Line Loss Calculation and Optimization in Low Voltage Lines with Photovoltaic Systems Using an Analytical Model and Quantum Genetic Algorithm

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Abstract: With the increasing integration of distributed photovoltaic (PV) generation into distribution networks, challenges such as power reverse flow and high line losses have emerged, leading to greater uncertainty in power systems. To address these issues, this paper presents an analytical model for calculating line losses in low-voltage distribution networks with PV generation, utilizing power flow calculations. A simulation model of a 15 node low-voltage network is developed using SIMULINK to validate the accuracy of the analytical model under the scenario of uniform load distribution (ULD). Additionally, a line loss optimization algorithm based on quantum genetic algorithms (QGA) is proposed for low-voltage distribution networks with distributed PV generation, along with an optimization model. The objective function of the optimization model is based on the reduction in line losses resulting from the integration of the PV system. The example results demonstrate the consistency between the line loss optimization using QGA and the analytical results, highlighting the significant advantages of QGA in terms of speed and accuracy. This research provides valuable insights for line loss optimization in low-voltage distribution networks with distributed PV generation and serves as a theoretical reference for future studies in this field.

Keywords: analytical model; distribution system; line loss; photovoltaic generation; quantum genetic algorithm

1 INTRODUCTION

With the increasing depletion of global energy resources and environmental problems [1], PV generation has rapidly developed due to its advantages of cleanliness and high efficiency [2], and it will become one of the main power supply methods in China for a long time to come. The access of new energy to the power grid can play an important role in reducing carbon emissions and promoting energy structure transformation [3]. But large-scale distributed PV connection into a power grid has a significant impact on the relay protection, voltage stability, power flow and line loss of the distribution network. After taking appropriate loss reduction measures, the power grid operating company can evaluate the application effect of loss reduction measures through the line loss results. Loss reduction has important practical significance for energy conservation and emission reduction, reducing investment in generation, transmission and distribution equipment, and lowering production costs for large industrial users. Existing research on PV access into distribution systems mainly focuses on modeling of distributed PVs and their impact on distribution networks, as well as site selection and capacity determination of distributed PVs [4, 5]. Abagiu et al. [6] analyzes the influencing factors of grid-connected PV power plants and establishes the simulation model. The result points out that when the connected and operated PV power plant reaches the rated voltage level, the power loss of the distribution network will decrease, and the minimum power loss can be achieved by controlling the terminal voltage of the transformer. L. F. et al. [7] studies the over voltages produced by a high PV-DG penetration, presents a review of the literature dedicated to mitigate these overvoltage problems, proposing the classification and definition of regulation devices and control schemes used. Sridevi [8] proposes a new method for finding the optimal location and scale of renewable distributed energy in distribution systems based on the loss sensitivity factor, and verifies the superiority of the method. KooiKamali et al. [9] analyzes a sample power grid in Malaysian Peninsula, including three load patterns: residential areas, commercial areas, and industrial areas. It determines the impact of interconnected distributed PV system on voltage distribution and power loss in the power grid. It also compares the power loss before and after installation of distributed PV system at different locations. The calculation and reducing of power system line loss is one of the major research areas of the power grid. After PV access to the power grid, the relationship between the PV active power and the total load of the distribution network is analyzed. The active power of the system tail load is also considered to determine the PV access position that minimizes line loss. Generally, the connection of different PV access to the system will lead to the increasing or decreasing of line loss [10, 11]. When the PV power generating capacity is greater than the power consumption, the line loss tends to increase at the minimum load condition. While in other cases, the line loss is often related to the PV penetration rate. Appropriately high percentage penetration of distributed PV in the power grid can reduce line loss, load loss, and transformer loss [12, 13]. Sayed and Takeshita [14] uses a unified power flow controller (UPFC) to control the phase angle of the controlled voltage. It proposes the condition that minimizes line