Valentin Goranko, *Temporal Logics.*

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The book *Temporal Logics* by Valentin Goranko is an edition featured in *Elements in Philosophy and Logic* series by Cambridge University Press. The Elements series provides an overview of many topics that link philosophy and logic, where authors present contemporary research results and their own insights into the issues.

Goranko's approach in *Temporal Logics* is methodical: a concise philosophical introduction to the topic to demonstrate the primary motivation for developing a further system, potential approaches depending on chosen ontological assumptions, the development of an axiomatic schema, its application, and possible extensions and practical implications. The book is intended for readers interested in philosophical discussion and formal reasoning about time.

From the very title of the book, we can see that there is not just one temporal logic but a cluster of different approaches and reflections on time, as the author demonstrates by acquainting us with a multitude of different approaches within the confined space of 90 pages of text. Towards the end of the book, the author mentions topics that he did not manage to cover and directs the reader to further texts, which he also does at the end of each thematic section.

Contemplation about time is not a product of contemporary modern logic; the earliest written records in Western philosophy can be found in Zeno's paradoxes. Zenon does not contemplate with the aim of understanding the nature of time itself but questions that everyday concept, and shows that, in certain aspects, it can be counterintuitive and even illogical. There was a current of advocates for the illogical nature of time. However, Goranko in his book demonstrates that arranging logical relations between events in time is quite possible, especially after the development of modal logic. In that context, we can try to define temporal logic as *(logical) reasoning about time.*

We are introduced to the Aristotelian example of *tomorrow's sea battle* from *On Interpretation*. Starting from the principle of bivalence, according to which every proposition must be either true or false — the proposition "There will be a sea battle tomorrow" must have one of these values at the moment of utterance. However, since tomorrow has not yet occurred, there is no way to determine this value, and Aristotle considers propositions about the future as lacking truth values. If we were to claim that such propositions must have truth values, we would implicitly assert that the future is already determined. It is precisely in this determinism that Goranko finds the motivation for developing temporal logic in the works of Arthur Prior, the pioneer of temporal logic, to whom he extensively refers.

Reasoning about time raises questions about the nature of time and how to approach its study and this greatly depends on the ontological approach we adopt. Goranko outlines two main approaches: the Instant–Based Model and the Interval–Based Model. In the former, the primitive terms are instants (moments) that we cannot precisely define. Rather, we consider having certain intuitive knowledge about them. The Instant–Based Model consists of a non–empty set of instants, T, and a binary precedence relation $\prec$, which we can formally express as T = $\langle$T, $\prec\rangle$. If $a \prec b$, a is the predecessor of b, and b is its successor. Different properties can be attributed to such a binary relation, such as reflexivity $\forall x(x \prec x)$ or non–reflexivity $\forall x \neg(x \prec x)$, the existence of an end $\exists x \neg \exists y(y \prec x)$ or its non–existence $\forall x \exists y(y \prec x)$, and, for example, density $\forall x \forall y(x \prec y \rightarrow \exists z(x \prec z \wedge z \prec y))$, which asserts that between any two moments, there is always (at least) one more moment.

Instant–Based Models are not suitable for events that persist over time; for that purpose, we use Interval–Based Models, which are ontologically richer and allow for more logical relations. It is shown that if we take an interval as the primitive term, it is possible to avoid Zeno's paradox with the arrow standing still in an instant. It is important to note that both models are interdefinable since an interval can be expressed as an ordered pair of instants, and an instant can be defined as a point interval where the beginning and end exist together.

According to Goranko, Arthur Prior wanted to view the timeless propositions from classical logics as temporal (tensed). Prior used simple temporal operators:

P: "It has at some past time been the case that..."

F: "It will at some future time be the case that..."

H: "It has always in the past been the case that..."

G: "It will always in the future be the case that..."

Operators *P* and *F* are called weak operators, while *H* and *G* are referred to as strong operators. They are interdefinable as $Hx = \neg P \neg x$ and $Gx = \neg F \neg x$, while additional operators *Always (A)* and *Sometimes (E)* are defined as $Ax = Hx \wedge x \wedge Gx$ and $Ex = Px \vee x \vee Fx$. Accordingly, the operators needed for the formal language of Prior's temporal logic TL are $\wedge$, $\neg$, *P*, and *F*. Through conjunction and negation, we define other logical truth–functional operators, and with the help of *P* and *F*, we define other temporal operators. Prior's TL allows the expression of different tenses in natural languages, as the combination of temporal operators can achieve more expressively demanding statements, such as *FGx* = "always true without exception." However, TL is still unsuitable for expressing continuous past tenses, for which the Interval–Based Model is more suitable, as mentioned.

