has been presented to determine natural frequencies of plates. The convergence studies are made. Frequency parameter is from below for different anisotropy ratios. Fundamental frequency was determined depending on the amount of supported area, spring coefficient and anisotropy. The effect of supported area ratio on fundamental frequency of plate is significiant for higher stiffness coefficients of springs for all anisotrpy ratios. The effect of the amount of supported area and damping coefficient on response curves is investigated. The effect of dampers increases when the supported area increases. The peak values of transmissibilities were plotted to determine where the minimum peak values were obtained. Numerical and graphical results have revealed the effects of supported area variations on the free and forced vibrations of anisotropic plate. It is believed that these novel results will be useful to designers in the various types of practical applications.

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ODREĐIVANJE REAKCIJE MIRNOG STANJA VISKOELASTIČNIH PODUPRTIH PRAVOKUTNIH, NAROČITO ORTOTROPNIH PLOČA KOJE SU PODUPRTE NA RAZLIČITIM PODRUČJIMA

SAŽETAK

U ovom radu govori se o utjecaju količine poduprtih područja na slobodna i forsirana vibracijska svojstva anizotropne ploče. Koristeći energiju koja se bazira na metodi konačne razlike, problem se modelira pomoću nekog elementa konačne razlike. Zbog važnosti osnovne frekvencije ploče, njezino mijenjanje se ispitivalo u odnosu na količinu poduprtog područja na ploči, te na mehanička svojstva materijala ploče kao i na transakcijski koeficijent opruga na potpornjima. Reakcija mirnog stanja viskoelastično poduprtih ploča numerički se ispitivala zbog različitih koeficijenata prigušenja i količine poduprtog područja. Dobiveni su numerički rezultati da bi se ispitalo djelovanje omjera sustava ploče. U numeričkim primjerima, parametri prirodne frekvencije i reakcije mirnog stanja na sinusoidno varirajuću silu određeni su za osnovni oblik. Rezultati koji pokazuju djelovanje omjera poduprtog područja na ploči pokazuju da je promjena omjera poduprtog područja sustava ploče vrlo značajna. Napravljene su studije konvergencije. Izneseni su i mnogi novi rezultati. Razmatrani problemi rješavaju se u okviru rada Kirchhoff-Love hipoteze.

Ključne riječi: *ortotropna ploča, viskoelastični oslonac, poduprta površina, koeficijent prigušenja, analiza vibracija.*