loss while ensuring that the voltage at each node is within the allowable voltage range of minimum loss. This method achieves the adjustment of node voltage under the condition of minimizing line loss. Khosravi et al. [15] proposes a network loss sharing method, which allocates network loss to load and distributed generation based on the contribution of active and reactive power injection of load and distributed generation in the power grid. This method is helpful for reducing loss and energy conservation in power generation and power grid dispatch. With the continuous development of intelligent algorithms, some algorithms such as serial firefly algorithm, bat search algorithm and improved sparrow search algorithm have been applied to the coordination, optimization, prediction and other aspects of power systems. Magham et al. [16] adopts a genetic algorithm (GA) combined with distributed generation characteristics such as power factor and distributed devices to optimize and control the voltage of distribution network. This method is applied to a distribution system with two PVs distributed generation.
units, obtaining a flatter voltage distribution. Yu et al. [17] combine metaheuristic SSA with conventional HC to propose a new hybrid SSA-HC algorithm that can deal with and adapt to extreme changing environments (PSC and gradual change irradiance) in PV systems. The results show that the proposed hybrid SSA-HC algorithm outperforms the SSA algorithm in terms of MPPT efficiency (nMPPT) by improving power output. Shouman et al. [18] investigates the impact of renewable energy penetration on power system operation using stochastic power dispatch. To solve the power dispatch problem, it employs a hybrid mean variance mapping optimization algorithm. The study reveals that renewable energy can lead to a decrease in the operating costs of thermal power units. Buluş and Zor [19] applies a new hybrid deep learning algorithm combining the group method of data handling network and gated loop device network to load forecasting, achieving the goal of reducing load forecasting errors. In the optimization of power system line loss, the research and application of artificial intelligence algorithms are currently research hotspots. Tirmikçi et al. [20] studies the advantages of using distribution static compensators and distributed power sources in reducing power losses in the IEEE 34-bus test system, and uses GA to optimize power losses. Wu et al. [21] applies a charging system search algorithm to determine the optimal PV capacity, minimizing line losses under relevant constraints, and studies the impact of increasing PV installed capacity on system steady-state and dynamic performance. In N. D. et al. [22] evolutionary computation techniques like particle swarm optimization and genetic algorithm are used to access the optimal DG allocation for minimization of power losses to facilitate the extensive integration of DG into distribution systems. Quantum genetic algorithm (QGA) as a new optimization algorithm has been given more attention in its applications in computational grid scheduling models [23], graph coloring [24], and other areas. However, it has not been extensively used in power network loss optimization. At the same time, there are certain limitations in the existing research on the combination of intelligent algorithms and power loss optimization in terms of real-time performance and accuracy. Therefore, improving the efficiency and accuracy of power loss optimization has become a current research hotspot and challenge. Based on the above background, this paper proposes a method for calculating the line loss of low-voltage distribution network with distributed PVs based on QGA. For the ULD system, the analytical models for calculation of the line loss before and after PV access are derived. To verify the effectiveness of the analytical model for calculating the line loss of a low-voltage distribution network with PVs, a simulation model of a low-voltage distribution network with 15 nodes containing PVs is established, and the distribution of line loss in typical scenarios is analyzed. The QGA is used to optimize the PV output to find the PV output that minimizes the line loss rate of the low-voltage distribution network system. The results are compared with those of the GA and the analytical model calculation to verify the feasibility of QGA in distribution network line loss calculation and optimization.

2 ESTABLISHMENT AND VALIDATION OF LINE LOSS ANALYTICAL MODEL

The voltage amplitude and phase information of each node could be obtained based on power flow calculation, and then the line losses of each branch and the total line losses are calculated. In this paper, an analytical model for line loss calculation is derived for the uniform load distribution (ULD) containing PV system.

2.1 Establishment of an Analytical Model for Line Loss Analysis

To derive the line loss calculation model for ULD, a low-voltage feeder system with $k$ nodes and $n-1$ branches is established, as shown in Fig. 1. The PV system is connected to the system at node $m$, with a voltage of $U_i$ at the head-end and a rated voltage of $U_N$ for the line.