The semantics of TL essentially involves the possible worlds semantics, often referred to as Kripke semantics. Possible worlds are temporal instants, and the truth of temporal propositions is observed in relation to them, with the accessibility relation defined through the precedence function. Although, in a modal framework, one could speak of two accessibility relations — before and after — in TL, they are reduced to one — the precedence relation, because if *y* comes after *x*, it is possible to simply say that *x* precedes *y*.

The axiomatic schema of Prior's TL, denoted as Kt, consists of a schema for classical propositional logic extended with the following axioms:

KG: $G(x \rightarrow y) \rightarrow (Gx \rightarrow Gy)$

KH: $H(x \rightarrow y) \rightarrow (Hx \rightarrow Hy)$

GP: $x \rightarrow GPx$

HF: $x \rightarrow HFx$

In addition to the classical *modus ponens* (*MP*) rule, in the inference rules of Kt, we also use two additional necessity rules:

MP: If $\vdash x \rightarrow z$ *and* $\vdash x$, *then* $\vdash z$.
NECG: If $\vdash x$, *then* $\vdash Gx$.
NECH: If $\vdash x$, *then* $\vdash Hx$.

The language of TL can be translated into the language of first–order logic if the precedence relation $\prec$ is replaced with a binary predicate *R*, where $x \prec y$ is then represented as xRy.

One of the crucial questions we face in *Temporal Logics* is the issue of the flow of time. A linear flow of time leaves the door open to a deterministic interpretation and the negation of free will, a defense against which is one of Prior's fundamental interests, primarily against Diodorus Cronus' Master Argument. The methodological counter–response to determinism is the so–called Branching Time, where time is not observed as linear but, at certain moments, begins to branch based on the possibilities available at that moment. If the past is unchangeable, it is not necessarily as it is; it could have been different. Past events (unchangeable concerning the present moment) represent only one actualization concerning the possibilities at that moment, and the same goes for future events: of all possible future temporal branches, one will actualize, but not out of necessity or determinism, thus keeping the future open.

The principle of branching time was proposed to Prior by Kripke in their mutual correspondence, and it is not surprising that it closely resembles the said possible worlds. Interestingly, such an approach to the flow of time can be found as early as in the thirteenth century in the works of William Ockham. He believed that God's foreknowledge of the future does not negate free will because, out of all possible futures (branches), the one that actualizes depends on our present choices. God, being outside of time, knows which one will actualize even before we make the choice, but this leaves freedom of will for humans. This is in contrast to Richard Lavenham from the fourteenth century, who believed that God's foreknowledge negates free will.

After reasoning about time, Goranko brings us back to Aristotle's proposition, "There will be a sea battle tomorrow", and analyzes its truth values according to the approaches he has shown in his book, using the operator "tomorrow" (*X*):

— In a linear deterministic flow of time, the truth value of *p* is already determined and necessary.
— Polyvalent logic, introduced by Polish philosopher Jan Łukasiewicz, incorporates the value 0.5, which can be defined as *undetermined*. Propositions like Xp or $\neg Xp$ have an undetermined value, leading to the undetermined value of $Xp \vee \neg Xp$.
— In branching time, there are multiple possible futures, and only one will be actualized. However, perspectives on the role of the present moment differ and thus the truth of $Xp \vee \neg Xp$ can differ.
— In Kripke's models, both Xp and $X\neg p$ are true, and the truth of non–modal parts p and $\neg p$ depends on the world they refer to, raising the question of the actualized (tomorrow's) world.

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Goranko also mentions his work with Ju and Grilleti (2018), where they provided a different formal logical solution to this problem. However, he does not go into details at this point.

In *Temporal Logics*, it was demonstrated that the development of temporal logic is motivated by some classical philosophical problems. However, its further evolution extends beyond that framework after its application in fields outside of philosophy. In modern times, it finds applications in linguistics, computer science, natural sciences, and more. Alongside this, specialized temporal logics are developed, many of which Goranko presents in his book. The past of temporal logic is known to us, and as for its future, only time will tell.

Goranko comes from a mathematical tradition, which is evident in his writing style: he prioritizes formal language and presents philosophical explanations as introductions to specific parts of formalism. The initial goal of linking logic and philosophy is fulfilled in a way that presents philosophy as the starting motivation for developing formal systems.

Although the author states that the exposition is mostly on a basic level, it is assumed that the reader has some prior knowledge of formal classical logic and propositional modal logic. By gathering such a substantial amount of content on temporal logic in one place, Goranko has provided a great service to individuals engaged in the same field. Although the book may be helpful to researchers already familiar with the subject matter, its primary application is likely to resonate more among students, particularly postgraduates, who are beginning more serious studies of logic. The book's format resembles the handbook format for a Temporal Logic course, and since it is more suitable for individuals inclined towards a formalistic–mathematical approach, expanding the literature is necessary for a comprehensive approach to philosophical issues, especially for individuals coming from mathematical traditions like the author himself, for whom this is the first encounter with philosophical issues serving as the foundation for formalism development.

Nino Stanojević