In Fig. 1, the load power of node $k$ and the corresponding line impedance of branch $i$ are denoted as:

$$
\tilde{S}_k = P_k + jQ_k \quad k \in [1, n]
$$

$$
Z_i = R_i + jX_i \quad i \in [1, n-1]
$$

where $\tilde{S}_k$, $P_k$, $Q_k$ represent the complex power, active power, and reactive power of the $k$th node of ULD. $Z_i$, $R_i$, $X_i$ represent the impedance, resistance, and reactance of the $i$th node, respectively.

While the system is ULD, except for the first node of the low-voltage feeder, the power consumption of all other nodes is equal:

$$
\tilde{S}_1 = \tilde{S}_2 = \ldots = \tilde{S}_n = \frac{1}{n-1} \tilde{S}
$$

$$
\tilde{S} = P + jQ = \sqrt{P^2 + Q^2}
$$

where $\tilde{S}$, $P$, $Q$ represent the complex power, apparent power, active power, and reactive power of each node under ULD.

If the PV system is not connected, the power loss of the $i$ branch is given by:

$$
\Delta P_i = (n-i)^2 \frac{S_{i+1}^2}{U_N^2} R_i \quad i \in [1, n-1]
$$

$$
\Delta Q_i = (n-i)^2 \frac{S_{i+1}^2}{U_N^2} X_i
$$
where $\Delta P$, $\Delta Q$, represent the total active power loss and reactive power loss of the ULD system without PV access.

For the entire feeder system, the system line loss is:

$$\Delta P_1 = \frac{S^2}{(n-1)^2 U_N^2} \left[ \sum_{i=1}^{n-1} (n-i)^2 R_i \right]$$

$$\Delta Q_1 = \frac{S^2}{(n-1)^2 U_N^2} \left[ \sum_{i=1}^{n-1} (n-i)^2 X_i \right]$$

(4)

where $\Delta P_1$, $\Delta Q_1$ respectively represent the total active power loss and reactive power loss of the system if the ULD system is connected without PV access.

As PVs are connected to node $m$, its output power is given by Eq. (5). The line loss of the $i$th branch is related to the location of the branch and the PV connection point.

$$\Delta P_{PV} = \frac{\left( (n-i)S_{i+1} - S_{PV} \right)^2}{U_N^2} R_{i}, \quad i \in [1, m-1]$$

$$\Delta Q_{PV} = \frac{\left( (n-i)S_{i+1} - S_{PV} \right)^2}{U_N^2} X_{i}, \quad i \in [1, m-1]$$

(5)

where $\Delta P_{PV}, \Delta Q_{PV}$ represent the active power loss and reactive power loss of each branch with PV connected to ULD.

At this point, the total line loss is:

$$\Delta P_{PV} = \sum \Delta P_{i, PV}$$

$$= \Delta P = \frac{S_{PV}^2}{U_N^2} \sum_{i=1}^{m-1} \left[ 2(n-i)S_{i+1} - S_{PV} \right] R_{i}$$

(6a)

$$\Delta Q_{PV} = \sum \Delta Q_{i, PV}$$

$$= \Delta Q = \frac{S_{PV}^2}{U_N^2} \sum_{i=1}^{m-1} \left[ 2(n-i)S_{i+1} - S_{PV} \right] X_{i}$$

(6b)

where $\Delta P_{PV}, \Delta Q_{PV}$ represent the total active power loss and reactive power loss of the ULD system and after PV access.

The line losses caused by the connection of PV are:

$$\Delta P_{loss} = \frac{S_{PV}^2}{U_N^2} \sum_{i=1}^{m-1} \left[ 2\frac{S(n-i) - S_{PV}}{n-1} \right] R_{i}$$

(7a)

$$\Delta Q_{loss} = \frac{S_{PV}^2}{U_N^2} \sum_{i=1}^{m-1} \left[ 2\frac{S(n-i) - S_{PV}}{n-1} \right] X_{i}$$

(7b)

where $\Delta P_{loss}, \Delta Q_{loss}$ represent the difference in active and reactive power loss under ULD before and after PV access condition.

Eq. (7) could be simplification, as shown in Eq. (8).

$$\Delta P_{loss} = \frac{\sum_{i=1}^{m-1} R_{i}}{U_N^2} \left( S_{PV} - \frac{2S}{n-1} \sum_{i=1}^{m-1} (n-i)R_{i} \right)$$

(8a)

$$\Delta Q_{loss} = \frac{\sum_{i=1}^{m-1} X_{i}}{U_N^2} \left( S_{PV} - \frac{2S}{n-1} \sum_{i=1}^{m-1} (n-i)X_{i} \right)$$

(8b)

In Eq. (8), the independent variable is the output of PV, and the dependent variable is the change of active power loss before and after the PV access, as shown in Fig. 2.

![Figure 2 Function diagram between PV output and power loss difference](image)

When the PV output is $S_{PV_{max}}$ in Eq. (9), the maximum value of the active power loss variation $\Delta P_{max}$ is obtained.

$$\tilde{S}_{PV_{max}} = \frac{-S}{n-1} \sum_{i=1}^{m-1} (n-i)R_{i}$$

(9)

$$\Delta P_{max} = \frac{\left( S_{PV} - \frac{2S}{n-1} \sum_{i=1}^{m-1} (n-i)R_{i} \right)^2}{2U_N(n-1)^2} \sum_{i=1}^{m-1} R_{i}$$

When the active power loss is 0 in Eq. (8), the value of PV output power variation $\tilde{S}_{PV_{L}}$ and $\tilde{S}_{PV_{R}}$ is obtained.
\[ S_{PV} = 0 \]
\[ \tilde{S}_{PV} = \frac{2S \sum_{i=1}^{n-1} (n-1)R_i}{(n-1) \sum_{i=1}^{m-1} R_i} \]  

(10)

where \( \tilde{S}_{PV\text{max}} \) presents the maximum power output of the PV system, if the change in active power loss is maximum with ULD. \( \tilde{S}_{PV_L} \) and \( \tilde{S}_{PV_R} \) represent the power output of the PV system when the change in active power loss is zero with ULD.

The change of active power loss shows a trend that first climbs up and then declines with the increasing of PV output. When the PV output corresponding to the zero point on the right is exceeded, the change in line loss becomes negative, and power back-flow occurs in the line.

2.2 Simulation Model Establishment

In the operation of the power system, line loss cannot be avoided, and it is important for the operation and management of the power system to estimate and control them reasonably. By using SIMULINK to establish a power system line model, precise calculation of line loss can be achieved. Taking a certain 15-node low-voltage distribution line as an example, the system is converted into a circuit model and a simulation model is established. The system topology is shown in Fig. 3.

The system in Fig. 3 is three-phase supplied. The cable used in the simulation is JKLYJ - 1 - 185, and the lengths of all the branches are different, resulting in different values of resistance, inductance, and capacitance. By setting parameters such as current and voltage for power supply and loads, the actual operation of the line can be simulated, so the line losses can be calculated. The load power from \( S_1 \) to \( S_{15} \) is the same, and a distributed PV generation is accessed into the node 8 of the line. The model is run using MATLAB 2018a, and the environment configured is an NVIDIA GTX1050 4G GPU, Intel Core i5 - 8400 CPU, and the operating system is Windows 10.

2.3 Validation of Line Loss Analytical Model

The output power of PV generally begins to increase gradually from 6:00 am, reaches its saturation point around 12:00 am, and then gradually decreases to zero at 6:00 pm. The variation of the total load of the system generally shows an increasing trend at first and then gradually decreases. The curves of the load of the power system and the 24 hour output of PV are shown in Fig. 4.

Based on the 24 hour power data of each node in the 15 node system and the PV 24 hour power output shown in Fig. 4, and combined with the theoretical analysis in the previous paper, two methods are set up to compare and calculate the 24 hour system line loss rate before and after PV access. The line loss rate results of the two methods are shown in Fig. 5.

Method A: SIMULINK simulation model with PV access.

Method B: Analytical model simulation model with PV access.

As shown in Fig. 5, the trend of the system line loss rate change is the same as the trend of the total load power change. The total load is the maximum at 7:00 pm, at which time the line loss rate calculated by both methods is also the maximum. The line loss is 19.91% for Method B and 16.49% for Method A. The difference in line loss rate between the two methods is not significant, and the minimum and maximum error are 0.12% and 3.42% respectively, and the average error is 0.865%.

3 SELECTION AND IMPLEMENTATION OF LINE LOSS OPTIMIZATION ALGORITHM

3.1 Mathematical Model of Line Loss Optimization

According to Eq. (6), it is known that the system line loss value will be reduced to a certain extent after the PV load is accessed to the ULD system, compared to the condition without PV access. The difference in line loss caused by this reduction is expressed as Eq. (7), and it is evident that the size of this difference determines the effectiveness of the line loss reduction after the PV access. After Eq. (7) is transformed into Eq. (8), the line loss difference is used as the objective function in optimization model of the line loss, taking the maximum value of the
objective function as the optimization target. The control variables of the function are the PV access node $m$ and the PV output $S_{PV}$. The change of the connection node $m$ mainly affects the sum of the line resistance, and the control variable constraint condition is given as:

$$
\begin{align*}
S_{PV} & > 0 \\
0 & < m < n
\end{align*}
$$

(11)

### 3.2 Selection of Line Loss Optimization Algorithm

When solving the line loss rate for larger networks with many system nodes, the analytical model increases the computational complexity. If the intelligent optimization algorithm is used and the PV access location is also considered as a variable, the optimal solution can be obtained directly, which improves the efficiency of solving the line loss. GA is an optimization method based on the rule of survival of the fittest. It mimics the natural selection and reproduction process by using selection, crossover, and mutation operations to search for the optimal solution and provide high-quality solutions. GA can solve problems without mathematical expressions, find the global optimal solution, and adapt to environmental changes for continuous operation. However, it requires appropriate representation and parameter adjustment for different problems to obtain the best solution. When the population size is large, the computational speed may be affected and parallel process is needed to accelerate the solution. Therefore, QGA has emerged. Compared with traditional GA, QGA has the ability to process data in parallel, which can converge faster. QGA combines GA with quantum computing’s probabilistic evolutionary algorithm by introducing the quantum state vector. By utilizing the properties of quantum states such as superposition, entanglement, and interference, the quantum bit (qubit) is used to describe individuals, and the optimal solution is obtained through quantum parallel computing. A qubit can be represented in three states during computation, namely the '0' state, the '1' state, and the superposition state of the two, which can be represented by

$$
|\Psi\rangle = \alpha |0\rangle + \beta |1\rangle
$$

(12)

where $\alpha$ and $\beta$ are two amplitude constants (complex numbers). The symbol "$|\Psi\rangle$" is called the "ket" notation, and it represents a quantum state. $|\Psi\rangle$ represents a quantum state, $|0\rangle$ represents the ground state of a qubit, and $|1\rangle$ represents the excited state of a qubit, which should meet the normalization conditions:

$$
|\alpha|^2 + |\beta|^2 = 1
$$

(13)

Eq. (13) can be written as Eq. (14).

$$
\cos^2 \theta + \sin^2 \theta = 1
$$

(14)

In QGA, probability amplitudes are used as chromosome encoding method.
4 EXAMPLE ANALYSIS

At the 7:00 am, the power consumption of each node in a ULD-PV system is selected as a typical state for optimization analysis. The input power of the ULD system and the total impedance of the branch are known.

4.1 Line Loss Analysis of Analytic Models in Different Scenarios

The temporal variation of PV output and load has a certain impact on line loss. Based on the aforementioned Method B, the line loss rate between the case without PVs access to the system and the original case is compared, as shown in Fig. 7.

As shown in Fig. 7, the overall trend of line loss rate will not change whether the PV is accessed in the ULD system. When PV generates power from 6:00 am to 6:00 pm, the line loss rate in the system without PV is higher than in the system with PV, and the average difference is 1.88%. The maximum PV output occurs at 12:00 am, with a line loss rate difference of 3.22% between the two cases. Although the system load is small at this time, with a line loss rate difference of 3.22% between the two cases.

When the PV output is small, at 6:00 am and 6:00 pm, the mean difference of line loss rate between the two cases is 0.04%. It is evident that the increase of the PV output will gradually reduce the line loss rate of the system compared to the condition without PV access. By changing the PV access location and solving the line loss rate for the 24 hour ULD system, the trend of line loss rate change is shown in Fig. 8.

4.2 Comparison of Optimization Algorithms

Based on Fig. 6, the optimal analysis of distribution network line loss is implemented by using QGA, which Eq. (8) is set as the fitness function in QGA and parameters such as population size and individual in the population are initialized. In order to better evaluate the performance of the online loss optimization results of QGA, GA is also used to optimize the low-voltage line loss, and the line loss optimization curves of the two methods are shown in Fig. 9. Fig. 9(a) shows the fitness curve of the entire iteration process, and Fig. 9(b) shows the fitness curve from the 10th iteration to the end.
From Fig. 9, it can be seen that using QGA to solve the optimal PV output, the initial solution is close to the optimal solution and can iterate to the optimal solution at a faster speed. The difference between GA initial solution and the optimal solution is close to 8 kW. After QGA reaches the optimal solution, GA still needs to perform nearly 8 more iterations. It is easy to know that QGA improves the speed of computing the maximum difference in line loss. By substituting the parameters at the typical time of 7:00 am into Eq. (8), the functional relationship between the optimal PV output and the difference in system line loss is obtained, and the maximum difference between the optimal PV output and the system line loss is calculated. At the same time, the optimization results of the three methods are substituted into the low-voltage power grid line loss calculation analysis model to obtain the corresponding line loss rate. The results of the three methods for line loss optimization are shown in Tab. 1.

4.3 Optimization Results Analysis

Considering that the analytical method must search for all connection locations to achieve the optimal PV output, if there are many network nodes, the calculation amount will be very high. Therefore, QGA is used to optimize all PV outputs throughout the time of the day, and the optimal PV output for all times is substituted into the analytical model to calculate the line loss rate. The calculation results are shown in Tab. 2. The results in Tab. 2 show that using QGA to optimize the PV output can achieve the goal of reducing the network line loss of the low-voltage power grid. Compared with the original scheme, the optimized PV output has an average line loss reduction of 26.99%, a maximum line loss reduction of 48.58% in 6:00 am, and a minimum line loss reduction of 19.29% in 10:00 am. Therefore, the effect of reducing loss with the optimized PV output is significant and has achieved the expected results.

5 CONCLUSIONS

This paper proposes a method for calculating the line loss of low-voltage distribution lines considering PV generation, derives a mathematical model for line loss calculation under ULD, establishes an analytical model for line loss calculation, and builds a simulation model of a 15-node low-voltage distribution system with ULD. Using QGA to optimize the analytical model, the optimal PV power output with the maximum difference in line loss is obtained, which results in the minimum system line loss rate. The following conclusions are drawn:

(1) The line loss of a distribution network with ULD varies quadratically with the PV power output. When the PV
power output gradually increases from zero, the change in line loss first increases and then decreases until it becomes negative.

(2) In terms of distribution network line loss calculation, QGA has faster convergence speed and higher optimization efficiency than GA. The model optimized by QGA has higher accuracy and can better reflect the actual line loss situation.

(3) By combining QGA with the analytical model, the optimal PV power output with the minimum line loss rate at a certain moment can be more accurately calculated. This provides an effective optimization method for calculating the line loss of low-voltage distribution lines with PV generation.

(4) When seeking the optimal PV integration point using the analytical model, it may not be possible to quickly find the optimal point when the distribution network has a large number of nodes. Therefore, we utilize quantum genetic algorithm to optimize the analytical model and efficiently locate the best PV integration position.

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